**CSCI8080 Final Exam Spring 2020**

**Problem 1 (10 points)**

Company TR’s employees are organized in a strict hierarchy with the CEO as the root of the tree. The children of a node N are all supervised by N.

Each employee E in TR is assigned a positive number, EV[E], that measures how valued he/she is, in TR. We want to find a set S of employees that with the total maximum T value with the following conditions: (i) The CEO is always in the set regardless of her value, and (ii) If an employee is in the set, then her immediate supervisor is not in the set.

Design an algorithm that computes S and T given the employee hierarchy of TR using a dynamic programming based approach.

Notes

* Probably going to start with only CEO and one other employee, build from there?

**Problem 2 (10 points)**

Let T be the minimum spanning tree of a graph G. Prove or disprove the following two statements.

1. T will not contain the maximum weighted edge on any cycle in G.
2. T will contain the minimum weighted edge of every cycle in G.

Notes

* MST = all vertices connected, without cycles, using minimum weight
* Cycle in G = all vertice
* A

**Problem 3 (5 points)**

Rewrite the Faster-APSP algorithm to include the predecessor matrix computation. Explain the modifications you made.

**Problem 4 (5 points)**

Suppose you are given a magic black box that can determine in polynomial time, given an arbitrary Boolean formula α, whether α is satisfiable.Describe and analyze a polynomial-timealgorithm that either computesa satisfying assignment for a given Boolean formula or correctly reportsthat no such assignment exists, using the magic black box as a subroutine.

**Problem 5 (10 points)**

We define the 2Sol-SAT problem as follows.

Input: A, an instance of a SAT formula (A is a conjunction of disjunctive clauses).

Output: 1 if A has at least two satisfying solutions, otherwise, 0.

(a) Show that SAT ≤p 2Sol-SAT.

(b) Show that the 2Sol-SAT problem is in NP.

Notes

* A
  + Map inputs, verify outputs match
* B
  + Show it can be verified in p-time

Answers

1. Slkd
2. In order to show that the 2Sol-SAT problem is in NP we need to show that it can be verified in polynomial time.

**Problem 6 (10 points)**

A Hamiltonian Cycle in a graph is a cycle that visits every vertex exactly once. DIRECTED-HAMILTONIANC problem checks to see if a directed graph contains a Hamiltonian cycle. UNDIRECTED-HAMILTONIANC problem does the same for undirected graphs.

1. Describe a polynomial-time reduction from UNDIRECTED-HAMILTONIANC to DIRECTED-HAMILTONIANC.
2. Describe a polynomial-time reduction from DIRECTED-HAMILTONIANC to UNDIRECTED-HAMILTONIAN PATH.

Notes

* Hamiltonian cycle = visits each vertex exactly once

Answers

1. A polynomial time reduction from UNDIRECTED-HAMILTONIANC to DIRECTED-HAMILTONIANC involves mapping the inputs of UNDIRECTED-HAMILTONIANC to the inputs of DIRECTED-HAMILTONIANC in polynomial time and then showing that the outputs match for both problems given the original input and the mapping function to the other problem.
   1. You can map the input of UNDIRECTED-HAMILTONIANC (a graph, G, with edges, E, and vertices, V) to UNDIRECTED-HAMILTONIANC (a graph, G’, with edges, E’, and vertices, V’) by simply taking every undirected edge (u, v) in E and creating a directed edge in both directions such that you end up with (u, v) and (v, u). The set of vertices would remain the same: G’ = {E’, V} such that E’ contains (u, v) and (v, u) for every edge (u, v) in the undirected edges in E. This can be done in polynomial time since you only need to iterate over the edges in E.
   2. DIRECTED-HAMILTONIANC will only return true given graph G’, as defined above, IFF UNDIRECTED-HAMILTONIANC will return true given graph G