In[1]:= $f[x\_, d\_, \sigma\_] := \frac{1}{\sqrt{2\pi}\,\sigma} \mathrm{Exp}\left[-\frac{(d-x)^2}{2\sigma^2}\right]$

```
In[2]:= prob[d_, σ_, x0_, xtrue_: 0] =
  Integrate[f[x, d, σ], {x, -x0, xtrue}] / Integrate[f[x, d, σ], {x, -x0, x0}]
```

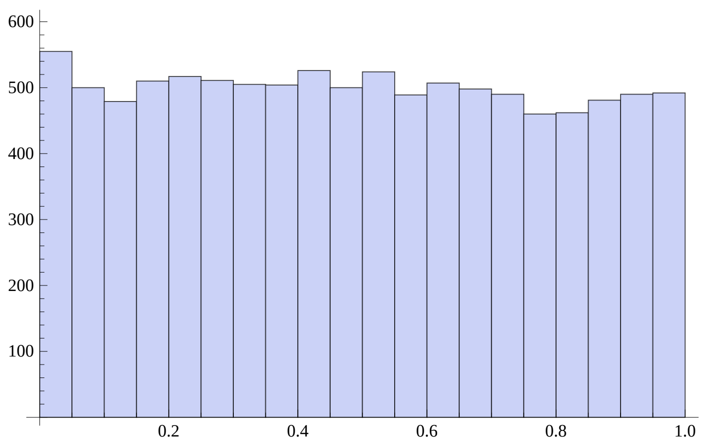
Out[2]= $$\frac{\mathrm{Erf}\left[\frac{d+x0}{\sqrt{2}\,\sigma}\right] - \mathrm{Erf}\left[\frac{d-xtrue}{\sqrt{2}\,\sigma}\right]}{-\mathrm{Erf}\left[\frac{d-x0}{\sqrt{2}\,\sigma}\right] + \mathrm{Erf}\left[\frac{d+x0}{\sqrt{2}\,\sigma}\right]}$$

```
In[3]:= prob[d, σ, x0]
```

Out[3]= $$\frac{-\mathrm{Erf}\left[\frac{d}{\sqrt{2}\,\sigma}\right] + \mathrm{Erf}\left[\frac{d+x0}{\sqrt{2}\,\sigma}\right]}{-\mathrm{Erf}\left[\frac{d-x0}{\sqrt{2}\,\sigma}\right] + \mathrm{Erf}\left[\frac{d+x0}{\sqrt{2}\,\sigma}\right]}$$

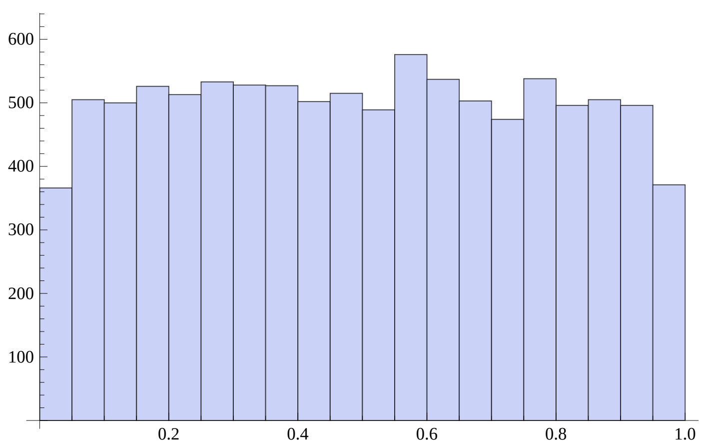
```
In[4]:= uniftest[σ_, x0_, N_] :=
   Histogram[prob[#, σ, x0] & /@ RandomVariate[NormalDistribution[0.0, σ], N]];
In[5]:= uniftest[0.01, 0.1, 10000]
```

Out[5]=



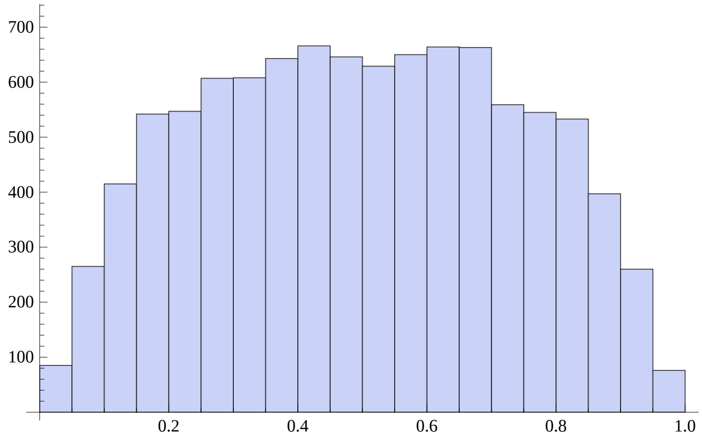

```
In[6]:= uniftest[0.04, 0.1, 10000]
```

Out[6]=



In[7]:=
```
uniftest[0.07, 0.1, 10 000]
```

Out[7]=



Test by putting in prior sampling of x

Cook, Gelman & Rubin (2006, Journal of Computational and Graphical Statistics) suggest a way of testing posterior distributions. Unlike the cases we just did, this requires sampling the true value from the prior distribution and then generating the data. In this case, P(x < x_true) is Uniform(0,1). Doing this below fixes the issues we saw above.

```
(* The function below draws a true value from -
   x0 to x0 and then a Gaussian with width σ).
```

In[34]:=
```
data2[σ_, x0_, N_] := MapThread[{#1, #1 + #2} &,
   {RandomReal[{-x0, x0}, N], RandomVariate[NormalDistribution[0.0, σ], N]}];
```

```
(* Now apply the test, taking into account the true value as well *)
```

In[37]:=
```
uniftest2[σ_, x0_, N_] := Histogram[prob[#[[2]], σ, x0, #[[1]]] & /@ data2[σ, x0, N]];
```

```
(* Note that this is now uniform! *)
```

In[42]:=
```
uniftest2[0.07, 0.1, 10 000]
```

Out[42]=