We are interested in recovering the halos and their masses, positions and velocities with the smallest time step necessary to preserve sufficient properties as mock catalogs.

1 MATCHING HALOS

First, we compare a halo by halo in different simulations. Since all the simulations use exactly the same initial conditions, if those approximated N-body simulations can recover the halos reasonably well (compared to a full N-body simulation), then we should find the same halo at the same position with the same mass in samples using different mass resolutions and time steps. The question here are how mass resolutions and time steps affect on halo properties (i.e., mass, position and velocity) and how many halos don't correspond to halos between different samples.

To spatially match the halos defined in different simulations, we first find a pair of halos from two different simulations, whose distance is the closest. Then, we set two conditions on those pairs to declare whether those paired halos are the same halos:

1) distance is smaller than $0.5h^{-1}$Mpc,

2) mass ratio is smaller than $10^{0.5}\mathrm{M}_{\odot}$.

Under the above conditions, more than 90% of halos in 450/5 found paired halos in 300/2 at redshift $z = 0.15$ with a fixed mass resolution, 256^3 particles (shown in Table 1). Both simulations have the same mass resolution, which is a 256^3 particles in a $256^3(h^{-1}\mathrm{Mpc})^3$ cubic box. We also checked how those conditions affect to fraction of matched halos by changing the numerical values for distance and mass ratio criteria. As shown in Table 1, changing a criterion on mass ratio did not change the fraction much, while the distance conditions does. This iimplies that deviation of halo masses on pairs is relatively small.

1.1 Mass Resolution

In this section, we examine how mass resolutions affect to halo properties. We use a $(256h^{-1}\mathrm{Mpc})^3$-cubic box and three different mass resolutions: 512^3, 256^3, and 128^3 particles with a fixed time steps (450/5). We take a simulation of 512^3 particles as a reference. Table 2 indicates fractions of unmatched halos in the sample of 512^3-particle mass resolution by matching with the samples of 256^3 and 128^3 particles. Since having a different mass resolution implies that halo samples have different lower mass limits, we only impose matching conditions on halos greater than $10^{12.5}\mathrm{M}_{\odot}$ for the case of 256^3-particles sample and greater than $10^{13.5}\mathrm{M}_{\odot}$ for the case of 128^3-particles sample. For matching between 512^3 and 256^3 partciles, the above matching conditions are enough to find pairs for most of halos. On the other hand, as indicated in Figure 2, separations of matched halos between 512^3 and 128^3 particles are bigger than $0.5h^{-1}$Mpc and the distance condition it too tight to find a pair. The right panel in Table 2 shows how unmatched fractions are changed by changing the distance condition. This suggests that dynamics for 128^3 and 512^3 are very different(?)...

Figure 1 shows number densities of unmatched halos as a function of halo mass at redshift $z = 0.15$. In matching, an algorithm takes a finer resolution sample as a reference and finds a matched halo from the other sample. Surprisingly, the number densities for bothe samples are almost the same.

Figure 2 shows distance between matched halos in two different samples. If two samples have the same halos at the same positions, then those distances go to zero. The simulation with 128^3 particles has more scatter and its distribution is not Gaussian. As indicated in the right panel, the mean separation for halos between 128^3 and 512^3 particles is about $1h^{-1}$Mpc. Note that, for the right panel, we only select halos whose masses are greater than $10^{14}\mathrm{M}_{\odot}$. For the simulation with 256^3 particles, the result improves siginificantly and the positions are matched within a few hundreds kpc. For halo masses, the results are shown in Figure 3 and both simulations with 256^3 and 128^3 particles well agree with the simulation with 512^3 particles.

Figure 4 indicates that difference on velocity magnitude. For the case between 512^3 and 256^3 particles, the scatter is relatively small. The mean and standard deviation are XXXX. We also checked angles between two velocity fields for a paired halos, and we fuond that 95% of halos between 512^3 and 256^3 particles agree within 0.3 radians, while 90% of halos between 512^3 and 128^3 particles agree within 0.6 radians.

1.2 Time steps

In this section, we examine how global steps and sub-cycles affect to halo properties. The goal here is to know the smallest global steps and sub-

[p]

mass[$M_\odot$]\distance [h^{-1}Mpc]	0.5	0.75	1.0
$10^{0.5}$	0.0915	0.0842	0.0842
$10^{0.75}$	0.0909	0.0825	0.0817
$10^{1.0}$	0.0909	0.0823	0.0763

Table 1. Fractions of unmatched halos (over matched halos) for the sample of 450/5 when we compare a halo by halo for 300/2 at redshift $z = 0.15$. Here, both simulations have the same mass resolution (i.e., 256^3 particles in the box of $(256h^{-1}\mathrm{Mpc})^3$). This table shows how the fraction is changed according to changing the matching conditions for distance and mass ratio.

[p]

mass-resolution\z	0.15	0.5	0.8
256^3	0.150	0.128	0.120
128^3	0.842	0.831	0.817

z\distance [h^{-1}Mpc]	0.5	1.0	2.0
0.15	0.842	0.260	0.061
0.5	0.831	0.260	0.066
0.8	0.817	0.213	0.053

Table 2. Left: Fractions of unmatched halos (over matched halos) for the sample of 512^3 particles when we compare a halo by halo for 256^3 and 128^3 particles with a fixed time step (450/5). For the comparison between 512^3 and 256^3 particles, we select halos greater than $10^{12.5}M_\odot$, while for the case of 512^3 and 128^3 particles, halos greater than $10^{13.5}M_\odot$ are selected. Right: For the comparison between 512^3 and 128^3 particles, we examined how the unmatched fractions are changed due to distance conditions.

cycles required to preserve necessary properties for mock catalogs.

Here, all the samples have the same mass resolution, 256^3 particles in the box of $(256h^{-1}\mathrm{Mpc})^3$. We use 450/5 (450 global steps and 5 sub-cycles) as a reference to other samples: 300/3, 300/2, 150/3, and 150/2.

Figure 5 shows positional differences for paired halos. This histogram indicates that global step has bigger effects on overall halo positions and difference on sub-cycles have negligible effects. Most of halos for 300 global steps have their center positions within 100 kpc by comparing with halo center positions for 450/5, while the simulations for 150 global steps have more scatter in the figure. Means and standard deviations for the histograms are shown in Table 3.

Figure 6 shows halo mass differences for paired halos with respect to halos from 450/5. Global steps affect on overall distribution properties (i.e., mean and standard deviation shown in Table 4), while sub-cycles affect on their amplitudes. This is because sub-cycles changes small-scale dynamics and it can cause a difference on number of halos declared through FOF, whose linking length is fixed to $b = 0.2$. In general, smaller step sizes make distribution of DM particles as halos more diffused (since bigger stepsizes cen't capture all the non-linearities that finer stepsizes generate) and there is a possibility that some of gathered DM particles are not considered as halos. In Manera et al. which approximates N-body simulations by using the second-order Lagrangian Perturbation Theory, they solved this problem by changing the linking length for FOF. -¿ Can we tune (or do we need to tune) linking lengths based on step sizes?

For halo velocities, the results are shown in Figure 7. The histogram is a function of velocity magnitude differences. For 150 global steps, the means slightly deviate from 0 and have negative values. This means that magnitude of velocity for 150 global steps is smaller than that for 450 global steps. One way to explain this is that because smaller time step halos are less dense, the potential wells at center of halos may be less deeper than the ones for larger step sizes. We also examined differences on velocity direction (orientation?). More than 98% of paired halos have angle differences (with respect to halos for 450/5) within 0.3 radians. This implies that orientation of velocities are well-preserved among the samples for different time steps.

At last, we show number density of un-

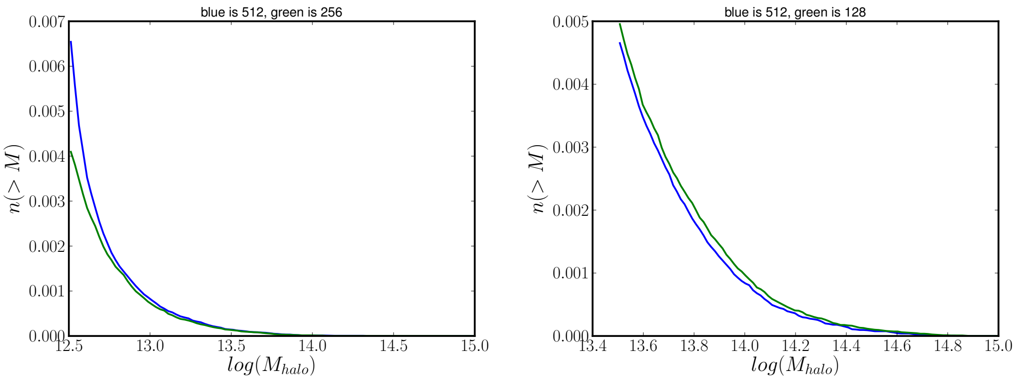


Figure 1. Unmatched halo number density for the simulation of 512^3 particles (blue in both panels) by comparing with 256^3 particles-simulation (left, green) and 128^3-particles simulation (right, green) at redshift $z = 0.15$.

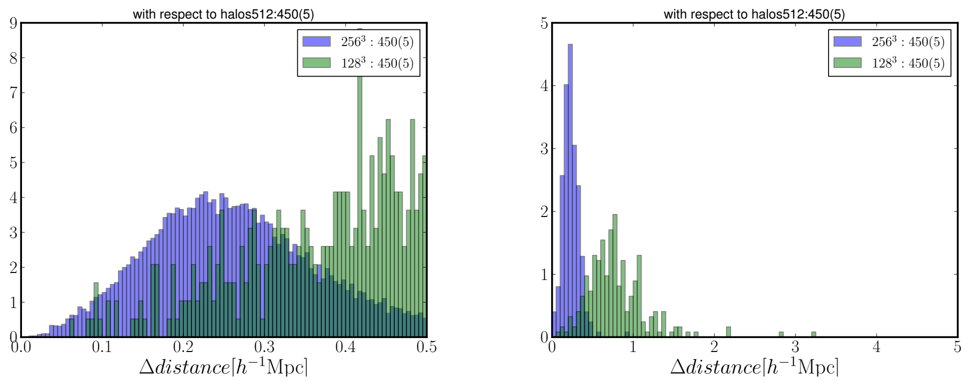


Figure 2. Left: Histograms of distance difference for matched halos with respect to halos from 512^3 particles with 450 global steps and 5 sub-cycles. Different colors correspond to different mass resolutions: 256^3 particles and 128^3 particles with a fixed time step (450/5) at redshift $z = 0.15$. Right: Histograms of distance difference for matched halos without iposing a distance condition. Halos are selected so that their masses are greater than $10^{14}\mathrm{M}_\odot$ at redshift $z = 0.15$.

matched halos in Figure 8 and Table 6. Table 6 shows fractions of unmatched halos for the sample of 450/5 compared with other time steps. Note that even though we only show the fractions for 450/5, the fractions for other time steps correspond to the ones for 450/5. As shown in Figure 8, number densities for unmatched halos for 450/5 and other time steps are almost the same. There are more unmatched halos for smaller halo masses shown in the upper panel of Figure 8, which compares 450/5 and 300/2 samples. Unmatched number densities for different step sizes are shown in the lower panel of Figure 8. As is clear, sub-cycles in the number of unmatched halos is negligible. Again, changing the matching conditions don't affect on the fraction of unmatched halos much and the fraction is always less than 10%.

***snapshots for unmatched halos whose mass is greater than $10^{14}\mathrm{M}_\odot$. There is a weird case between 450_5 and 300_2...

2 OBSERVABLE/STATISTICS

Goal: To correctly describe the large-scale distribution of these galaxies. ⇒ Need to correctly locate DM halos in the simulations and estimate their masses.

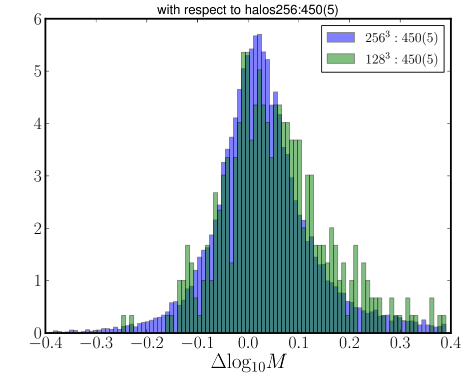


Figure 3. Histograms of log-based mass difference for different mass resolutions with respect to 512^3 particles at redshift $z = 0.15$. Histograms are normalized and all simulations have the same time step, 450/5. Colors correspond to 256^3 particles (blue) and 128^3 particles (green).

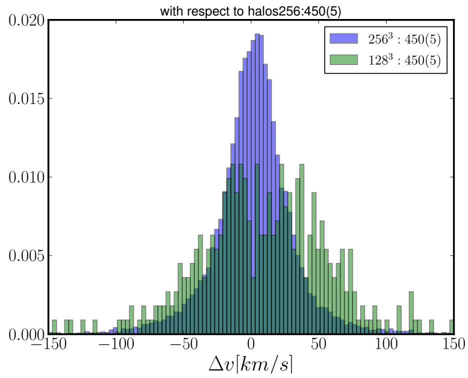


Figure 4. Histograms of velocity magnitude difference for matched halos with respect to halos from 512^3 particles at redshift $z = 0.15$. Different colors correspond to different mass resolutions: 256^3 particles (blue) and 128^3 particles (green). All the simulations use the same step size, 450/5. Note that angle difference of velocity vectors for 95% of matched halos in the 256^3-particle sample are within 0.3 radians and 90% in the 128^3-particle sample are within 0.6 radians.

2.1 Mass Function

* Why do we care about mass functions?: White 2002 "mass function is one of the most fundamental predictions of a theory of structre formation."

* If we have samples with different mass functions, what does it mean? and what kind of problems are caused by that?

* What does it physically (observationally?) mean to have the same mass function?

* I am not sure how Manera et al. assigned masses to halos...apparently, they were using analytic mass functions for this...:(

* relation between $n(M)$ and HOD.

* ingredients of mass function: cosmology, initial conditions (how important is initial condition??), what other physical processes??

In this section, we examine how mass resolutions and time steps affect to mass functions. Note that all the number densities (as a function of halo mass) are mean of 100 samples generated through the bootstrap method.

2.1.1 Mass Resolution

* Analytic comparison

In this section, we investigate effects of mass resolutions on mass function and number density

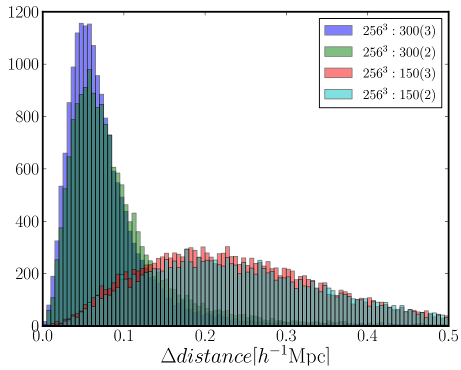


Figure 5. Histograms of distance difference for matched halos with respect to halos from 256^3 particles with 450 global steps and 5 sub-cycles. Different colors correspond to different simulation step sizes: 300 global steps with 3 sub-cycles (blue) and 2 sub-cycles (green), and 150 global steps with 3 sub-cycles (red) and 2 sub-cycles (cyan). All the simulations use 256^3 particles at redshift $z = 0.15$.

[H]

z=0.15	mean $[h^{-1}$Mpc]	std$[(h^{-1}$Mpc$)^2]$
300/3	0.078	0.056
300/2	0.092	0.068
150/3	0.233	0.106
150/2	0.237	0.107

z=0.5	mean $[h^{-1}$Mpc]	std$[(h^{-1}$Mpc$)^2]$
300/3	0.078	0.053
300/2	0.089	0.062
150/3	0.196	0.095
150/2	0.202	0.097

z=0.8	mean $[h^{-1}$Mpc]	std$[(h^{-1}$Mpc$)^2]$
300/3	0.068	0.050
300/2	0.080	0.059
150/3	0.199	0.096
150/2	0.204	0.097

Table 3. Means and standard deviations for positional differences in Figure 5.

z=0.15	mean	std
300/3	-0.005	0.067
300/2	-0.011	0.075
150/3	-0.035	0.074
150/2	-0.059	0.087

z=0.5	mean	std
300/3	-0.007	0.069
300/2	-0.019	0.075
150/3	-0.047	0.082
150/2	-0.078	0.091

z=0.8	mean	std
300/3	-0.009	0.069
300/2	-0.026	0.077
150/3	-0.065	0.084
150/2	-0.100	0.094

Table 4. Means and standard deviations for halo mass differences in log-based halo mass (with base 10) in Figure 6.

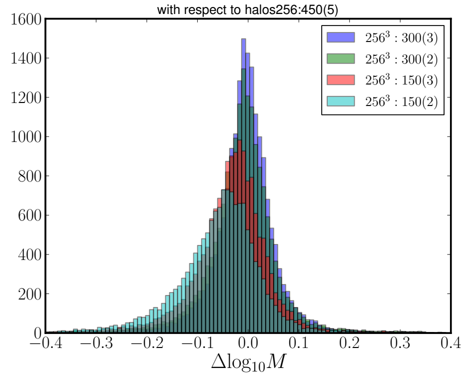
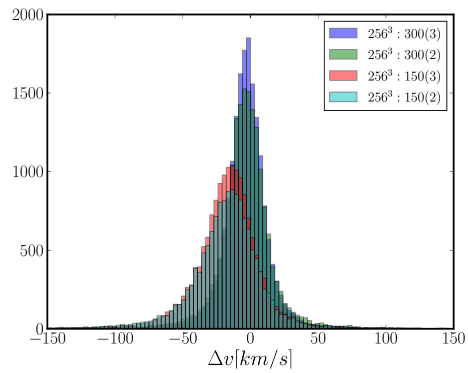


Figure 6. Histograms of log-based mass difference of different simulation step sizes with respect to the one with 450 global steps and 5 sub-cycles. Histograms are not normalized and all halos are from the simulations with 256^3 particles. For each of simulations: 300 global steps with 3 sub-cycles (blue) and 2 sub-cycles (green), and 150 global steps with 3 sub-cycles (red).



[p]

Figure 7. Histograms of velocity magnitude difference for matched halos with respect to halos from 256^3 particles with 450 global steps and 5 sub-cycles. Different colors correspond to different simulation stepsizes: 300 global steps with 3 sub-cycles (blue) and 2 sub-cycles (green), and 150 global steps with 3 sub-cycles (red) and 2 sub-cycles (cyan). All the simulations use 256^3 particles at redshift $z = 0.15$. Note that angle difference of velocity vectors for 98% of matched halos are within 0.3 radians.

[p]

time step/z	0.15	0.5	0.8
300/3	0.099	0.107	0.120
300/2	0.134	0.154	0.182
150/3	0.219	0.233	0.285
150/2	0.299	0.331	0.387

Table 6. Fractions of unmatched halos (over matched halos) for the sample of 450/5 when we compare a halo by halo for other time steps shown in the table with a fixed mass resolution (256^3 particles).

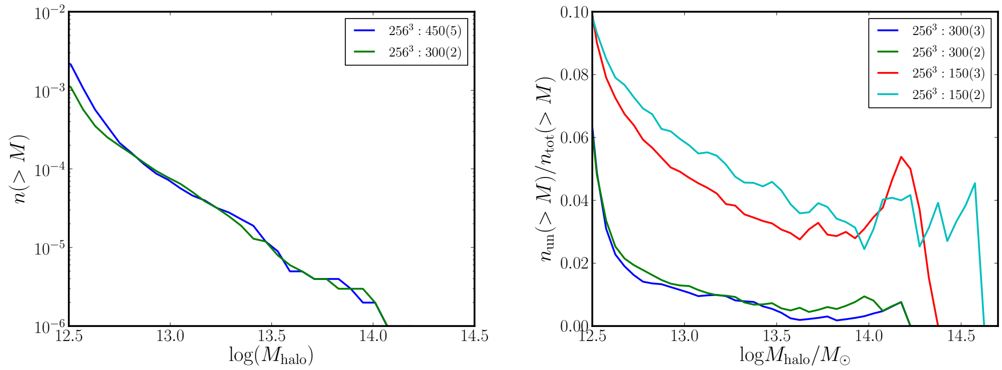


Figure 8. Upper: Unmatched halo number density for the simulation of 300 global steps and 2 sub-cycles matching with 450 global steps and 5 sub-cycles. Both are from 256^3 particle simulations. Lower: Ratio of unmatched halo number densities (which are the same as the ones in the left plot) with respect to the corresponding total number densities. Both plots are at redshift $z = 0.15$. I am not sure why there are more unmatched halos above $10^{14}M_\odot$ for 150/2.

[t]

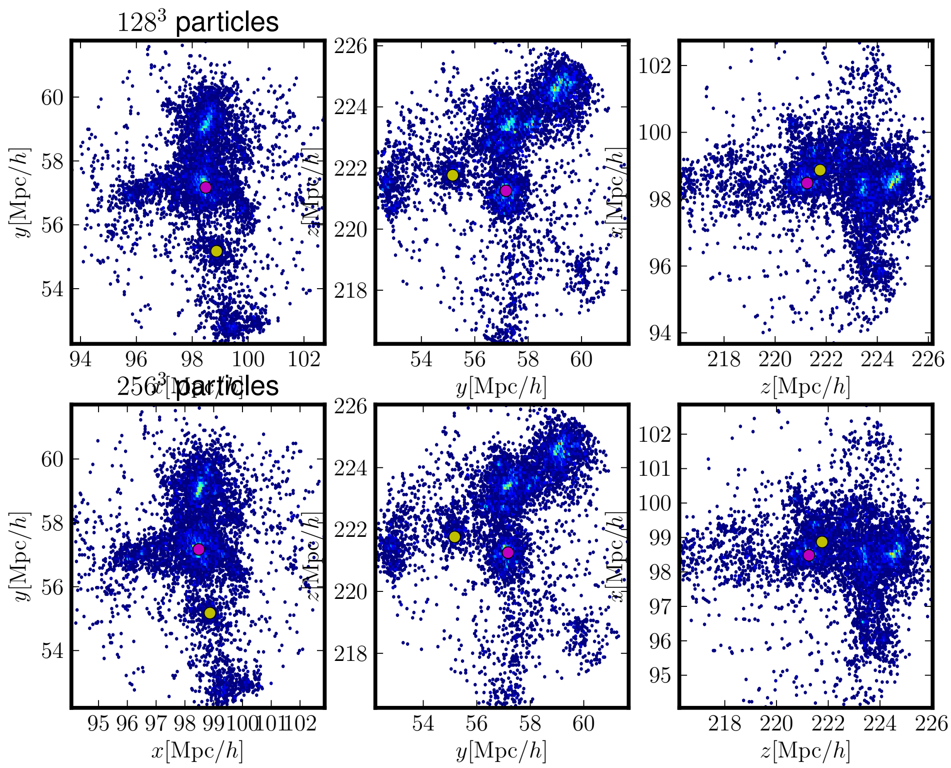


Figure 9. Snapshot of halos between 450/5 and 300/2 which don't agree completey. All the surrounding halos in 300/2 have distance more than $2h^{-1}$Mpc from an unmatched halo in 450/5.

z=0.15	mean [km/s]	std [(km/s)^2]
300/3	-3.54	23.27
300/2	-3.90	25.65
150/3	-17.77	28.83
150/2	-17.94	29.30

z=0.5	mean [km/s]	std [(km/s)^2]
300/3	-4.36	33.23
300/2	-4.07	34.54
150/3	-25.30	40.72
150/2	-26.34	42.26

z=0.8	mean [km/s]	std [(km/s)^2]
300/3	-6.37	39.96
300/2	-6.66	42.77
150/3	-26.11	51.25
150/2	-27.83	54.28

Table 5. Means and standard deviations for halo velocity differences in Figure 7.

of halos (as a function of mass). For the case of 128^3 particles simulation, a mass of one particle is XXX, which implies that we cannot have halos smaller than about $10^{13.5}\mathrm{M}_\odot$ through FOF. This is because FOF requires a certain number of particles clustered to declare a halo. The lowest mass halo for 256^3 particles simulations is about $10^{12.5}\mathrm{M}_\odot$. Figure 10 shows number densities as a function of halo mass at redshift $z = 0.15$ and $z = 0.8$. The colored regions indicate number densities for 512^3 particles with 450/5 with errors. The reason that number densities for 128^3 particles below $M_{halo} < 10^{13.5}\mathrm{M}_\odot$ are constant is because there are no halos smaller than $10^{13.5}\mathrm{M}_\odot$ for the case of 128^3 particles. Otherwise, all three simulations have almost the same number densities, though they start slightly deviating on large halo mass end. I don't understand the disagreement at $z = 0.8$ is bigger than that for $z = 0.15$:(

2.1.2 Time Steps

Here, we check the same thing as 2.1.1 for varying time steps with a fixed mass reslution of 256^3 particles. Figure 11 compares 450/5 and 300/2 with

Figure 14. Cross power spectra between halos and matter?

512^3 particles mass resolution and 450/5 time steps. The colored regions indicate number densities of 512^3 : 450/5 with errors. Both 450/5 and 300/2 for 256^3 particles are well within the errors. Figure 12 shows the ratios between 512^3 : 450/5 and 256^3 particles with different time steps. For the case of 450 and 300 global time steps, the agreement is better than 10%.

2.2 Halo power spectra

From proposal: "A large enough number for the global time steps is important to resolve the large scale structure correctly and recover, e.g. the power spectrum correctly on large scales while the sub-cycles mainly influence the inner structure of halos and the power spectrum on the smallest scale."

What we want to know here is quantitative effects of mass resolutions and time steps on power spectra.

* One advantage for this mock catalogs over Manera et al. 2012 is that this does not require changing linking length.

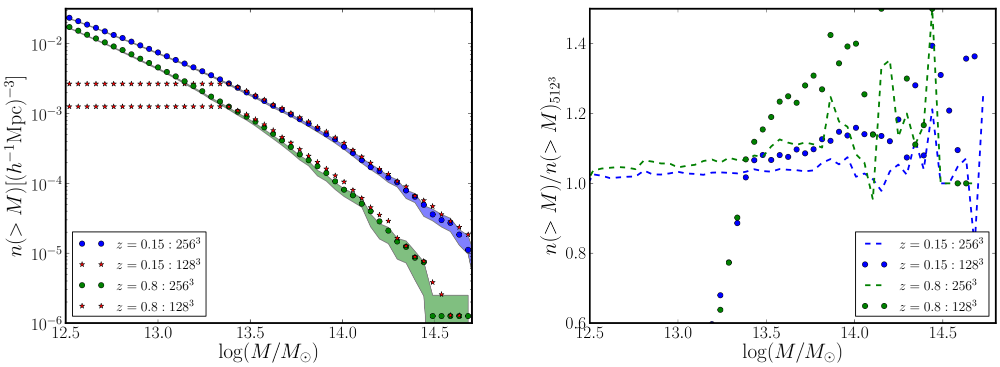
2.2.1 Mass Resolution

*only auto power spectra since I don't have DM particle samples for 512^3 particles resolution. Are there any meanings to calculate cross-power with 256^3 particles?

"Before doing the ratio, a Poisson shot noise contribution of $1/n$, where n is a number density of halos, have been subtracted from the power, as it is common under the approximation of Poisson sampling.

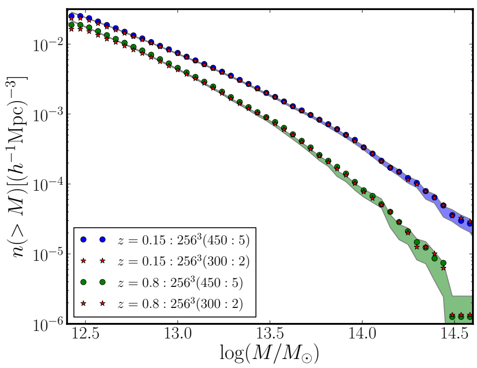
2.2.2 Time Steps

In this section, we examine effects of time steps on power spectra, which is a statistical observable (?). In Figure 15, we show various auto power spectra for 450/5 and 300/2 at redshift $z = 0.15$, $z = 0.5$, and $z = 0.8$ on the left panels and ratios of auto power spectra for different global steps and sub-cycles at redshift $z = 0.15$ on the right panels. Power spectra on the upper panels are subtracted a Poisson shot noise of $1/n$, where n is the nuber density of halos. The lower panels are



[p]

Figure 10. Comparison of mass functions (should I call this number density?) for different simulations at redshift $z = 0.15$ and $z = 0.8$. The shaded regions indicate the upper limit and the lower limit of mass functions for the simulation with 512^3 particles and 450 steps with 5 sub-cycles. The mean and standard deviations are calculated from bootstrap method. We compare them with the simulations of 256^3 particles with 450 global steps and 5 sub-cycles (described as 256^3 with circles in the figure) and 128^3 particles with 450 global steps and 5 sub-cycles (described as 128^3 with red stars in the figure).

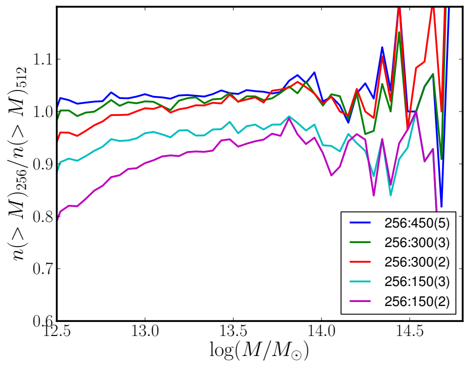


[p]

Figure 11. Comparison of mass functions (should I call this number density?) for different simulations at redshift $z = 0.15$ and $z = 0.8$. The shaded regions indicate the upper limit and the lower limit of mass functions for the simulation with 512^3 particles and 450 steps with 5 inner(?) steps. The mean and standard deviations are calculated from bootstrap method. We compare them with the simulations of 256^3 particles with 450 global steps and 5 sub-cycles (described as $256^3(450 : 5)$ with circles in the figure) and with 300 global steps and 2 sub-cycles (described as $256^3(300 : 2)$ with red stars in the figure).

the same as the upper panels without a Poisson shot noise subtraction. Note that halos selected here has a mass treshhold of $10^{12.5}\mathrm{M}_{\odot}$. As shown in Figure 15, the agreement between 450/5 and other time steps is within 10%.

In Figure 16, we computed the cross power spectra between halos and the matter field at redshift $z = 0.15$. For the matter field, we used the one for 256^3 particles mass resolution with 450/5 time steps. On the left panel, we show three different mass bins and compare 300/2 (circles) and 450/5 (solid line) simulations. They agree remarkably well. On the right panel, we take ratios between the cross power spectrum for 450/5 time steps and the cross for other time steps. Halos selected for the right panel has a mass from $10^{12.5}\mathrm{M}_{\odot}$ to $10^{13.0}\mathrm{M}_{\odot}$. We see that all the ratios are within 5% accuracy except 150/2 simulations on small scales. Note that those ratios are equivalent to the ratios of the halo biases, which implies that biases for different time steps are almost the same.



[p]

Figure 12. Ratios of mass functions of different time steps with 256^3 particles, compared to 512^3 particles with 450 global steps and 5 sub-cycles, at redshift $z = 0.15$. Different colors are corresponding to different time steps. The legend in the plot has the same format as the one in Figure 1. This plot shows that the simulations with 450 and 300 global steps agree well with the simulation with 512^3 particles.

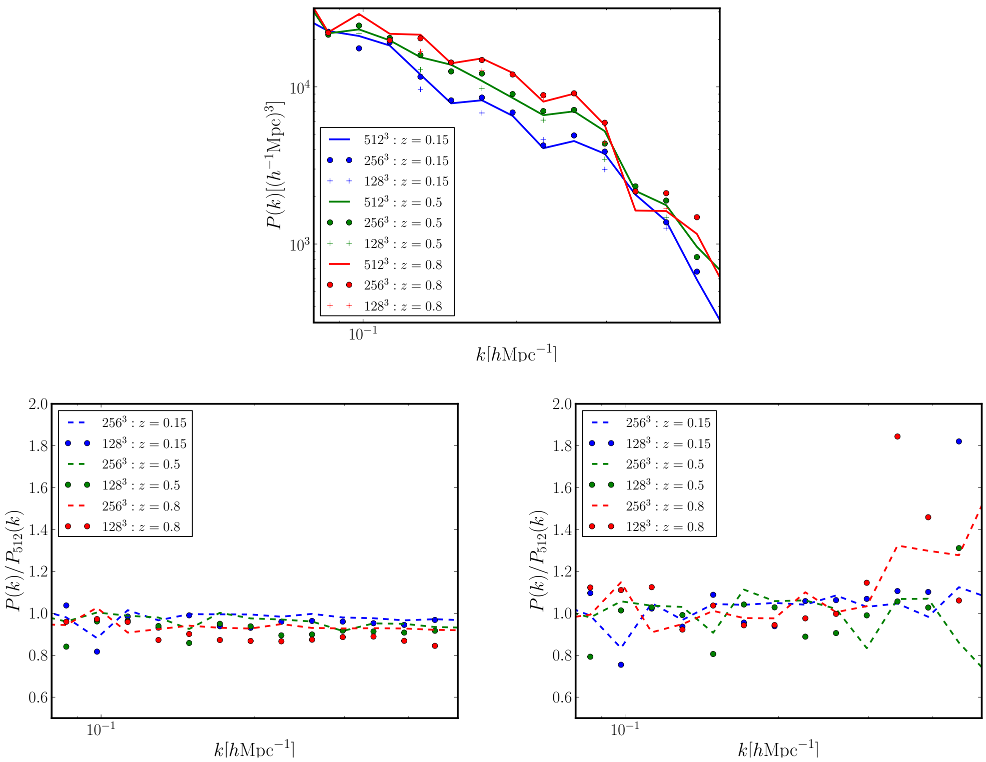


Figure 13. Halo auto power spectra for different mass resolutions (512^3, 256^3, and 128^3 particles in the box of $(256h^{-1}\mathrm{Mpc})^3$) with a fixed time step of 450/5. The sample of halos is equivalent to a mass thresholds of $M = 10^{13.5}\mathrm{M}_\odot$.

[t]

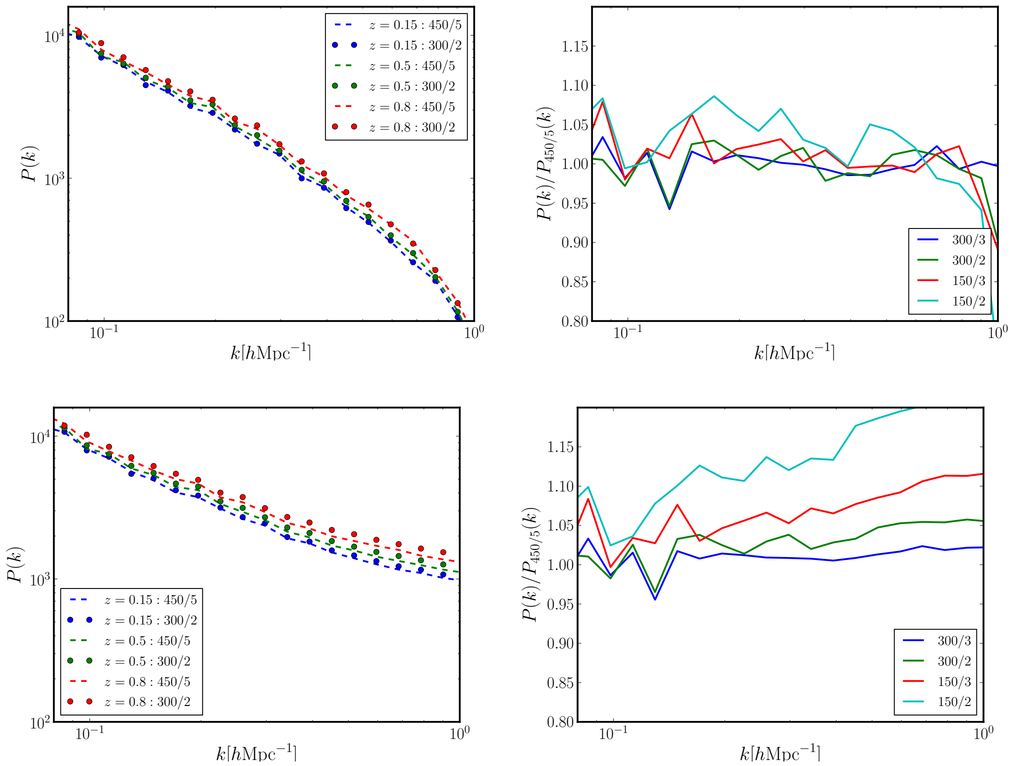


Figure 15. Left: Halo auto power spectra for different redshift $z = 0.15$ (blue), $z = 0.5$ (green), and $z = 0.8$ (red), which compare 450/5 (dashed line) and 300/2 (circles). Right: Ratios of halo auto power spectra between 450/5 and other time steps at redshift $z = 0.15$. All the samples used here has the same mass resolution, 256^3 particles in a $(256h^{-1}\mathrm{Mpc})^3$ cubic box. The upper panels are subtracted a Poisson shot noise, while the lower panels are without a Poisson shot noise subtraction. The sample of halos is equivalent to a mass thresholds of $M = 10^{12.5}\mathrm{M}_{\odot}$.

2.2.3 Question 1: mass selected samples v.s. matched halo samples

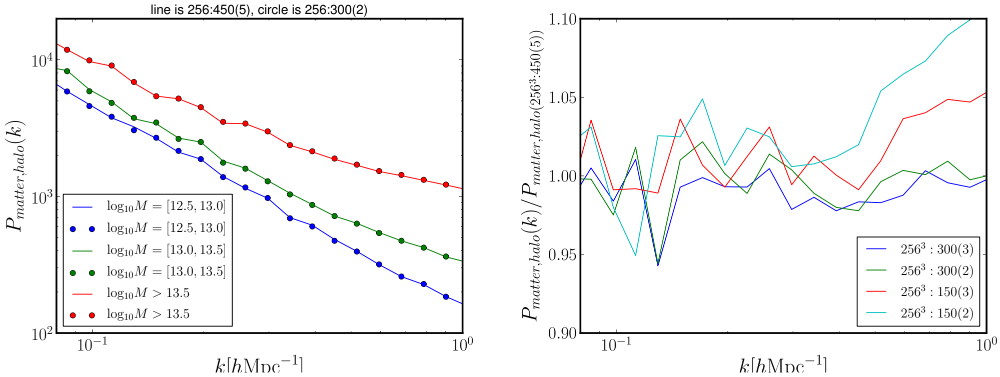
• Comparison of matched halo samples and mass-sliced samples: what the comparison indicates is that corresponding halos don't have the same masses. -¿What kind of problems do we have by having this issue?-¿$b(M)$ may be diffrent for different simulations.

3 OBSERVABLE BOX

What we want to check/know here are:

1) What redshift can we use linear-shifting?,

2) What redshift-step size is required to preserve dynamics in simulations?



[p]

Figure 16. Left: Cross power spectra between halos and DM particles at redshift $z = 0.15$. DM particles are taken from the simulation of 256^3 particles with 450 global steps and 5 sub-cycles. Different colors indicate different halo mass slices: $\log_{10}\mathrm{M} \in [12.5, 13.0]$ (blue), $\log_{10}\mathrm{M} \in [13.0, 13.5]$ (green), and $\log_{10}\mathrm{M} > 13.5$ (red). Lines are the simulations with 450 global steps and 5 sub-cycles, and circles are the ones with 300 global steps and 2 sub-cycles. Right:Ratios of cross power spectra for different simulations with respect to the cross power spectra with 450 global steps and 5 sub-cycles.

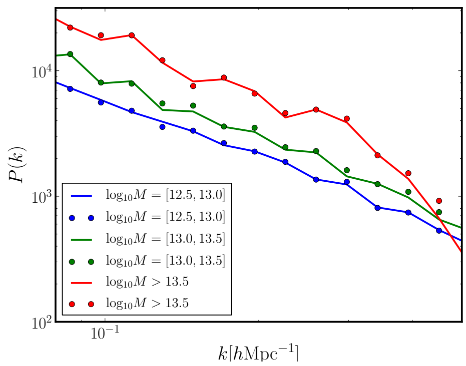


Figure 17. Auto power spectra for different halo mass bins, which compare 450/5 (solid line) and 300/2 (circles) at redshift $z = 0.15$.

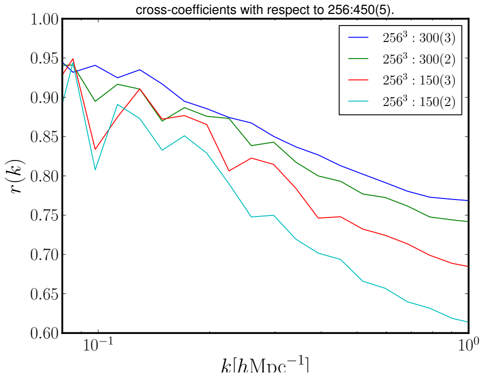


Figure 18. Cross power coefficients between 450/5 and other time steps at redshift $z = 0.15$. Halos used here have a mass from $10^{12.5}\mathrm{M}_{\odot}$ to $10^{13.0}\mathrm{M}_{\odot}$.

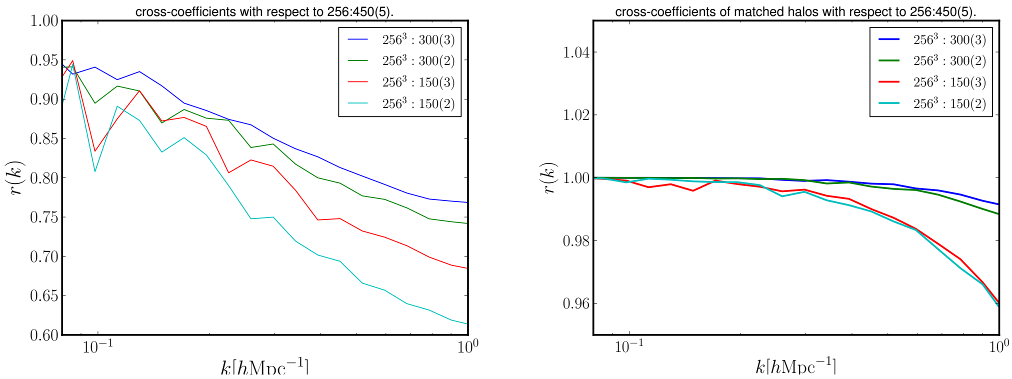


Figure 19. Cross-power coefficients for mass-selected halos (left) and for matched halos (right) at redshift $z = 0.15$. Both cross power coefficients compare the 450/5 simulation and other time steps. Halos for 450/5 have a mass from $10^{12.5}\mathrm{M}_{\odot}$ to $10^{13.0}\mathrm{M}_{\odot}$.

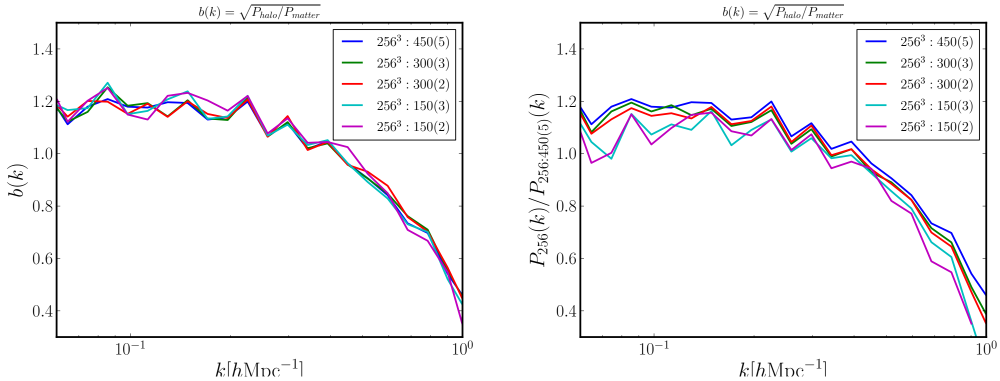


Figure 20. Halo Biases for different stepsizes with 256^3 particles at redshift $z = 0.15$. Left panel is halo biases for samples which are selected based on mass, and right panel is for samples which are corresponding to halos of 450 global steps and 5 sub-cycles. Different colors indicate different stepsizes and mass range for halo samples is from $10^{12.5}\mathrm{M}_\odot$ to $10^{13.0}\mathrm{M}_\odot$.