

Đáp số Bài tập Chương 3

3.1. (a) $a = 2, b = -1$; (b) $a = b$. **3.2.** (a) 12, (b) $16\ln 2$, (c) $1/27$, (d) 5. **3.3.** (a) 0, (b) $n(n+1)/2$.

3.4. (a) $f'(0-0) = -1, f'(0+0) = 1, f(x)$ không khả vi tại $x = 0$; (b) $f'(0-0) = f'(0+0) = 0, f(x)$ khả vi tại $x = 0$; (c) $f'(0-0) = -1, f'(0+0) = 1, f(x)$ không khả vi tại $x = 0$; (d) $f'(1-0) = -1, f'(1+0) = 1, f(x)$ không khả vi tại $x = 1$.

3.5. (a) $\frac{4x}{1+x^4}$, (b) $x^x x^{x^x} (\ln^2 x + \ln x + 1/x)$, (c) $x^x a^{x^x} \ln a (\ln x + 1)$, (d) $a^x x^{a^x} (\ln a \ln x + 1/x)$,
(e) $x^{a-1} x^{x^a} (a \ln x + 1)$, (f) $2|x|$, (g) $1/\sqrt{x^2+1}$, (h) $(\sin x)^{\tan x} [1 + \ln(\sin x)/\cos^2 x]$, (i) $e^x \arctan e^x$.

3.6. (a) $x/(x+1)$, (b) $1/(1+e^x)$, (c) $x^3/2y^2$. **3.7.** (a) $-\frac{\sin t + \sin at}{\cos t + \cos at}$, (b) $\cot \frac{t}{2}$, (c) $\frac{1}{2(t+1)^2}$,
(d) $\frac{1-1/e^{at}}{e^{at}}$.

3.8. (a) $-\frac{e^x - y2^{xy} \ln 2}{e^y - x2^{xy} \ln 2}$, (b) $\frac{y}{x}$, (c) $-\sqrt{\frac{y}{x}}$, (d) $\frac{x+y}{x-y}$. **3.9.** $\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$. **3.10.** $-1/3$.

3.14. (a) $y^{(n)}(0) = \begin{cases} 0 & \text{khí } n = 2k \\ (-1)^k (2k)! & \text{khí } n = 2k+1 \end{cases}$, (b) $y^{(n)}(1) = (n-1)!$.

3.15. $\left(\frac{b}{c} - \frac{ad}{c^2}\right) \frac{(-1)^n n!}{(x+d/c)^{n+1}}$, (a) $\frac{ad}{c^2} \frac{(-1)^{n+1} n!}{(x+d/c)^{n+1}}$, (b) $\frac{b}{c} \frac{(-1)^n n!}{(x+d/c)^{n+1}}$, (c) $\frac{n!}{(1-x)^{n+1}}$, (d) $\frac{(-1)^n n!}{(1+x)^{n+1}}$,
(e) $\frac{(-1)^n n!}{x^{n+1}}$.

3.16. (a) $\frac{n!}{2a} \left[\frac{1}{(a-x)^{n+1}} + \frac{(-1)^n}{(a+x)^{n+1}} \right]$, (b) $\begin{cases} x(2-x)/(1-x)^2 & \text{khí } n = 1 \\ n!/(1-x)^{n+1} & \text{khí } n \geq 2 \end{cases}$,
(c) $(-1)^{n+1} n! \left[\frac{1}{(x-1)^{n+1}} + \frac{1}{x^{n+1}} \right]$, (d) $(-1)^{n+1} n! \left[\frac{1}{(x-2)^{n+1}} - \frac{1}{(x-1)^{n+1}} \right]$,
(e) $\frac{(-1)^n n!}{4} \left[\frac{1}{(x+2)^{n+1}} - \frac{3}{(x-2)^{n+1}} \right]$, (f) $(-1)^{n+1} n! \left[\frac{1}{(x-1)^{n+1}} + \frac{1}{(x+1)^{n+1}} \right]$,
(g) $\frac{(-1)^n \cdot a \cdot b^n \cdot (2n-1)!!}{2^n} \frac{1}{(bx+c)^n \sqrt{bx+c}}$, (h) $(-1)^{n-1} (n-1)! \left[\frac{1}{(x+2)^n} - \frac{1}{(x-1)^n} \right]$,
(i) $(-1)^n (2-x)e^{-x}$, (k) $n! e^x \sum_{k=0}^n \frac{x^k}{k!}$.

3.17. (a) $a^n \sin\left(ax + \frac{n\pi}{2}\right)$, (b) $a^n \cos\left(ax + \frac{n\pi}{2}\right)$,
(c) $\frac{(a+b)^n}{2} \sin\left[(a+b)x + \frac{n\pi}{2}\right] + \frac{(a-b)^n}{2} \sin\left[(a-b)x + \frac{n\pi}{2}\right]$,
(d) $\frac{(a+b)^n}{2} \cos\left[(a+b)x + \frac{n\pi}{2}\right] + \frac{(a-b)^n}{2} \cos\left[(a-b)x + \frac{n\pi}{2}\right]$,
(e) $\frac{(a+b)^n}{2} \cos\left[(a+b)x + \frac{n\pi}{2}\right] - \frac{(a-b)^n}{2} \cos\left[(a-b)x + \frac{n\pi}{2}\right]$,
(f) $-2^{n-1} \cos\left(2x + \frac{n\pi}{2}\right)$, (g) $2^{n-1} \cos\left(2x + \frac{n\pi}{2}\right)$, (h) $-2^{2n-3} \left(4x + \frac{n\pi}{2}\right)$.

3.18. (a) $\sqrt{(a^2+b^2)^n} e^{ax} \sin(bx + n\varphi)$, (b) $\sqrt{(a^2+b^2)^n} e^{ax} \cos(bx + n\varphi)$ với $\varphi = \arctan \frac{b}{a}$.

$$\mathbf{3.19.} \text{ (a) } [(ax^2 + bx + c)d^2 - an(n-1)]d^{n-2} \sin\left(dx + \frac{n\pi}{2}\right) - (2ax + b)d^{n-1}n \cos\left(dx + \frac{n\pi}{2}\right),$$

$$\text{ (b) } [(ax^2 + bx + c)d^2 - an(n-1)]d^{n-2} \cos\left(dx + \frac{n\pi}{2}\right) + (2ax + b)d^{n-1}n \sin\left(dx + \frac{n\pi}{2}\right),$$

$$\text{ (c) } d^{n-2}e^{dx}[(ax^2 + bx + c)d^2 + nd(2ax + b) + n(n-1)a],$$

$$\text{ (d) } (-1)^{n-1}(n-1)!a^n \left[\frac{1}{(ax+b)^n} + \frac{1}{(ax-b)^n} \right]$$