Decentralization and consumer welfare with substitutes or complements

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December 12, 2021

Abstract

A vertically integrated producer (VIP) may decentralize the final price decision to its downstream unit. This often occurs when the VIP supplies downstream rivals. Our paper studies a setting where the VIP competes in prices with a single downstream rival. It shows that: (i) when products are substitutes, decentralization decreases final prices and benefits consumers, despite restoring a double margin on the downstream unit's sales; and (ii) in contrast, when products are complements, decentralization increases the downstream unit's final price while decreasing the rival's price, and overall harms consumers.

Keywords: decentralization, substitutes, complements
JEL classification: L14, L22, L42

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1 Introduction

The common wisdom is that a centralized decision maker privy to all pertinent information would generate greater profits than those achieved under a decentralized structure. However, vertically integrated producers (VIPs) that supply and compete with downstream rivals often have their downstream units acting independently of the whole structure. For example, empirical studies of the media market in Israel (Gilo and Spiegel, 2011) and of the television industry in the United States (Crawford *et al.*, 2018) find that the VIP's downstream units take decisions that only partially internalize the effect on the upstream sales.

In particular, it is puzzling why a VIP would let, or even deliberately enable, its downstream unit to set the final price independently, as the latter would fail to consider the price effect on upstream sales. Such decentralization would moreover restore a double margin on the downstream unit's sales, which would mean reduced profits for the VIP. Our paper explores this puzzle by comparing the decentralized equilibrium outcomes with the benchmark centralized equilibrium outcomes where the whole structure quotes the final price.

We model an industry composed of a VIP supplying a downstream rival under price competition. We consider the widely used Singh and Vives (1984)'s linear demand with product substitutes or complements. We find that the decentralized VIP raises the rival's input price irrespective of substitutes or complements, while it quotes a distinctive input price to its downstream unit to boost its revenues on the downstream market. The incentives to divert sales through discriminatory pricing is the main cause of concern in antitrust policy and has recently been emphasized in the FTC and DoJ (2020) Vertical Merger Guidelines. Our results show that decentralized input prices benefit the VIP under price competition irrespective of substitutes or complements. However, they only benefit consumers with substitutes, and harm them with complements.

Intuitively, a centralized VIP sees its downstream unit internalize the effect of its final price on the VIP's upstream sales to the other firm. This prompts the latter to compete less with substitutes - by increasing its price and diverting demand to the rival - or, in contrast, to collaborate more with complements - by decreasing its price and attracting demand to both firms. As a consequence, the VIP inefficiently sets a low input price to encourage competition or collaboration from the other firm. This is the Chen incentive (Moresi and Schwartz, 2021; Chen, 2001). Decentralization eliminates this incentive and redirects competition or collaboration in favor of the downstream unit. This boosts downstream profits, to the detriment of upstream sales, and overall the VIP benefits from a more efficient balance of profits.

Even though decentralization restores a double margin with substitutes, the pro-competitive effect due to the elimination of the Chen incentive overcomes the anti-competitive effect of the former. Overall, prices decrease which benefits consumers. By contrast, decentralization with complements provides a subsidy to the downstream unit, which is lower than the Chen incentive (in that case, this incentive encouraged the downstream unit to decrease its price). As a consequence, the downstream unit's price increases and offsets the decrease of the other firm's price, which harms consumers.

The remainder of the paper is as follows. Section 2 briefly explains our contribution to the existing literature. Section 3 introduces the model. Sections 4 and section 5 derive the analysis with demand substitutes and complements. Finally, section 6 discusses some extensions of the model and section 7 concludes. All proofs are in the Appendix.[1]

2 Related literature

The existing literature claims that a centralized VIP, which also supplies a downstream rival, faces a problem of effectively balancing upstream and downstream profits (Arya *et al.*, 2008; Moresi and Schwartz, 2017). In particular, Arya *et al.* (2008) shows that decentralization improves the VIP's balance of profits under Cournot competition and thus is more profitable than centralization. However, it harms consumers. Interestingly, Moresi and Schwartz (2017) points out with a specific numerical example (after having studied the centralized VIP's incentive) that decentralization may benefit consumers under Bertrand competition.

Our paper contributes to this literature by deriving the equilibrium under decentralization and Bertrand competition using a linear demand where products can be either substitutes or comple-

[1] At the exception of the proofs of Section 6 which are in the Supplementary Appendix.

ments. We generalize Moresi and Schwartz (2017)'s result with product substitutes. In particular, we find that, even though decentralization restores an additional margin on the downstream unit's sales, it triggers lower final prices and indeed benefits consumers. For demand complements, we establish that decentralization remains more profitable for the VIP but yields a higher price from the downstream unit, which offsets the decrease of the other firm's price, therefore harming consumers.

3 The model

We assume a vertically integrated producer (VIP) and a downstream rival. The VIP consists of two entities, an upstream unit and a downstream unit. The upstream unit is the monopoly producer of a key input to the downstream unit (D_1) and an independent downstream rival (D_2). The inputs are essential for downstream production and each unit of output requires one unit of the input. We assume the firms bear neither production nor transformation costs.

D_1 and D_2 respectively sell horizontally-differentiated products to a representative consumer at prices p_1 and p_2. The representative consumer holds the linear demand function *à la Singh and Vives (1984)*. Formally, for product $i \in \{1, 2\}$, the consumer's demand is $q_i(p_i, p_j) = \frac{\alpha}{1+\gamma} - \frac{1}{1-\gamma^2}p_i + \frac{\gamma}{1-\gamma^2}p_j$, where $j \in \{1, 2\}$, $j \neq i$, $\gamma^2 < 1$, and $\alpha > 0$. Products are substitutes when $\gamma > 0$, complements when $\gamma < 0$ and unrelated when $\gamma = 0$. With the inverse demand function $p_i(q_i, q_j) = \alpha - q_i - \gamma q_j$, $\forall i = 1, 2, j \neq i$, we find that the consumer surplus writes $CS = [(q_1)^2 + 2\gamma q_1 q_2 + (q_2)^2]/2$.

With this basic setting, we seek to compare the outcomes under centralization and decentralization, and investigate the role of product substitution and complementarity.

4 Results for susbstitutes

The typical view is that a centralized decision maker privy to all pertinent information can generate greater profits than those achieved under a decentralized structure. Decentralization would for example restore an inefficient double margin on the downstream unit's sales, which would mean reduced profits for the VIP. On the other hand, Moresi and Schwartz (2017) and Arya *et al.* (2008) claim that a centralized VIP, which also supplies a downstream rival, faces a problem of effectively balancing upstream and downstream profits. This section shows that decentralization improves the VIP's balance of profits even though it does restore double marginalization.

4.1 Centralization

Under centralization, the VIP supplies inputs to the downstream rival D_2 at per-unit price w_2. The timing of the game is as follows: (1) the VIP charges the input price w_2; and (2) the VIP and the rival set prices p_1 and p_2. Profits are made. We use the Sub-game Perfect Nash Equilibrium (SPNE) concept to solve the game.

We use backward induction to find the SPNE. At stage 2, the firms maximize their profits $V(p_1, p_2)$ and $\pi_2(p_2, p_1)$ with respect to the prices. Remember that $V(p_1, p_2)$ includes the downstream firm's profits. Formally, the profits write:

$$V(p_1, p_2) = p_1.q_1(p_1, p_2) + w_2.q_2(p_1, p_2) \tag{1}$$

$$\pi_2(p_1, p_2) = (p_2 - w_2)q_2(p_1, p_2) \tag{2}$$

The first term to the right of the equal sign in Eq. (1) reflects the VIP's profits from its downstream sales, just as Eq. (2) reflects D_2's profits from its downstream sales. The second term to the right of the equal sign in Eq. (1) captures the VIP's profit from selling the inputs to the rival. The VIP thus considers the effect of a change in the price p_1 on the upstream sales $q_2(p_1, p_2)$.

Comment: firms' price decisions are *strategic complements* with *substitutes* ($\gamma > 0$), irrespective of centralization or decentralization.[2]

The firms' optimization problems yield the following sub-game equilibrium prices:

$$p_1^C(w_2) = \frac{\alpha(1-\gamma)(2+\gamma) + 2\,C_1(w_2) + \gamma w_2}{4-\gamma^2} \quad ; \quad p_2^C(w_2) = \frac{\alpha(1-\gamma)(2+\gamma) + 2w_2 + \gamma\,C_1(w_2)}{4-\gamma^2} \tag{3}$$

where $C_1(w_2) = \gamma w_2$ reflects the opportunity cost of supplying the rival. Intuitively, the VIP anticipates that for each new quantity q_1 obtained through a decrease of its price p_1, there will be

[2]From eq. 1 2 & 4, we find that $\partial^2 V/\partial p_2 \partial p_1 = \partial^2 \pi_2/\partial p_1 \partial p_2 = \partial^2 \pi_1/\partial p_2 \partial p_1 = \gamma/(1-\gamma^2)$.

some quantities which are diverted from the rival's demand q_2 and will not be sold at price w_2 (see Moresi and Schwartz (2017) for more explanation). With our linear demand, the diversion ratio, $DR_{21} = -(\partial q_2/\partial p_1)/(\partial q_1/\partial p_1)$, simplifies to the level of product substitution γ. That is why, the opportunity cost neatly writes as $C_1(w_2) = \gamma w_2$.[3] Note that the opportunity cost is positive and therefore softens competition, to the detriment of the VIP (refer to coefficient before $C_1(w_2)$).

At stage 1, the VIP anticipates the sub-game prices and thus maximizes its profit $V(p_1^C(w_2), p_2^C(w_2), w_2)$ with respect to w_2. Lemma 1 displays the equilibrium outcomes.

Lemma 1. *Under centralization, the equilibrium input price is:*

$$w_2^C = \frac{\alpha}{2} - \frac{\alpha(1-\gamma)\gamma^2}{2(8+\gamma^2)}$$

and the final prices, firms' profits and consumer surplus, respectively, are

$$p_1^C = \frac{\alpha(4-\gamma)(2+\gamma)}{2(8+\gamma^2)}\ , \quad p_2^C = \frac{\alpha[2(6+\gamma^2)-\gamma(4+\gamma^2)]}{2(8+\gamma^2)}\ , \quad V^C = \frac{\alpha^2(\gamma+2)\left(\gamma^2-\gamma+6\right)}{4(\gamma+1)\left(\gamma^2+8\right)}$$
$$\pi_2^C = \frac{\alpha^2(1-\gamma)\left(\gamma^2+2\right)^2}{(\gamma+1)\left(\gamma^2+8\right)^2},\ \textit{and} \quad CS^C = \frac{\alpha^2\left(5\gamma^5+\gamma^4+24\gamma^3+36\gamma^2+16\gamma+80\right)}{8(\gamma+1)\left(\gamma^2+8\right)^2}$$

The VIP does not foreclose the rival as long as products remain imperfect substitutes ($\gamma < 1$).

Lu *et al.* (2007) shows that, compared with a total separation structure, vertical integration decreases the rival's input price and the downstream prices, which increases competition and benefits consumers. This happens because the pro-competitive *elimination of the double margin effect* (vertical integration eliminates the upstream unit's margin on the downstream unit's sales) overcomes the anti-competitive *raising rival cost effect* (vertical integration prompts the VIP to limit the rival's efficiency by raising its input price).

At equilibrium, the VIP nevertheless remains too concerned by upstream profits, to the detriment of downstream profits. This upstream-downstream profits inefficient balance relates to the

[3]Formally, the expression of the opportunity cost comes from the VIP's first order condition as follows: $\frac{dV}{dp_1} = \frac{dq_1}{dp_1}p_1 + q_1 + w_2\frac{dq_2}{dp_1} = q_1 + \frac{dq_1}{dp_1}\left(p_1 - \left[w_2\left(-\frac{dq_2/dp_1}{dq_1/dp_1}\right)\right]\right)$.

VIP internalizing the reduced demand from the rival upon a decrease of its own downstream price. This is what Moresi and Schwartz (2021) refers to as the anti-competitive *Chen incentive* (Chen, 2001). This *Chen incentive* materializes into the downstream prices as the opportunity cost of supplying the rival $C_1(w_2)$, which limits the VIP's downstream unit efficiency in the downstream competition. As a consequence, the VIP quotes an inefficiently low input price.

4.2 Decentralization

Under decentralization, the VIP continues to charge the downstream rival a per-unit price w_2 but it now also charges its downstream unit a per-unit price w_1. The downstream unit and the rival observe these input prices (more formally, contracts are *observable*) and then compete in prices, respectively setting prices p_1 and p_2.

Decentralization can be achieved by hiring a manager and writing a contract that would incentivize the downstream unit to maximize only the downstream profits. Another way would be to have minority outside shareholders i.e., the shareholders spin off the control rights of the downstream unit to outside shareholders and become passive majority shareholders (O'Brien and Salop, 1999). Last, decentralization can be viewed as organizational frictions between units (Arya *et al.*, 2008)

We solve the game using the SPNE concept. By backward induction, at the price competition stage, the downstream unit and the rival respectively set the downstream prices p_1 and p_2 to maximize their profits $\pi_1(p_1, p_2)$ and $\pi_2(p_2, p_1)$. Note that the downstream unit maximizes only the downstream profit π_1 and in particular does not consider the upstream unit's sales to the rival. Formally, and in contrast to Eq. (1), its objective function is:

$$\pi_1 = (p_1 - w_1).q_1(p_1, p_2) \tag{4}$$

The firms' optimization problems yield the following new sub-game equilibrium prices:

$$p_1^D(w_1, w_2) = \frac{\alpha(2-\gamma-\gamma^2) + 2w_1 + \gamma w_2}{4-\gamma^2} \;\; ; \;\; p_2^D(w_1, w_2) = \frac{\alpha(2-\gamma-\gamma^2) + 2w_2 + \gamma w_1}{4-\gamma^2} \tag{5}$$

We note that the term w_1 has replaced the term $C_1(w_2)$. In words, the downstream unit's input price w_1 has replaced the VIP's opportunity cost of supplying the rival. Decentralization thus has two opposite effects on downstream competition. On the one hand, decentralization enables the VIP to eliminate the inefficiency of the *Chen incentive* which has a pro-competitive effect since w_2 has a lesser effect on the downstream competition. On the other hand, it restores the VIP's margin on its downstream unit's sales, which has an anti-competitive effect. Besides these two effects, the new margin also provides the VIP with an additional independent instrument (w_1) to affect the downstream competition. In contrast to total separation, the VIP internalizes the effect of this instrument on the upstream sales to the rival when it sets this instrument.

At the contracting stage, the VIP maximizes its expected profit $V(p_1^D(w_1, w_2), p_2^D(w_1, w_2), w_1, w_2)$ with respect to w_1 and w_2. We obtain the equilibrium input prices w_1^D and w_2^D displayed in Lemma 2 below. Proposition 1 then compares the values with those under centralization.

Lemma 2. *Under decentralization, the equilibrium input prices are*

$$w_1^D = \gamma \left(\frac{\alpha}{2} - \frac{\alpha(1-\gamma)}{4} \right) \quad ; \quad w_2^D = \frac{\alpha}{2} \tag{6}$$

Proposition 1. *With substitutes, decentralization increases the rival's input price ($w_2^D \geq w_2^C$) but sets the downstream unit's input price lower than the centralized opportunity cost ($w_1^D \leq \gamma w_2^C$). Inequalities are strict except when the goods are unrelated ($\gamma = 0$).*

The VIP now controls the value of the downstream unit's downstream price through the value of w_1. In particular, it can simulate the Chen incentive by setting $w_1 = C_1(w_2^C)$. Since the Chen incentive inefficiently reduces the downstream unit's efficiency, the VIP sets the new margin w_1 below this value, which increases the downstream unit's efficiency and distorts competition in favor of the latter. In other words, the anti-competitive effect triggered by the double margin only mitigates the pro-competitive effect triggered by the elimination of the Chen incentive. This boosts the VIP's downstream profits, to the detriment of upstream sales $((dq_2^D/dw_1) = (dq_2^D/dC_1(w_2)) > 0)$.

On the other hand, the elimination of the Chen incentive also reduces the effect of the rival's input price w_2 upon the downstream firms' prices (i.e. pass-through rates). Even though the downstream unit's reduced responsiveness is greater than that of the rival (refer to the coefficient before the opportunity cost term in Eq. (3)), the VIP's upstream sales responsiveness to w_2 remains the same $((dq_2^D/dw_2) = (dq_2^C/dw_2))$. As the rival's input price was inefficiently low due to the Chen incentive, the elimination of the latter emboldens the VIP to raise the rival's input price and mitigates the loss on upstream sales through a higher margin. Moresi and Schwartz (2017) points out that this incentive is only superficially reminiscent of the RRC and is not driven by a desire to foreclose the rival.

By substituting the input price values in the sub-game price strategies, we find p_1^D and p_2^D and the rest of the equilibrium outcomes q_1^D, q_2^D, V^D and π_2^D.

Lemma 3. *Under decentralization, the equilibrium prices, profits and consumer surplus are:*

$$p_1^D = \frac{\alpha}{2}\ , \quad p_2^D = \frac{\alpha(3-\gamma)}{4}\ , \quad V^D = \frac{\alpha^2(\gamma+3)}{8(\gamma+1)}\ , \quad \pi_2^D = \frac{\alpha^2(1-\gamma)}{16(\gamma+1)}\ , \ \textit{and}\ CS^D = \frac{\alpha^2(3\gamma+5)}{32(\gamma+1)}.$$

Proposition 2. *With substitutes, decentralization decreases the final prices ($p_1^D \leq p_1^C$ and $p_2^D \leq p_2^C$). Inequalities are strict except when the goods are unrelated ($\gamma = 0$).*

The replacement of the Chen incentive by the new margin tends to decrease the downstream unit's price and therefore also the rival's downstream price. As the new margin is set at a lower level than the Chen incentive, the pro-competitive effect overcomes the anti-competitive effect. On the other hand, the superficial RRC increases the rival's input price, which tends to increase the rival's downstream price (anti-competitive effect). Overall, the replacement of the Chen incentive overcomes the superficial RRC; hence the decreasing prices.

We furthermore find that the downstream firm's price and the rival's input price equal the input prices under total separation ($w^{NI} = \alpha/2$, see appendix). This reflects the fact that decentralization enables the VIP to act as if it were a Stackelberg leader on the downstream market: the input price w_1 acts as a commitment device to control p_1 and makes its downstream unit the leader on the downstream market ($p_1^D = w_1^{NI}$) while the rival is a the follower $w_2^D = w_2^{NI}$. Lu *et al.* (2007)'s appendix 4 proves the equivalence.

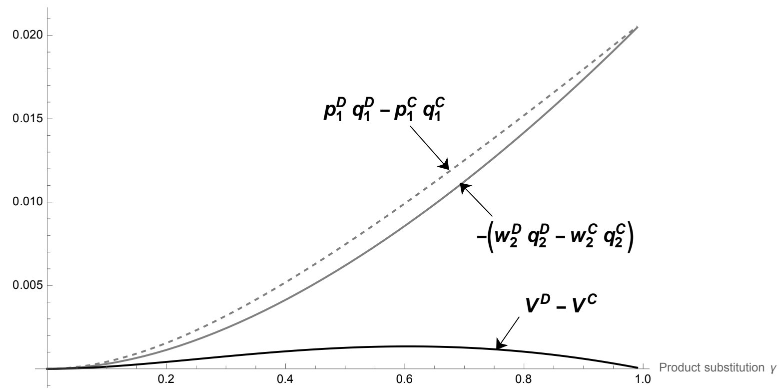

Figure 1: Comparative statics (substitutes)
(Note: we consider $\alpha = 1$ and $0 \leq \gamma \leq 0.99 < 1$)

To sum up, decentralization enables the VIP to boost the downstream profits at the expense of upstream profits. Figure 1 provides a decomposition of the variation of the VIP profits. It shows that the increase in downstream revenues ($p_1^D q_1^D > p_1^C q_1^C$) overcomes the decrease in upstream profits from the rival sales ($w_2^D q_2^D < w_2^C q_2^C$). Ultimately, the VIP is better off since the upstream-downstream profits balance has improved.

Proposition 3. *With substitutes, decentralization increases the VIP's profit, decreases the rival's profit and increases consumer surplus.*

Our propositions generalize the results of Moresi and Schwartz (2017) to a linear demand function. It addition, they show that the gains for the VIP depend on the length of product substitution. Bear in mind that the objective of the VIP is to boost downstream profits. Starting at unrelated products ($\gamma = 0$), the VIP cannot gain from decentralization as products are independent and the Chen incentive is already muted. When products are more substitutable ($0 < \gamma < 1$), the VIP can replace the Chen incentive and benefit from decentralization even though upstream profits diminish. Above a certain substitution degree, competition is very fierce so downstream margins are short. The effect of the replacement of the Chen incentive is softened relative to the superficial RRC which becomes more pronounced. As a result, the VIP's behavior is closer to that under centralization. When $\gamma \to 1$, the VIP forecloses D_2 in both structures and is therefore indifferent.

5 Results for complements

The outcomes are formally the same as under substitutes. However, product complementarity yields $\gamma < 0$ which modifies our results and conclusions. In particular, note that firms' price reactions become *strategic substitutes* with *complements* ($\gamma < 0$), irrespective of centralization or decentralization.[4] As the interaction between the downstream firms D_1 and D_2 has changed, such that a decrease of one firm's price yields an increase of both firms' demand, we rename D_2 the *complementor*[5] of D_1 instead of its *rival*.

5.1 Examples of vertical structures with product complements

In real life, the relation between the products is often fuzzy and greatly depends on the market definition. It can happen that products actually turn out to be demand complements.

Consider Microsoft which owns Windows OS and produces Surface laptops in-house. Microsoft also supplies its rival, Acer, a desktop computer manufacturer. Individuals are likely to consider these two products as substitutes. However, it is likely that firms and families actually consider Surface laptops and Acer computers as complements. The set of consumers studied (groups vs. individuals) thus affects the relation between the products.

Another example comes from Google which owns Android OS and produces Pixel smartphones in-house. Google also supplies the smartphone manufacturer Samsung. The firms' smartphones are *per se* imperfect substitutes, but a greater number of users on one device may increase the valuation of the other device (e.g. network exclusive apps, ...). Accounting for such a relation (network effects) may turn these gross substitutes into net complements (see the formal demand example below). Intuitively, if the number of users on one device increases the valuation of the other device then a decrease in the price of one device increases the demand of the two devices.[6] The type of goods studied (standard goods vs. network goods) thus also affects the relation between products.

[4] From eq. 1 2 & 4, we find that $\partial^2 V/\partial p_2 \partial p_1 = \partial^2 \pi_2/\partial p_1 \partial p_2 = \partial^2 \pi_1/\partial p_2 \partial p_1 = \gamma/(1-\gamma^2)$.

[5] We borrow the term from Hagiu *et al.* (2020)

[6] A recent trial showed that Google was able to manipulate the product differentiation between the final devices (through a restriction of the default app) and henceforth this could have affected the relation between these goods.

Network effect example. *By implementing network effects* $(1/2)\mu(q_1+q_2)^2$*, where* μ *denotes the strength of network effects, in Bowley's utility function (which is the origin of the demand functions used in the paper), we obtain the following gross utility function* $U(q_1,q_2) = m + \alpha(q_1 + q_2) - (\beta(q_1^2+q_2^2)+2\gamma' q_1 q_2)/2 + (1/2)\mu(q_1+q_2)^2$ *where* γ' *is the gross degree of substitution between the goods.* m*,* α *and* β *are positive parameters. Such utility gives the following linear inverse demand* $p_i = \alpha - (\beta-\mu)q_i - (\gamma'-\mu)q_j,\ \forall i,j.$[7] *For a given level of network effects* $\mu \in [0,1]$*, take* $\beta = 1+\mu$ *then we get the inverse demand used in the paper that is* $p_i = \alpha - q_i - \gamma q_j$ *where* $\gamma = \gamma' - \mu$*. Goods are substitutes when product substitution is stronger than network effects* $\gamma > 0$*, and complements otherwise* $\gamma < 0$*.*

5.2 Centralization

Appendix shows that, compared with total separation, vertical integration decreases the complementor's input price, increases the complementor's downstream price, and decreases the VIP's downstream price. Overall, the decrease of the VIP's downstream price overcomes the increase of the complementor's price, and as a result the consumers are better off.

As with substitutes, the VIP still inefficiently balances upstream and downstream profits at equilibrium. The VIP again focuses too much on upstream sales. However, the reason is now that its downstream unit internalizes the *increased* complementor's demand when the VIP's downstream price decreases. Indeed, in contrast to substitutes, the diversion ratio is now negative (formally, $\gamma < 0$ implies $DR_{21} = \gamma < 0$), which means that the Chen incentive materealizes into the downstream prices as an opportunity benefit of supplying the rival ($C_1(w_2) = \gamma w_2 < 0$). In other words, the VIP's downstream unit feels subsidized for decreasing its price.

This implies that the VIP decreases its downstream price too much, whereas the rival increases its downstream price too much. Or equivalently, the VIP collaborates too much by attracting

[7] Existence of solution. The second order condition requires the Hessian matrix to be definite semi-negative. This happens when minor determinant is negative while whole determinant is positive. For a level μ and parameter $\beta = 1+\mu$, the minor determinants are $\frac{\partial^2 U}{\partial q_i^2} = -1 < 0$ and the whole determinant is $\frac{\partial^2 U}{\partial q_i^2}\frac{\partial^2 U}{\partial q_j^2} - \frac{\partial^2 U}{\partial q_j \partial q_i}\frac{\partial^2 U}{\partial q_i \partial q_j} = 1^2 - (\gamma'-\mu)^2 = (1+\gamma'-\mu)(1-\gamma'+\mu)$. The whole determinant is positive as long as $\mu \leq 1 \leq 1+\gamma'$.]

demand for both firms, using its downstream unit. Once again, the VIP quotes an inefficiently low input price to impede an excess of collaboration on the VIP's side.

5.3 Decentralization

We find the following distinctive results.

Proposition 4. *With complements, decentralization increases the rival's input price ($w_2^D \geq w_2^C$) and sets the downstream unit's subsidy lower than the centralized opportunity benefit ($0 > w_1^D \geq \gamma w_2^C$). Inequalities are strict except when the goods are unrelated ($\gamma = 0$).*

As with substitutes, the VIP now controls the value of the downstream unit's downstream price through the value of w_1. Hence, it can again simulate the Chen incentive by setting $w_1 = C_1(w_2^C)$. However, in contrast to substitutes, the Chen incentive inefficiently makes the downstream unit decrease its price because it feels subsidized when its downstream price decreases. The downstream unit collaborates too much by attracting demand to both firms. It prompts the VIP to set w_1 below this opportunity benefit - in absolute value -, which reduces the perceived subsidy and thus increases the downstream unit's price while decreasing the complementor's price ($|w_1| \leq |C_1(w_2^C)|$). The VIP makes its downstream unit collaborate less and distorts collaboration in favor of the latter, to the detriment of upstream sales ($(dq_2^D/dw_1) = (dq_2^D/dC_1(w_2)) < 0$).

On the other hand, as with substitutes, the elimination of the Chen incentive reduces the effect of the complementor's input price w_2 upon the downstream firms' prices, and the asymmetric reduced responsiveness does not modify the VIP's upstream sales response when w_2 increases. As the rival's input price was inefficiently low due to the Chen incentive, it emboldens the VIP to raise the rival's input price, and mitigates the loss on upstream sales through a higher margin. In that case, it is clearer that this incentive is not driven by a desire to foreclose a rival.

Proposition 5. *With complements, decentralization increases the downstream unit's price ($p_1^D \geq p_1^C$) but decreases the complementor's price ($p_2^D \leq p_2^C$). Inequalities are strict except when the goods are unrelated ($\gamma = 0$).*

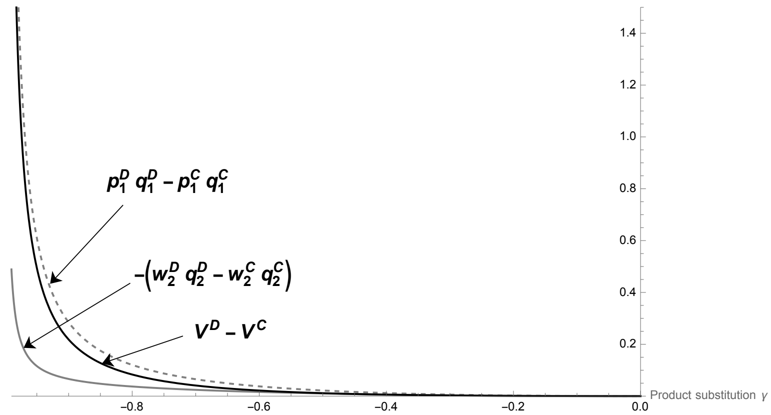

Figure 2: Comparative statics (complements)

In contrast to substitutes, the downstream unit's final price increases, as the replacement of the Chen incentive by the lower subsidy indeed tends to increase the downstream unit's price. But due to price substitution, it diminishes the complementor's downstream price. This effect distorts collaboration in favor of the downstream unit, since the downstream unit drives demand away and forces the complementor to attract demand. On the other hand, the superficial RRC increases the complementor's input price, which tends to increase the complementor's downstream price while reducing the downstream unit's price. This effect distorts collaboration in the opposite direction: the complementor drives demand away while the downstream unit attracts demand.

Overall, the decentralized VIP again pushes collaboration in favor of its downstream unit - hence forcing the complementor to attract more demand. As with subsititutes, this happens because decentralization enables the VIP to act as if it was a Stackelberg leader downstream, by committing to set a certain input price w_1.

To sum up, decentralization enables the VIP to boost the downstream profits, to the detriment of upstream profits. Figure 1 provides a decomposition of the variation of the VIP profits. It shows that the increase in downstream revenues ($p_1^D q_1^D > p_1^C q_1^C$) again overcomes the decrease in upstream profits from the rival sales ($w_2^D q_2^D < w_2^C q_2^C$). Ultimately, the VIP is better off. However, and in contrast to substitutes, the consumers are worse off due to the rise of the downstream unit's price, which overcomes the decrease of the complementor's price. These results extend Moresi and Schwartz (2017)'s analysis which mostly remained at the centralization equilibrium outcome.

Proposition 6. *With complements, decentralization increases the VIP's profit, decreases the complementor's profit and decreases consumer surplus.*

The gains for the VIP furthermore depend on the length of product complementarity. In contrast to substitutes, the VIP's gains from decentralization always increases with respect to product complementarity. We need to bear in mind that the objective of the VIP is to boost downstream profits, to the detriment of upstream profits. Starting at unrelated goods ($\gamma = 0$), the VIP cannot gain from decentralization, as products are independent. When products are more complements ($0 > \gamma > 1$), the monopolist can replace the Chen incentive and benefit from decentralization even though upstream profits diminish. When complementarity is strong, collaboration is great: the replacement of the Chen incentive distorts collaboration relative to the superficial RRC even more. As it entails relatively fewer upstream losses than downstream gains, the VIP's behavior is further to that under centralization.

6 Extensions

6.1 Private transfer to the downstream unit

In any real-world setting the competition enforcement agencies would be suspicious as to why the downstream unit's input price is being made public, and might suspect collusion. On the other hand, the downstream unit would likely know the input price that the VIP was charging D_2, and this would likely appear to be "natural" and not suspicious to the competition agencies. This section studies the informational structure where D_2 is unable to observe w_1, but knows that the downstream unit observes w_2. We use the Perfect Bayesian Equilibrium concept to solve this variant of the game where information is imperfect.

By backward induction, at the competition stage, the firms maximize their profits taking the anticipated price of the other firm as given. Their reaction functions write $R_i(w_i, p_j^{a_i})$, where $p_j^{a_i}$ denotes D_i's anticipation on D_j's reaction. D_2 observes only w_2 and thus forms beliefs about the downstream unit's input price. Assuming passive beliefs, D_2 thinks w_1 sticks to the equilibrium level w_1^*. Though D_2 observes only w_2, it knows that D_1 observes the two input prices and that the latter knows that D_2 reacts only to w_2 while believing that $w_1 = w_1^*$. Formally, it yields $R_2(w_2, R_1(w_1^*, p_2^{a_1})) = p_2^M(w_1^*, w_2)$.[8] By contrast, D_1 observes the two input prices, and knows that D_2 observes only w_2 but still expects D_1 to observe both prices. Formally, it yields $R_1(w_1, R_2(w_2, p_1^{a_2})) = p_1^M(w_1, w_2)$.

$$p_1^M(w_1, w_2) = \frac{\alpha(2-\gamma-\gamma^2) + 2w_1 + \gamma w_2}{4-\gamma^2} \;\; ; \;\; p_2^M(w_1^*, w_2) = \frac{\alpha(2-\gamma-\gamma^2) + 2w_2 + \gamma w_1^*}{4-\gamma^2} \tag{7}$$

At the contracting stage, the VIP maximizes its expected profits $V(p_1^M(w_1, w_2), p_2^M(w_2, w_1^*), w_2)$ with respect to w_1 and w_2. Then, considering that D_2's belief about w_1^* is correct at equilibrium $w_1^* = w_1^M$, we obtain the optimal values w_1^M, w_2^M, p_1^M and p_2^M displayed in Appendix. The next proposition summarizes our findings.

Lemma 4. *When the VIP hides w_1 from D_2, it prefers to foreclose the other firm irrespective of products being substitutes or complements.*

In a nutshell, the VIP cannot commit on an input price w_1 that would set D_2's expectations on the downstream unit's behavior, and therefore cannot distort the downstream interactions in favor of its downstream unit with this instrument. Moreover, the expectations of D_2 are no longer linked to the Chen incentive and, in particular to the input price w_2. Therefore, the VIP cannot recover the centralized profits by just setting $w_1 = C_1(w_2)$ (remember that this price is not observed by D_2). Ultimately, the VIP cannot improve the upstream-downstream balance of profits and is worse off. We find that the harm on the upstream-downstream balance of profits is so hard that the VIP even prefers to foreclose D_2 so as to secure the monopoly profit on D_1's side.

Proposition 7. *In order to make decentralization more profitable than centralization, it is crucial that D_2 observes the downstream unit's input price w_1.*

[8] When D_2 does not know that D_1 observes w_2, it extends its passive beliefs to D_1's behavior. Formally, it yields $R_2(w_2, p_1^{a_2})$. In that case, we get the same equilibrium as under secret contracts (see Supplementary Appendix).

6.2 Opportunism

Another real-world setting is that the VIP secretly renegotiates the contracts with its partners. This section elaborates on the bilateral incentive and the possibility of the upstream unit renegotiating the input prices with each downstream firm once at the decentralization equilibrium.

Consider that the VIP has made the two offers to the firms. The VIP then expects the downstream unit to react to a change of input price w_1, taking D_2's behavior as given ($p_2 = p_2^D$). Formally, this means that D_1's reaction function is $R_1(w_1, p_2^D)$, because the secret change in w_1 is unobservable to D_2, which therefore cannot react accordingly. A similar argument applies to D_2, which takes $p_1 = p_1^D$ as given. Formally, this means that D_2's reaction function is $R_2(w_2, p_1^D)$.

We then implement these reactions into the VIP's profit function, derivate the latter with respect to w_1 and w_2, and evaluate the sign of the derivatives at the decentralized optimal input prices w_1^D and w_2^D. We obtain the following results: (i) the VIP would like to raise w_1 above w_1^D with substitutes ($\partial V/\partial w_1(w_1^D) > 0$), whereas it wants to decrease w_1 below w_1^D under complements ($\partial V/\partial w_1(w_1^D) < 0$); and (ii) the VIP does not wish to modify w_2 with respect to w_2^D, irrespective of the type of goods ($\partial V/\partial w_2(w_2^D) = 0$). Therefore, there is only an incentive to bilaterally renegotiate on the downstream unit's side.

Intuitively, the VIP wishes to set the input price essentially to encourage D_2 to act in favor of its downstream unit by committing to offer a certain input price, even though it harms upstream sales. Once this is incentive is set and benefits the downstream unit, the VIP is thus better off secretly modifying the input price to reduce the harm on upstream sales. The VIP moreover only wants to bilaterally renegotiate the contract with its downstream unit because the other input price is already set optimally to alleviate the loss on upstream sales, once the effect of the replacement of the Chen incentive occurs.

Nevertheless, the downstream unit still has to decide whether to accept the renegotiated input price. We find that it prefers to obtain an input price below w_1^D, irrespective of substitutes or complements ($\partial \pi_1/\partial w_1(w_1^D) < 0$). Intuitively, it only accepts a new input price that increases its efficiency. It is thus likely to refuse the renegotiated price with substitutes, but accept it with complements. Therefore, the following proposition arises.

Proposition 8. *The VIP is willing to renegotiate only the downstream unit's input price, and the downstream unit accepts the VIP's renegotiated input price only when products are complements.*

6.3 Secret contracts with passive beliefs

Even if the VIP has decided to delegate and thereby decentralize the pricing decision to its downstream unit, it is hard to see why it would find it worthwhile to prevent its downstream unit from knowing the price that it charges the downstream rival. As total secrecy is therefore unlikely, we therefore relegate the computations of secret contracts to the Appendix. Overall, we find that decentralization under secret contract harms the VIP's profits.

By a similar argument as in sub-section 6.1, the VIP cannot improve the balance of profits due to the unobservability of w_1 and is worse off. However, the harm on the balance of profits is reduced when the downstream unit also does not observe the other firm's input price. As a result, and in contrast to sub-section 6.1, the VIP does not foreclose the other firm.

This result holds true when firms hold passive beliefs. The results with two other beliefs (*symmetric beliefs* and *wary beliefs* (McAfee and Schwartz, 1994)) are relegated to the Supplementary Appendix as they overall yield to the similar conclusion.

6.4 Non-linear contracts

All our results lie on the assumption that the VIP makes linear contracts under decentralization. We assume so because linear pricing yields more interesting results and is empirically more relevant for our analysis than non-linear contracts.[9] Nonetheless, we discuss below what would occur if we assumed non-linear contracts.

The first thing to notice is that non-linear contracts, in the decentralization scheme and public contracts, enable the VIP to earn the profits of a multi-product monopolist. The VIP simply has to offer the same input prices to both firms so as to induce the latter to set the multi-product monopoly

[9]Empirically, Crawford and Yurukoglu (2012) finds that payments between distributors and content providers rarely or negligibly include fixed monetary transfers and Dobson and Waterson (2007) argues that negotiations between vertically-related firms typically occur infrequently (e.g., annually) which makes challenging to set fixed fees.

prices. The VIP can then capture the firms' entire profits through the fixed fees. Decentralization with public contracts is therefore preferred to centralization: the monopolist cannot earn more than the multi-product monopolist profit.

Turning to secret contracts, we again find that the opportunism issue remains present and centralization remains more profitable than decentralization. Nonetheless, Gilo and Yehezkel (2020) shows that non-linear contracts genuinely afford firms the possibility to vertically collude when interactions are repeated. Such a pattern may make decentralization more profitable than centralization under secret contracting. However, the study of repeated interactions exceeds the focus of our paper and we thus leave it to future research.

Asymmetric transformation costs

We derive our main results restricting the setting to symmetric firms. This restriction has the great advantage of simplifying the exposition and the explanations. However, in real life, firms are likely to differ widely in their ability to transform or use the input product. We briefly provide intuition for the effect of such an alternative hypothesis on the main results.

When the rival becomes less efficient, the VIP's downstream unit is already more competitive with substitutes (or less collaborative with complements). The VIP earns less from being a greater competitor (or softer collaborator) and therefore earns less from decentralization. In contrast, when the rival becomes more efficient, the VIP earns more from decentralization for the opposite reason.

7 Conclusion

A vertically integrated producer (VIP) supplying inputs to downstream rivals often prefers to leave some independence to its integrated units. Decentralization of the final pricing decision is one way to implement such independence. This new scheme enables the downstream unit to be fully responsible for the final pricing decision while the VIP confines itself to offer supply contracts.

Our paper demonstrates that the VIP prefers decentralization over centralization as long as the contract with its downstream unit is disclosed to the rival. Decentralization with public contracts enables the VIP to act on the rival's anticipation about the downstream unit's behavior. We also show that decentralization, though creating an additional margin, reduces the final prices and benefits consumers with substitutes. In contrast, decentralization with complements increases final prices and harms consumers without creating any additional margin.

Acknowledgements

I am indebted to Editor Lawrence J. White and two anonymous referees for their suggestions that have drastically enhanced the paper. I am grateful to Eric Avenel, Olivier Bonroy, Christos Constantatos, Adrien Hervouet, Yassine Lefouili, Emmanuel Petrakis and Thibaud Vergé for the thoughtful discussions on previous drafts. All errors are mine.

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Appendices

A Proofs.

Proof of Lemma 1. We look for the SPNE in pure strategies of the benchmark game. We use backward induction to solve this game. At the competition stage, the VIP and the rival maximize respectively $V(p_1, p_2) = p_1 q_1(p_1, p_2) + w_2 q_2(p_1, p_2)$ and $\pi_2(p_2, p_1) = (p_2 - w_2) q_2(p_2, p_1)$ with respect to p_1 and p_2. This gives the following first order conditions $\text{FOC}_{p_1} : \alpha(\gamma - 1) + 2p_1 - \gamma(p_2 + w_2) = 0$ and $\text{FOC}_{p_2} : \alpha\gamma - \alpha - \gamma p_1 + 2p_2 - w_2 = 0$. The linear demand specification implies that the second order conditions are satisfied ($\text{SOC}_{p_i} : \frac{2}{\gamma^2 - 1} < 0$, $\forall i$). The FOC system delivers the sub-game pricing strategies $p_1^C(w_2)$ and $p_2^C(w_2)$ of equation 3.

At the contracting stage, the VIP accounts for sub-game strategies. We thus substitute the sub-game strategies into the VIP's profit function leading to $V(w_2) = p_1^C(w_2) q_1(p_1^C(w_2), p_2^C(w_2)) + w_2 q_2(p_2^C(w_2), p_1^C(w_2))$ and maximize this expression with respect to the input price w_2. We obtain the following first order condition $\text{FOC}_{w_2} : \alpha\left(\gamma^4 + \gamma^3 + 8\gamma + 8\right) - 2(\gamma + 1)\left(\gamma^2 + 8\right) w_2 = 0$. The second order condition is satisfied $\text{SOC}_{w_2} : -\frac{2\left(\gamma^2+8\right)}{\left(\gamma^2-4\right)^2} < 0$. By isolating w_2 in the FOC, we obtain the equilibrium input price set by the VIP under centralization w_2^C.

Finally, we substitute w_2 by w_2^C into the sub-game pricing strategies to obtain the equilibrium prices $p_1^C = p_1^C(w_2^C)$ and $p_2^C = p_2^C(w_2^C)$. It remains to substitute these equilibrium prices into the demand functions, profit functions and consumer surplus to get the outcomes of Lemma 1. □

Proof of Lemma 2 and 3. We look for the SPNE in pure strategies of the game with decentralization. We use backward induction to solve the game. At the competition stage, the independent unit and the rival maximize respectively $\pi_1(p_1, p_2) = (p_1 - w_1) q_1(p_1, p_2)$ and $\pi_2(p_2, p_1) = (p_2 - w_2) q_2(p_2, p_1)$ with respect to p_1 and p_2. This gives the following first order conditions $\text{FOC}_{p_1} : \alpha\gamma - \alpha + 2p_1 - \gamma p_2 - w_1 = 0$ and $\text{FOC}_{p_2} : \alpha\gamma - \alpha - \gamma p_1 + 2p_2 - w_2 = 0$. The second order conditions are again satisfied ($\text{SOC}_{p_i} : \frac{2}{\gamma^2 - 1} < 0$, $\forall i$). The FOC system delivers the sub-game pricing strategies $p_1^D(w_1, w_2)$ and $p_2^D(w_1, w_2)$ of equation 7.

At the contracting stage, the VIP accounts for the sub-game strategies so that the VIP's expected profit function is $V(w_1,w_2) = p_1^D(w_1,w_2)q_1(p_1^D(w_1,w_2),p_2^D(w_1,w_2)) + w_2q_2(p_2^D(w_1,w_2),p_1^D(w_1,w_2))$. The VIP maximizes this expression with respect to the input prices w_1 and w_2. We obtain the following first order conditions $\mathrm{FOC}_{w_1}: \gamma\left(\alpha\gamma\left(\gamma^2+\gamma-2\right)-4w_2\right)-4\left(\gamma^2-2\right)w_1=0$, and FOC_{w_2} : $\alpha\left(\gamma^2\left(-\gamma^2+\gamma+8\right)-8\right)-4\gamma w_1+2\left(\gamma^4-7\gamma^2+8\right)w_2=0$. The second order conditions require the Hessian matrix to be definite semi-negative. We find $\partial^2V/\partial w_1^2 = -\frac{4\left(2-\gamma^2\right)}{(1-\gamma^2)(\gamma^2-4)^2} < 0$, $\partial^2V/\partial w_2^2 = -\frac{2\left(\gamma^4-7\gamma^2+8\right)}{(1-\gamma^2)(4-\gamma^2)^2} < 0$, and $(\partial^2V/\partial w_1^2)(\partial^2V/\partial w_2^2) - (\partial^2V/\partial w_1\partial w_2)(\partial^2V/\partial w_2\partial w_1) = \frac{8}{(1-\gamma^2)(4-\gamma^2)^2} > 0$ meaning that the Hessian matrix is definite semi-negative and the SOC is always satisfied. By solving the FOC system, we find the equilibrium input prices w_1^D and w_2^D, displayed in Lemma 2.

Finally, we substitute these input prices by their equilibrium values into the sub-game pricing strategies to obtain the equilibrium prices $p_1^D = p_1^D(w_1^D,w_2^D)$ and $p_2^D = p_2^D(w_1^D,w_2^D)$. It remains to substitute these equilibrium prices into the demand functions, profit functions and consumer surplus to get the equilibrium outcomes of Lemma 3. □

Proof of Proposition 1 and 4. Lemma 1 and 2 provide the equilibrium input prices. By rewriting these values, we find:

$$w_2^D = \frac{\alpha}{2} \geq \frac{\alpha}{2} - \frac{\alpha(1-\gamma)\gamma^2}{2(8+\gamma^2)} = w_2^C \ ; \quad |w_1^D| = |\gamma\frac{\alpha(1+\gamma)}{4}| \leq |\gamma(\frac{\alpha(1+\gamma)}{4} + \frac{\left(8-\gamma^2\right)\alpha(1-\gamma)}{4\left(\gamma^2+8\right)})| = |\gamma w_2^C| \quad \square$$

Proof of Proposition 3 and 6. Lemma 1 and 3 provide the equilibrium profits and consumer plus. We just have to compute the differences between firms' profits and consumer surplus under the two organizational schemes. They are as follows:

$$V^D - V^C = \frac{\alpha^2(1-\gamma)\gamma^2}{8(\gamma+1)\left(\gamma^2+8\right)} \geq 0 \quad , \quad \pi_2^D - \pi_2^C = \frac{3\alpha^2(\gamma-1)\gamma^2\left(5\gamma^2+16\right)}{16(\gamma+1)\left(\gamma^2+8\right)^2} \leq 0$$

$$CS^D - CS^C = \frac{\alpha^2\gamma\left(\gamma\left(\gamma\left(-17\gamma^2+\gamma-48\right)-64\right)+128\right)}{32(\gamma+1)\left(\gamma^2+8\right)^2} \geq (<)\, 0 \quad \text{when} \quad \gamma \geq (<)\, 0 \quad \square$$

B Computations.

Computations of the equilibrium under total separation. We look for the SPNE in pure strategies of the game where the upstream unit is totally separated from the downstream unit. We use backward induction to solve this game. At the competition stage, the downstream unit and the rival maximize respectively $\pi_1(p_1,p_2)=(p_1-w_1)q_1(p_1,p_2)$ and $\pi_2(p_2,p_1)=(p_2-w_2)q_2(p_2,p_1)$ with respect to p_1 and p_2. This gives the following first order conditions $\text{FOC}_{p_1}: \alpha\gamma-\alpha+2p_1-\gamma p_2-w_1=0$ and $\text{FOC}_{p_2}: \alpha\gamma-\alpha-\gamma p_1+2p_2-w_2=0$. The linear demand specification implies that the second order conditions are satisfied ($\text{SOC}_{p_i}: \frac{2}{\gamma^2-1}<0, \forall i$). The FOC system delivers the sub-game pricing strategies $p_1^{NI}(w_1,w_2)=\frac{\alpha\left(2-\gamma^2-\gamma\right)+2w_1+\gamma w_2}{4-\gamma^2}$ and $p_2^{NI}(w_1,w_2)=\frac{\alpha\left(2-\gamma^2-\gamma\right)+2w_2+\gamma w_1}{4-\gamma^2}$.

At the contracting stage, the upstream unit accounts for sub-game strategies. We thus substitute the sub-game strategies into the upstream unit profit function *under separation* $\pi_U(w_1,w_2)=w_1q_1(p_1^{NI}(w_1,w_2),p_2^{NI}(w_1,w_2))+w_2q_2(p_2^{NI}(w_1,w_2),p_1^{NI}(w_1,w_2))$ and maximize this expression with respect to the input prices w_1 and w_2. We obtain the following first order conditions $\text{FOC}_{w_1}: -\alpha\left(\gamma^2+\gamma-2\right)+2\gamma w_2+2\left(\gamma^2-2\right)w_1=0$ and $\text{FOC}_{w_2}: -\alpha\left(\gamma^2+\gamma-2\right)+2\gamma w_1+2\left(\gamma^2-2\right)w_2=0$. The second order conditions require the Hessian matrix to be definite semi-negative. We find $\partial^2V/\partial w_1^2=\partial^2V/\partial w_2^2=-\frac{2\left(2-\gamma^2\right)}{\gamma^4-5\gamma^2+4}<0$, and $(\partial^2V/\partial w_1^2)(\partial^2V/\partial w_2^2)-(\partial^2V/\partial w_1\partial w_2)(\partial^2V/\partial w_2\partial w_1)=\frac{4}{\gamma^4-5\gamma^2+4}>0$ meaning that the Hessian matrix is definite semi-negative and the SOC is always satisfied. By isolating w_1 and w_2 in the FOCs, we obtain the equilibrium input prices set by the VIP under centralization $w_1^{NI}=w_2^{NI}=\frac{\alpha}{2}$.

Last but not least, we substitute w_1 and w_2 by w_1^{NI} and w_2^{NI} into the sub-game prices to obtain the equilibrium prices $p_1^{NI}=p_2^{NI}=\frac{\alpha(3-2\gamma)}{2(2-\gamma)}$. It remains to substitute these equilibrium prices into the demand functions, profit functions and consumer surplus to get the other outcomes. In particular, we have $\pi_U^{NI}=\frac{\alpha^2}{2(2-\gamma)(\gamma+1)}$, $CS^{NI}=\frac{\alpha^2}{4(2-\gamma)^2(\gamma+1)}$.

Finally, we find:

$$\begin{aligned}
p_1^C - p_1^{NI} &= -\frac{\alpha(1-\gamma)(8-(4-3\gamma)\gamma)}{2(2-\gamma)\left(\gamma^2+8\right)} \leq 0 \\
p_2^C - p_2^{NI} &= -\frac{\alpha(1-\gamma)\gamma(4-(1-\gamma)\gamma)}{2(2-\gamma)\left(\gamma^2+8\right)} \leq 0\ (>0) \quad \text{if} \quad \gamma > 0\ (\gamma < 0) \\
w_2^C - w2^{NI} &= -\frac{\alpha(1-\gamma)\gamma^2}{2\left(\gamma^2+8\right)} \leq 0 \\
V^C - V^{NI} &= \frac{\alpha^2(1-\gamma)\left(\gamma^3+4\gamma+8\right)}{4(2-\gamma)(\gamma+1)\left(\gamma^2+8\right)} \geq 0 \\
CS^C - CS^{NI} &= \frac{\alpha^2}{8(\gamma+1)}\left(\frac{\gamma(\gamma(5\gamma+6)(6-(1-\gamma)\gamma)+16)+80}{\left(\gamma^2+8\right)^2} - \frac{2}{(2-\gamma)^2}\right) \geq 0
\end{aligned}$$

$\square$

Computations for Explanations below P3 and P6. We obtain the pass-through rates by deriving the subgame pricing strategies under each scheme. Let us remind that Eq (3) provides the expression under centralization while Eq (7) provides the ones under decentralization. We find:

$$\frac{dp_1^C}{dw_2} = \frac{dp_1^D}{dw_2} + \frac{2\gamma}{4-\gamma^2} \quad ; \quad \frac{dp_2^C}{dw_2} = \frac{dp_2^D}{dw_2} + \frac{\gamma^2}{4-\gamma^2} \quad ; \quad \frac{dp_1^C}{dC_1(w_2)} = \frac{dp_1^D}{dw_1}$$

Using the demand function we have $dq_i/dw_j = [1/(1-\gamma^2)][-(dp_i/dw_j)+\gamma(dp_j/dw_j)]$ and $dq_i/dw_i = [1/(1-\gamma^2)][-(dp_i/dw_i)+\gamma(dp_j/dw_i)]$. This gives

$$\frac{dq_1^C}{dw_2} = -\frac{\gamma}{4-\gamma^2} \quad ; \quad \frac{dq_1^D}{dw_2} = \frac{\gamma}{(1-\gamma^2)(4-\gamma^2)} \quad ; \quad \frac{dq_2^C}{dw_2} = \frac{dq_2^D}{dw_2} = -\frac{2-\gamma^2}{(1-\gamma^2)(4-\gamma^2)}$$

$$\frac{dq_1^D}{dw_1} = -\frac{2-\gamma^2}{4-\gamma^2} \quad ; \quad \frac{dq_2^D}{dw_1} = \frac{\gamma}{(1-\gamma^2)(2+\gamma)}$$

Last, we display below the decomposition of VIP profit variations.

$$p_1^D q_1^D - p_1^C q_1^C = \frac{3\alpha^2\gamma^2\left(\gamma^3+8\right)}{8(\gamma+1)\left(\gamma^2+8\right)^2} \geq 0 \quad , \quad w_2^D q_2^D - w_2^C q_2^C = \frac{\alpha^2\gamma^2\left(\gamma\left(-4\gamma^2+\gamma-8\right)-16\right)}{8(\gamma+1)\left(\gamma^2+8\right)^2} \leq 0 \quad \square$$