

1. Quicksort is a divide and conquer algorithm. It works by picking a pivot element from an array and then dividing

the remaining elements into two smaller arrays (sub-arrays) based on whether each element is greater or less than the

pivot element. After the creation of the two sub arrays, the arrays are then sorted in a recursive manner.

The worst-case scenario for the quicksort method occurs when the pivot element is chosen as the smallest or largest

element in the array. In this case, the algorithm will make $n-1$ recursive calls due to the unbalanced partitions, and each

sub-array will be of size $n-1$. Consequently there is a total of $n-1$ levels of recursion, and each sub-array will be of

size $n-1$. Thus, this leads to a worst-case complexity of $O(n^2)$

Let $X(k)$ be the worst-case time complexity of a quicksort algorithm for an array of size k .

Then,

$X(k) = X(k-1) + X(0) + O(k)$ where $X(k-1)$ is the time taken to divide the array into smaller sub-arrays

$X(0)$ is the time taken to divide the array with zero elements and,

$O(k)$ is the time taken for the dividing/partitioning step

By solving this, it is determined that the quicksort time complexity is $O(k^2)$.

2. Array of 16 elements:

[0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15]

(i) Choose an element within the array to be your pivot.

We chose element 0 as our pivot element

(ii) Divide the array into 2 sub-arrays.

[0] and [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15]

(iii) Once you have 2 sub-arrays, recursively sort them.