Insurance: Mathematics and Economics 58 (2014) 174–184

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

A health insurance pricing model based on prevalence rates: Application to critical illness insurance

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ARTICLE INFO

Article history:
Received October 2013
Received in revised form
May 2014
Accepted 19 July 2014
Available online 30 July 2014

JEL classification:
G22
I13

MSC:
IM12
IB13

Keywords:
Multiple state models
Transition intensities
Gompertz–Makeham
Prevalence rates
Critical illness insurance

ABSTRACT

The Italian health insurance market is currently undersized. The paucity of assured data and the discontinuous statistical surveys carried out by the National Institute of Statistics (ISTAT) represent one of the main obstacles to the insurance market development. The paper sets forth a parametric model to estimate technical basis for health insurance policies when data are limited and only aggregated information on mortality and morbidity is available. The probabilistic framework is based on a multiple state continuous and time inhomogeneous Markov model. We provide an estimate of transition intensities from the healthy state to the sickness state when only prevalence rates of sickness are available, according to an extension and modification of the methodology proposed in Olivieri (1996) for Long Term Care insurance. We assume that mortality intensity of both healthy and sick lives is modelled by two independent Gompertz–Makeham models.

1. Introduction

Throughout the world there are various types of health insurance products and there are many different traditions in actuarial calculation. Health insurance in the form of Critical Illness (CI) or Long Term Care (LTC) is generally structured by multiple state models allowing us to represent the evolution of a given insurance policy as a sample path of a time-continuous (or time-discrete) inhomogeneous Markov chain. In the insurance literature, the Markovian multiple state model first appeared in Amsler (1968) and Hoem (1969). When data on time spent in a sickness/disability state are available, the extension to semi-Markov processes could be considered (see e.g. Janssen, 1966; Waters, 1989). However, the available Italian National statistics on health do not usually allow us to apply them. For a comprehensive survey of actuarial modelling of CI and LTC insurance see Pitacco (1995) and Haberman and Pitacco (1999); for Markov and semi-Markov models in health insurance see Christiansen (2012).

One of the main critical aspects of the application of Markov models to health insurance, and even more of semi-Markov, deals with the estimation of transition intensities or probabilities that requires a consistent data set where transitions between states of a reference population are collected. Therefore, as stated by Pitacco (2012) "in actuarial practice [...] simplified calculation procedures are often used for pricing and reserving". Pitacco provides a classification of calculation methods based on the format of statistical data, e.g., methods based on the probability of becoming ill (inception annuity models) and methods based on average time spent in sickness (Manchester Unity model).

Modelling the transition intensities of the policyholders of health insurance contracts is a key issue that has a huge impact on insurance premiums. The problem has been addressed by some authors, e.g. Cordeiro (2002), Czado and Rudolph (2002), and Helms et al. (2005). In the context of time-discrete Markov models, the problem concerns the estimation of transition probabilities instead of transition intensities. In this context Dash and Grimshaw (1993) derive a suitable technical basis from UK statistics for assured lives.

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http://dx.doi.org/10.1016/j.insmatheco.2014.07.005
0167-6687/

In actuarial practice an accurate model for health insurance required a wide range of statistical data. Nevertheless, the Italian health insurance market is undersized (Swiss Re, 2012) and national health statistics are scanty and usually referred to the general population. The paper focuses on the pricing of health insurance contracts structured on a multiple state model when there is lack of data and only aggregated information on mortality and morbidity is available. Our approach can be applied when sickened/disabled data are only of "prevalence rates" type, i.e. provide the proportion of people who currently are sickness/disabled in a population, allowing us to estimate only the probability of being ill/disabled. Prevalence rates are easier to collect than incidence rates that refer to the annual number of people who have a new case of the condition (illness/disability). These two measures are very different. For example, a chronic disease can have a low incidence but high prevalence, because the prevalence is the cumulative sum of past year incidence rates. A short-duration curable condition can have a high incidence instead, but low prevalence. However, under specific assumptions, incidence rates can be estimated by using prevalence rates. Assuming that the random pattern of states is a homogeneous continuous Markovian process, our model provides an estimate of transition intensities from the healthy state to the sickened/disabled one, according to an extension of the methodology proposed in Olivieri (1996) for LTC. Despite the assumptions made by Olivieri (1996), where the mortality intensity of disabled is proportional to the mortality intensity of healthy, we assume that mortality intensity of both healthy and sick lives can be represented by two independent Gompertz–Makeham (GM) models.

The paper is organized as follows. In Section 2 we define the structure of multiple state models in continuous time for CI insurance as well as pure premium rates for typical CI coverage. In Section 3 we describe the framework of transition intensities when data on prevalence rates of sickness instead of incidence rates are available. Section 4 focuses on the pricing of a CI insurance. We first provide transition probabilities formulas in the case of GM models for mortality intensities by cause, and then we solve the corresponding premium rates equation in continuous time. In Section 5 an application to Italian data is reported for CI insurance while quantitative results are described in Section 6. Finally, conclusions and further researches are discussed in Section 7.

2. Critical illness insurance: multiple state model and pure premium rates

A CI insurance or dread disease (DD) insurance provides the policyholder with a lump sum if the insured individual catches a serious illness included within a set of diseases specified by the policy conditions. The most common diseases are heart attack, coronary artery disease requiring surgery, cancer, and stroke.

Let $[0, T]$ be a fixed finite time horizon and $\{S(t)\}_{t\in[0,T]}$ a Markovian process describing the development of a single policy in continuous time, CI is modelled by a multiple state model with state space $S = \{1 = \text{healthy/active}, 2 = \text{ill/dread disease sufferer}, 3 = \text{dead due to critical illness}, 4 = \text{dead due to other causes}\}$ and a set of transitions according to Fig. 2.1, assuming $S(0) = 1$.

Let x ($x \geq 0$) be the age of entry and $S(t)$ the state occupied by the policyholder at time t. The transition probabilities of a policyholder being in state j at age $x+t$, given that the policyholder is in state i at age x, are defined as follows:

$$ {}_tp_x^{ij} = \mathbb{P}\{S(x+t) = j | S(x) = i\} \quad t \in [0, T], i, j \in S, i \neq j. \tag{2.1} $$

And the corresponding transition intensities:

$$ \mu^{ij}(x) = \lim_{t\to 0}\left(\frac{{}_tp_x^{ij}}{t}\right) \quad t \in [0, T], i, j \in S, i \neq j. \tag{2.2} $$

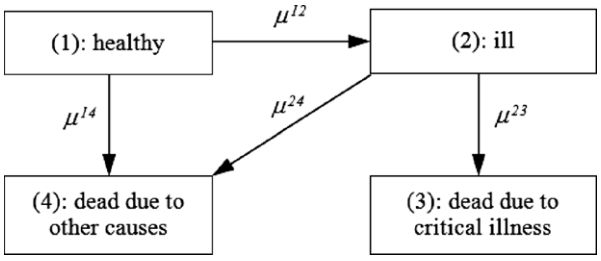

Fig. 2.1. Set of states and set of transitions for CI benefits.

While the probability of a policyholder being in state i at age x to remain in the same state up to age $x + t$ is:

$$ {}_tp_x^{ii} = \mathbb{P}\{S(x+z) = i \text{ for all } z \in [0, T], S(x) = i\}. \tag{2.3} $$

Given the transition intensities of the multiple state model, the corresponding probabilities are solutions of the Kolmogorov forward differential equations (see e.g. Haberman and Pitacco, 1999). The transition probabilities (2.1) and (2.3) according to the model in Fig. 2.1 are:

$$ {}_tp_x^{11} = \exp\left\{-\int_0^t \left[\mu^{12}(x+u) + \mu^{14}(x+u)\right] du\right\} \tag{2.4} $$

$$ {}_tp_x^{12} = \int_0^t \left[{}_up_x^{11}\mu^{12}(x+u){}_{t-u}p_{x+u}^{22}\right] du \tag{2.5} $$

$$ {}_tp_x^{22} = \exp\left\{-\int_0^t \left[\mu^{23}(x+u) + \mu^{24}(x+u)\right] du\right\}. \tag{2.6} $$

We disregard the possibility of recovery from the diseased state (i.e. we do not consider the transition $2 \to 1$).

The CI coverage is usually offered combined with a life insurance (or an endowment) as a rider benefit (when serious illness occurs, the insurance company pays in advance the sum insured in the case of death) or a supplementary benefit (when serious illness occurs, the insurance company pays an additional benefit to the sum insured in the case of death). Depending on the type of CI coverage we can define different formulas to calculate pure premium rates, for example:

- Stand-alone CI policy with an N duration limit, where the sum insured is payable upon occurrence of one of the diseases specified by the policy conditions (no waiting period is considered):

$$ {}_{(1)}\bar{A}_{x:N}^{(DD)} = \int_0^N {}_tp_x^{11}\mu^{12}(x+t)v(0,t)dt \tag{2.7} $$

  where $v(0, t) = \exp(-\delta t)$ is the value at time 0 of a monetary unit to be paid at time t, i.e. the discount factor (where δ is the force of interest).
- CI full acceleration benefit of a term life insurance with N duration limit: when serious illness occurs, the insurance company pays in advance the sum insured in the case of death (in this example set to 1 monetary unit), if not, the insured amount is paid if the policyholder dies before the expiry of the contract:

$$ {}_{(2)}\bar{A}_{x:N}^{(DD)} = \int_0^N {}_tp_x^{11}\left[\mu^{12}(x+t) + \mu^{14}(x+t)\right] v(0,t)dt. \tag{2.8} $$

  The previous equation can be rewritten as follows:

$$ {}_{(2)}\bar{A}_{x:N}^{(DD)} = {}_{(1)}\bar{A}_{x:N}^{(DD)} + \int_0^N {}_tp_x^{11}\mu^{14}(x+t)v(0,t)dt. \tag{2.9} $$

3. The framework of transition intensities

Our approach estimates transition intensities when there is a lack of data and only aggregated information on mortality and morbidity are available. This approach is particularly useful in countries where national health statistics are sparse, non-continuous and aggregated by age groups. First, we have to assign a function to the mortality intensities of the model; second a function to the transition intensities from healthy to ill is defined. The first issue is tackled by assuming that mortality intensities $\mu^{14}(x)$ and $\mu^{23}(x)$ are described by two independent Gompertz–Makeham (GM) functions. For $\mu^{24}(x)$ we have no data about the mortality following illness for causes other than CI. We apply the approach proposed by Dash and Grimshaw (1993) based on the assumption that the mortality of ill people from causes other than CI exceeds the mortality of healthy ones by an extra mortality of γ, i.e.:

$$\mu^{24}(x) = \mu^{14}(x)(1+\gamma). \tag{3.1}$$

Let us define GM general formula of order (r, s) as:

$$GM(r,s) = \sum_{h=1}^{r} \alpha_h x^{h-1} + \exp\left(\sum_{k=1}^{s} \beta_k x^{k-1}\right) \tag{3.2}$$

where r is the polynomial order and s is the polynomial order of the exponential, while α and β are vectors of non-negative parameters.

Once defined the mortality intensity functions, one shall determine transition intensity from healthy to ill. When "incidence rates" of sickness are available, the $\mu^{12}(x)$ can be directly estimated from data. Conversely, if only "prevalence rates" of sickness are available a method based on the probability of being sick could be implemented (see Haberman and Pitacco, 1999). Set a reference initial age, x_0, where all the policyholders are healthy ($S(x_0) = 1$), the prevalence rates of sickness, f_x (where: $x = x_0+s, s > 0$), can be considered as the probability of being ill at age x for a life aged x_0:

$$f_x = \mathbb{P}\left[S(x) = 2 \mid (S(x) = 1 \vee S(x) = 2) \wedge S(x_0) = 1\right]. \tag{3.3}$$

Therefore, the following equation holds (Haberman, 1984):

$$f_{x_0+s} = \frac{{}_sp^{12}_{x_0}}{{}_sp^{11}_{x_0} + {}_sp^{12}_{x_0}} \quad (s > 0). \tag{3.4}$$

Eq. (3.4) links prevalence rates to probabilities p^{11}_x and p^{12}_x used to calculate premium rates of the insurance coverage here considered (see Eqs. (2.4)–(2.6)). Following Olivieri (1996), we suppose that transition intensity from healthy to ill can be described by a piecewise constant function (for $k = 0, 1, \ldots, n-2$) as follows:

$$\mu^{12}(x) = \begin{cases} 0 & x \le x_0 \\ \sigma_{k+1} & x_k < x \le x_{k+1} \\ \sigma_n & x_{n-1} < x \end{cases} \tag{3.5}$$

where n is equal to the number of prevalence rates available from statistical data.

The use of piecewise constant transition intensities has been proposed by Jones (1994). However it would be preferable to have transition intensities varying between each year of age.

To solve the transition probability Eq. (2.5), we need to fix the parameters r and s of GM. Approximated solutions can be obtained for $r \le 1$ and $s \le 2$, nevertheless in the following we show a solution for $r = 0$ and $s = 2$, knowing that these values may not be suitable for all mortality experiences (see e.g. Brink, 2010), but allow us to avoid over-parameterization in the case of scarce data. This assumption is usually consistent with the empirical evidence emerging from many data sets on health and disability. GM models are generally used for graduation of mortality data. The choice concerning parameters' order must be made carefully, as different graduated rates will result from different values of r and s.

We suppose that $\mu^{14}(x) \approx GM^{14}(0, 2)$. In formula, we have:

$$\mu^{14}(x) = \exp\left(\beta^h_1 + \beta^h_2 x\right) \quad \text{with } \beta^h_1, \beta^h_2 > 0 \tag{3.6}$$

where β^h_1, β^h_2 are the GM parameters for healthy people (h). While the mortality for critical illness is assumed to follow a $GM^{23}(0, 2)$ model, where β^{dd}_1, β^{dd}_2 are the GM parameters for dread disease sufferers (dd):

$$\mu^{23}(x) = \exp\left(\beta^{dd}_1 + \beta^{dd}_2 x\right) \quad \text{with } \beta^{dd}_1, \beta^{dd}_2 > 0. \tag{3.7}$$

4. The pricing of a CI insurance in a GM context

4.1. Transition probabilities estimation

Under the assumptions made in Eqs. (3.5) and (3.6) and considering Eq. (2.4), the probability to remain in state 1 until time t is:

$${}_tp^{11}_x = \exp\left\{-\int_0^t \left[\sigma_{k+1} + e^{\beta^h_1+\beta^h_2(x+u)}\right] du\right\}$$
$$\text{for } k = 0, 1, \ldots, n-2, x_k < x \le x_{k+1} \text{ and } t \le x_{k+1} - x$$
$$\text{and for } k = n-1, x > x_{n-1} \text{ and } \forall t. \tag{4.1}$$

Fixed $\dot\beta^h_1 = \exp(\beta^h_1)$, the solution to the previous equation is:

$${}_tp^{11}_x = \exp\left\{-\sigma_{k+1}t - \frac{\dot\beta^h_1}{\beta^h_2}\left[e^{\beta^h_2(x+t)} - e^{\beta^h_2 x}\right]\right\}$$
$$\text{for } k = 0, 1, \ldots, n-2, x_k < x \le x_{k+1} \text{ and } t \le x_{k+1} - x$$
$$\text{and for } k = n-1, x > x_{n-1} \text{ and } \forall t. \tag{4.2}$$

Using Eq. (2.6), we derive the probability to remain in state 2 until time t, where $\dot\beta^{dd}_1 = \exp(\beta^{dd}_1)$:

$${}_tp^{22}_x = \exp\left\{-\frac{\dot\beta^h_1}{\beta^h_2}(1+\gamma)\left[e^{\beta^h_2(x+t)} - e^{\beta^h_2 x}\right]\right.$$
$$\left. - \frac{\dot\beta^{dd}_1}{\beta^{dd}_2}\left[e^{\beta^{dd}_2(x+t)} - e^{\beta^{dd}_2 x}\right]\right\} \quad \forall x, t. \tag{4.3}$$

From Eq. (2.5), under assumptions in Eq. (3.5), (3.6) and (3.7), we obtain:

$${}_tp^{12}_x = \exp\left\{-\frac{\dot\beta^h_1}{\beta^h_2}\left[(1+\gamma)e^{\beta^h_2(x+t)} - e^{\beta^h_2 x}\right]\right.$$
$$\left. - \frac{\dot\beta^{dd}_1}{\beta^{dd}_2}\left[e^{\beta^{dd}_2(x+t)}\right]\right\}\sigma_{k+1}$$
$$\cdot \int_0^t \exp\left\{-\sigma_{k+1}u + \frac{\dot\beta^h_1}{\beta^h_2}\gamma\left[e^{\beta^h_2(x+u)}\right] + \frac{\dot\beta^{dd}_1}{\beta^{dd}_2}\left[e^{\beta^{dd}_2(x+u)}\right]\right\} du$$
$$\text{for } k = 0, 1, \ldots, n-2, x_k < x \le x_{k+1} \text{ and } t \le x_{k+1} - x$$
$$\text{and for } k = n-1, x > x_{n-1} \text{ and } \forall t. \tag{4.4}$$

To solve Eq. (4.4) we assume the following approximations according to a Taylor series expansion:

$$\exp\left(e^{\beta^{(i)}_2 u}\right) \cong \exp\left[e^{\beta^{(i)}_2 \frac{t}{2}} + e^{\beta^{(i)}_2 \frac{t}{2}}\beta^{(i)}_2\left(u - \frac{t}{2}\right)\right]$$
$$\text{for } i = h, dd \tag{4.5}$$

hence, Eq. (4.4) becomes:

$${}_tp^{12}_x \cong \left[\exp\left(\frac{\dot\beta^h_1}{\beta^h_2}e^{\beta^h_2 \cdot x}\left(1 - e^{\beta^h_2 \cdot t}(1+\gamma)\right)\right.\right.$$
$$\left.\left. + \gamma\frac{\dot\beta^h_1}{\beta^h_2}e^{\beta^h_2\left(x+\frac{t}{2}\right)}\left(1 - \beta^h_2 \cdot \frac{t}{2}\right)\right)\right]$$

$$\cdot \left[\exp\left(-\frac{\dot\beta_1^{dd}}{\beta_2^{dd}}e^{\beta_2^{dd}(x+t)} + \frac{\dot\beta_1^{dd}}{\beta_2^{dd}}e^{\beta_2^{dd}(x+\frac{t}{2})}\left(1-\beta_2^{dd}\cdot\frac{t}{2}\right)\right)\right]$$
$$\cdot \frac{\sigma_{k+1}\cdot\left[\exp\left(-\sigma_{k+1}+\gamma\dot\beta_1^h e^{\beta_2^h(x+\frac{t}{2})}+\dot\beta_1^{dd}e^{\beta_2^{dd}(x+\frac{t}{2})}\right)t-1\right]}{-\sigma_{k+1}+\gamma\dot\beta_1^h e^{\beta_2^h(x+\frac{t}{2})}+\dot\beta_1^{dd}e^{\beta_2^{dd}(x+\frac{t}{2})}}. \tag{4.6}$$

Therefore, when prevalence rates, f_x, are available, by substituting Eqs. (4.2) and (4.6) in Eq. (3.4), it is possible to obtain an estimate of the unknown parameters σ_{k+1} for all $k = 0, 1, \ldots, n-1$, via an iterative approach, starting from the initial age group (x_0, x_1).

Our model can be extended to LTC insurance, characterized by a three-states Markov model, as illustrated in Appendix A.

4.2. Pure premium rates

With reference to CI coverage described in Section 2 in the following we solve Eqs. (2.7) and (2.9). Depending on the covered age period $(x, x+N)$, integral in formula (2.7) should be obtained as the sum of sub-integrals each defined on the sub-intervals $(0, y_1], (y_1, y_2], \ldots, (y_{k+1}, y_{k+2}], \ldots, (y_{n-1}, N]$ with $y_{k+1} = x_{k+1} - x$, and set $y_0 = 0$, $y_n = N$, as shown in Eq. (4.7). This is a consequence of the hypothesis of piecewise constant function describing the transition intensity, $\mu^{12}(x)$, on each sub-interval.

$${}_{(1)}\bar A_{x:N}^{(DD)} = \sum_{k=0}^{n-1}\int_{y_k}^{y_{k+1}} {}_tp_x^{11}\mu^{12}(x+t)v(0,t)dt \tag{4.7}$$

Using the approximated formula (4.5), the generic integral in (4.7) defined on the sub-interval $(y_k, y_{k+1}]$ has the following solution:

$$\int_{y_k}^{y_{k+1}} {}_tp_x^{11}\mu^{12}(x+t)v(0,t)dt$$
$$\cong \exp\left\{\frac{\dot\beta_1^h}{\beta_2^h}e^{\beta_2^h x} - \frac{C_k}{\beta_2^h}\left(1-\beta_2^h\left(\frac{y_{k+1}+y_k}{2}\right)\right)\right\}$$
$$\cdot \frac{\left\{e^{-y_k(C_k+\delta+\sigma_{k+1})} - e^{-y_{k+1}(C_k+\delta+\sigma_{k+1})}\right\}\sigma_{k+1}}{C_k+\delta+\sigma_{k+1}} \tag{4.8}$$

where $C_k = \dot\beta_1^h \exp\left(\beta_2^h\left(x+\frac{y_k+y_{k+1}}{2}\right)\right)$.

Similar to the previous case, we split the integral in formula (2.9) into the sum of sub-integrals $\int_{y_k}^{y_{k+1}} {}_tp_x^{11}\mu^{14}(x+t)v(0,t)dt$. Using the approximated formula (4.5), the generic sub-integral on $(y_k, y_{k+1}]$ has the following solution:

$$\int_{y_k}^{y_{k+1}} {}_tp_x^{11}\mu^{14}(x+t)v(0,t)dt$$
$$\cong \exp\left\{\beta_2^h x + \frac{\dot\beta_1^h e^{\beta_2^h x} - C_k\left(1-\beta_2^h\left(\frac{y_{k+1}+y_k}{2}\right)\right)}{\beta_2^h}\right\}$$
$$\cdot \frac{\left\{e^{-y_k\left(C_k+\delta+\sigma_{k+1}-\beta_2^h\right)} - e^{-y_{k+1}\left(C_k+\delta+\sigma_{k+1}-\beta_2^h\right)}\right\}\dot\beta_1^h}{C_k+\delta+\sigma_{k+1}-\beta_2^h}. \tag{4.9}$$

5. Numerical application

Starting from data collected by Italian National health surveys (Istituto Nazionale di Statistica ISTAT, 2008, 2009), our model is applied to CI coverage in order to estimate a technical basis for pricing purposes. We refer to National statistics because CI products are relatively new on the Italian market and insurance data are still too scarce and inconsistent to be used in actuarial calculation. As stated in the introduction, our approach is suitable to situation where health statistics are sparse, non-continuous and aggregated by age groups.

Table 5.1
Prevalence rates of sickness (people reporting chronic conditions by disease, age group and gender). Italy, 2005 (rates per 1000).
Source: ISTAT.

Prevalence rates (per 1000)		
Age group	Males	Females
15–24	11.80	23.20
25–34	28.70	54.30
35–44	85.60	116.30
45–54	220.80	285.90
55–64	512.20	555.40
65–69	701.40	765.60
70–74	822.20	871.80
75–79	939.60	946.20

Table 5.2
Mortality rates by age group, males and main causes. Italy, 2009 (rates per 1000).
Source: Authors' processing of data from ISTAT.

Age group	C00–D48	E00–E90	I00–I99	K00–K93	Total
15–24	0.06	0.01	0.03	0.00	0.10
25–34	0.10	0.01	0.07	0.01	0.19
35–44	0.26	0.03	0.22	0.07	0.57
45-54	1.09	0.10	0.66	0.22	2.06
55–64	3.82	0.29	1.89	0.41	6.41
65–69	7.78	0.59	3.93	0.71	13.01
70–74	11.53	1.01	6.88	1.11	20.53
75–79	17.56	1.78	14.15	1.75	35.23

5.1. Data set

(a) People reporting chronic conditions by disease, gender and age group. Italy, year 2005 (Istituto Nazionale di Statistica ISTAT, 2008).
(b) Mortality rates by age group, gender, main causes and year of death. Italy, year 2009 (Istituto Nazionale di Statistica ISTAT, 2009).
(c) Mortality table by age and gender. Italy, year 2009 (downloadable from the ISTAT website).

Serious diseases to be included in CI insurances are selected considering their mortality incidence on the population. To calculate the mortality incidence of diseases we reclassified mortality rates by causes according to the International Classification of Disease — version 10 (ICD-10), published by the World Health Organization (WHO, 2007). We consider the following four categories of disease as "critical illnesses", i.e. those whose incidence on the total mortality is around 65%–90%: Neoplasms (ICD code C00–D48), Endocrine diseases (ICD code E00–E90), Diseases of the circulatory system (ICD code

I00–I99) and Diseases of the digestive system (ICD code K00–K93).

Table 5.1 shows prevalence rates of sickness (data of type (a)), while Tables 5.2 and 5.3 show the mortality rates by main causes of sickness (data of type (b)) for males and females, respectively. Please note that the last column in Tables 5.2 and 5.3 includes the mortality rates of all the four categories of disease considered.

5.2. Transition intensities estimation

To estimate transition intensities $\mu^{14}(x)$ and $\mu^{23}(x)$ we should consider the mortality rates of both healthy and ill lives by age and gender. However, since the mortality rates of ill lives collected by ISTAT belong to 5–10 year age group, we first construct an abridged multistate life table. To help understanding of the definitions

Table 5.3
Mortality rates by age group, females and main causes. Italy, 2009 (rates per 1000).
Source: Authors' processing of data from ISTAT.

Age group	C00–D48	E00–E90	I00–I99	K00–K93	Total
15–24	0.04	0.01	0.02	0.00	0.07
25–34	0.09	0.01	0.03	0.01	0.13
35–44	0.31	0.01	0.07	0.03	0.43
45–54	1.08	0.05	0.22	0.07	1.42
55–64	2.44	0.16	0.62	0.17	3.39
65–69	4.16	0.37	1.61	0.38	6.52
70–74	5.78	0.71	3.51	0.63	10.63
75–79	8.28	1.43	8.55	1.17	19.43

presented below, the following notation is used to denote the composition of the population belonging to the generic age group $(x, x+n)$, where n is the length of the interval:

- ${}_nL_x$: expected number of survivors;
- ${}_nL_x^{(1)}$: expected number of healthy lives;
- ${}_nL_x^{(2)}$: expected number of sick lives.

According to this notation, the observed prevalence rates for the generic age group $(x, x+n)$, ${}_nf'_x$, can be expressed as: ${}_nf'_x = \frac{{}_nL_x^{(2)}}{{}_nL_x}$.

Denoting as ${}_nD_x^{ij}$ the expected number of transitions from i to j between ages x and $x+n$ for an insured individual who survives to age $x+n$, the mortality rates of sick lives between ages x and $x+n$, can be obtained as ${}_nM_x^{23} = \frac{{}_nD_x^{23}}{{}_nL_x^{(2)}}$.

Note that data of type (b) represent the mortality rates due to CI of the whole population and they are not referred to the portion of population affected by CI; in formula: ${}_nm_x^{23} = \frac{{}_nD_x^{23}}{{}_nL_x}$. By combining data of type (a) and (b) ${}_nM_x^{23}$ can be obtained as:

$$ {}_nM_x^{23} = \frac{{}_nm_x^{23}}{{}_nf'_x}. \tag{5.1} $$

Besides, considering the paucity of information about the relationship between the mortality rate of healthy lives and mortality rates of sick lives for causes other than CI, we set $\gamma = 0$ in Eq. (3.1). Therefore, the following relation holds:

$$ {}_nM_x^{14} = \frac{{}_nD_x^{14}}{{}_nL_x^{(1)}} = \frac{{}_nD_x^{24}}{{}_nL_x^{(2)}}. \tag{5.2} $$

According to Eq. (5.2) and using the relationship ${}_nD_x - {}_nD_x^{23} = {}_nD_x^{14} + {}_nD_x^{24}$, we calculate the mortality rate of healthy lives between ages x and $x+n$, as follows:

$$ {}_nM_x^{14} = {}_nM_x - {}_nm_x^{23} \tag{5.3} $$

where ${}_nM_x$ is the mortality rate of lives between ages x and $x+n$. We now assume that the force of mortality remains constant over each age group $(x, x+n)$. This assumption allows us to estimate the transition probabilities of the model since it implies that:

$$ \mu^{14}(\xi) = {}_nM_x^{14} \quad \text{for } \xi \in (x, x+n) \tag{5.4} $$

$$ \mu^{23}(\xi) = {}_nM_x^{23} \quad \text{for } \xi \in (x, x+n). \tag{5.5} $$

According to the assumptions made in Eqs. (3.6) and (3.7) we graduate $\mu^{14}(x)$ and $\mu^{23}(x)$ with two $GM(0,2)$ functions. The values of GM parameters, obtained by using MMSE estimation, are reported in Table 5.4.

5.3. Prevalence rates adjustment

Prevalence rates of ill individuals collected by ISTAT count the number of people reporting chronic diseases including people affected by more than one chronic disease at the same time. This may result in a multiple count mainly concerning the older age groups. In fact, the number of people claiming to be suffering from at least three diseases increases significantly with increasing age (Istituto Nazionale di Statistica ISTAT, 2008).

Table 5.4
Parameters of the GM models of healthy and sickness mortality. Year 2009.

Gender	Parameters of p_x^{14}		Parameters of p_x^{23}	
	$\dot\beta_1^h$	$\dot\beta_2^h$	$\dot\beta_1^{dd}$	$\dot\beta_2^{dd}$
Male	0.000096	0.051384	0.002755	0.031450
Female	0.000016	0.067573	0.000903	0.038857

Table 5.5
Adjusted prevalence rates of sickness (people reporting chronic conditions by disease, age group and gender). Italy, 2005 (rates per 1000).
Source: Authors' processing of data from ISTAT.

Prevalence rates (per 1000)		
Age group	Males	Females
15–24	11.76	23.05
25–34	28.44	53.43
35–44	83.97	112.46
45–54	212.30	265.17
55–64	465.08	481.53
65–69	600.75	616.31
70–74	682.01	668.23
75–79	751.68	715.33

It is impossible to completely eliminate multiple counting of individuals within our data set. However, ISTAT collects data on the frequency of cases reporting at least three chronic diseases, ${}_n\rho_x$ for the generic age group $(x, x+n)$, and states that about half of them have at least one serious chronic disease. Therefore, we can adjust prevalence rates removing people who have three or more chronic diseases including at least a serious one. The adjusted prevalence rates, ${}_nf_x$, are then estimated according to the following equation (results are reported in Table 5.5):

$$ {}_nf_x = {}_nf'_x \left(1 - \frac{{}_n\rho_x}{2}\right) \tag{5.6} $$

where $\frac{{}_n\rho_x}{2}$ is the frequency of people who have three or more chronic diseases including at least a serious one.

To define a demographical basis for CI insurance we only have prevalence rates data limited to year 2005; therefore we assume them to be constant over time.

6. Numerical results

For pricing purposes it is suitable to arrange a multiple life table for each reference age of the coverage. Consequently, we have to calculate the values of parameters $\sigma_1, \sigma_2, \ldots, \sigma_n$ (see Eq. (3.5)) and then define the transition intensity $\mu^{12}(x)$ by age. To this aim we propose three different approaches based on:

- raw prevalence rates (*approach 1*);
- graduated prevalence rates with a cubic spline function (*approach 2*);
- raw prevalence rates and then applying the graduation (with a cubic spline function) directly to the resulting transition intensities (*approach 3*).

The first approach estimates $\mu^{12}(x)$ for age groups starting from data in Table 5.5, via the iterative procedure described in Section 4.1 (see Table 6.1). In this way, the multiple life table for each reference age is based on the assumption that $\mu^{12}(x)$ is a piecewise constant function.

The second approach consists of the graduation of prevalence rates reported in Table 5.5 using a cubic spline, in order to obtain

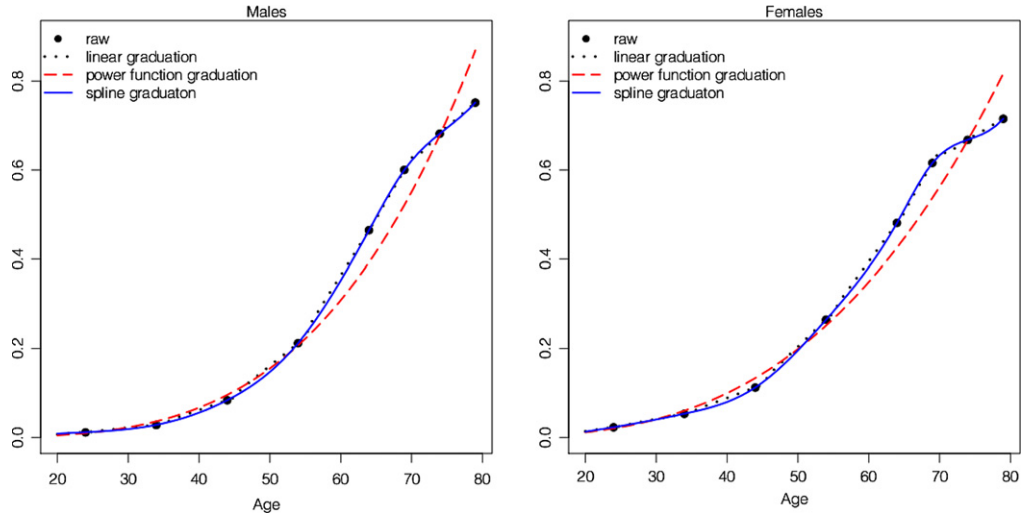

Fig. 6.1. Raw and graduated prevalence rates (cubic spline, power function, linear functions).

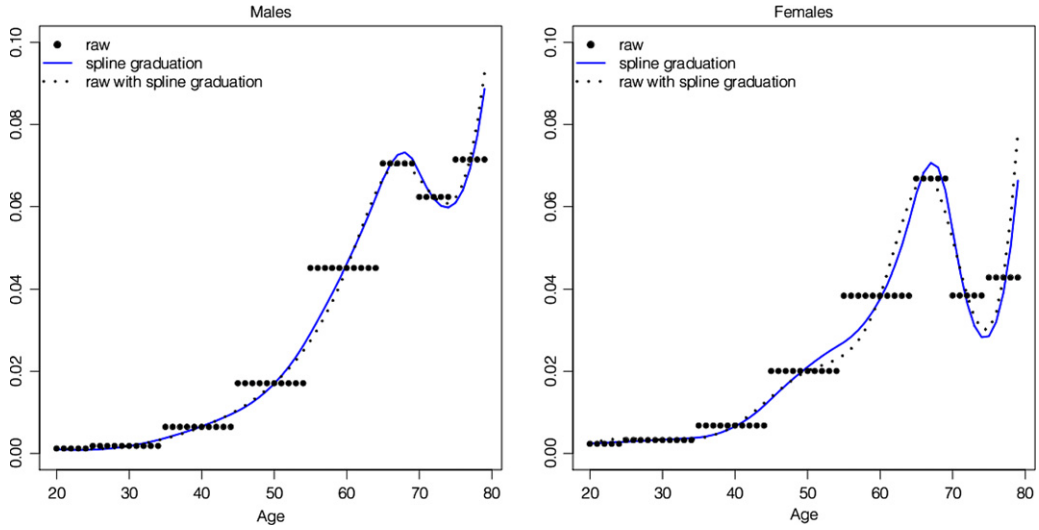

Fig. 6.2. Transition intensities from healthy to ill by age.

Table 6.1
Values of $\sigma_1, \sigma_2, \ldots, \sigma_n$ by age group and gender (rates per 1000).

σ_{k+1} (per 1000)		
Age group	Males	Females
15–24	1.2161	2.3571
25–34	1.8513	3.2756
35–44	6.4626	6.8111
45–54	17.1806	20.1482
55–64	45.2849	38.5437
65–69	70.9479	67.1382
70–74	62.9193	38.8376
75–79	72.1920	43.3726

an age-specific rate. Therefore, on each age interval $(x_r, x_{r+1}]$ for $r = 0, 1, \ldots, n-1$, prevalence rates are modelled as:

$$f_x = a_r(x - x_r)^3 + b_r(x - x_r)^2 + c_r(x - x_r) + d_r \quad \text{for } x_r \le x < x_{r+1}. \tag{6.1}$$

Since prevalence rates strongly increase upon increasing age, alternative approximations, but less valuable than cubic spline in terms of fitting, may consist in a power function ($f_x = cx^d$; $c, d > 0$) or in a linear interpolation for each age group r ($r = 0, 1, 2, \ldots, n-1$) ($f_x = a_r + b_r x$; for $x_r \le x < x_{r+1}$; $a_r, b_r > 0$).

Finally, the last approach is based on the graduation of transition intensities obtained with *approach 1*.

The approaches proposed here produce similar results in terms of probabilities but more relevant effects on pure premium rates, as notable in the next section. Quantitative results are reported starting from an insurance minimum entry age equal to 20. Fig. 6.1 shows both raw and graduated prevalence rates, where the graduation methods used are: a cubic spline, a power function and a linear function for each age group. The estimates of transition intensities are shown in Fig. 6.2 for males and females according to the proposed approaches.

With respect to the values labelled "raw with spline graduation" – indicating values of $\mu^{12}(x)$ obtained with *approach 3* – it should be noted that those for ages 78–79 has been extrapolated according to a cubic spline function with knots placed at the central value of the last two classes (i.e. at ages 72, 77). Although the increasing trend seems to be confirmed by the "spline graduation" (*approach 2*), extrapolated data provides results that are not fully convincing and not fully supported by raw data. Therefore, they should be considered with caution.

In Fig. 6.2 we can observe that transition intensity does not turn out to be a strictly non-decreasing function. The outcomes show a decreasing transition intensity between age groups 65–69 and

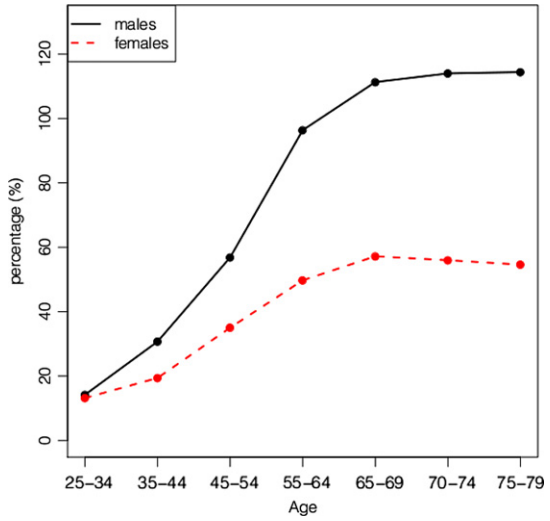

Fig. 6.3. Growth rate of prevalence rates.

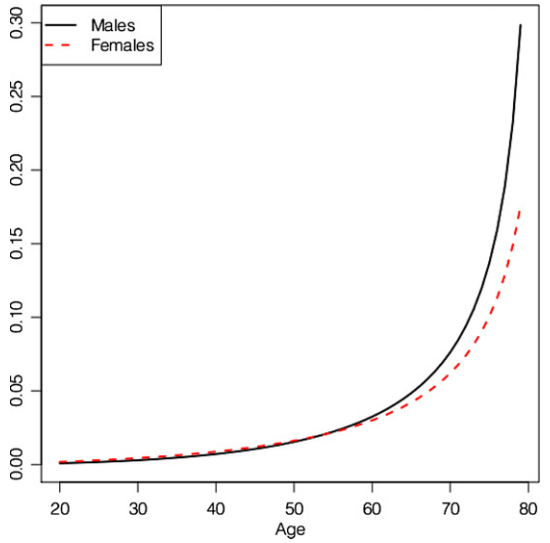

Fig. 6.4. Transition intensities from healthy to ill by age obtained from graduated (with a power function) prevalence rates.

70–74, which might seem counterintuitive. This behaviour is both due to the prevalence rates and to the mortality rates of healthy and sick people used in the numerical application. In particular, it mainly depends on the different size of the increase observed on prevalence rates between age groups 65–69 and 70–74.

To appreciate the prevalence rates increment we calculate the growth rate, $\frac{f'_{x_{r+1}} - f'_{x_1}}{f'_{x_1}} \cdot \frac{1}{x_{r+1} - x_1}$ for $r = 1, 2, \ldots, 6$. Values are shown in Fig. 6.3. The growth rate of prevalence rates shows a change in the trend between the age groups 65–69 and 70–74 both for males and females. In our model this change produces a reduction of transition intensity between the aforementioned age groups. Therefore, we can assert that our model properly depicts the information provided by prevalence rates taking into account mortality rates of both healthy and sick individuals, as well.

For example, if we graduate the prevalence rates using a power function (see Fig. 6.1), the growth rate assumes a monotone increasing trend and the model produces morbidity rates that increase with age, as shown in Fig. 6.4. It is worth noting that the graduated prevalence rates compared to the raw ones are lower for the age groups 55–64 and 65–69 and highest for the age group 75–79, while they are very similar for the other age groups (see Fig. 6.1). The power graduation of prevalence rates implies that the transition intensities for ages higher than 70 increase rapidly and reach values six times higher than the ones obtained with *approach 1*.

The assumption that $\mu^{12}(x)$ is a piecewise constant function allows us to obtain estimates consistent with the observed prevalence rates. However, if one focuses solely on the increasing trend of $\mu^{12}(x)$ till the age group 65–69, he/she may be induced to relax this assumption and introduce e.g. an exponential function such as a GM(0, 2). According to our data, an exponential approach may be only supported till the age group 65–69, but it is questionable to represent trend for higher ages. Anyhow, it can be considered an effective alternative approach when supported by data, but it will imply to modify the probabilistic framework in a "full GM" (see Appendix B).

We show in Fig. 6.5 the values of p_x^{14} and p_x^{23} by age and gender, calculated through *approach 2*. Other approaches proposed produce similar results without appreciable differences. Both the mortality transition probabilities are increasing with age, but the mortality of ill people due to CI is – on average by age – almost ten times higher than the mortality of healthy individuals.

Once arranged a multiple life table for each age covered it is trivial to calculate the pure premiums rates according to formula

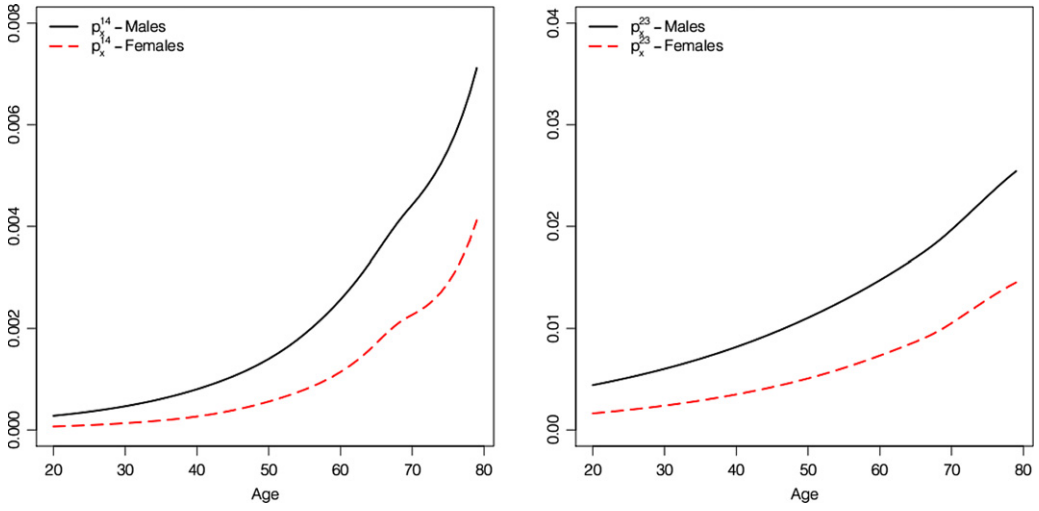

Fig. 6.5. Transition probabilities p_x^{14} and p_x^{23} by gender. *Approach 2*.

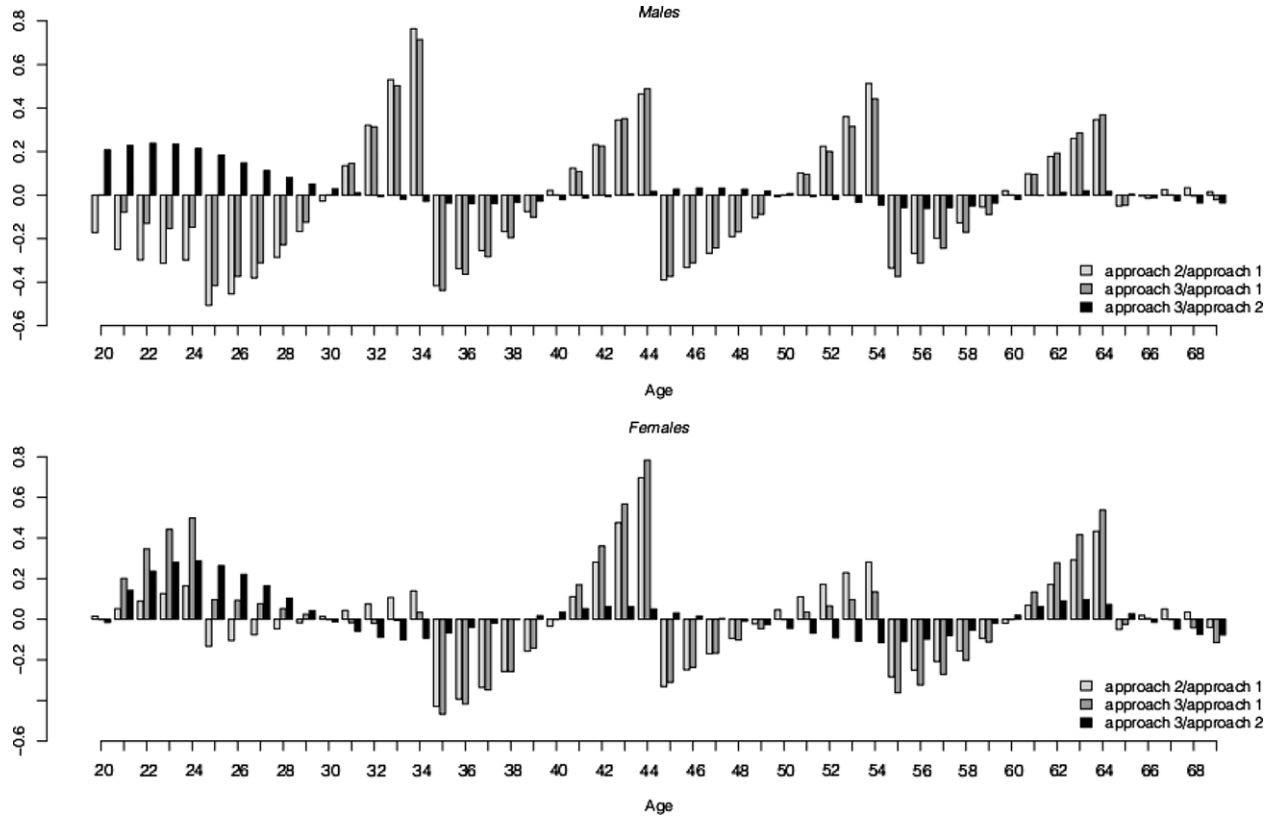

Fig. 6.6. Percentage variation of natural premium rates calculated by approaches 2 and 3 compared with approach 1 and by approach 3 compared with approach 2.

Table 6.2
Single premium rates (per 1000 €) of a stand-alone and full acceleration benefit CI. Approach 1. Year 2009.

Age	Males		Females	
	Stand-alone	Full acceleration	Stand-alone	Full acceleration
20	13.60	16.71	24.91	25.69
30	35.12	40.22	43.27	44.79
40	94.67	102.79	106.25	109.07
50	215.48	227.54	214.95	220.02
60	368.78	386.51	340.54	349.30

Table 6.3
Single premium rates (per 1000 €) of a stand-alone and full acceleration benefit CI. Approach 2. Year 2009.

Age	Males		Females	
	Stand-alone	Full acceleration	Stand-alone	Full acceleration
20	9.25	12.38	24.84	25.63
30	31.47	36.60	35.79	37.32
40	85.75	93.98	103.19	106.03
50	209.51	221.84	208.36	213.54
60	396.99	414.41	366.20	374.81

(4.8) and (4.9). Tables 6.2–6.4 show the values of single premiums for both stand-alone and full acceleration benefit CI insurance with a ten-year duration, obtained by the three approaches proposed herein. Different from what has been highlighted in Fig. 6.2, the choice of a specific approach to arrange a multiple life table has a significant impact on premium rates, especially comparing *approach 1* to the other ones.

Keeping in mind that a term insurance policy premium can be expressed as a weighted average of the natural premiums, we have investigated the percentage variations of natural premiums (see Fig. 6.6) in the stand-alone CI case (results for full acceleration benefit are similar and thus not reported).

Table 6.4
Single premium rates (per 1000 €) of a stand-alone and full acceleration benefit CI. Approach 3. Year 2009.

Age	Males		Females	
	Stand-alone	Full acceleration	Stand-alone	Full acceleration
20	10.73	13.86	29.08	29.87
30	30.73	35.87	34.27	35.80
40	86.87	95.09	105.50	108.34
50	202.88	215.32	193.36	198.61
60	396.28	413.78	375.45	384.12

Focusing on a specific age-group (e.g. ages 30–40), the single premium rate for a ten-year policy duration on a male policyholder aged exactly 30 years is 35.12 €, 31.47 € and 30.73 € depending on the selected approach; Table 6.5 shows the corresponding natural premiums and the transition intensities $\mu^{12}(x)$ by age.

As obviously expected, the estimated natural premiums are strictly related to the transition intensities' trend by age, and the differences on premiums are attributable to graduation adopted by *approaches 2* and *3*. In particular, in the considered example the *approach 3* produces – on average – lower premiums due to the greater convexity as compared with the other two approaches.

7. Conclusions and further research

In this paper we propose a multiple state model to determine the set of probabilities required for premium calculation of a generic CI coverage using GM(0, 2) functions to model mortality intensities. Closed solutions for transition probabilities and premium rates are provided when transition intensities from healthy to ill are derived by means of prevalence rates rather than incidence rates. The proposed method can be extended to the case of $GM(r, s)$ with $r = 1$ and $s = 2$, i.e. when the mortality of healthy or the mortality of ill individuals, or both, follows a $GM(1, 2)$. The

Table 6.5
Natural premium rates (per 1000 €) of a stand-alone benefit CI vs Transition Intensities from healthy to ill (per 1000).

Age	Natural premiums			Transition intensities $\mu^{12}(x)$		
	Approach 1	Approach 2	Approach 3	Approach 1	Approach 2	Approach 3
30	1.7864	1.7370	1.7864	1.8467	1.7954	1.8467
31	1.7864	2.0273	2.0468	1.8467	2.0965	2.1168
32	1.7863	2.3592	2.3458	1.8467	2.4411	2.4272
33	1.7862	2.7337	2.6840	1.8467	2.8303	2.7787
34	1.7862	3.1519	3.0618	1.8467	3.2656	3.1718
35	6.1903	3.6115	3.4799	6.4445	3.7446	3.6074
36	6.1901	4.0991	3.9385	6.4445	4.2537	4.0860
37	6.1897	4.6124	4.4383	6.4445	4.7903	4.6083
38	6.1894	5.1527	4.9797	6.4445	5.3563	5.1751
39	6.1891	5.7217	5.5630	6.4445	5.9535	5.7869
40	6.1887	6.3211	6.1887	6.4445	6.5838	6.4445

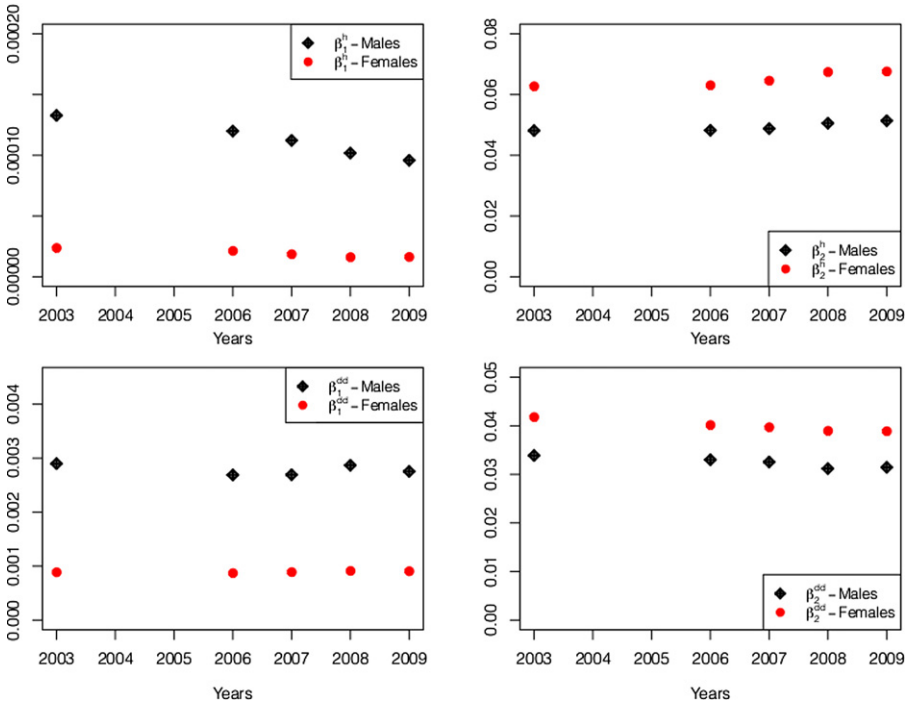

Fig. 7.1. Values of parameters of p_x^{14} and p_x^{23} by year and gender.

cases of $GM(r, s)$ with $r > 1$ and $s > 3$ are very complex to implement and they generate over-parameterized models.

The choice of the approach we suggested has been motivated by the poor quality of data set currently available in Italy. Generally, the choice of a model depends on both the type of benefits provided by the policy and the availability of data.

The future perspective of CI insurance is linked to a number of factors that may affect the number of sicknesses, the number of deaths due to CI and the definition of CI itself. For example, a better prognosis, new cures for old diseases and population ageing affect the number of ill individuals, the number of deaths and also the amount insured. In this context, the use of projected tables considering the uncertainty inherent in the mortality of ill people and in the incidence rates of disease would be more suitable.

In our model the construction of a projected multiple decrement table requires the implementation of a GM-based projection model. There were attempts to include mortality trends in the (generalized) GM model by making parameters time-dependent (see, e.g., Korn et al., 2006). In particular, there are two papers focusing on this topic: Wetterstrand (1981) that proposes a Gompertz-based projection model and Korn et al. (2006) that propose a stochastic version of the GM model incorporating mortality trends by making the parameters time-dependent. The numerical application developed by Korn et al. suggests that "a one factor model would be sufficient to explain the randomness in the evolution of mortality rates over time".

Following Korn et al. (2006, 2010), our model could be implemented to produce a stochastic set of transition probabilities. To this aim we have investigated the GM time-dependency parameters (see Fig. 7.1), using data for the other available years 2003, 2006, 2007 and 2008. Data refer only to mortality rates while no data are available for prevalence rates.

The trend appears quite regular in all the parameters suggesting to use a projection model in order to forecast probabilities of death for both healthy and ill individuals. However, the current Italian national health statistics used in the paper, collected only data for five years, much too scanty and not allowing us to implement a robust stochastic GM model.

Appendix A. Multiple state model: the long term care insurance (LTC) case

A LTC insurance provides financial support for insured people who are in need of nursing or medical care. It usually provides life annuities for the duration of disability. LTC policies are commonly modelled by multiple state models, and the state space usually consists of the states active, dead, and the corresponding levels of

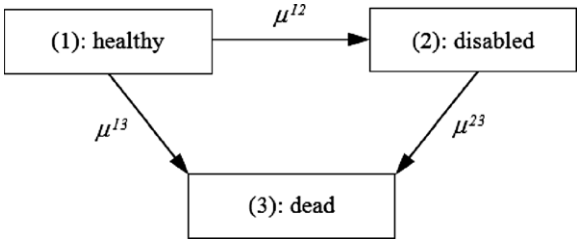

Fig. A.1. Set of states and set of transitions for LTC benefits.

frailty. We consider one level of disability; thus we have a state space of $S = \{1 = \text{healthy/active}, 2 = \text{disabled}, 3 = \text{dead}\}$ and a set of transitions according to Fig. A.1. We assume $S(0) = 1$. We disregard the possibility of recovery from the disabled state due to the usually chronic feature of disability for the elderly.

The transition probabilities of the model are defined as:

$$ {}_tp_x^{11} = \exp\left\{-\int_0^t \left[\mu^{12}(x+u) + \mu^{13}(x+u)\right] du\right\} \tag{A.1} $$

$$ {}_tp_x^{12} = \int_0^t \left[{}_up_x^{11}\mu^{12}(x+u)\,{}_{t-u}p_{x+u}^{22}\right] du \tag{A.2} $$

$$ {}_tp_x^{22} = \exp\left\{-\int_0^t \mu^{23}(x+u)du\right\}. \tag{A.3} $$

A.1. The framework of transition intensities

The assumptions made for the LTC coverage are similar to those for CI coverage. We suppose that mortality intensity of both healthy and disabled lives can be modelled by a GM model, where mortality rates exponentially increase with age. Thus, we suppose that $\mu^{13}(x)$ and $\mu^{23}(x)$ are described by two independent $GM(0, 2)$ models:

$$ \mu^{i3}(x) = \exp\left(\beta_1^i + \beta_2^i x\right) \quad \text{with } \beta_1^i, \beta_2^i > 0 \text{ for } i = h, d \tag{A.4} $$

where β_1^h, β_2^h and β_1^d, β_2^d are the GM parameters for healthy (h) and disabled people (d), respectively. Similar to the case of CI insurance, we suppose that transition intensity from healthy to disabled can be described by a piecewise constant function; hence Eq. (3.5) is still valid.

A.2. Transition probabilities estimation

Under the assumptions made in Eqs. (3.5) and (A.4) and considering Eq. (A.1), we assume the probability to remain in state 1 until time t, ${}_tp_x^{11}$ the same solution exposed in Eq. (4.2). From Eq. (A.3), we can calculate the probability to remain in state 2 until time t, where $\dot\beta_1^d = \exp(\beta_1^d)$ as follows:

$$ {}_tp_x^{22} = \exp\left[-\frac{\dot\beta_1^d}{\beta_2^d}\left(e^{\beta_2^d(x+t)} - e^{\beta_2^d x}\right)\right] \quad \forall x, t. \tag{A.5} $$

From Eq. (A.2) – that it is equal to Eq. (2.5) – we obtain the following expression for transition probability under the assumptions made in Eqs. (3.5) and (A.4):

$$ \begin{aligned} {}_tp_x^{12} &= \exp\left\{\frac{\dot\beta_1^h}{\beta_2^h}\left(e^{\beta_2^h x}\right) - \frac{\dot\beta_1^d}{\beta_2^d}\left[e^{\beta_2^d(x+t)}\right]\right\}\sigma_{k+1} \\ &\cdot \int_0^t \exp\left\{-\sigma_{k+1}u + \frac{\dot\beta_1^h}{\beta_2^h}\left[e^{\beta_2^h(x+u)}\right] + \frac{\dot\beta_1^d}{\beta_2^d}\left[e^{\beta_2^d(x+u)}\right]\right\} du \\ &\text{for } k = 0, 1, \ldots, n-2, x_k < x \le x_{k+1} \text{ and } t \le x_{k+1} - x \\ &\text{and for } k = n-1, x > x_{n-1} \text{ and } \forall t. \end{aligned} \tag{A.6} $$

A solution to Eq. (A.6) is obtained by assuming the approximations in Eq. (4.5):

$$ \begin{aligned} {}_tp_x^{12} &\cong \left[\exp\left(\frac{\dot\beta_1^h}{\beta_2^h}e^{\beta_2^h x} - \frac{\dot\beta_1^h}{\beta_2^h}e^{\beta_2^h(x+\frac{t}{2})}\left(1 - \beta_2^h\cdot\frac{t}{2}\right)\right)\right] \\ &\cdot \left[\exp\left(-\frac{\dot\beta_1^d}{\beta_2^d}e^{\beta_2^d(x+t)} + \frac{\dot\beta_1^d}{\beta_2^d}e^{\beta_2^d(x+\frac{t}{2})}\left(1 - \beta_2^d\cdot\frac{t}{2}\right)\right)\right] \cdot \\ &\cdot \frac{\sigma_{k+1}\cdot\left[\exp\left(-\sigma_{k+1} - \dot\beta_1^h e^{\beta_2^h(x+\frac{t}{2})} + \dot\beta_1^d e^{\beta_2^d(x+\frac{t}{2})}\right)t - 1\right]}{-\sigma_{k+1} - \dot\beta_1^h e^{\beta_2^h(x+\frac{t}{2})} + \dot\beta_1^d e^{\beta_2^d(x+\frac{t}{2})}}. \end{aligned} \tag{A.7} $$

Therefore, when prevalence rates, f_x, are available, by substituting Eqs. (4.2) and (A.7) in Eq. (3.4), we can obtain an estimate of the unknown parameters σ_{k+1} for all $k = 0, 1, \ldots, n-1$, via an iterative approach, starting from the initial age group (x_0, x_1).

Appendix B. Full GM

Assuming that transition intensity from healthy to ill state is described by a GM(0,2), we have:

$$ \mu^{12}(x) = \exp\left(\beta_1^{hdd} + \beta_2^{hdd}x\right) = \dot\beta_1^{hdd}\cdot\exp\left(\beta_2^{hdd}x\right) \tag{B.1} $$

where $\dot\beta_1^{hdd} = \exp(\beta_1^{hdd})$ and $\beta_1^{hdd}, \beta_2^{hdd} > 0$ are the GM parameters of transition intensity from healthy to ill state, while the solution to Eqs. (4.1) and (4.4) become, respectively:

$$ \begin{aligned} {}_tp_x^{11} &= \exp\left\{-\int_0^t \left[\dot\beta_1^{hdd}\cdot\exp(\beta_2^{hdd}(x+u)) + \dot\beta_1^h\cdot\exp(\beta_2^h(x+u))\right] du\right\} \\ &= \exp\left\{-\frac{\dot\beta_1^{hdd}}{\beta_2^{hdd}}\left[e^{\beta_2^{hdd}(x+t)} - e^{\beta_2^{hdd}x}\right] - \frac{\dot\beta_1^h}{\beta_2^h}\left[e^{\beta_2^h(x+t)} - e^{\beta_2^h x}\right]\right\} \end{aligned} \tag{B.2} $$

$$ \begin{aligned} {}_tp_x^{12} &\cong \left[\exp\left(\frac{\dot\beta_1^h}{\beta_2^h}e^{\beta_2^h x}\left(1 - e^{\beta_2^h t}(1+\gamma)\right) + \gamma\frac{\dot\beta_1^h}{\beta_2^h}e^{\beta_2^h(x+\frac{t}{2})}\left(1 - \beta_2^h\cdot\frac{t}{2}\right)\right)\right] \\ &\cdot \left[\exp\left(\beta_2^{hdd}x + \frac{\dot\beta_1^{hdd}}{\beta_2^{hdd}}e^{\beta_2^{hdd}x} - \frac{\dot\beta_1^{hdd}}{\beta_2^{hdd}}e^{\beta_2^{hdd}\cdot(x+\frac{t}{2})}\left(1 - \beta_2^{hdd}\cdot\frac{t}{2}\right)\right)\right] \\ &\cdot \left[\exp\left(-\frac{\dot\beta_1^{dd}}{\beta_2^{dd}}e^{\beta_2^{dd}(x+t)} + \frac{\dot\beta_1^{dd}}{\beta_2^{dd}}e^{\beta_2^{dd}(x+\frac{t}{2})}\left(1 - \beta_2^{dd}\cdot\frac{t}{2}\right)\right)\right] \\ &\cdot \frac{\dot\beta_1^{hdd}\cdot\left[\exp\left(\beta_2^{hdd} - \dot\beta_1^{hdd}e^{\beta_2^{hdd}(x+\frac{t}{2})} + \gamma\dot\beta_1^h e^{\beta_2^h(x+\frac{t}{2})} + \dot\beta_1^{dd}e^{\beta_2^{dd}(x+\frac{t}{2})}\right)t - 1\right]}{\beta_2^{hdd} - \dot\beta_1^{hdd}e^{\beta_2^{hdd}(x+\frac{t}{2})} + \gamma\dot\beta_1^h e^{\beta_2^h(x+\frac{t}{2})} + \dot\beta_1^{dd}e^{\beta_2^{dd}(x+\frac{t}{2})}}. \end{aligned} \tag{B.3} $$

The values of GM parameters of $\mu^{12}(x)$ can be obtained by using MMSE estimation minimizing the mean square error of the fitted values obtained according to Eq. (3.4).

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