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Critical illness insurance pricing with stochastic interest rates model

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Critical illness insurance pricing with stochastic interest rates model

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Abstract. In this paper, we will explain critical illness insurance calculations with stochastic interest rates. The survival model is multiple states, while the interest rate is the Cox-Ingersoll-Ross stochastic interest rate model. In determining the survival model, we use the prevalence and mortality rates of certain critical diseases, such as neoplasms, endocrine diseases, and diseases of the digestive system. Furthermore, the Monte Carlo simulation will be used to simulate the possibility of interest rate pathways in determining critical illness insurance premiums.

1. Introduction
Health insurance is a type of insurance product that specifically guarantees the health costs or care of the insurance members if they fall ill or have an accident. At present, many health insurance products have been developed. For example, health insurance for sickness, health insurance for critical illness, health insurance for long-term care, and others.

In Actuarial science, many methods have been developed to calculate assumptions. The method used to calculate insurance premiums is adjusted to the type of insurance product. However, the insurance product uses the Markov multi-status chain model in its calculations. Insurance models with the Markov multi-status chain model are described in [1].

Most previous research requires data that is quite difficult to obtain. In this study, we will continue critical illness research conducted by Baione and Levantesi [2], in this paper we will calculate critical illness insurance premiums with data that is quite easy to obtain, namely prevalence rates. The mortality intensities used in this paper was upgraded to the Gompertz-Makeham model, and the stochastic interest rate model used in this paper is the interest rate of the Cox-Ingersoll-Ross (CIR) model. The CIR interest rate was first introduced in [3] to calculate bond prices by paying attention to interest rates that always change from year to year. Using Monte Carlo simulation [4] for the stochastic model described it would be compared to critical illness insurance premiums with stochastic interest rates and constant interest rates.

2. Stochastics Interest Rates
Cox on [3] explain a model of stochastic interest rates that have a model

$$dr(t) = \alpha(\beta - r(t))dt + \sigma\sqrt{r(t)}dW(t) \tag{1}$$

where $W(t)$ is a Wiener process with α, β and σ are its positive parameters.

The interest rate of the CIR model will move around the balance point so that it can be said that interest rates will move in a limited range. The CIR stochastic differential equation has expectations and variance as

$$E[r(T)|r(t)] = r(t)e^{-\alpha(T-t)} + \beta(1 - e^{-\alpha(T-t)})$$

$$Var[r(T)|r(t)] = \frac{r(t)}{\alpha}\sigma^2\left(e^{-\alpha(T-t)} - e^{-2\alpha(T-t)}\right) + \sigma^2\frac{\beta}{2\alpha}\left(1 - e^{-\alpha(T-t)}\right)^2$$

To generate CIR interest rates, we must determine the parameters at the CIR interest rate first. The CIR interest rate parameters are explained in [5] and it has been shown that these parameters have consistent and normal asymptotic properties.

$$\hat{\alpha} = -\frac{1}{\Delta t}\ln\left(\frac{n\sum_{k=1}^n r_{k-1}r_k - \sum_{k=1}^n r_{k-1}\sum_{k=1}^n r_k}{n\sum_{k=1}^n r_{k-1}^2 - \left(\sum_{k=1}^n r_{k-1}\right)^2}\right)$$

$$\hat{\beta} = \frac{\bar{r}_k - e^{-\hat{\alpha}\Delta t}\bar{r}_{k-1}}{1 - e^{-\hat{\alpha}\Delta t}}$$

$$\hat{\sigma}^2 = \frac{\sum_{k=1}^n(\eta_0 + \eta_1 r_{k-1})(r_k - (\gamma_1 r_{k-1} + \gamma_0))^2}{\sum_{k=1}^n(\eta_0 + \eta_1 r_{k-1})^2}.$$

The CIR model in Eq. (1) is a one-factor stochastic differential equation. Analytical solutions to this equation are very difficult to do even when stochastic calculations are used, so a numerical approach to the equation must be done. For this reason, Euler scheme is explained in [6]. By taking $t_{k+1} - t_k = 1$ then the CIR interest rate will be obtained as follows

$$r(t_{k+1}) = r(t_k) + \alpha(\beta - r(t_k)) + \sigma\sqrt{r(t_k)}Z_{k+1} \qquad (2)$$

where $r(t_0) = r_0$ and Z_{k+1} is a normal standard distribution.

3. Critical Illness Insurance
3.1. Multiple State Model of CI Covers
Critical illness insurance provides benefits in the form of sum assured, if the policy holder is diagnosed with a critical illness in accordance with the insurance contract. Examples of insurance in critical illness insurance are heart attacks, coronary hearts that require surgery, cancer and stroke.

The model that will be used in critical illness insurance is explained in [1]. Suppose that $[0, T]$ as a finite time interval and $\{Y(t)\}_{t\in[0,T]}$ is a Markov process of a single policy in continuous time, so the critical illness model can be modeled in a multiple state model with status space

$$Y = \{1 : \text{healthy};$$
$$2 : \text{critical illness};$$
$$3 : \text{death due to critical illness};$$
$$4 : \text{death due to other causes}\}.$$

and depicted in Figure 1 assuming $Y(0) = 1$.

3.2. Transition Probabilities of the CI Model
To calculate insurance prices, transition probabilities are needed in the calculation. In this section, the transition probabilities needed in the critical illness insurance model above will be explained.
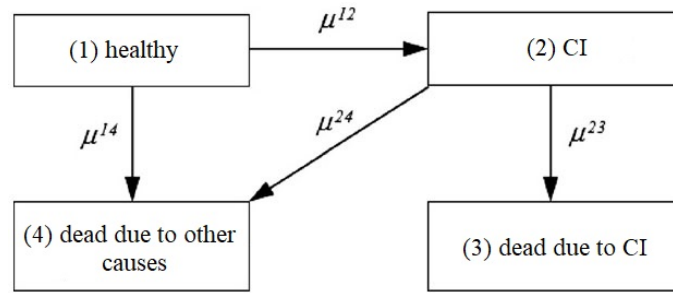
Figure 1. Multiple state model for critical illness covers

For example, given $x \geq 0$ as the age entered in $Y(t)$ as the status occupied by the insured at time t. The transition probabilities of an insured is in the status of j when the time is $x + t$, given the insured is in the status of i when the age of x, is defined as

$$ {}_t p_x^{ij} = P\{Y(x+t) = j | Y(x) = i\} \quad t \in [0,T], i, j \in Y, i \neq j \tag{3} $$

and the insured transition probabilities remains in the i status from the age of x to the age of $x + t$ expressed as

$$ {}_t p_x^{ii} = P\{Y(x+z) = i \text{ for all } z \in [0,T] | Y(x) = i\}. $$

The transition intensities for Eq. (3) is expressed as

$$ \mu^{ij}(x) = \lim_{t \to 0} \frac{{}_t p_x^{ij}}{t} \quad t \in [0,T], i, j \in Y, i \neq j. $$

Using forward kolmogorov obtained transition probabilities

$$ {}_t p_x^{11} = \exp\left\{ -\int_0^t \left(\mu_{x+u}^{12} + \mu_{x+u}^{14} \right) du \right\} \tag{4} $$

$$ {}_t p_x^{12} = \int_0^t {}_u p_x^{11} \mu_{x+ut-u}^{12} p_{x+u}^{22} du \tag{5} $$

$$ {}_t p_x^{14} = \int_0^t {}_u p_x^{11} \mu_{x+u}^{14} dt + \int_0^t {}_u p_x^{12} \mu_{x+u}^{24} du \tag{6} $$

$$ {}_t p_x^{22} = \exp\left\{ -\int_0^t \left(\mu_{x+u}^{23} + \mu_{x+u}^{24} \right) du \right\} \tag{7} $$

3.3. Coverage of CII
The benefits of CII are used to cover medical expenses and provide protection against possible loss of income. Ordinary insurance policy in a fixed time, and is included in the waiting time so that benefits are paid when the diagnosis falls within the insurance period.

In this paper, two types of insurance benefits are explained:

(i) A stand-alone cover only includes a CII benefit; the insurance policy ceases imeediately after the payment of the sum assured. This insurance is valid for n years, where sum assured is payable upon occurence of one of the disease specified by the policy conditions (no waiting period is considered):

$$ {}_1 A_{x:\overline{n}|}^{dd} = \sum_{t=0}^{n-1} b v^{t+1} {}_t p_x^{11} p_{x+t}^{12} $$

(ii) A CII benefit can constitute a rider benefit for a life insurance cover, in particular a term insurance providing a banefit in the case of death. Critical illness with full acceleration benefit will provide benefits with sum assured in cases of death or when the insured experiences critical illness. The single premium for this type of insurance is written as

$$_2A^{dd}_{x:\overline{n}|} = {}_1A^{dd}_{x:\overline{n}|} + \sum_{t=0}^{n-1} bv^{t+1}{}_tp^{11}_x p^{14}_{x+t}$$

4. Estimation of Transition Probabilites and Transition Intensities

The important thing in modeling health insurance with multiple state model is determining the transition intensities. In this section, we will discuss the form of transition intensities needed in the multi-status model of critical illness. First, for the intensities of the death transition μ^{14} and μ^{23}, it is assumed to follow the function of Gompertz-Makeham (GM), which is

$$\mu(x) = A + Bc^x \tag{8}$$

where $A \geq -B$, $B > 0$ and $c > 1$, which can be rewritten to

$$\mu(x) = \alpha + \exp(\beta_1 + \beta_2 x). \tag{9}$$

So that,

$$\mu^{14}(x) = \alpha^h + \exp(\beta_1^h + \beta_2^h x) \tag{10}$$

$$\mu^{23}(x) = \alpha^{dd} + \exp(\beta_1^{dd} + \beta_2^{dd} x) \tag{11}$$

with the h index the parameters for the transition intensities from healthy status to death due to other causes, and with an index of dd the parameter for the transition intensities from dread disease to death status due to critical illness.

Furthermore, for the transition intensities μ^{24} and μ^{12} still use the same assumption in the previous study, namely death due to other causes of critical illness status is greater than γ than from status healthy, that is

$$\mu^{24}(x) = (1 + \gamma)\mu^{14}(x). \tag{12}$$

and the transition intensities from healthy status to critically ill status is expressed as a piecewise constant function, i.e.

$$\mu^{12}(x) = \begin{cases} 0 & x \leq x_0 \\ \sigma_{k+1} & x_k < x \leq x_{k+1} \\ \sigma_n & x_{n-1} < x \end{cases} \tag{13}$$

4.1. Transition Probabilites

After defining the form of transition intensities in the CI model, it is necessary to explain the form of the transition probabilities ${}_tp^{11}_x, {}_tp^{12}_x$, and ${}_tp^{22}_x$ to help estimate the transition intensities parameters described earlier.

First, based on Eq. (4), (10), and (13), obtained

$$_tp^{11}_x = \exp\left\{ -(\sigma_{k+1} + \alpha^h)t - \frac{\dot{\beta}_1^h}{\beta_2^h}\left(e^{\beta_2^h(x+t)} - e^{\beta_2^h x} \right) \right\} \tag{14}$$

with $\dot{\beta}_1^h = e^{\beta_1^h}$. Second, based on Eq. (7), (11), and (12), obtained

$$_tp_x^{22} = \exp\left\{-(\alpha^{dd} + (1+\gamma)\alpha^h)t - \frac{\dot{\beta}_1^{dd}}{\beta_2^{dd}}\left(e^{\beta_2^{dd}(x+t)} - e^{\beta_2^{dd}x}\right)\right.$$
$$\left. -(1+\gamma)\frac{\dot{\beta}_1^h}{\beta_2^h}\left(e^{\beta_2^h(x+t)} - e^{\beta_2^h x}\right)\right\} \tag{15}$$

with $\dot{\beta}_1^{dd} = e^{\beta_1^{dd}}$. Third, to calculate $_tp_x^{12}$ used a Taylor series expansion

$$\exp\left(e^{\beta_2 u}\right) \approx \exp\left\{e^{\beta_2 \frac{t}{2}} + e^{\beta_2 \frac{t}{2}}\beta_2\left(u - \frac{t}{2}\right)\right\}.$$

So,

$$_tp_x^{12} \approx \exp\left\{\frac{\dot{\beta}_1^h}{\beta_2^h}e^{\beta_2^h x}\left(1 - (1+\gamma)e^{\beta_2^h t}\right) - (\alpha^{dd} + (1+\gamma)\alpha^h)t - \frac{\dot{\beta}_1^{dd}}{\beta_2^{dd}}e^{\beta_2^{dd}(x+t)}\right\}$$
$$\times \exp\left\{\gamma\frac{\dot{\beta}_1^h}{\beta_2^h}e^{\beta_2^h(x+\frac{t}{2})}\left(1 - \beta_2^h\frac{t}{2}\right) + \frac{\dot{\beta}_1^{dd}}{\beta_2^{dd}}e^{\beta_2^{dd}(x+\frac{t}{2})}\left(1 - \beta_2^{dd}\frac{t}{2}\right)\right\}\sigma_{k+1}$$
$$\times \frac{\exp\left\{\left(\gamma\alpha^h + \alpha^{dd} - \sigma_{k+1} + \gamma\dot{\beta}_1^h e^{\beta_2^h(x+\frac{t}{2})} + \dot{\beta}_1^{dd}e^{\beta_2^{dd}(x+\frac{t}{2})}\right)t\right\} - 1}{\gamma\alpha^h + \alpha^{dd} - \sigma_{k+1} + \gamma\dot{\beta}_1^h e^{\beta_2^h(x+\frac{t}{2})} + \dot{\beta}_1^{dd}e^{\beta_2^{dd}(x+\frac{t}{2})}}. \tag{16}$$

4.2. Transition Intensities

After defining the intensity of the transition intensities in the previous discussion, then the parameters must be determined from the intensity of the transition μ^{14}, μ^{12}, and μ^{24}. To determine the values of these parameters, three types of data are needed, i.e.

(i) Prevalence Rate.
   Prevalence rates data based on gender and age group obtained from Dr. Sardjito Hospital, Yogyakarta. 2018. (Table 1).

(ii) Mortality due to the dread disease based on gender and age group obtained from Dr. Sardjito Hospital, Yogyakarta. 2018. (Table 2).

(iii) Mortality Table based on gender and age. Indonesia, year 2011.

Data included in the type of disease above only refers to the events that occurred in Dr. Sardjito Hospital and the population used are residents of Yogyakarta. The types of diseases included in the data are neoplasms, endocrine diseases, diseases of the digestive system.

Before estimating all the transition intensities, we will explain some new notations in the calculation. Notations are used for age groups between x and $x+n$. $_nL_x$: expected number of live individual, $_nL_x^{(1)}$: number of individual hopes in healthy status, $_nL_x^{(2)}$: number of individual expectations is in the sick state, $_nD_x^{ij}$: expected number of individual that move from i status to j status..

Based on the notation described earlier, the first type of data, the prevalence rates of critical illness can be stated as

$$_nf_x = \frac{_nL_x^{(2)}}{_nL_x},$$

the number of expectations of individuals living in sickness status divided by the number of expectations of individuals living in the age between x and $x+n$. In addition, it can be stated

Table 1. Prevalence rates of sickness (per 1000)

Age group	Male	Female
15 - 19	4.797461	3.614721
20 - 24	6.387844	4.923378
25 - 29	5.740381	4.434018
30 - 34	7.325084	4.996752
35 - 39	9.856916	6.159742
40 - 44	13.960483	8.201313
45 - 49	20.680287	11.399328
50 - 54	26.044417	15.094834
55 - 59	29.881056	21.037604
60 - 64	30.346919	23.552647
65 - 69	27.065504	22.937438
70 - 74	23.304459	19.306358
75 - 79	18.957868	13.759746

Table 2. Mortality rates due to the dread disease (per 1000)

Age group	Male	Female
15 - 19	0.059319	0.062323
20 - 24	0.108531	0.072285
25 - 29	0.095806	0.088680
30 - 34	0.089429	0.104266
35 - 39	0.134252	0.120640
40 - 44	0.194519	0.194202
45 - 49	0.341509	0.388031
50 - 54	0.470235	0.470508
55 - 59	0.851605	0.718065
60 - 64	0.995723	0.640817
65 - 69	1.055747	0.737974
70 - 74	1.082784	0.789981
75 - 79	0.964426	0.530070

that the death rate due to critical illness that occurs in populations that have contracted critical illness for ages between x and $x + n$ as

$$_nM_x^{23} = \frac{_nD_x^{23}}{_nL_x^{(2)}}. \tag{17}$$

Furthermore, in the second type of data, the mortality rate due to critical illness only represents the mortality rate that occurs in the entire population and does not refer to a portion of the people that has only got a critical illness so it can be written as

$$_nm_x^{23} = \frac{_nD_x^{23}}{_nL_x}. \tag{18}$$

By using the first and the second type of data, then we have

$$_nM_x^{23} = \frac{_nm_x^{23}}{_nf_x}, \tag{19}$$

i.e mortality for the age group between x and $x + n$. Assuming the intensity of the transition from a dread disease state to death due to critical illness state is constant for age between x and $x + n$, obtained

$$\mu^{23}(\xi) = {_nM_x^{23}} \quad, \xi \in (x, x + n).$$

So to estimate the value of μ^{23} can be done by doing a regression of nonlinear equations

$$\alpha^{dd} + \exp(\beta_1^{dd} + \beta_2^{dd}\xi) = \frac{_nm_x^{23}}{_nf_x} \quad, \xi \in (x, x + n) \tag{20}$$

to get the $\alpha^{dd}, \beta_1^{dd},$ and β_2^{dd}.

To determine μ^{14}, it is defined

$$_nf_x' = 1 - {_nf_x} \tag{21}$$

and $_nD_x$ as the death that occurs in the age group between x and $x + n$, i.e

$$_nD_x = {}_nD_x^{14} + {}_nD_x^{24} + {}_nD_x^{23}, \tag{22}$$

then

$$_nM_x^{14} = \frac{_nM_x - {}_nm_x^{23}}{_nf'_x + (1 + \gamma)_nf_x} \tag{23}$$

with $_nM_x$ is the age group mortality between x and $x + n$ which can be calculated from the third type of data, namely mortality table. Therefore, in the same way, we obtain

$$\mu^{14}(\xi) = {}_nM_x^{14} \quad , \xi \in (x, x + n).$$

So to estimate the value of μ^{14} can be done by doing a regression of nonlinear equation

$$\alpha^h + \exp(\beta_1^h + \beta_2^h \xi) = \frac{_nM_x - {}_nm_x^{23}}{_nf'_x + (1 + \gamma)_nf_x} \quad , \xi \in (x, x + n) \tag{24}$$

to get the values α^h, β_1^h, and β_2^h.

To determine the intensity of the transition from healthy status to illness status can use the relationship of prevalence and transition probabilities. Suppose that N is the number of population in a healthy state, then

$$_nf_x = \frac{_np_x^{12}}{_np_x^{11} + {}_np_x^{12}}. \tag{25}$$

Because Eq. (25) above is a function with parameter $\alpha^h, \beta_1^h, \beta_2^h, \alpha^{dd}, \beta_1^{dd}, \beta_2^{dd}$, and σ_{k+1} for $k = 0, 1, 2, \ldots n - 1$, then the value σ_{k+1} for $k = 0, 1, 2, \ldots n - 1$ can be obtained using $\alpha^{dd}, \beta_1^{dd}$, and β_2^{dd} from Eq. (20), and the values α^h, β_1^h, and β_2^h which are obtained from Eq. (24).

5. Computation Result

5.1. Generating CIR interest rate

By using Commercial Bank deposit rates from 2007-2019, the CIR interest rate parameters are obtained as in Table 3.

Table 3. CIR Interest Rate Parameters

| Parameter	Estimation
α	0.50315905
β	0.07505371
σ	0.05120608

5.2. Parameter CII

To calculate the transition intensities μ^{23} and μ^{14}, we can use OLS method to obtain parameters in Eq. (20) and (24). Due to limited information about the value of γ, it is assumed that $\gamma = 0$. The parameter results for the transition intensities of the μ^{23} and μ^{14} are explained as in the Table 4 and 5. Based on the parameters that have been obtained, the intensity of the transition μ^{23} can be described in Figure 2 for male and Figure 3 for female and the intensity of the

Table 4. Transition intensities μ^{23} parameters

Parameter	Male	Female
α^{dd}	0.00968820	-0.01190690
β_1^{dd}	-6.92369092	-3.77599198
β_2^{dd}	0.04922975	0.01144945

Table 5. Transition intensities μ^{14} parameters

| Parameter	Male	Female
α^{h}	0.00000888	0.00003596
β_1^{h}	-9.56508168	-10.04324363
β_2^{h}	0.08740237	0.08784453
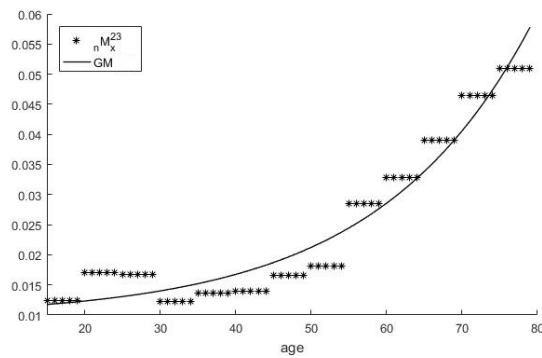
Figure 2. Transition intensities μ^{23} for male
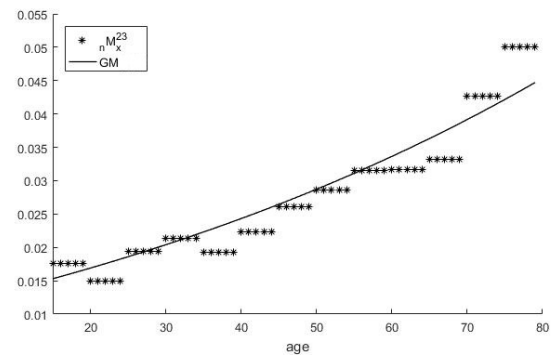
Figure 3. Transition intensities μ^{23} for female
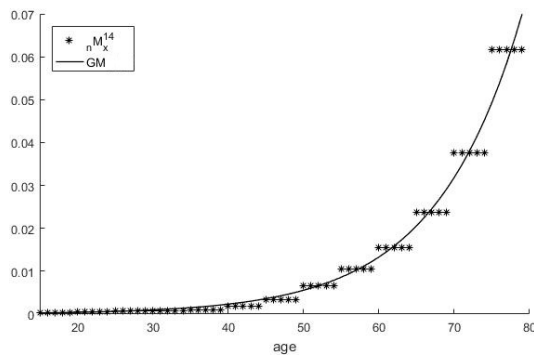
Figure 4. Transition intesities μ^{14} for male
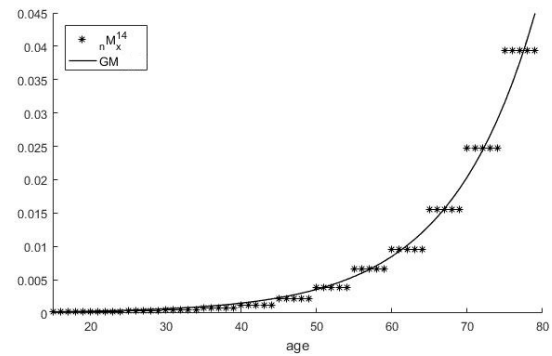
Figure 5. Transition intensities μ^{14} for female

transition μ^{24} can be described in Figure 4 for male and Figure 5 for female. The transition intensities from healthy status to critically ill status, ie μ^{12} can be obtained by solving the Eq. 25. The transition intensities μ^{12} is shown in Table 6.

After obtaining the required transition intensity, we can calculate the transition probabilities of a person aged x years remaining in a healthy state for one year with Eq. (14), the transition probabilities of moving from healthy to sick status with Eq. (16), and the transition probabilities of a healthy person dies not because of critical illness with Eq. (6). The results of the estimation of the transition probabilities are explained in Figure 6, 7, and 8.

Table 6. Transition intensitas μ^{12} parameters

Parameter	Male	Female
σ_1	0.00099426	0.00074284
σ_2	0.00132932	0.00102028
σ_3	0.00119983	0.00096527
σ_4	0.00154252	0.00104246
σ_5	0.00209825	0.00134766
σ_6	0.00301919	0.00174261
σ_7	0.00457811	0.00261079
σ_8	0.00595331	0.00335694
σ_9	0.00714701	0.00473316
σ_{10}	0.00774604	0.00556211
σ_{11}	0.00760314	0.00571572
σ_{12}	0.00756784	0.00520125
σ_{13}	0.00767183	0.00424227
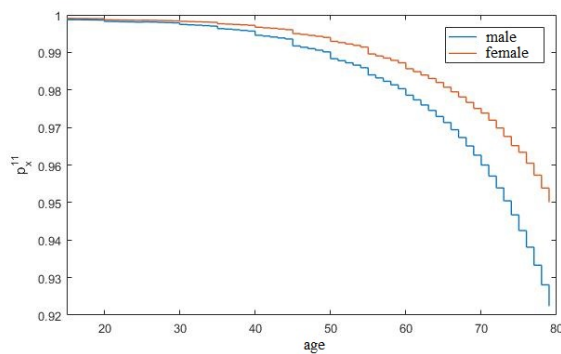
Figure 6. Transition probabilites p_x^{11}
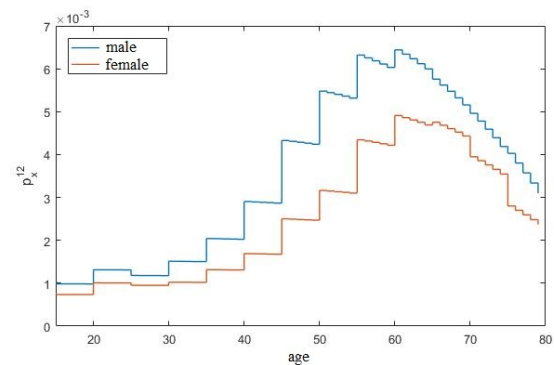
Figure 7. Transition probabilites p_x^{12}
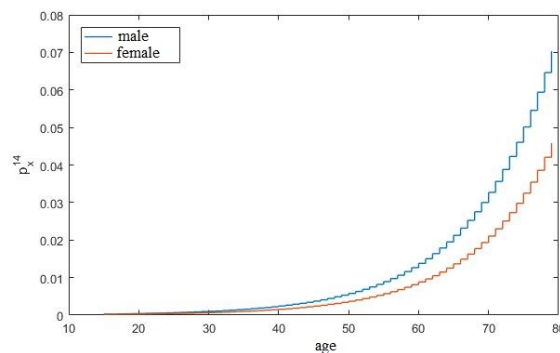
Figure 8. Transition probabilites p_x^{14}

5.3. Premium Critical Illness Insurance

After obtaining the required transition probability value, single premium critical illness insurance can be calculated. To get premium with CIR interest rates, Monte Carlo simulation with $10,000$ repetition is used and calculated as the average of the premiums obtained for each simulation. For

a constant interest rate, the average CIR interest rate will be used, which is $r = \beta = 0.07505371$. The benefits used in the calculation are assumed to be worth $1,000$ both the benefits of critical illness insurance and term life insurance. The results of calculations and comparisons of constant premium rates and CIR are shown in Tables 7 and 8.

Table 7. Premium of CII. Gender Male. (per 1000)

Age	CIR interest rates		Constant interest rates	
	$_{(1)}A^{dd}_{x:\overline{25}|}$	$_{(2)}A^{dd}_{x:\overline{25}|}$	$_{(1)}A^{dd}_{x:\overline{25}|}$	$_{(2)}A^{dd}_{x:\overline{25}|}$
15	13.76947392	21.31805109	13.74652137	21.27879518
20	16.73870193	28.28095344	16.70963033	28.22699813
25	20.32070770	37.93699777	20.28149656	37.85998086
30	27.07932057	53.76936396	27.02709346	53.66031808
35	35.44355927	75.54401613	35.37838313	75.39448795
40	44.46292805	104.06740545	44.38860914	103.86975637
45	52.22000326	139.69431659	52.14353708	139.44080780
50	55.37685833	181.93324478	55.30546550	181.61338889

Table 8. Premium of CII. Gender Female. (per 1000)

Age	CIR interest rates		Constant interest rates	
	$_{(1)}A^{dd}_{x:\overline{25}|}$	$_{(2)}A^{dd}_{x:\overline{25}|}$	$_{(1)}A^{dd}_{x:\overline{25}|}$	$_{(2)}A^{dd}_{x:\overline{25}|}$
15	10.20924443	15.31460189	10.19286919	15.28763115
20	11.97965397	19.64538618	11.96070855	19.61041935
25	13.67739681	25.27985918	13.65330563	25.23144726
30	16.98951261	34.57894774	16.95825488	34.51085289
35	22.27492134	48.88343277	22.23381660	48.78686564
40	28.56231871	68.60100831	28.51135371	68.46733009
45	35.29694790	95.01693526	35.23960294	94.83787864
50	39.91477419	128.06030904	39.85648893	127.82579222

6. Conclusion and further research

This paper discusses the application of multiple state models in critical illness health insurance with prevalence rates such as [2], and develops the form of the intensity of death transitions into the Gompertz-Makeham model. Also, a single premium calculation with a stochastic interest rate is applied. The stochastic interest rate model used is CIR, and the yield obtained is a single premium calculated at the CIR interest rate having a higher price than that calculated with a constant interest rate. The use of stochastic interest rates can be considered considering the prevailing interest rates every year.

In further research, other models can be applied to determine the intensity of death. Also, research can be developed into other types of health insurance models, such as health insurance for long-term care, minor illness health insurance, etc., and can be applied to other stochastic interest rates models.

References

[1] Haberman S and Pitacco E 1999 *Actuarial Models for Disability Insurance* (USA: CHAPMAN & HALL/CRC)

[2] Baione F and Levantesi S 2014 A health insurance pricing model based on prevalence rates: Application to critical illness insurance *Insurance: Mathematics and Economics* **58** 174-84

[3] Cox J C, Ingersoll J E and Ross S A 1985 A Theory of the Term Structure of Interest Rates *Econometrica* **53** 385-407

[4] Graham C and Talay D 2013 *Stochastic Modelling and Applied Probability* (Berlin: Springer)

[5] Overbeck L and Rydén 1997 Estimation in the cox-ingersoll-ross model *Econometric Theory* **13** 430-61

[6] Kloeden P E and Platen E 1992 *Numerical Solution of Stochastic Differential Equations* (New York: Springer)

[7] Haberman S 1984 Decrement tables and the measurement of morbidity: II *Journal of the Institute of Actuaries* **111** 73-86

[8] Alim K 2019 Penentuan harga asuransi penyakit kritis dengan suku bunga Cox-Ingersoll-Ross berdasarkan angka prevalensi UGM: Master Theses