Math 281 HW 7.

Consider two CI's for μ_1 & μ_2. For testing $\mu_1 = \mu_2$, one consider the testing strategy: if two CI's overlap then not reject H_0, otherwise reject H_0. What is the effective significance level?

Consider the standard z-test and the sample sizes are equal; with variance known.

How would you modify the test if $H_1: \mu_1 > \mu_2$ is one-sided?

Without loss of generality, assume $CI_1 = [\hat{\mu}_1 - R, \hat{\mu}_1 + R]$ and $CI_2 = [\hat{\mu}_2 - R, \hat{\mu}_2 + R]$. Since they have same sample size and variance, under H_0, $\hat{\mu}_1 \overset{d}{=} \hat{\mu}_2$.

The test is to reject H_0 if $CI_1 \cap CI_2 = \emptyset$, equivalently,

$|\hat{\mu}_1 - \hat{\mu}_2| \geq 2R$. If the level of CI's are α, then

$$P(|\hat{\mu}_1 - \hat{\mu}_2| \geq 2R) = P(|Z_1 - Z_2| \geq 2 Z_{1-\frac{\alpha}{2}})$$

where Z_1, Z_2 are iid $N(0,1)$ and $Z_{1-\frac{\alpha}{2}}$ is the $(1-\frac{\alpha}{2})^{th}$ quantile of standard normal.

Note that $Z_1 - Z_2 \sim N(0,2)$, therefore

$$P(|Z_1 - Z_2| \geq 2 Z_{1-\frac{\alpha}{2}}) = P(|Z_1| \geq \sqrt{2}\, Z_{1-\frac{\alpha}{2}}) < \alpha.$$

Therefore the effective level is less than α.

If $H_1: \mu_1 > \mu_2$ is interested, construct one-sided CI's for μ_1 & μ_2 such that μ_1 has a CI of type $(\underline{\hat{\mu}}_1, +\infty)$ and μ_2 has CI like $(-\infty, \hat{\bar{\mu}}_2)$. If the two CI's does not overlap then reject the H_0.