## Chương 3

**3.1.** (a) $\frac{ab(a^2+ab+b^2)}{3(a+b)}$,(b) 24, (c) 0. **3.2.** (a) $\frac{40\sqrt{5}}{3}$, (b) $\frac{4(2+\sqrt{2})}{3}$ (c) $\frac{\pi a^3}{2}$. **3.3.** 32.

**3.4.** (a) $\frac{1}{4}\left(\frac{3\sqrt{3}-1}{2}+\frac{3}{4}\ln\frac{3+2\sqrt{3}}{3}\right)$, (b) $2\pi\sqrt{a^2+b^2}\left(a^2+\frac{4\pi^2}{3}b^2\right)$, (c) $60\pi$, (d) $\frac{2\pi a^3}{3}$.

**3.5.** (a) 1, (b) $\frac{\sqrt{2}}{2}\left[2\pi\sqrt{1+4\pi^2}+\ln\left(2\pi+\sqrt{1+4\pi^2}\right)\right]$. **3.6.** $\frac{e^2+1}{4}$.

**3.7.** (a) $-\frac{32}{3}$, (b) 4, (c) $\frac{4}{3}$, (d) $-2\pi a^2$, (e) 0, (f) 0. **3.8.** (a) 1, (b) $\frac{17}{15}$, (c) $\frac{4}{3}$.

**3.9.** (a) $\frac{1}{30}$, (b) $\frac{3\pi}{2}$, (c) 0. **3.10.** (a) 4, (b) $-\frac{\pi a^3}{8}$, (c) 3.

**3.11.** (a) Không phụ thuộc vào đường lấy tích phân với điều kiện $xy \neq 0$, (b) $\frac{(\pi^2+16)^2}{16\pi}-4$.

**3.12.** m = n = 1, m = 1 và n tùy ý, m tùy ý và n = 1. **3.13.** m = n = 2. **3.14.** m = 1.

**3.15.** $u(x,y)=(3x^2+y)e^y$. **3.16.** $u(x,y)=x^2y^3+\sin y+C$.

**3.17.** (a) $-\pi$, (b) $2+e-\frac{1}{e}$ và $\frac{1}{e}-e-\pi-2$, (c) $\frac{2-4\sqrt{2}}{3}$, (d) $a^2-\frac{2a^3}{3}$, (e) $2-e^2$

**3.18.** (a) $\frac{\pi a^5}{8}$, (b) $\frac{1}{\sqrt{3}}$, (c) $\frac{5}{4}$. **3.19.** (a) $\frac{4\pi}{15}$, (b) $\frac{7\pi}{12}$, (c) $\frac{3}{2}$. **3.20.** (a) $\frac{7}{30}$, (b) $\frac{4}{3}$. **3.21.** $2\sqrt{2}\pi$.

## Chương 4

**4.1.** (a) $\frac{1}{1+x^2}+\frac{1}{1+y^2}=C$, (b) $\ln\left|\frac{x}{y}\right|-\frac{1}{x}-\frac{1}{y}=C\ (xy\neq 0)$, (c) $\ln\left|\tan\frac{y}{2}\right|+2\cos y-x=C\ (y\neq 0)$,

(d) $\ln\left|\tan\frac{y}{2}\right|+2\sin x=C\ (y\neq 0)$, (e) $\left|\tan\frac{y}{2}\right|=e^C\left|\left(1+\tan\frac{y}{2}\right)\left(1-\tan\frac{x}{2}\right)\right|$, (f) $x+\cot\frac{x-y}{2}=C$,

(g) $x+\frac{(x-y)^2}{2}=C$, (h) $x+\frac{(x-y)^2}{2}=C$.

**4.2.** (a) $\sqrt{1+x^2}+\sqrt{1+y^2}=1+\sqrt{2}$, (b) $y=\sqrt[3]{3\arctan e^x-\frac{3\pi}{4}}$, (c) $|\ln y|=\left|\tan\frac{x}{2}\right|$,

(d) $y=2\frac{x^2+2x-1}{-x^2+2x+1}$.

**4.3.** (a) $\left|y^2+2xy-x^2\right|=e^C$, (b) $y+\sqrt{x^2+y^2}=e^C x^2$, (c) $\sqrt{\left|y^2-x^2\right|}=e^C$, (d) $(x^2+y^2=e^C\left|\frac{y}{x}\right|$,

(e) $\sqrt{\left|y^2+6y+2yx-2x-x^2+1\right|}=e^C$, (f) $\frac{y'}{2}=\left(\frac{y+2}{x+y-1}\right)^2$, (g) $\ln|y+2|+2\arctan\frac{y+2}{x-3}=C$,

(h) $\left|xy\cos\frac{y}{x}\right|=e^C$.

**4.4.** (a) $y(x)=\left(C-\frac{\ln\left|x+\sqrt{x(x-1)}\right|}{2}\right)\frac{1}{\sqrt{x(x-1)}}$ nếu $x<0$ hoặc $x>1$,

$y(x)=\left(C-\frac{\arcsin(2x-1)}{2}\right)\frac{1}{\sqrt{x(1-x)}}$ nếu $0<x<1$ ; (b) $\frac{1}{x}+C\left(|x|+\frac{1}{|x|}\right)$, (c) $y(x)=\frac{C+x^2/2}{e^{x^2}}$,

(d) $y(x) = (x + C)(x^2 + 1)$, (e) $x(y) = \frac{y^2}{2} + C|y|^3$, (f) $y(x) = x^2 \arctan x - x \ln\sqrt{1 + x^2} + C|x|$,

(g) $y = 1/2$, (h) $y = \frac{\ln\left(x + \sqrt{1 + x^2}\right)}{\sqrt{1 + x^2}}$.

**4.5.** (a) $u(x, y) = \frac{x^2}{2} + x + xy - \frac{y^3}{3} + 3y$, (b) $u(x, y) = x^4 + 3x^2y^2 + y^3$, (c) $u(x, y) = \ln\left|\frac{y}{x}\right| + \frac{xy}{y - x}$,

(d) $\ln|x + y| + \frac{y}{x + y}$, (e) $u(x, y) = \sin\frac{y}{x} - \cos\frac{x}{y} + x - \frac{1}{y}$, (f) $u(x, y) = x^3(1 + \ln y) - y^2$.

**4.6.** (a) $\alpha(x) = e^x$ và $u(x, y) = (x^2y + y^3/3)e^x$, (b) $\alpha(y) = 1/y^2$ và $u(x, y) = x\left(\frac{x}{2} + \frac{1}{y}\right)$, (c) $\alpha(y) = 1/y^2$ và $u(x, y) = \frac{x^3}{3} + \frac{x}{y} + y$, (d) $\alpha(x) = 1/x^2$ và $u(x, y) = \frac{y^3}{3} + \frac{y}{x} + x$, (e) $\alpha(y) = 1/y^2$ và $u(x, y) = x^2 - 3xy - 7/y$.

**4.7.** (a) $y = \frac{1}{2 + Ce^x}$, (b) $y^2 = \frac{1}{Ce^{x^2} + x^2 + 1}$, (c) $y = \frac{1}{1 + Cx + \ln x}$, (d) $\sqrt{y} = \frac{1}{(\ln\sqrt{|x|} + C)x^2}$,

(e) $y^4 = \frac{x^2}{C - 4x^5}$, (f) $y^3 = \frac{1}{C|x|^3 - 6x^4}$, (g) $\sqrt{y} = \frac{e^{\frac{x}{2}}}{2} + \frac{C}{e^{\frac{x}{2}}}$.