MATH 281 A

Question I. $X_1, \ldots, X_n \sim$ iid unif $[\theta - 1, \theta]$.

(a) Find with explanation an unbiased estimator $\hat{\theta}$ of θ which is a function of $X_{(1)}$. (You may state and use the "broken-stick" theorem if you prefer to avoid calculus.)

(b) By finding, and proving the superiority of, another unbiased estimator of θ dominating $\hat{\theta}$ in (a), show that $\hat{\theta}$ is not UMVU.

(c) Since $\hat{\theta}$ is not UMVUE $X_{(1)}$ must be either not complete or not sufficient. (i) What theorem does this negative result follow from? (ii) Give a direct proof from the primitive definition of completeness or sufficiency that $X_{(1)}$ fails to be complete sufficient. (Don't make any reference to minimal sufficiency, which we barely studied.)

(d) [unconnected to (b) & (c)] Explain whether your estimator $\hat{\theta}$ in (a) is a symmetric function of the data.

Question II. A Poisson point process of constant intensity λ is followed for one time unit, and $X \sim \text{Pois}(\lambda)$ (the number of "events") is observed. We need an estimate of the probability that no "events" would occur if the process were watched for 3 time units. This probability is of course $e^{-3\lambda}$.

(a) Find the UMVUE for $e^{-3\lambda}$ based on the single observation X.

(b) Tabulate its value for a few small values of X, and comment on why it seems peculiar, compared to say the MLE (ie. plug-in estimator) of $e^{-3\lambda}$.

(c) Prove directly that your UMVUE has finite variance.