**CSCI8080 Final Exam Spring 2020**

**Problem 1 (10 points)**

Company TR’s employees are organized in a strict hierarchy with the CEO as the root of the tree. The children of a node N are all supervised by N.

Each employee E in TR is assigned a positive number, EV[E], that measures how valued he/she is, in TR. We want to find a set S of employees that with the total maximum T value with the following conditions: (i) The CEO is always in the set regardless of her value, and (ii) If an employee is in the set, then her immediate supervisor is not in the set.

Design an algorithm that computes S and T given the employee hierarchy of TR using a dynamic programming based approach.

**Problem 2 (10 points)**

Let T be the minimum spanning tree of a graph G. Prove or disprove the following two statements.

1. T will not contain the maximum weighted edge on any cycle in G.
2. T will contain the minimum weighted edge of every cycle in G.

**Problem 3 (5 points)**

Rewrite the Faster-APSP algorithm to include the predecessor matrix computation. Explain the modifications you made.

**Problem 4 (5 points)**

Suppose you are given a magic black box that can determine in polynomial time, given an arbitrary Boolean formula α, whether α is satisfiable.Describe and analyze a polynomial-timealgorithm that either computesa satisfying assignment for a given Boolean formula or correctly reportsthat no such assignment exists, using the magic black box as a subroutine.

**Problem 5 (10 points)**

We define the 2Sol-SAT problem as follows.

Input: A, an instance of a SAT formula (A is a conjunction of disjunctive clauses).

Output: 1 if A has at least two satisfying solutions, otherwise, 0.

(a) Show that SAT ≤p 2Sol-SAT.

(b) Show that the 2Sol-SAT problem is in NP.

**Problem 6 (10 points)**

A Hamiltonian Cycle in a graph is a cycle that visits every vertex exactly once. DIRECTED-HAMILTONIANC problem checks to see if a directed graph contains a Hamiltonian cycle. UNDIRECTED-HAMILTONIANC problem does the same for undirected graphs.

1. Describe a polynomial-time reduction from UNDIRECTED-HAMILTONIANC to DIRECTED-HAMILTONIANC.
2. Describe a polynomial-time reduction from DIRECTED-HAMILTONIANC to UNDIRECTED-HAMILTONIAN PATH.