Decentralization and welfare with substitutes or complements

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Abstract

A vertically integrated producer (VIP) may decentralize the final price decision to its downstream unit. This often occurs when the VIP supplies downstream rivals. Our paper studies a setting where the VIP competes in prices with a unique downstream rival. It shows that (i) when products are substitutes, decentralization triggers lower final prices and benefits consumers despite creating an additional margin, (ii) in contrast, when products are complements, decentralization induces the downstream unit to set a higher price without creating an additional margin, which overall harms consumers.

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JEL classification: L14, L22, L42

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1 Introduction

The downstream unit of a vertically integrated producer (VIP) may act independently of the whole structure. This often occurs when the VIP supplies and competes with downstream rivals. For example, empirical studies of the media market in Israel and of the television industry in the United States show that the VIP's downstream units take decisions only partially internalizing the effect on the upstream sales (Gilo and Spiegel, 2011; Crawford *et al.*, 2018).

A VIP can actually voluntarily decentralize some decisions. Our paper focuses on decentralization of the final pricing decision where a VIP enables its downstream unit to independently set the final price. Arya *et al.* (2008b) demonstrates that such a strategy harms consumers under Cournot competition. Interestingly, Moresi and Schwartz (2017), which overall studies a VIP's incentive at a non-decentralized state, suggests through a numerical example that the reverse could occur under Bertrand competition.

Our paper fully derives the equilibrium with decentralization under Bertrand competition and confirms Moresi and Schwartz (2017)'s intuition. In particular, it shows that, though decentralization creates an additional margin on the VIP's side, it triggers lower final prices and indeed benefits consumers. Our paper also extends the analysis to demand complements. It establishes that, this time, decentralization triggers a higher price by the downstream unit without creating an additional margin. This higher price shrinks quantities and harms consumers.

This happens because, without decentralization, the downstream unit integrates the impact of a change of its price on the VIP's sales to the rival. This behavior is expected by the rival which sets its strategy accordingly. Decentralization enables the VIP to distort the downstream unit behavior and thus act on the rival's expectation. This capacity nonetheless relies on the observability of the supply contracts. Without contract observability, the paper shows that decentralization does not benefit the VIP.

The remainder of the paper is as follows. Section 2 introduces the model. Section 3 and section 4 derive the analysis with demand substitutes and complements. Section 5 studies secret renegotiations. Finally, section 6 discusses some extensions of the model and section 7 concludes.

2 The benchmark situation: centralization

2.1 The model

We assume a vertically integrated producer (VIP) and a downstream rival. They respectively sell horizontally differentiated products denoted good 1 and good 2 to a representative consumer at prices p_1 and p_2. The representative consumer has the following linear demand function *à la Singh and Vives (1984)* for good $i \in \{1, 2\}$:

$$q_i(p_i, p_j) = \frac{\alpha}{1+\gamma} - \frac{1}{1-\gamma^2} p_i + \frac{\gamma}{1-\gamma^2} p_j \tag{1}$$

with $j \in \{1, 2\}$, $j \neq i$, $\gamma^2 < 1$, and $\alpha > 0$. Good 1 and good 2 are substitutes when $\gamma > 0$, complements when $\gamma < 0$ and unrelated when $\gamma = 0$.

The VIP supplies inputs to the downstream rival D_2 at per-unit price w_2 whereas it supplies its downstream unit freely. The inputs are essential for downstream production and each unit of output requires one unit of the input. We assume the firms bear no production nor transformation costs.

Under centralization, the timing of the game is as follows: (1) the VIP charges the input price w_2, and (2) the VIP and the rival set prices p_1 and p_2. Profits are made. Given the final outputs, the VIP's profits and the rival's profits respectively are

$$V = p_1.q_1 + w_2.q_2 \tag{2}$$

$$\pi_2 = (p_2 - w_2)q_2 \tag{3}$$

The first term to the right of the equal sign in Eq. (2) reflects VIP's profit from its downstream sales, just as Eq. (3) reflects D_2's profit from its downstream sales. The second term to the right of the equal sign in Eq. (2) captures VIP's profit from selling the inputs to the rival.

Last, from the demand function, we find the inverse demand function $p_i(q_i, q_j) = \alpha - q_i - \gamma q_j$,

$\forall i = 1, 2, j \neq i$, leading us to the following expression of the consumer surplus.

$$CS = \frac{(q_1)^2 + 2\gamma q_1 q_2 + (q_2)^2}{2}$$

We use the Sub-game Perfect Nash Equilibrium (SPNE) concept to solve the game.

2.2 The equilibrium outcomes

Using backward induction, we find that the firms respectively set the downstream prices p_1 and p_2 to maximize their profits $V(p_1, p_2)$ and $\pi_2(p_2, p_1)$ giving the following sub-game pricing strategies:

$$p_1^C(w_2) = \frac{\alpha(1-\gamma)(2+\gamma) + 2\,C_1(w_2) + \gamma w_2}{4-\gamma^2} \quad ; \quad p_2^C(w_2) = \frac{\alpha(1-\gamma)(2+\gamma) + 2w_2 + \gamma\,C_1(w_2)}{4-\gamma^2} \tag{4}$$

where $C_1(w_2) = \gamma w_2$. This latter term actually has the following interpretation.

Consider substitutes ($\gamma > 0$), then the term refers to the marginal *opportunity cost* incurred by the VIP for supplying the rival, $C_1(w_2) = \gamma w_2 \geq 0$. Intuitively, centralization leads the VIP to anticipate that part of the sales obtained through a decrease in its price p_1 are diverted from the rival (due to demand substitutes). Formally, the VIP anticipates the effect of the positive diversion ratio, $DR_{21} = -(\partial q_2/\partial p_1)/(\partial q_1/\partial p_1) = \gamma > 0$, which implies that for each new quantity q_1 obtained through a decrease of p_1, γ quantities come from the rival's demand q_2 and will now not be sold at price w_2.

For complements ($\gamma < 0$), the term refers to the marginal *opportunity benefit* incurred by the VIP for supplying the rival, $C_1(w_2) = \gamma w_2 \leq 0$. Intuitively, centralization leads the VIP to anticipate that part of the sales obtained through a decrease in its price p_1 also divert new sales towards the rival (due to demand complements). Formally, the VIP anticipates the effect of the negative diversion ratio, $DR_{21} = \gamma < 0$, which implies that for each new quantity q_1 obtained through a decrease of p_1, γ new quantities go to the rival's demand q_2 and will now be sold at price w_2.

The VIP integrates the sub-game strategies into its profit leading to $V(p_1^C(w_2), p_2^C(w_2), w_2)$. The VIP then maximizes this expected profit with respect to w_2. Lemma 1 displays the equilibrium input price and outcomes.

Lemma 1. *Under centralization, the equilibrium input price is:*

$$w_2^C = \frac{\alpha(8+\gamma^3)}{2(8+\gamma^2)}$$

and the final prices, VIP and rival's profits and consumer surplus, respectively, are

$$p_1^C = \frac{\alpha(4-\gamma)(2+\gamma)}{2(8+\gamma^2)}\ , \quad p_2^C = \frac{\alpha[2(6+\gamma^2)-\gamma(4+\gamma^2)]}{2(8+\gamma^2)}$$
$$V^C = \frac{\alpha^2(\gamma+2)\left(\gamma^2-\gamma+6\right)}{4(\gamma+1)\left(\gamma^2+8\right)}, \quad \pi_2^C = \frac{\alpha^2(1-\gamma)\left(\gamma^2+2\right)^2}{(\gamma+1)\left(\gamma^2+8\right)^2}, \ \textit{and}$$
$$CS^C = \frac{\alpha^2\left(5\gamma^5+\gamma^4+24\gamma^3+36\gamma^2+16\gamma+80\right)}{8(\gamma+1)\left(\gamma^2+8\right)^2}$$

These results are in line with those by Arya *et al.* (2008a). Therefore, like these authors, we find that the VIP does not foreclose the rival under centralization as long as products are not perfect substitutes or complements ($\gamma^2 < 1$).

3 Decentralization with substitutes

In this section, we first analyse the usual framework of Bertrand competition with demand substitutes. This allows us to grasp the main mechanisms at stake.

3.1 The new assumption

Under decentralization, the VIP continues to charge the downstream rival a per-unit price w_2 but it now also charges a per-unit price w_1 to its downstream unit. The downstream unit and the rival then compete in prices and respectively set prices p_1 and p_2. Importantly, the downstream unit now has an objective function that does not consider upstream sales and therefore differs from the VIP's profit (see Eq. (2)). Formally, the objective function is

$$\pi_1 = (p_1 - w_1).q_1 \tag{5}$$

Remark. Firms' price decisions are *strategic complements* with *substitutes* ($\gamma > 0$), irrespective of centralization or decentralization.[1]

Decentralization, and therefore the new objective function, can be achieved by hiring a manager and writing a contract that would incentivize it to maximize only the downstream profits. Another way would be to have minority outside shareholders i.e., the shareholders spin off the control rights of the downstream unit to outside shareholders and become passive majority shareholders (O'Brien and Salop, 1999). Last, note that decentralization can also be viewed as organizational frictions between units (Arya *et al.*, 2008b)

3.2 The equilibrium outcomes

At the price competition stage, the firms respectively set the downstream prices p_1 and p_2 to maximize their profits $\pi_1(p_1, p_2)$ and $\pi_2(p_2, p_1)$. We suppose that contracts are observable so that each firm observes its rival's input price. The new sub-game pricing strategies are:

$$p_1^D(w_1, w_2) = \frac{\alpha(2-\gamma-\gamma^2) + 2w_1 + \gamma w_2}{4-\gamma^2} \quad ; \quad p_2^D(w_1, w_2) = \frac{\alpha(2-\gamma-\gamma^2) + 2w_2 + \gamma w_1}{4-\gamma^2} \tag{6}$$

The pivotal change is that the input price level w_1 replaces the 'opportunity cost' term $C_1(w_2)$. This has two consequences. The rival now anticipates that (i) its input price offer w_2 has a lesser effect upon the downstream unit's decision ; and (ii) the downstream unit accounts for the new input price w_1 for the diversion of sales.

At the contracting stage, the VIP maximizes its expected profit $V(p_1^D(w_1, w_2), p_2^D(w_1, w_2), w_1, w_2)$ with respect to w_1 and w_2. We obtain the equilibrium input prices w_1^D and w_2^D displayed in Lemma 2 below. Proposition 1 then compares the values with those under centralization (all proofs are in the Appendix).

Lemma 2. *Under decentralization, the equilibrium input prices are*

$$w_1^D = \gamma\frac{\alpha(1+\gamma)}{4} \quad ; \quad w_2^D = \frac{\alpha}{2} \tag{7}$$

[1]From eq. 2 3 & 5, we find that $\partial^2 V/\partial p_2 \partial p_1 = \partial^2 \pi_2/\partial p_1 \partial p_2 = \partial^2 \pi_1/\partial p_2 \partial p_1 = \gamma/(1-\gamma^2)$.

Proposition 1. *With substitutes, decentralization increases the rival's input price* ($w_2^D \geq w_2^C$) *and decreases the downstream unit's input price with respect to the centralized opportunity cost* ($w_1^D \leq \gamma w_2^C$)*. Inequalities are strict except when the goods are unrelated* ($\gamma = 0$)*.*

The intuition for the changes is as follows. First, as we noticed earlier, decentralization makes the rival anticipate that its input price w_2 has a lesser effect upon the downstream unit's decision. This happens because the rival anticipates that such a rise does not discourage the downstream unit to divert sales anymore. Formally, we find that the rival increases its price by a lesser proportion following a rise in its input price w_2 ($dp_2^C/dw_2 > dp_2^D/dw_2$) and the same applies to the downstream unit ($dp_1^C/dw_2 > dp_1^D/dw_2$). Consequently, we indeed find that a rise in w_2 by the VIP now diverts sales to the downstream unit ($\partial q_1^D/\partial w_2 > 0 > \partial q_1^C/\partial w_2$).

On the other hand, decentralization makes the rival expect the downstream unit to react to w_1 in place of $C_1(w_2) = \gamma w_2$. Interestingly, this new reaction follows the same rate as under centralization with respect to the opportunity cost ($\partial p_2^C/\partial C_1(w_2) = \partial p_2^D/\partial w_1$). An input price w_1 below the opportunity cost $C_1(w_2^C) = \gamma w_2^C$ makes the rival know that diverting sales from the latter to the downstream unit is now more interesting than under centralization. This enables the VIP to credibly raise w_2 and divert sales from the rival to its downstream unit.

By substituting the input price values in the sub-game price strategies, we find p_1^D and p_2^D and the rest of the equilibrium outcomes q_1^D, q_2^D, V^D and π_2^D.[2]

Lemma 3. *Under decentralization, the equilibrium prices, profits and consumer surplus are:*

$$p_1^D = \frac{\alpha}{2}, \quad p_2^D = \frac{\alpha(3-\gamma)}{4}$$
$$V^D = \frac{\alpha^2(\gamma+3)}{8(\gamma+1)}, \quad \pi_2^D = \frac{\alpha^2(1-\gamma)}{16(\gamma+1)}, \ \textit{and} \ CS^D = \frac{\alpha^2(3\gamma+5)}{32(\gamma+1)}.$$

Compared with centralization, the VIP now diverts more sales from the rival to the downstream unit (greater competitor), with fewer margins on downstream sales (lower final price) but higher margins on upstream sales (higher input price). It also obliges the rival to decrease its price to

[2] Note that if the VIP would have set w_1 at the level of the previous equilibrium opportunity cost ($w_1^D = \gamma w_2^D$), it would have trigger the same final outcome as under centralization, essentially because the rival anticipation about diversion of sales would have remained the same.

attract demand. Figure 1 provides a decomposition of the variation of the VIP profits. It shows that the increase in downstream revenues ($p_1^D q_1^D > p_1^C q_1^C$) overcomes the decrease in upstream profits from the rival sales ($w_2^D q_2^D < w_2^C q_2^C$). In the end, the VIP is better off.

Interestingly, decentralization creates an additional margin but actually enables the VIP to use the new margin to decrease the final prices. This occurs because the VIP is better off setting a margin on the downstream unit sales which is lower than the value of opportunity cost integrated by the downstream unit under centralization. The lower prices lead to larger quantities which benefit consumers.

Comparison of pass-through rates
$dp_2^D/dw_2 < dp_2^C/dw_2$
$dp_1^D/dw_2 < dp_1^C/dw_2$
$\partial p_2^D/\partial w_1 = \partial p_2^C/\partial C_1(w_2)$
$\partial q_1^D/\partial w_2 > 0 > \partial q_1^C/\partial w_2$

Table 1: Comparison (substitutes)

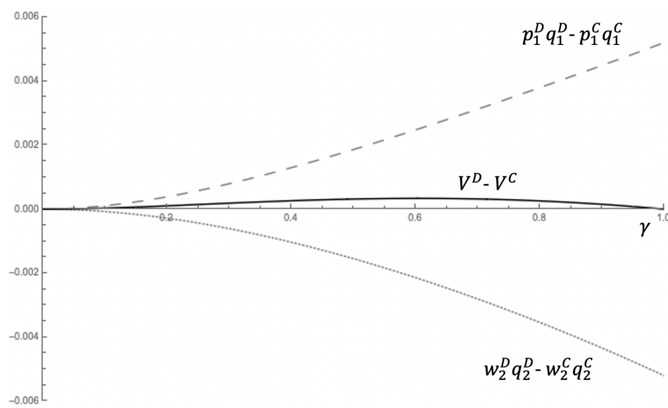

Figure 1: Comparative statics (substitutes)

Proposition 2. *With substitutes, decentralization increases the VIP's profit, decreases the rival's profit and increases consumer surplus.*

The propositions generalize the results by Moresi and Schwartz (2017) to a linear demand function. It addition, it shows that the gains for the VIP depend on the length of product substitution. Starting at unrelated goods ($\gamma = 0$), the VIP cannot divert sales and thus does not gain from fiercer competition. When products are more substitutable ($0 < \gamma < 1$), the monopolist can divert sales due to competition and thus benefit from decentralization. Nonetheless, above some substitution degree, competition is very fierce and diversion of sales hard so that it becomes less pronounced and decentralization becomes less profitable.

4 Decentralization with complements

In real life, the relation between the goods is often fuzzy and also greatly depends on the market definition. In particular, it can happen that goods actually are demand complements. The next section provides some illustrations and the following section details the changes with respect to substitutes.

4.1 Motivation

On the one hand, the set of consumers studied (groups vs. individuals) can affect the relation between the goods. Consider Microsoft which owns Windows OS and produces in-house Surface laptops. Microsoft also supplies its rival Acer, a fixed computer manufacturer. Individuals are likely to consider these two goods as substitutes. However, it is likely that firms and families consider Surface laptops and Acer computers rather as complements.

The type of goods studied (standard goods vs. network goods) can also affect the product relation. Consider Google which owns Android OS and produces in-house Pixel smartphones. Google also supplies the smartphone manufacturer Samsung. The firms' smartphones are *per se* imperfect substitutes. Nonetheless, note that the number of users on one device may increase the valuation of the other device (e.g. network exclusive apps, ...). Accounting for such a relation (network effects) may turn these gross substitutes into net complements (see the demand example below). Intuitively, if the number of users on one device increases the valuation of the other device then a decrease in the price of one device increases the demand of the two devices.[3]

Network effect example. *By implementing network effects* $(1/2)\mu(q_1+q_2)^2$*, where* μ *denotes the strength of network effects, in Bowley's utility function (which is the origin of the demand functions used in the paper), we obtain the following gross utility function* $U(q_1,q_2) = m + \alpha(q_1 + q_2) - (\beta(q_1^2+q_2^2) + 2\gamma' q_1 q_2)/2 + (1/2)\mu(q_1+q_2)^2$ *where* γ' *is the gross degree of substitution between the goods.* m*,* α *and* β *are positive parameters. Such utility gives the following linear inverse*

[3] A recent trial showed that Google was able to manipulate the product differentiation between the final devices (through a restriction of the default app) and henceforth this could have affected the relation between these goods.

demand $p_i = \alpha - (\beta - \mu)q_i - (\gamma' - \mu)q_j$, $\forall i, j$.[4] *For a given level of network effects* $\mu \in [0, 1]$, *take* $\beta = 1 + \mu$ *then we get the inverse demand used in the core paper that is* $p_i = \alpha - q_i - \gamma q_j$ *where* $\gamma = \gamma' - \mu$. *Goods are substitutes when product substitution is stronger than network effects* $\gamma > 0$, *and complements otherwise* $\gamma < 0$.

Remark. Note that firms' price reactions become *strategic substitutes* with *complements* ($\gamma < 0$), irrespective of centralization or decentralization.[5]

4.2 The equilibrium outcomes

The expression of equilibrium outcomes with complements are the same as with substitutes. However, the values of the equilibrium outcomes with complements differ from those with substitutes. In particular, we find the following distinctive result.

Proposition 3. *With complements, decentralization increases the rival's input price* ($w_2^D \geq w_2^C$) *and the downstream unit's input price with respect to the centralized opportunity cost* ($0 > w_1^D \geq \gamma w_2^C$). *Inequalities are strict except when the goods are unrelated* ($\gamma = 0$).

The intuition is as follows. Remind that decentralization has two effects on the rival's anticipation. As for substitutes, the reactions soften because the rival anticipates that a rise of w_2 has a lesser effect upon the downstream unit's decision. The rival increases its price by a lesser proportion following a rise in the its input price ($dp_2^C/dw_2 > dp_2^D/dw_2$) and the downstream unit decreases its price by a lesser proportion following a change in w_2 ($|dp_1^C/dw_2| > |dp_1^D/dw_2|$). This modifies the impact of a change in w_2 on the downstream unit's quantities. In contrast to substitutes, a rise in w_2 now diverts sales away from both firms ($\partial q_1^D/\partial w_2 < 0 < \partial q_1^C/\partial w_2$).

On the other hand, the rival again reacts to w_1 in place of $C_1(w_2)$ and at the same rate as under centralization. This is similar to substitutes. In contrast, the VIP has to set a subsidy to mimic the value of $C_1(w_2)$ because remember that $C_1(w_2) = \gamma w_2$ is negative with complements. Actually,

[4]Existence of solution. The second order condition requires the Hessian matrix to be definite semi-negative. This happens when minor determinant is negative while whole determinant is positive. For a level μ and parameter $\beta = 1 + \mu$, the minor determinants are $\frac{\partial^2 U}{\partial q_i^2} = -1 < 0$ and the whole determinant is $\frac{\partial^2 U}{\partial q_i^2}\frac{\partial^2 U}{\partial q_j^2} - \frac{\partial^2 U}{\partial q_j \partial q_i}\frac{\partial^2 U}{\partial q_i \partial q_j} = 1^2 - (\gamma' - \mu)^2 = (1 + \gamma' - \mu)(1 - \gamma' + \mu)$. The whole determinant is positive as long as $\mu \leq 1 \leq 1 + \gamma'$.]

[5]From eq. 2 3 & 5, we find that $\partial^2 V/\partial p_2 \partial p_1 = \partial^2 \pi_2/\partial p_1 \partial p_2 = \partial^2 \pi_1/\partial p_2 \partial p_1 = \gamma/(1 - \gamma^2)$.

the VIP sets an input subsidy $|w_1|$ below the opportunity benefit $|C_1(w_2^C)|$ making the rival know that diverting new sales to the latter is now less interesting for the downstream unit than under centralization. This enables the VIP to credibly raise w_2 and divert sales away from both firms.

Compared with centralization, the VIP now diverts sales away from the rival and the downstream unit (worse collaborator). It increases the downstream unit's final price and the input price of the rival (higher margins) which nonetheless obliges the rival to decrease its price with respect to centralization to attract demand. Figure 2 provides a decomposition of the variation of the VIP's profits. It shows that, as for substitutes, the increase in downstream revenues ($p_1^D q_1^D > p_1^C q_1^C$) overcomes the decrease in upstream profits from the rival sales ($w_2^D q_2^D < w_2^C q_2^C$). In the end, the VIP is again better off.

Comparison of pass-through rates
$dp_2^D/dw_2 < dp_2^C/dw_2$
$\lvert dp_1^D/dw_2\rvert < \lvert dp_1^C/dw_2\rvert$
$\partial p_2^D/\partial w_1 = \partial p_2^C/\partial C_1(w_2)$
$\partial q_1^D/\partial w_2 < 0 < \partial q_1^C/\partial w_2$

Table 2: Comparison (complements)

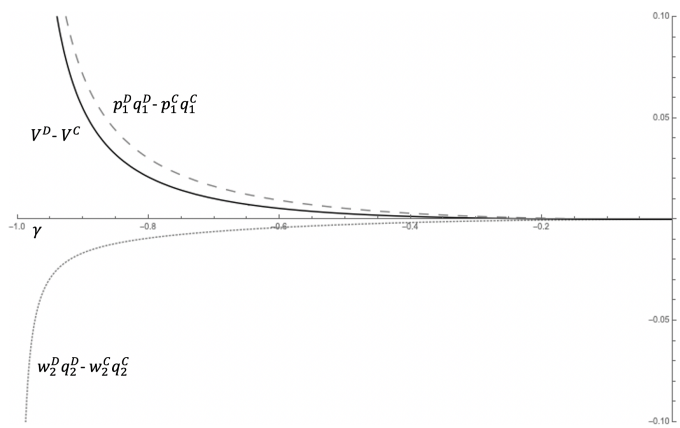

Figure 2: Comparative statics (complements)

Interestingly, this time, decentralization does raise the VIP's final price but without creating an additional margin. This occurs because centralization makes the downstream unit integrate an opportunity benefit of supplying the rival which leads the VIP to use decentralization to set a subsidy instead of a margin. This subsidy is nonetheless lower than the opportunity benefit and thus increases the final price. Also, the lesser collaboration by the VIP leads to less quantities which harms consumers.

Proposition 4. *With complements, decentralization increases the VIP's profit, decreases the rival's profit and decreases consumer surplus.*

These additional propositions complement those by Moresi and Schwartz (2017) which remain at the centralization equilibrium outcome. In addition, we again find that the gains for the VIP depend on the length of product differentiation. Starting at unrelated goods ($\gamma = 0$), the VIP cannot divert sales and thus does not gain from weak collaboration. When products become complements ($1 > \gamma > 0$), the monopolist can divert sales away by a weaker collaboration and benefit from decentralization. As complementarity increases, the diversion of sales becomes easier putting more pressure on the rival to decrease its price and decentralization is more beneficial to the VIP.

5 Opportunism and secret contracting

We have supposed in the previous sections that supply contracts are observable but it may not always be the case. Besides, even if contracts are observable, the firms can fear secret renegotiations by the VIP. We explore the issue of such opportunism in the next section and then derive results under secret contracting.

5.1 The opportunistic incentive

Consider that the VIP has made the two offers to the firms and now tries to renegotiate with its downstream unit. The VIP expects the downstream unit to react only to the input price w_1, taking the rival's behavior as given. This happens because the secret change in w_1 is unobservable to the rival and therefore it cannot react in consequence. Formally, the downstream unit's reaction is:

$$p_1^O(w_1, p_2^D) = \frac{\alpha(1-\gamma) + \gamma p_2^D + w_1}{2} \tag{8}$$

The VIP then implements this new behaviour in its expected profit function. In order to optimize this new profit function, we find that it would like to raise w_1 above w_1^D with substitutes ($\partial V / \partial w_1(w_1^D) > 0$) but decrease w_1 below w_1^D with complements ($\partial V / \partial w_1(w_1^D) < 0$).

The downstream unit still has to decide whether to accept the new offer. Actually, it prefers to obtain an input price below w_1^D, irrespective of substitutes or complements ($\partial \pi_1 / \partial w_1(w_1^D) < 0$). It is thus likely to reject the renegotiated price with substitutes but accept it with complements. To sum up, on the downstream unit's side, renegotiation is more likely to occur with complements than substitutes.

By a symmetric reasoning on the rival's side, we find that the VIP does not wish to modify w_2 with respect to w_2^D irrespective of the type of goods ($\partial V/\partial w_2(w_2^D) = 0$). Therefore, there is no incentive to bilaterally renegotiate on the rival's side. Intuitively, the VIP only is likely to renegotiate the contract with its downstream unit because the contract was designed to act on the rival's behavior (its expectation on how likely sales are going to be diverted) which enables the increase of its margins on the rivals' sales. Once the rival's behavior is set, the VIP would like to divert more or less sales than it wanted the rival to expect. It thus wants to set a new contract to its downstream unit without modifying the margin on the rival's sales.

5.2 Secret contracts with passive beliefs

Because we find an opportunistic incentive with complements, we now derive the equilibrium when contracts are secret. By secret contracts we mean that neither firm 1 nor firm 2 is able to observe the rival's offer when setting the final prices. In other words, contracts are *interim unobservable*.[6] Given that information is imperfect, we use the Perfect Bayesian Equilibrium (PBE) concept with passive beliefs to solve the game.

Passive beliefs implies that each firm thinks that the rival always sets the equilibrium price irrespective of the offer it gets from the VIP.[7] It also implies that each firm only takes into account its offer when setting its price. The firms' sub-game pricing strategies are similar to the one in equation 8 and its symmetric expression for the rival, except for p_j^D which is replaced by p_j^S where p_j^S denotes the equilibrium price of firm $j \neq i$ under secret contracts.

At the contracting stage, the VIP maximizes its expected profits $V(p_1^S(w_1, p_2^S), p_2^S(w_1, p_1^S), w_1, w_2)$ with respect to w_1 and w_2 taking into account the reaction functions and the passive beliefs of the downstream firms. We obtain the optimal values w_1^S, w_2^S, p_1^S and p_2^S. In line with Rey and

[6] Under linear contracts, *interim observable* and *observable* contracts are equivalent. Thus we do not consider interim observability, and the reader can refer to the section with public contracts to get the outcomes.

[7] More technically, passive beliefs imply that following any unexpected (out-of-equilibrium path) offer, a firm expects the rival to get the expected (on-the-equilibrium path) offer. This restriction on off-path beliefs is necessary to obtain a unique PBE. The "contract equilibrium" concept is another concept that avoids the off-path belief issue, but it gives equivalent results and is a weaker equilibrium concept than the PBE with passive beliefs (see discussion in Rey and Verge (2004)).

Verge (2004), we find that the equilibrium exists only for a certain range of product differentiation ($\gamma^2 < \frac{2\sqrt{2}}{3}$, see appendix for details). Lemma 4 summarizes the findings and Proposition 5 compares the VIP's choice with respect to the one under centralization.

Lemma 4. *Under decentralization and secret contracts, provided $\gamma^2 < \frac{2\sqrt{2}}{3}$, the equilibrium input prices are*

$$w_1^S = \gamma \frac{\alpha(\gamma+2)^2}{8(\gamma+1)} \ , \quad w_2^S = \frac{\alpha(\gamma+2)^2}{8(\gamma+1)}$$

and the final prices, profits and consumer surplus, respectively, are

$$p_1^S = \frac{\alpha}{2} + \frac{\alpha\gamma}{8(\gamma+1)} \quad ; \quad p_2^S = \frac{3\alpha}{4} - \frac{\alpha\gamma(\gamma+2)}{8(\gamma+1)}$$
$$V^S = \frac{3\alpha^2}{8} - \frac{\alpha^2\gamma(17\gamma+16)}{64(\gamma+1)^2} \ , \quad \pi_2^S = \frac{\alpha^2(1-\gamma)}{16(\gamma+1)} \ , \textit{and} \quad CS^S = \frac{5\alpha^2}{32} - \frac{\alpha^2\gamma(15\gamma+16)}{128(\gamma+1)^2}$$

Proposition 5. *Compared with centralization, decentralization with unobservable contracts increases the rival's input price ($w_2^S \geq w_2^C$). On the other hand, decentralization increases the downstream unit's input price/subsidy ($|w_1^S| \geq |\gamma w_2^C|$). Inequalities are strict except when the goods are unrelated, $\gamma = 0$.*

The great contrast with decentralization and public contracts is that the VIP cannot distort the rival's expectation through w_1 anymore. The rival's anticipation of how the downstream unit is likely to divert sales is pinned down by its passive beliefs.

We focus the explanation on complements since it is in that case that opportunism may arise. We have seen that the rival can expect a secret renegotiation so that the input subsidy to the downstream unit would actually be higher than the one publicly announced. This means that the rival expects the downstream unit to attract more sales to itself and the rival than publicly supposed. It incentivizes the rival to set a higher price (due to demand complements). Also, the rival's reaction to its input price is very soft with secret contracts ($dp_2^S/dw_2 < dp_2^D/dw_2 < dp_2^C/dw_2$) which incentivizes the VIP to greatly increase its margin from the rival's sales ($w_2^S > w_2^D > w_2^C$). On the other hand, the VIP indeed increases the subsidy to its downstream unit ($|w_1^S| > |C_1(w_2)| > |w_1^D|$) which therefore does divert new sales to the rival.

Compared with centralization, the VIP now attracts sales towards the rival and the downstream unit (better collaborator). It decreases the downstream unit's final price but increases the input price of the rival (higher margin on rival's sales) which incentivizes the rival to increase its price. Overall, the VIP's downstream profits decrease ($p_1^S q_1^S < p_1^C q_1^C$) whereas the VIP's profits from upstream sales to the rival increase ($w_2^S q_2^S > w_2^C q_2^C$). Note that these changes contrast those under public contracting. In the end, the gains from upstream profits do not compensate the loss from downstream profits and the VIP is worse off.

Proposition 6. *Compared with centralization, decentralization with unobservable contracts (i) decreases the VIP's profit and (ii) decreases the rival's profit.*

It is worth noting that the rival profit under secret contracts and public contracts are the same. The same applies to the quantities sold by the latter.

Several effects explain that pattern. On the one hand, the rise of the subsidy ($|w_1^S| > |w_1^D|$) implies that the downstream unit attracts quantities to both firms which benefits the rival's demand. Expecting this effect ($p_1^S < p_1^D$), the rival increases its price. The VIP further increases the rival price by increasing its margin ($w_2^S > w_2^D$). The price increase harms the rival's demand. Actually, these two effects exactly balance each other so that quantities remain the same ($q_2^S = q_2^D$). Moreover, the VIP increases the rival's input price at the same rate as the rival increases its price due to the rival's expectation ($w_2^S - w_2^D = p_2^S - p_2^D$). Therefore, the rival's margin also remains the same. In the end, the rival's profits do not change with respect to observable contracts.

6 Discussion

In this section, we discuss to what extent the assumptions of our model are important for our findings. To do that, we derive and compare equilibrium outcomes under alternative assumptions.

6.1 Secret contracts with alternative beliefs

In the section with secret contracts, we restrict the analysis to passive beliefs in line with the recent development in this literature. We discuss below what would occur with the two other canonical beliefs that are *symmetric beliefs* and *wary beliefs* (McAfee and Schwartz, 1994).

Symmetric beliefs. Symmetric beliefs imply that upon receiving an out-of-equilibrium offer a firm thinks the other firm gets the exact same offer. Formally, they suppose $w_2 = w_1 = w$. Let us remember that in our setting, each firm knows that the VIP integrates the profits of its downstream unit while only accounts for the input margin on the rival's sales. In other words, the firms should expect some asymmetry in the input prices $w_2 \neq w_1$. It thus seems unlikely that the firms spontaneously develop symmetric beliefs in our setting.

Nevertheless, a regulator could specify an arms-length clause into the VIP's contracts, obliging the latter to offer the same price to the two firms. With such a clause, we find that the VIP still prefers to remain centralized with substitutes. However, it can prefer decentralization with complements, provided complementarity is sufficiently strong.

For complements, the core change that potentially reverses our results is that there is no possibility to subsidize the downstream unit anymore. This incidentally reflects in the pass-through rate of the downstream unit's price which becomes positive ($dp_1^A/dw > 0 > dp_1^C/dw_2$). In other words, the VIP must set an input price to its downstream unit obliging the latter to increase its price with respect to centralization. On the other hand, the VIP decreases the rival input price. In the end, collaboration is weaker and becomes profitable for the VIP only when complementarity is sufficiently strong so that it balances the absence of subsidy.

Wary beliefs. Wary beliefs imply that upon receiving an offer, a firm expects this offer to be the best offer of the VIP given the other firm's beliefs. Formally, each firm i thinks the rival gets $w_j^*(w_i) = \underset{w_j}{\operatorname{argmax}}\{\pi_U\}$. The great advantage of wary beliefs is to get around the equilibrium existence issue that arises with passive beliefs (when $\gamma^2 > \frac{2\sqrt{2}}{3}$). The great disadvantage is the complexity which makes the computations much less tractable.

Following Rey and Verge (2004) and Gaudin (2019), we derive the equilibrium outcomes with wary beliefs assuming that they take the form of polynomial beliefs. Likewise, we find that wary beliefs also exhibit the opportunism issue encountered with passive beliefs. This opportunism issue leads to the same general result as the one with passive beliefs: the VIP is worse off decentralizing.

6.2 Non-linear contracts

All our results lie on the assumption that the VIP makes linear contracts under decentralization. We assume so because linear pricing triggers more interesting results and is empirically more relevant for our analysis than non-linear contracts.[8] Nonetheless, we discuss below what would occur if we assume non-linear contracts.

The first thing to notice is that non-linear contracts, in the decentralization scheme and public contracts, enable the VIP to earn the profit of a multi-product monopolist. The VIP simply has to set the same input prices to both firms so as to induce the latter to set the multi-product monopoly prices. The VIP can then capture the firms' entire profits through the fixed fees. Decentralization with public contracts thus is at least preferred to centralization: the monopolist cannot earn more than the multi-product monopolist profit.

Turning to secret contracts, we again find that the opportunism issue remains present and centralization remains more profitable than decentralization. Nonetheless, Gilo and Yehezkel (2020) shows that non-linear contracts genuinely procure firms the possibility to vertically collude when interactions are repeated. Such a pattern may make decentralization more profitable than centralization under secret contracting. However, the study of repeated interactions exceeds the focus of our paper and we thus leave it to future research.

Asymmetric transformation costs

We derive our main results restricting the setting to symmetric firms. This restriction has the great advantage of simplifying the exposition and the explanations. However, in real life, firms are likely to be heterogenous in their ability to transform or use the input product. We briefly provide

[8] Empirically, Crawford and Yurukoglu (2012) finds that payments between distributors and content providers rarely or negligibly include fixed monetary transfers and Dobson and Waterson (2007) argues that negotiations between vertically-related firms typically occur infrequently (e.g., annually) which makes challenging to set fixed fees.

intuition for the effect of such an alternative hypothesis on the main results.

When the rival becomes less efficient, the VIP's downstream unit is already more competitive with substitutes (or less collaborative with complements). The VIP earns less from being a greater competitor (or softer collaborator) and therefore earns less from decentralization. In contrast, when the rival becomes more efficient, the VIP earns more from decentralization for the reverse reason.

7 Conclusion

A vertically integrated producer (VIP) supplying inputs to downstream rivals often prefers to leave some independence to its integrated units. Decentralization of the final pricing decision is one way to implement such independence. This new scheme enables the downstream unit to be fully responsible for the final pricing decision while the VIP confines itself to offer supply contracts.

Our paper demonstrates that the VIP prefers decentralization over centralization as long as the contract with its downstream unit is disclosed to the rival. Decentralization with public contracts enables the VIP to act on the rival's anticipation about the downstream unit's behavior. We also exhibit that decentralization, though creating an additional margin, diminishes the final prices and benefits consumers with substitutes. In contrast, decentralization with complements increases final prices and harms consumers without creating any additional margin.

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Appendices

A Proofs

Proof of Lemma 1. We look for the SPNE in pure strategies of the benchmark game. We use backward induction to solve this game. At the competition stage, the VIP and the rival maximize respectively $V(p_1, p_2) = p_1 q_1(p_1, p_2) + w_2 q_2(p_1, p_2)$ and $\pi_2(p_2, p_1) = (p_2 - w_2) q_2(p_2, p_1)$ with respect to p_1 and p_2. This gives the following first order conditions $\text{FOC}_{p_1} : \alpha(\gamma - 1) + 2p_1 - \gamma(p_2 + w_2) = 0$ and $\text{FOC}_{p_2} : \alpha\gamma - \alpha - \gamma p_1 + 2p_2 - w_2 = 0$. The linear demand specification implies that the second order conditions are satisfied ($\text{SOC}_{p_i} : \frac{2}{\gamma^2 - 1} < 0$, $\forall i$). The FOC system delivers the sub-game pricing strategies $p_1^C(w_2)$ and $p_2^C(w_2)$ of equation 4.

At the contracting stage, the VIP accounts for sub-game strategies. We thus substitute the sub-game strategies into the VIP's profit function leading to $V(w_2) = p_1^C(w_2) q_1(p_1^C(w_2), p_2^C(w_2)) + w_2 q_2(p_2^C(w_2), p_1^C(w_2))$ and maximize this expression with respect to the input price w_2. We obtain the following first order condition $\text{FOC}_{w_2} : \alpha\left(\gamma^4 + \gamma^3 + 8\gamma + 8\right) - 2(\gamma + 1)\left(\gamma^2 + 8\right) w_2 = 0$. The second order condition is satisfied $\text{SOC}_{w_2} : -\frac{2\left(\gamma^2+8\right)}{\left(\gamma^2-4\right)^2} < 0$. By isolating w_2 in the FOC, we obtain the equilibrium input price set by the VIP under centralization w_2^C.

Finally, we substitute w_2 by w_2^C into the sub-game pricing strategies to obtain the equilibrium prices $p_1^C = p_1^C(w_2^C)$ and $p_2^C = p_2^C(w_2^C)$. It remains to substitute these equilibrium prices into the demand functions, profit functions and consumer surplus to get the outcomes of Lemma 1. $\square$

Proof of Lemma 2 and 3. We look for the SPNE in pure strategies of the game with decentralization. We use backward induction to solve the game. At the competition stage, the independent unit and the rival maximize respectively $\pi_1(p_1, p_2) = (p_1 - w_1) q_1(p_1, p_2)$ and $\pi_2(p_2, p_1) = (p_2 - w_2) q_2(p_2, p_1)$ with respect to p_1 and p_2. This gives the following first order conditions FOC_{p_1} : $\alpha\gamma - \alpha + 2p_1 - \gamma p_2 - w_1 = 0$ and $\text{FOC}_{p_2} : \alpha\gamma - \alpha - \gamma p_1 + 2p_2 - w_2 = 0$. The second order conditions are again satisfied ($\text{SOC}_{p_i} : \frac{2}{\gamma^2 - 1} < 0$, $\forall i$). The FOC system delivers the sub-game pricing strategies $p_1^D(w_1, w_2)$ and $p_2^D(w_1, w_2)$ of equation 6.

At the contracting stage, the VIP accounts for the sub-game strategies so that the VIP's expected profit function is $V(w_1,w_2) = p_1^D(w_1,w_2)q_1(p_1^D(w_1,w_2),p_2^D(w_1,w_2))+w_2q_2(p_2^D(w_1,w_2),p_1^D(w_1,w_2))$. The VIP maximizes this expression with respect to the input prices w_1 and w_2. We obtain the following first order conditions $\text{FOC}_{w_1}: \gamma\left(\alpha\gamma\left(\gamma^2+\gamma-2\right)-4w_2\right)-4\left(\gamma^2-2\right)w_1=0$, and FOC_{w_2} : $\alpha\left(\gamma^2\left(-\gamma^2+\gamma+8\right)-8\right)-4\gamma w_1+2\left(\gamma^4-7\gamma^2+8\right)w_2=0$. The second order conditions require the Hessian matrix to be definite semi-negative. We find $\partial^2V/\partial w_1^2 = -\frac{4\left(2-\gamma^2\right)}{(1-\gamma^2)(\gamma^2-4)^2} < 0$, $\partial^2V/\partial w_2^2 = -\frac{2\left(\gamma^4-7\gamma^2+8\right)}{(1-\gamma^2)(4-\gamma^2)^2} < 0$, and $(\partial^2V/\partial w_1^2)(\partial^2V/\partial w_2^2) - (\partial^2V/\partial w_1\partial w_2)(\partial^2V/\partial w_2\partial w_1) = \frac{8}{(1-\gamma^2)(4-\gamma^2)^2} > 0$ meaning that the Hessian matrix is definite semi-negative and the SOC is always satisfied. By solving the FOC system, we find the equilibrium input prices w_1^D and w_2^D, displayed in Lemma 2.

Finally, we substitute these input prices by their equilibrium values into the sub-game pricing strategies to obtain the equilibrium prices $p_1^D = p_1^D(w_1^D,w_2^D)$ and $p_2^D = p_2^D(w_1^D,w_2^D)$. It remains to substitute these equilibrium prices into the demand functions, profit functions and consumer surplus to get the equilibrium outcomes of Lemma 3. □

Proof of Proposition 1 and 3. Lemma 1 and 2 provide the equilibrium input prices. By rewriting these values, we find:

$$w_2^D = \frac{\alpha}{2} \geq \frac{\alpha}{2}\frac{(8+\gamma^3)}{(8+\gamma^2)} = w_2^C\ ;\ \ |w_1^D| = |\gamma\frac{\alpha(1+\gamma)}{4}| \leq |\gamma(\frac{\alpha(1+\gamma)}{4}+\frac{\left(8-\gamma^2\right)\alpha(1-\gamma)}{4\left(\gamma^2+8\right)})| = |\gamma w_2^C| \quad \square$$

Proof of Proposition 2 and 4. Lemma 1 and 3 provide the equilibrium profits and consumer plus. We just have to compute the differences between firms' profits and consumer surplus under the two organizational schemes. They are as follows:

$$V^D - V^C = \frac{\alpha^2(1-\gamma)\gamma^2}{8(\gamma+1)\left(\gamma^2+8\right)} \geq 0 \quad , \quad \pi_2^D - \pi_2^C = \frac{3\alpha^2(\gamma-1)\gamma^2\left(5\gamma^2+16\right)}{16(\gamma+1)\left(\gamma^2+8\right)^2} \leq 0$$

$$CS^D - CS^C = \frac{\alpha^2\gamma\left(\gamma\left(\gamma\left(-17\gamma^2+\gamma-48\right)-64\right)+128\right)}{32(\gamma+1)\left(\gamma^2+8\right)^2} \geq (<)\, 0 \quad \text{when} \quad \gamma \geq (<)\, 0 \quad \square$$

Computations for Explanations below P2 and P4. We obtain the pass-through rates by deriving the subgame pricing strategies under each scheme. Let us remind that Eq (4) provides the expression under centralization while Eq (6) provides the ones under decentralization. We find:

$$\frac{dp_1^C}{dw_2} = \frac{dp_1^D}{dw_2} + \frac{2\gamma}{4-\gamma^2} \quad ; \quad \frac{dp_2^C}{dw_2} = \frac{dp_2^D}{dw_2} + \frac{\gamma^2}{4-\gamma^2} \quad ; \quad \frac{dp_1^C}{dC_1(w_2)} = \frac{dp_1^D}{dw_1}$$

Using the demand function we have $dq_1/dw_2 = [1/(1-\gamma^2)][\gamma(dp_2/dw_2) - (dp_1/dw_2)]$. This gives

$$\frac{dq_1^C}{dw_2} = -\frac{\gamma}{4-\gamma^2} \;\; ; \;\; \frac{dq_1^D}{dw_2} = \frac{\gamma}{(1-\gamma^2)(4-\gamma^2)}$$

Last, we display below the decomposition of VIP profit variations.

$$p_1^D q_1^D - p_1^C q_1^C = \frac{3\alpha^2\gamma^2\left(\gamma^3+8\right)}{8(\gamma+1)\left(\gamma^2+8\right)^2} \geq 0 \quad , \quad w_2^D q_2^D - w_2^C q_2^C = \frac{\alpha^2\gamma^2\left(\gamma\left(-4\gamma^2+\gamma-8\right)-16\right)}{8(\gamma+1)\left(\gamma^2+8\right)^2} \leq 0 \quad \square$$

Computations for Opportunism. Lemma 2 and 3 provide the equilibrium outcomes with public contracts. We here analyse whether the VIP prefers to secretly deviate with the downstream unit. This implicitly means that the downstream unit reacts only to the new input price taken the rival's strategy as given. Formally, the downstream sub-game reaction is displayed by Eq (8). The VIP integrates this reaction into its expected profit function $V(p_1^O(w_1), w_1)$ and maximizes it with respect to w_1. We obtain $dV/dw_1 = \frac{\alpha^2(\gamma(\gamma(11-(\gamma-2)\gamma)+16)-24)+16(w_1)^2-16\alpha\gamma w_1}{64(\gamma^2-1)}$. Evaluated at the public contract equilibrium we find $(dV/dw_1)(w_1^D) = \alpha\gamma/(8\gamma+8)$ which takes the sign of γ.

Following the same method, we find $(dV/dw_2)(w_2^D) = 0$ on the rival's side.

Proof of Lemma 4. We look for the PBNE in pure strategies of the game with decentralization under secret contracts. We use backward induction to solve the game.

At the downstream competition stage, the independent unit and the rival maximize respectively $\pi_1(p_1, p_2^S) = (p_1 - w_1)q_1(p_1, p_2^S)$ and $\pi_2(p_2, p_1^S) = (p_2 - w_2)q_2(p_2, p_1^S)$ with respect to p_1 and p_2 and given passive beliefs about the other firm's strategy p_2^S and p_1^S. Passive beliefs imply that, upon receiving out-of-equilibrium offer, beliefs are the same as on the equilibrium path (they are passive). This gives the following first order conditions $\text{FOC}_{p_1} : \alpha\gamma - \alpha + 2p_1 - \gamma p_2^S - w_1 = 0$ and $\text{FOC}_{p_2} : \alpha\gamma - \alpha - \gamma p_1^S + 2p_2 - w_2 = 0$. The second order conditions are satisfied ($\text{SOC}_{p_i} : \frac{2}{\gamma^2-1} < 0$,

$\forall i$). The FOC system delivers the sub-game pricing strategies $p_1^S(w_1, p_2^S)$ and $p_2^S(w_2, p_2^S)$. The first one is displayed in equation 8 by replacing p_i^D by p_i^S and the second is just the symmetric expression.

At the contracting stage, the VIP accounts for sub-game strategies so that the VIP's expected profit function is $V(w_1, w_2) = p_1^S(w_1, p_2^S)q_1(p_1^S(w_1, p_2^S), p_2^S(w_2, p_1^S)) + w_2 q_2(p_1^S(w_1, p_2^S), p_2^S(w_2, p_2^S))$. The VIP maximizes this expression with respect to the input prices w_1 and w_2. We obtain the following first order conditions $\text{FOC}_{w_1} : \gamma(\alpha(\gamma - 1) - \gamma p_1^S + 2p_2^S - 3w_2) + 2w_1 = 0$, and $\text{FOC}_{w_2} : \alpha\left(3\gamma^2 - \gamma - 2\right) + 2\gamma p_1^S - 3\gamma(\gamma p_2^S + w_1) + 4w_2 = 0$. We check the second order condition through the Hessian matrix. We get $\partial^2 V/\partial w_1^2 = -\frac{1}{2(1-\gamma^2)} < 0$, $\partial^2 V/\partial w_2^2 = -\frac{1}{1-\gamma^2} < 0$, and $(\partial^2 V/\partial w_1^2)(\partial^2 V/\partial w_2^2) - (\partial^2 V/\partial w_1 \partial w_2)(\partial^2 V/\partial w_2 \partial w_1) = \frac{8-9\gamma^2}{16(1-\gamma^2)^2} > 0$ whenever $|\gamma| < \frac{2\sqrt{2}}{3}$ meaning that the SOC is satisfied (the Hessian matrix is definite semi-negative) only when $|\gamma| < \frac{2\sqrt{2}}{3}$.

The input prices are equilibrium input prices only for low substitutes/complements such that $|\gamma| < \frac{2\sqrt{2}}{3}$. By solving the system composed of the two previous FOC and the fact that equilibrium strategies match equilibrium beliefs $p_i^S(w_i, p_j^S) = p_i^S$, $\forall i \neq j = 1, 2$, we find the equilibrium input prices w_1^S and w_2^S joint with the equilibrium price beliefs p_1^S and p_2^S.

Finally, we substitute these equilibrium price beliefs, which also correspond to the equilibrium prices by construction, into the demand functions, profit functions and consumer surplus to get the last equilibrium outcomes of Lemma 4. □

Proof of Proposition 5. Lemma 1 and 4 provide the equilibrium input prices. By rewriting these values, we find:

$$w_2^S = \frac{\alpha(8+\gamma^3)}{2(8+\gamma^2))} + \frac{3\alpha\gamma^2\left(4-\gamma^2\right)}{8(\gamma+1)\left(\gamma^2+8\right)} \geq \frac{\alpha(8+\gamma^3)}{2(8+\gamma^2))} = w_2^C \;;\;\; |\gamma w_2^C| \leq |\gamma w_2^S| = |w_1^S| \quad \square$$

Proof of Proposition 6. Lemma 1 and 4 provide the equilibrium profits and consumer plus. We just have to compute the differences between firms' profits and consumer surplus under the two organizational schemes. They are as follows:

$$V^S - V^C = -\frac{9\alpha^2\gamma^4}{64(\gamma+1)^2\left(\gamma^2+8\right)} \leq 0 \quad , \quad \pi_2^S - \pi_2^C = -\frac{3\alpha^2(1-\gamma)\gamma^2\left(5\gamma^2+16\right)}{16(\gamma+1)\left(\gamma^2+8\right)^2} \leq 0 \quad \square$$

Supplementary Appendix of *'Decentralization and welfare with substitutes or complements'*

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Proof for Perfect Bayesian Equilibrium with Symmetric beliefs (or arms-length pricing). Symmetric beliefs or arms-length pricing imply that beliefs about the rival's input price offer is the same as one's firm own offer. Formally, upon receiving any input price w_i, each firm i thinks the other gets $w_j = w_i$. For simplification, we thus suppose $w_j = w_i = w$.

Implementing this new rule into the firms' profits gives $\pi_1(p_1, p_2) = (p_1 - w)q_1(p_1, p_2)$ and symmetrically $\pi_2(p_1, p_2) = (p_2 - w)q_2(p_2, p_1)$. This induces the following first order conditions $\text{FOC}_{p_1} : \alpha\gamma - \alpha + 2p_1 - \gamma p_2 - w = 0$ and $\text{FOC}_{p_2} : \alpha\gamma - \alpha - \gamma p_1 + 2p_2 - w = 0$. Note that the linear demand specification still enables the second order conditions to be satisfied $\text{SOC}_{p_1} : \frac{2}{\gamma^2-1} < 0$ and $\text{SOC}_{P_2} : \frac{2}{\gamma^2-1} < 0$. The FOC system delivers the sub-game pricing strategies $p_1^A(w) = p_2^A(w) = \frac{\alpha(1-\gamma)+w}{2-\gamma}$.

At the input pricing stage, the VIP accounts for these sub-game strategies. We thus substitute the sub-game strategies into the VIP profit function leading to $V(w) = p_1^A(w)q_1(p_1^A(w), p_2^A(w)) + wq_2(p_2^A(w), p_1^A(w))$ and maximize this expression with respect to the input price w. We obtain the following first order condition $\text{FOC}_w : 2(\alpha + (\gamma - 3)w) = 0$. The second order condition is satisfied $\text{SOC}_w : -\frac{2(3-\gamma)}{(2-\gamma)^2(\gamma+1)} < 0$. By isolating w in the FOC, we obtain the equilibrium input price set by the VIP under centralization:

$$w^A = \frac{\alpha}{3-\gamma}$$

Substituting this price into the subgame pricing strategies, profit functions and consumer surplus leads to the following equilibrium outcome.

$$p_i^A = \frac{\alpha(2-\gamma)}{3-\gamma} \quad ; \; q_i^A = \frac{\alpha}{(3-\gamma)(\gamma+1)} \;, \forall i$$
$$V^A = \frac{\alpha^2}{(3-\gamma)(\gamma+1)} \quad ; \; \pi_2^A = \frac{\alpha^2(1-\gamma)}{(3-\gamma)^2(\gamma+1)} \quad , \text{and } \; CS^A = \frac{\alpha^2}{(3-\gamma)^2(\gamma+1)}$$

By comparing with equilibrium outcomes under centralization displayed by Lemma 3, we find:

$$V^A - V^C = -\frac{\alpha^2\left(4 - \gamma^2((\gamma-2)\gamma+5)\right)}{4(3-\gamma)(\gamma+1)\left(\gamma^2+8\right)} \geq 0 \quad \text{iff } \; \gamma \leq \gamma' \approx -0.752$$

$\square$

Proof for Perfect Bayesian Equilibrium with Wary beliefs. Wary beliefs imply that firm i upon receiving input price offer w_i thinks that the VIP offers the optimal price $W_j(w_i)$ to firm $j \neq i$.

Equilibrium conditions.

Formally, it means that firm i sets $P_i(w_i)$ which maximizes $\pi_i(p_i|P_j(W_j(w_i)), w_i)$ where $W_j(w_i)$ maximizes the VIP's profit $V(w_j|w_i, P(w_i), P(w_j))$. The symmetric reasoning applies to firm j. Note that in our setting, the VIP integrates the downstream unit's profits into its profits function so that there is some asymmetry between the beliefs. Formally, we find the following optimal conditions for prices and beliefs:

$$P_1(w_1)\ :\ 2P_1(w_1) - \gamma P_2(W_2(w_1)) = \alpha(1-\gamma) + w_1 \tag{$\mathcal{P}_1$}$$

$$P_2(w_2)\ :\ 2P_2(w_2) - \gamma P_1(W_1(w_2)) = \alpha(1-\gamma) + w_2 \tag{$\mathcal{P}_j$}$$

$$W_2(w_1)\ :\ (\gamma P_1(w_1) - W_2(w_1))P_2'(W_2(w_1)) + \alpha(1-\gamma) - P_2(W_2(w_1)) + \gamma P_1(w_1) = 0 \tag{$\mathcal{W}_2$}$$

$$W_1(w_2)\ :\ \gamma w_2 - P_1(W_1(w_2)) + \alpha(1-\gamma) - P_1(W_1(w_2)) + \gamma P_2(w_2) = 0 \tag{$\mathcal{W}_1$}$$

We now need to solve the system formed by the equations above. Following the supplementary appendices by Rey and Verge (2004) and Gaudin (2019), we consider polynomial solutions (henceforth polynomial wary beliefs) such that $W_i(w_j) = \sum_{k=0}^{n_i} \mu_{i,k}(w_j)^k$ and $P_i(w_i) = \sum_{k=0}^{m_i} \theta_{i,k}(w_i)^k$, $\forall i$.

Wary beliefs are Affine beliefs.

First, we show that any polynomial solution is affine. Consider equation ($\mathcal{P}_1$), then

$$\underbrace{2P_1(w_1)}_{\text{degree}=m_1} - \underbrace{\gamma P_2(W_2(w_1))}_{\text{degree}=m_2 n_2} = \underbrace{\alpha(1-\gamma) + w_1}_{\text{degree}=1}$$

Three cases can arise:

1. $m_1 < m_2 n_2$. It implies $m_1 = 0$ and $m_2 = n_2 = 1$. Eq. ($\mathcal{P}_1$) implies $-\gamma\theta_{2,1}\mu_{2,1} = 1$ which contradicts $-2\theta_{2,1}\mu_{2,1} = 0$ implied by Eq. ($\mathcal{W}_2$).

2. $m_1 > m_2 n_2$. It implies $m_1 = 1$ and ($m_2 = 0$ or $n_2 = 0$). Suppose $m_2 = 0$, then Eq ($\mathcal{P}_1$) implies

$2\theta_{1,1} = 1$ which contradicts $\gamma\theta_{1,1} = 0$ implied by Eq. ($\mathcal{W}_2$). Instead, suppose $n_2 = 0$. Let Eq ($\mathcal{K}_i'$) be the derivative of Eq ($\mathcal{K}_i$) and Eq ($\mathcal{K}_i''$) the derivative of Eq ($\mathcal{K}_i'$), $\forall \mathcal{K} = \mathcal{P}, \mathcal{W}$ and $\forall i = 1, 2$. Eq ($\mathcal{P}_1'$) implies $\theta_{1,1} = 1/2$. Eq ($\mathcal{P}_2''$) joint with the latter result implies $4P_2'' = \gamma W_1''$ which contradicts $W_1'' = \gamma P_2''$ implied by Eq ($\mathcal{W}_i''$) and $\theta_{1,1} = 1/2$ unless n_1 and m_2 are strictly inferior to 2.

3. $m_1 = m_2 n_2 \geq 1$. It implies either all degrees are lower than one, or ($m_2 = m_1 \equiv m \geq 2$ and $n_1 = n_2 = 1$). Suppose the latter situation is true then Eq ($\mathcal{W}_2$) implies

$$\underbrace{(\gamma P_1(w_1) - W_2(w_1))P_2'(W_2(w_1))}_{\text{degree}=2m-1\geq 3} + \underbrace{\alpha(1-\gamma) - P_2(W_2(w_1)) + \gamma P_1(w_1)}_{\text{degree}\leq m} = 0$$

which contradicts $m \geq 2$. This shows that polynomial solutions must be affine.

Equilibrium outcomes.

Substituting the expressions of the affine solutions into the system of equations gives the value of the parameters of these affine solutions. We use Mathematica to find such values (the file is available upon request). Let w_i^b denote the equilibrium input price to firm i then at equilibrium we have $w_i^b = \mu_{j,0} + \mu_{j,1} w_i^b$, $\forall i$ and we find

$$w_1^b = \frac{\alpha(\gamma-2)\gamma(\gamma+2)^2((\gamma-2)\gamma+4)\left(5\gamma^2-8\right)}{\left(\gamma^2+8\right)\left(\gamma(\gamma(\gamma(\gamma(5\gamma+8)+2)-40)-16)+32\right)}$$

$$w_2^b = \frac{\alpha(\gamma-2)(\gamma+2)^2((\gamma-2)\gamma+4)\left(5\gamma^2-8\right)}{\left(\gamma^2+8\right)\left(\gamma(\gamma(\gamma(\gamma(5\gamma+8)+2)-40)-16)+32\right)}$$

Note that these input prices do differ ($w_1^b = \gamma w_2^b$) for any $\gamma \neq 0$, and this happens because the VIP internalizes the downstream unit's profit into its objective function. Using the same method, we find the equilibrium prices and thus the equilibrium profit V^b. By comparing with the benchmark result, we have

$$V^b - V^C = -\frac{\alpha^2((\gamma-2)\gamma+4)^2(\gamma(\gamma(\gamma(\gamma(5\gamma-2)+2)+16)-16)-32)^2}{4(\gamma-2)^2\left(\gamma^2+8\right)\left(\gamma(\gamma(\gamma(\gamma(5\gamma+8)+2)-40)-16)+32\right)^2} \leq 0 \quad \square$$

Proof of Subgame Perfect Nash Equilibrium with Non-linear contracts. We first derive the equilibrium under centralization, then the one under decentralization and finally compare the outcomes.

◇ Centralization

The game changes as follows: the VIP now charges a fixed fee $f_2 \in \Re$ to the rival in addition to the linear input price w_2.

We look for the SPNE in pure strategies. We use backward induction to solve the game. At the downstream competition stage, the VIP and the rival maximize respectively $V(p_1, p_2) = p_1 q_1(p_1, p_2) + w_2 q_2(p_1, p_2) + f_2$ and $\pi_2(p_1, p_2) = (p_2 - w_2) q_2(p_1, p_2) - f_2$ with respect to p_1 and p_2. This gives the same first order conditions as before $\text{FOC}_{p_1} : \alpha(\gamma - 1) + 2p_1 - \gamma(p_2 + w_2) = 0$ and $\text{FOC}_{p_2} : \alpha\gamma - \alpha - \gamma p_1 + 2p_2 - w_2 = 0$. This essentially happens because the fixed fee is sunk at this stage. Obviously, the second order conditions remain satisfied $\text{SOC}_{p_1} : \frac{2}{\gamma^2-1} < 0$ and $\text{SOC}_{P_2} : \frac{2}{\gamma^2-1} < 0$. The FOC system delivers the sub-game pricing strategies $p_1^{TC}(w_2) = p_1^C(w_2)$ and $p_2^{TC}(w_2) = p_1^C(w_2)$ which are displayed in equation 4.

In addition, we find that the rival accepts the contract whenever $\pi_2(w_2, f_2) \geq 0$ leading to $f_2 \leq (p_2^{TC}(w_2) - w_2) q_2(p_1^{TC}(w_2), p_2^{TC}(w_2))$.

At the input pricing stage, the VIP accounts for the sub-game strategies. We thus substitute the sub-game strategies into the VIP's profit leading to $V(w_2, f_2) = p_1^{TC}(w_2) q_1(p_1^{TC}(w_2), p_2^{TC}(w_2)) + w_2 q_2(p_1^{TC}(w_2), p_2^{TC}(w_2)) + f_2$ and maximize this expression with respect to the input price w_2 and the fixed fee f_2. We focus on equilibria where the rival accepts the offer. Since the VIP's profit is increasing with the fixed fee, the VIP sets the fixed fee that extracts all the rival's profit.

It then sets the input price w_2 so as to maximize the following simplified expected profit $V(w_2) = p_1^{TC}(w_2) q_1(p_1^{TC}(w_2), p_2^{TC}(w_2)) + p_2^{TC}(w_2) q_2(p_1^{TC}(w_2), p_2^{TC}(w_2))$. We obtain the following first order condition $\text{FOC}_{w_2} : \alpha\gamma(\gamma + 2)^2 - 2\left(5\gamma^2 + 4\right) w_2 = 0$. Once more, note that the linear demand specification enables the second order condition to be satisfied $\text{SOC}_{w_2} : -\frac{2\left(5\gamma^2+4\right)}{\left(4-\gamma^2\right)^2} < 0$. By isolating

w_2 in the FOC, we obtain the equilibrium input price set by the VIP under centralization:

$$w_2^{TC} = \frac{\alpha\gamma(\gamma+2)^2}{2(5\gamma^2+4)}$$

Finally, we substitute w_2 by its equilibrium value into the sub-game pricing strategies to obtain the equilibrium prices $p_1^{TC} = p_1^{TC}(w_2^{TC})$ and $p_2^{TC} = p_2^{TC}(w_2^{TC})$. It remains to substitute these equilibrium prices into the demand functions, profit functions to get the last equilibrium outcomes. In particular, the VIP's profit is:

$$V^{TC} = \frac{\alpha^2\left(\gamma^3+9\gamma^2+8\right)}{4(\gamma+1)\left(5\gamma^2+4\right)}$$

$\diamond$ Decentralization with observable contracts

The game changes as follows: (i) the VIP now charges a fixed fee $f_1 \in \Re$ and a linear input price w_1 to its downstream unit ; (ii) the downstream unit does not integrate the VIP's upstream profits from the rival's sales.

Once more, we look for the SPNE in pure strategies of the alternative game with decentralization. We again use backward induction to solve the game. At the downstream competition stage, the downstream unit and the rival maximize respectively $\pi_1(p_1,p_2) = (p_1-w_1)q_1(p_1,p_2) - f_1$ and $\pi_2(p_1,p_2) = (p_2-w_2)q_2(p_1,p_2) - f_2$ with respect to p_1 and p_2. This gives the same first order conditions as without non-linear contracts as the fixed fees are sunk at this stage. We have FOC_{p_1} : $\alpha\gamma-\alpha+2p_1-\gamma p_2-w_1=0$ and $\mathrm{FOC}_{p_2} : \alpha\gamma-\alpha-\gamma p_1+2p_2-w_2=0$. Therefore the second order conditions remain satisfied $\mathrm{SOC}_{p_1} : \frac{2}{\gamma^2-1} < 0$ and $\mathrm{SOC}_{P_2} : \frac{2}{\gamma^2-1} < 0$. The FOC system delivers the sub-game pricing strategies $p_1^{TD}(w_1,w_2) = p_1^D(w_1,w_2)$ and $p_2^{TD}(w_1,w_2) = p_2^D(w_1,w_2)$ of equation 6.

In addition, we find that the rival accepts the contract whenever $\pi_2(w_2,f_2) \geq 0$ leading to $f_2 \leq (p_2^{TC}(w_1,w_2)-w_2)q_2(p_2^{TC}(w_1,w_2),p_1^{TC}(w_1,w_2))$. The same applies to the downstream unit and $f_1 \leq (p_1^{TC}(w_1,w_2)-w_1)q_1(p_1^{TC}(w_1,w_2),p_2^{TC}(w_1,w_2))$. However, if the downstream unit rejects the offer then the VIP nevertheless internalizes the downstream unit's profit. This happens because the downstream unit is vertically integrated and is independent solely to set the final price. By internalyzing, the downstream unit's profit, which also includes the fixed fee f_1, the VIP makes f_1 irrelevant in its contracting strategy.

At the input pricing stage, the VIP accounts for sub-game strategies which leads to $V(w_1, w_2, f_2) = p_1^{TD}(w_1, w_2)q_1(p_1^{TD}(w_1, w_2), p_2^{TD}(w_1, w_2)) + w_2 q_2(p_1^{TD}(w_1, w_2), p_2^{TD}(w_1, w_2)) + f_2$, for any f_1. The VIP maximizes this expression with respect to the input prices w_1 and w_2, and the fixed fee f_2. We focus on equilibria where the rival accepts the offer. Because the profit is increasing in f_2, the VIP sets the fixed fee to extract all the rival's profit.

The VIP's profit expression simplifies to $V(w_1, w_2) = p_1^{TD}(w_1, w_2)q_1(p_1^{TD}(w_1, w_2), p_2^{TD}(w_1, w_2)) + p_2^{TD}(w_1, w_2)q_2(p_1^{TD}(w_1, w_2), p_2^{TD}(w_1, w_2))$. The VIP then sets w_1 and w_2 to maximize this expression. We obtain the following first order conditions $\text{FOC}_{w_1} : \alpha(\gamma-1)(\gamma+2)^2\gamma + \left(8-6\gamma^2\right)w_1 - 2\gamma^3 w_2 = 0$, and $\text{FOC}_{w_2} : \alpha(\gamma-1)(\gamma+2)^2\gamma - 2\gamma^3 w_1 + \left(8-6\gamma^2\right)w_2 = 0$. Note that the Hessian matrix is definite semi-negative (we find $\partial^2 V/\partial w_1^2 = \partial^2 V/\partial w_2^2 = -\frac{8-6\gamma^2}{(1-\gamma^2)(4-\gamma^2)^2} < 0$ and $(\partial^2 V/\partial w_1^2)(\partial^2 V/\partial w_2^2) - (\partial^2 V/\partial w_1 \partial w_2)(\partial^2 V/\partial w_2 \partial w_1) = \frac{4}{(1-\gamma^2)(4-\gamma^2)^2} > 0$). By solving the FOC system, we find the equilibrium input prices set by the VIP under decentralization

$$w_1^{TD} = w_2^{TD} = \frac{\alpha\gamma}{2}$$

Finally, we substitute these input prices by their equilibrium values into the sub-game pricing strategies to obtain the equilibrium prices $p_1^{TD} = p_1^{TD}(w_1^{TD}, w_2^{TD})$ and $p_2^{TD} = p_2^{TD}(w_1^{TD}, w_2^{TD})$. It remains to substitute these equilibrium prices into the demand functions and then profit functions to get the last equilibrium outcomes. In particular, the VIP's profit is:

$$V^{TD} = \frac{\alpha^2}{2(\gamma+1)}$$

Comparison with centralization.

Finally, we compare the equilibrium profits and find:

$$V^{TD} - V^{TC} = \frac{\alpha^2(1-\gamma)\gamma^2}{4(\gamma+1)\left(5\gamma^2+4\right)} \geq 0$$

$\diamond$ Decentralization with unobservable contracts (passive beliefs)

The game modifies as follows: the VIP makes bilateral offers that are unobservable by the third party. We suppose the firms hold passive beliefs.

We now look for the PBNE in pure strategies. We again use backward induction to solve the game. At the downstream competition stage, the downstream unit and the rival maximize respectively $\pi_1(p_1, p_2) = (p_1 - w_1)q_1(p_1, p_2^{TS}) - f_1$ and $\pi_2(p_1, p_2) = (p_2 - w_2)q_2(p_2, p_1^{TS}) - f_2$ with respect to p_1 and p_2, and given their passive beliefs p_2^{TS} and p_1^{TS}. This gives the same first and second order conditions as without non-linear contracts as the fixed fees are sunk at this stage. We find $p_1^{TS}(w_1) = p_1^S(w_1)$ and $p_2^{TS}(w_2) = p_2^S(w_2)$.

The rival accepts the contract whenever $\pi_2(w_2, f_2) \geq 0$ which leads to $f_2 \leq (p_2^{TC}(w_2) - w_2)q_2(p_1^{TC}, p_2^{TC}(w_2))$. Importantly, note that the demand function uniquely depends on w_2. This happens because when accepting the fixed fee, the rival uses its belief about the downstream unit's strategy. Similarly, the downstream unit accepts the fixed fee whenever $f_1 \leq (p_1^{TC}(w_1) - w_1)q_1(p_1^{TC}(w_1), p_2^{TC})$ which only depends on w_1. But like the observable case, even if the downstream unit rejects the VIP's offer, the VIP still internalizes the whole downstream unit's profit (including f_1). The fixed fee f_1 is thus unsound.

At the input pricing stage, the VIP accounts for sub-game strategies. We thus substitute the sub-game strategies into the VIP profit function leading to $V(w_1, w_2, f_2) = p_1^{TS}(w_1)q_1(p_1^{TS}(w_1), p_2^{TS}(w_2)) + w_2q_2(p_2^{TS}(w_2), p_1^{TS}(w_1)) + f_2$ and maximize this expression with respect to the input prices w_1 and w_2, and the fixed fee f_2. We focus our analysis on equilibria where the rival accepts the offer. Because the profit is increasing in f_2, the VIP sets the fixed fee to extract all the rival's profit.

The profit simplifies to $V(w_1, w_2) = p_1^{TS}(w_1)q_1(p_1^{TS}(w_1), p_2^{TS}(w_2)) + w_2q_2(p_2^{TS}(w_2), p_1^{TS}(w_1)) + (p_2^{TS}(w_2) - w_2)q_2(p_S^{TS}(w_2), p_1^{TS})$. The VIP then sets w_1 and w_2 to maximize this expression. We obtain the following first order conditions $\text{FOC}_{w_1} : 2w_1 + \gamma(2p_1^{TS} - 3w_2 - \alpha(1-\gamma) - p_1^{TS}\gamma) = 0$, and $\text{FOC}_{w_2} : 2w_2 + 4\gamma p_1^{TS} - 3\gamma(w_1 + \alpha(1-\gamma) + \gamma p_2^{TS}) = 0$. Once more, we check the second order condition. We get $\partial^2 V/\partial w_1^2 = \partial^2 V/\partial w_2^2 = -\frac{1}{2(1-\gamma^2)} < 0$. Moreover, we find $(\partial^2 V/\partial w_1^2)(\partial^2 V/\partial w_2^2) - (\partial^2 V/\partial w_1 \partial w_2)(\partial^2 V/\partial w_2 \partial w_1) = \frac{4-9\gamma^2}{16(1-\gamma^2)^2} > 0$ whenever $|\gamma| < \frac{2}{3}$ meaning that the SOC is satisfied (the Hessian matrix is definite semi-negative) only when $|\gamma| < \frac{2}{3}$.

By solving the system composed of the two previous FOC and the fact that equilibrium strategies match equilibrium beliefs $p_i^{TS}(w_i, p_j^{TS}) = p_i^{TS}$, $\forall i \neq j = 1, 2$, we find the equilibrium input prices set by the VIP under decentralization w_1^{TS} and w_2^{TS} joint with the equilibrium price beliefs p_1^{TS} and p_2^{TS}. In particular, we have

$$w_1^{TS} = \frac{\alpha\gamma^2(\gamma+2)}{4(\gamma+1)} \text{ and } w_2^{TS} = \frac{\alpha\gamma(\gamma+2)}{4(\gamma+1)}$$

Interestingly, note that due to the asymmetry led by the unsoundness of f_1, the input prices differ $w_1^{TS} = \gamma w_2^{TS}$ and are higher than zero. This contrasts the usual outcomes without integration.

It remains to substitute these equilibrium prices into the demand functions, profit functions to get the last equilibrium outcomes. In particular, the VIP's profit is:

$$V^{TS} = \frac{\alpha^2\left(-\gamma^2+8\gamma+8\right)}{16(\gamma+1)^2}$$

Comparison with unobservable contracts.

Finally, we compare the equilibrium profits and find:

$$V^{TS} - V^{TC} = -\frac{9\alpha^2\gamma^4}{16(\gamma+1)^2\left(5\gamma^2+4\right)} \leq 0 \quad \square$$

Proof of Subgame Perfect Equilibrium with Cost asymmetry. We now assume that the downstream bears a per-unit transformation cost denoted c_1 while the rival bears a per-unit transformation cost denoted c_2. We further suppose that $\alpha_i = \alpha - c_i$ denotes firm i's efficiency. In particular, $\alpha_1 \geq \alpha_2$ means the downstream unit is more efficient than the downstream rival whereas $\alpha_1 < \alpha_2$ means the reverse.

$\diamond$ Centralization

The equilibrium with asymmetric transformation costs has been derived by Arya *et al.* (2008).

We display their results below. The equilibrium input price is:

$$w_2^C = \frac{\alpha_2}{2} - \frac{(\alpha_2 - \alpha_1\gamma)\gamma^2}{2\,(\gamma^2+8)}$$

and the final prices, profits and consumer surplus, respectively, are

$$\begin{aligned}
&p_1^C = c_1 + \frac{(8-\gamma^2)\alpha_1 + 2\gamma\alpha_2}{2(8+\gamma^2)}\ , \quad p_2^C = c_2 + \frac{2(6+\gamma^2)\alpha_2 - \gamma(4+\gamma^2)\alpha_1}{2(8+\gamma^2)} \\
&V^C = \frac{(8-3\gamma^2-\gamma^4)(\alpha_1)^2 + 4(\alpha_2)^2 - 8\gamma\alpha_1\alpha_2}{4(8-7\gamma-\gamma^4)}\ , \ \ \pi_2^C = \frac{(2+\gamma^2)^2(\alpha_2-\gamma\alpha_1)^2}{(1-\gamma^2)(8+\gamma^2)^2}, \text{ and} \\
&CS^C = \frac{(64-23\gamma^4-5\gamma^6)(\alpha_1)^2 + 4(4+5\gamma)(\alpha_2)^2 - 4(16+\gamma^2-\gamma^4)\gamma\alpha_1\alpha_2}{8(1-\gamma^2)(8+\gamma^2)^2}
\end{aligned}$$

Likewise, we find that the VIP does not foreclose the rival under centralization as long as $\gamma < \alpha_2/\alpha_1$.

$\diamond$ Decentralization (observable contracts)

Following the same procedure as with symmetric firms, we find the equilibrium input prices:

$$w_1^D = \gamma\frac{1}{4}(\alpha_2 + \alpha_1\gamma) \text{ and } \ w_2^D = \frac{\alpha_2}{2}\ ,$$

which leads to the following equilibrium prices, profits and consumer surplus:

$$\begin{aligned}
&p_1^D = c_1 + \frac{\alpha_1}{2}\ , \quad p_2^D = c_2 + \frac{3\alpha_2 - \alpha_1\gamma}{4} \\
&V^D = \frac{(\alpha_1)^2\left(2-\gamma^2\right) - 2\alpha_1\alpha_2\gamma + (\alpha_2)^2}{8\,(1-\gamma^2)}\ , \quad \pi_2^D = \frac{(\alpha_2-\alpha_1\gamma)^2}{16\,(1-\gamma^2)}\ , \text{ and} \\
&CS^P = \frac{(\alpha_1)^2\left(4-3\gamma^2\right) - 2\alpha_1\alpha_2\gamma + (\alpha_2)^2}{32\,(1-\gamma^2)}.
\end{aligned}$$

$\diamond$ Comparison between centralization and decentralization (observable contracts)

<u>*General result.*</u>

By comparing the VIP's profits under the two schemes we find

$$V^D - V^C = \frac{\gamma^2(\alpha_2 - \alpha_1\gamma)^2}{8\left(-\gamma^4 - 7\gamma^2 + 8\right)}$$

This confirms our results for the asymmetric case: the VIP still benefits from decentralization. However, the gain now depends on the VIP's relative downstream efficiency.

Effect of cost asymmetry on the VIP's gains.

Compare with the symmetric case, we easily see that the main difference is the term $(\alpha_2 - \alpha_1\gamma)^2$ in the expression of gains displayed above. Three cases arise:

1. Case $\alpha_1 = \alpha$. Suppose $\alpha_1 = \alpha$ and $\alpha_2 = \epsilon\alpha$, we then find $(\alpha_2 - \alpha_1\gamma)^2 = (\alpha(\epsilon - \gamma))^2$. If $\epsilon > 1$ then the rival is more efficient than the downstream unit and the reverse holds for $\epsilon < 1$. Importantly, when $\epsilon = 1$, we are back to the symmetric case. We thus have to compare the expression $(\alpha(\epsilon - \gamma))^2$ when $\epsilon \neq 1$ to the expression $(\alpha(1 - \gamma))^2$ to obtain the impact of asymmetric firm on the VIP's gains from decentralization.

   $$(\alpha(\epsilon - \gamma))^2 > (\alpha(1 - \gamma))^2$$

   Remind that the VIP foreclose the rival when $\gamma > \frac{\alpha_2}{\alpha_1} = \epsilon$, we thus analyse only the cases where $\epsilon > \gamma$. This implies that the above inequality simplifies to

   $$\epsilon > 1$$

   We find the VIP earns more when the downstream unit is more efficient than the rival ($\alpha_2 > \alpha_1 = \alpha$) but earns less when the reverse occurs ($\alpha_2 < \alpha_1 = \alpha$).

2. Case $\alpha_2 = \alpha$. Same result just take $\alpha_1 = \frac{1}{\epsilon}\alpha_2$, and consider again cases where $\gamma < \epsilon$.

3. Case $\alpha_2 \neq \alpha_1 \neq \alpha$. Same result just take $\alpha_2 = \epsilon\alpha_1$ and $\alpha_1 = \epsilon'\alpha$, and consider cases where $\gamma < \epsilon$.

$\square$

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