Threshold public good games with model uncertainty

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(Very preliminary, do not cite)

April 17, 2022

Abstract

Various collective action problems can be described as discrete public good games with a threshold. In such games, a public good is provided when total contributions exceed a threshold. Yet, the latter is often not known with certainty because (i) several models might compete to predict its true value - layer of model uncertainty, and (ii) each model only predicts the true value up to some precision degree - layer of risk. We derive equilibria when players are either aware or ignorant about which model to pick (model certainty vs. model uncertainty). We find that when the contribution cost is sufficiently low the ignorant players contribute more than the aware players. In addition, ignorance triggers a "ratchet effect" weakly impeding contributions to fall below a minimum.

Keywords: public good ; threshold ambiguity; ambiguity attitude

JEL classification: C70, D81, H41

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1 Introduction

Discrete public good games are simplified representations of many collective action scenarios. For example, neighbors may decide to petition to build a public project; fishers may restrict fishing to enable fish reproduction; and countries may engage to reduce their carbon emissions. These games posit that a public good is provided if the number of contributors equals or exceeds a threshold. Interestingly, this threshold is often uncertain due to the existence of multiple predictive models. Continuing with our examples, the neighbors may not know the public official who will notice their petition and its subjective threshold number of petitioners; fishing organizations may propose various fish reproduction models; and scientists may propose various climate change models.[1]

The uncertainty triggered by the presence of multiple models is known as *model uncertainty* (Berger and Marinacci, 2020). This concept distinguishes the layers of (i) *risk*, in which the uncertainty is about the possible true threshold values within a given probability model; and (ii) *model uncertainty*, in which the uncertainty is about which alternative probability model one should use to assign probabilities. The economic literature provides no prediction about the effect of this second layer of uncertainty on the level of contributions in collective actions. This is surprising as this topic is of extreme importance for public authorities which face more and more this multiplicity of predictive models due to the progress of information technologies. One reason behind this literature gap lies in the difficulty to theorize how players navigate model uncertainty and therefore to what extent they contribute.

The present study precisely tackles this issue. It proposes to build on the framework with arbitral uncertainty by Çelen and Özgür (2018) to frame *model uncertainty*. More precisely, we assume there is a set of models predicting the true contribution threshold of a public good game. Each model is characterized by an expected threshold and a degree of precision. Furthermore, the players cannot infer which model is the correct one. Using the widespread and empirically founded assumption that players are uncertainty averse, our framework yields the players to build their own predictive model picking the models characteristics that would trigger the worst-case outcomes.

[1] Refer to the following site by the Food and Agricultural Organization to know more about the diversity of fishing restrictions in the world `http://www.fao.org/3/AC865E/AC865E05.htm`. Regarding climate change models, Figure 5 in appendix illustrates the multiplicity of models.

We first find that provided the contribution cost is sufficiently low, model uncertainty increases contributions. Intuitively, the players facing model uncertainty believe to have to reach a greater expected threshold level of contributions than those facing model certainty. This boosts contributions. On the other hand, these players also believe that the (non-additive) probabilities of having thresholds substantially greater than the expected one increase whereas the (non-additive) probabilities of having thresholds slightly greater than the expected one decrease. This second effect boosts contributions when the contribution cost is sufficiently low. Model uncertainty thus increases contributions with respect to model certainty provided the contribution cost is sufficiently low.

Interestingly, model uncertainty also triggers a "ratchet effect" that could mitigate the fall of contributions when the contribution cost is higher. Indeed, suppose that the certain model is the one with the worst expected threshold and the lowest precision. In that case, model uncertainty still yields the players to believe the (non-additive) probabilities of having thresholds slightly lower than this one increase whereas the (non-additive) probabilities of having thresholds substantially lower decrease. As a result, contributions stick to greater levels of contribution cost.

The closest papers to ours are the theoretical works by McBride (2006) and by Kishishita and Ozaki (2020) about uncertainty in threshold public good games. In contrast to our paper, the two studies assume a unique threshold distribution (i.e. a unique predictive model). McBride (2006) then shows that higher risk (mean-preserving spread) increases contributions when the cost-benefit ratio of the public good is sufficiently low. Kishishita and Ozaki (2020) shows that higher ambiguity (less confidence in the unique threshold distribution) decreases contributions. Our analysis shows that when ambiguity arises as a second layer of uncertainty, it increases the equilibrium maximal number of contributors when players are ambiguity averse. This nuances K&O's result when ambiguity arises on unique model predicting the threshold value. In addition, our set-up displays a ratchet effect which impedes - but does not prevent - the minimum number of non-unanimous contributions to fall upon uncertainty (ambiguity). This nuances McBride's result that this number might fall upon a rise of uncertainty (mean-preserving spread).

The remainder of the paper is as follows. Section 2 reminds the benchmark framework of the literature with only the first layer of uncertainty (i.e. risk). Section 3 then expands the framework to the second layer of uncertainty (i.e. model uncertainty), and derives the new results. Section 4 compares our results. Finally, Section 5 concludes. All proof are in the appendix.

2 Model Certainty

The Framework. Following McBride (2006), we consider a discrete public good game with n players with $2 < n < \infty$. We denote the set of all players by $\mathcal{I} = \{1, 2, ..., n\}$. Player $i \in \mathcal{I}$ takes action $a_i \in \mathcal{A}_i = \{0, 1\}$ where $a_i = 1$ means he contributes while $a_i = 0$ means he does not. The cost of contribution is $c \in (0, \infty)$, and the benefit of the public good provision is $v \in (0, \infty)$. The public good is successfully provided when the sum of the contributions exceeds a threshold $s^* \in S = \{0, 1, ..., n + 1\}$. Formally, it requires $\sum_{i \in \mathcal{I}} a_i \geq s^*$. Note that if $s^* = n + 1$, then even if all the players contribute the public good is not provided. At the opposite if $s^* = 0$, then players do not even have to contribute.

The true contribution threshold s^* is pinned down by a *model* which is a normal cumulative distribution F with mean s and variance σ^2. We denote the associated probability density function by f. Note that we will illustrate our results using continuous distributions to simplify the graphical understanding, yet, the reader must remind that the analysis is discrete and only considers discrete transformations. Suppose each player i's beliefs about the contributions of the other players simplify to $x - 1 = \sum_{j \neq i} a_j$, then the expected utility of player i associated with the model $F(.|s, \sigma)$ denotes

$$\mathcal{U}_i^{F(|s,\sigma)}(a_i, x-1) = \begin{cases} vF(s^* \leq x|s, \sigma) - c & \text{if } a_i = 1 \\ vF(s^* \leq x-1|s, \sigma) & \text{otherwise} \end{cases} \tag{1}$$

The present public good problem is a n-player normal form game $<\mathcal{I}, (\mathcal{U}_i^F)>$, where each player i has the strategy space $\mathcal{A}_i = \{0, 1\}$, set of prior beliefs F, and an expected utility function $\mathcal{U}_i^F$, which represents preferences of player i given its expectations about the model. We solve the game using the Nash equilibrium concept and focus on pure strategy equilibria.

The Results. Given all the relevant information about the other players' contributions and the threshold level, player i contributes when its expected utility from contributing is greater than the one when he does not: $\mathcal{U}_i^F(1, x-1) \geq \mathcal{U}_i^F(0, x-1) \Leftrightarrow vF(s^* \leq x|s,\sigma) - c \geq vF(s^* \leq x-1|s,\sigma)$. Denote $f(x|s,\sigma) = F(s^* \leq x|s,\sigma) - F(s^* \leq x-1|s,\sigma)$ the probability of player i of being pivotal given other players' contributions $x-1$, then player i contributes if and only if this probability is greater than the cost-benefit ratio of contributing:

$$f(x|s,\sigma) \geq \frac{c}{v} \tag{2}$$

Let us denote C^* the number of players contributing at equilibrium, then we retrieve the existence property of at least one of these equilibria by McBride (2006) and Kishishita and Ozaki (2020).

$$C^* = \begin{cases} 0 & \text{if } f(1|s,\sigma) < \frac{c}{v} \\ x \in \{1, ..., n-1\} & \text{if } f(x|s,\sigma) \geq \frac{c}{v} \text{ and } f(x+1|s,\sigma) < \frac{c}{v} \\ n & \text{if } f(n|s,\sigma) \geq \frac{c}{v} \end{cases} \tag{3}$$

Consequently, our Lemma 1 provides the similar characterization of equilibria as in the cited literature. In particular, note that non-unanimous equilibria arise only on the downward-slopping side of the probability distribution function.

Lemma 1. *With model certainty, the following characterization of pure equilibria arises: (i)* ***(No contribution equilibrium)****: $C^* = 0$ constitutes an equilibrium if and only if $f(1|s,\sigma) < \frac{c}{v}$; (ii)* ***(Non-unanimous equilibrium)****: $C^* = x \in \{1, ..., n-1\}$ constitutes an equilibrium if and only if $f(x|s,\sigma) \geq \frac{c}{v}$ and $f(x+1|s,\sigma) < \frac{c}{v}$; (iii)* ***(Unanimous equilibrium)****: $C^* = n$ constitutes an equilibrium if and only if $f(n|s,\sigma) \geq \frac{c}{v}$; (iv)* ***(Existence)****: There always exists at least one equilibrium.*

The present section assumes the players only know one model predicting the true threshold. It

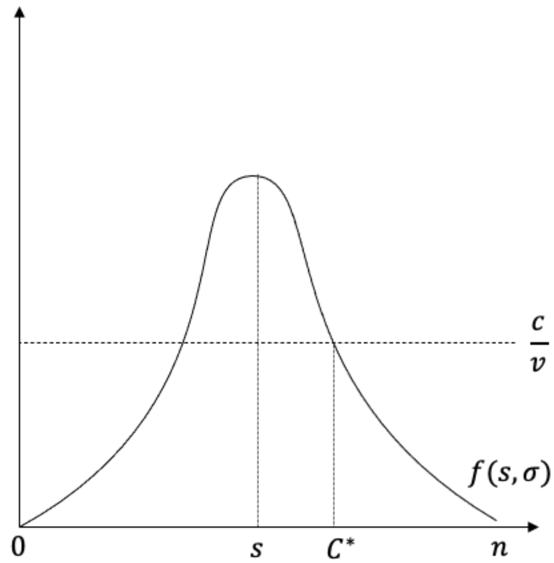

Figure 1: Illustration of non-unanimous equilibrium

can also be that the players are able to discriminate among many models and thus picked one.[2] The players could operate such drastic selection provided they have the ability, knowledge or experience to discriminate between models. Next section suppose the players face many models but are ambiguous about which one to pick due to lack of ability, knowledge or experience.

3 Model uncertainty

In this section, we build on Çelen and Özgür (2018)'s *arbitral uncertainty* to introduce *model uncertainty* in our framework. Specifically, we suppose a set of conceivable models about s^*, which is commonly known by the players. We denote such set by $\mathcal{F} := \{F(.|s,\sigma) : \underline{s} \leq s \leq \overline{s}\,,\ \underline{\sigma} \leq \sigma \leq \overline{\sigma}\}$ and call it a *media*.[3] Both s and σ have natural interpretations. The parameter s is a model predicted threshold while σ gives information about the associated degree of precision. Therefore, the set $[\underline{s}, \overline{s}]$ captures the extent of different models predictions while $[\underline{\sigma}, \overline{\sigma}]$ captures variety of the

[2]Note that one could also suppose that the players are rational and use Lebesgue's principle of insufficient reason to form their expectations over the models. This principle asserts that in face of total ignorance of a probability distribution one must assume uniform probability distribution. This would create a compound lottery $F(.|\hat{s}, \hat{\sigma})$. One condition for the results to follow the same pattern as under awareness is that such compound lottery is unimodal.

[3]Figure 5 gives an empirical illustration of such a set of models (though distributions are not all Gaussian, most of them are unimodal)

models precisions.

The previous section and thus literature assumed players were able to pick one model among the media. We now assume players are ambiguous about which model to pick. The players could not operate such selection provided they lack the ability, knowledge or experience to discriminate between the models displayed in the media. To decide whether to contribute, we suppose players only consider the worst possible outcomes. This assumption is in line with Çelen and Özgür (2018), and, moreover, the experimental literature has shown that upon facing ambiguity, subjects display ambiguity aversion.

More formally, we suppose players hold a common attitude towards uncertainty and hold Min-expected utility. In other words, every player thinks Nature minimizes its expected utility. The preferences of player $i \in \mathcal{I}$ is thus represented by expected utility:

$$\mathcal{U}_i^{\mathcal{F}}(a_i, x-1) = \min_{F \in \mathcal{F}} \mathcal{U}_i^F(a_i, x-1) \tag{4}$$

Note that the expected utility of contributing or not, $\mathcal{U}_i^F(a_i, k)$, is increasing in the probability that the threshold is reached, $F(s^* \leq k|s, \sigma)$ with $k \in \{x-1, x\}$. Therefore, if Nature aims to minimize the utility, it boils down to minimize this probability. For any $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, a rise of the mean threshold s diminishes the probability. Therefore, $\overline{s}$ minimizes the probability. On the other hand, for any $s \in [\underline{s}, \overline{s}]$, a rise of σ flattens the probability around the mean threshold s. As a result, it diminishes the probability if $s < k$, whereas increases it otherwise (except at the exact point $s = k$ due to symmetry). Appendix details and illustrates the reasoning. Lemma 2 summarizes our findings.

Lemma 2. *Given belief about the contribution of the other players $x-1$, Nature's best response to player i's choice is as follows. (i) For any $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, the unique minimizer to (1) is $\arg\min_{F \in \mathcal{F}} U_i^F(x) = F(.|\overline{s}, \sigma)$. (ii) For any $s \in [\underline{s}, \overline{s}]$, denote $k = \{x-1, x\}$, then the unique minimizer to (1) is $\arg\min_{F \in \mathcal{F}} U_i^F(x) = F(.|s, \underline{\sigma})$ when $s < k$ and the unique minimizer to (1) is $\arg\min_{F \in \mathcal{F}} U_i^F(x) = F(.|s, \overline{\sigma})$ when $s > k$, the minimizer is not unique otherwise when $s = k$ (by symmetry).*

Taking nature's behavior into account, player i's expected utility simplifies to:

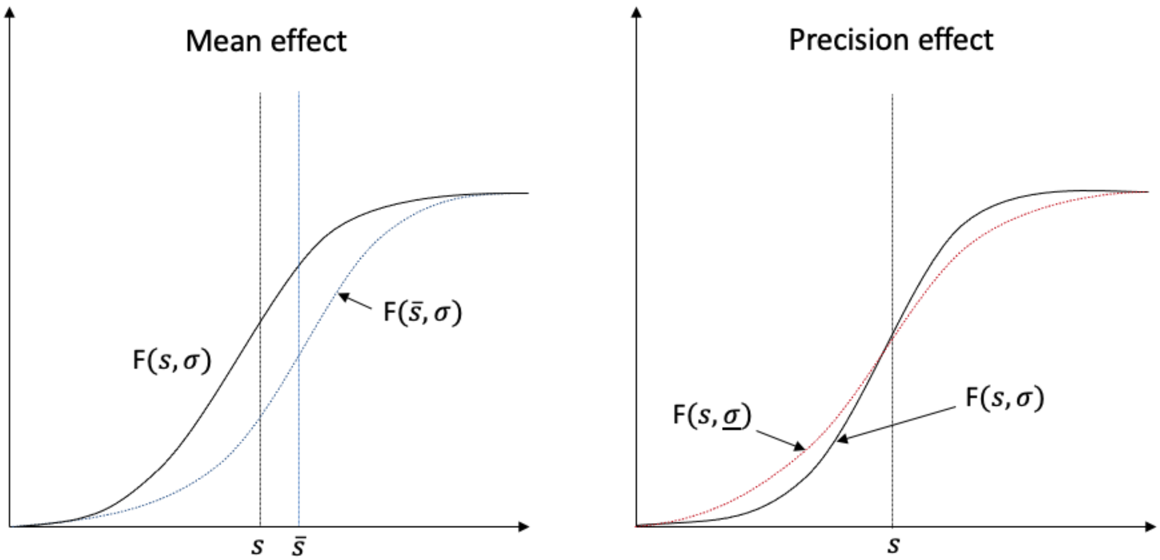

Figure 2: Illustration of Nature choice

$$\mathcal{U}_i^{\mathcal{F}}(a_i, x-1) = \begin{cases} vF_U(s^* \leq x|\overline{s}, \sigma) - c & \text{if } a_i = 1 \\ vF_U(s^* \leq x-1|\overline{s}, \sigma) & \text{otherwise} \end{cases} \tag{5}$$

where $F_U(s^* \leq k|\overline{s}, \sigma) = F(s^* \leq k|\overline{s}, \overline{\sigma})$ if $s \leq k$, and $F(s^* \leq k|\overline{s}, \underline{\sigma})$, otherwise.

The Results. By the same reasoning as in the previous section, player i contributes whenever its expected utility from contributing is greater than that of not contributing. As we saw the expected utility depends on the location of $x-1$, x and $\overline{s}$. We find that player i contributes whenever $f_U(x|\overline{s}, \sigma) \geq \frac{c}{v}$ where

$$f_U(x|\overline{s}, \sigma) = \begin{cases} f_U(x|\overline{s}, \underline{\sigma}) & \text{when } x-1 < x \leq s \\ f_U(x|\overline{s}, \overline{\sigma}) & \text{when } s \leq x-1 < x \end{cases} \tag{6}$$

Let us denote C_U^* the number of players contributing at equilibrium, we then retrieve our previous characterization of equilibria except that $f(x)$ is now pinned down by player i's ambiguity aversion.

$$C_U^* = \begin{cases} 0 & \text{if } f(1|\overline{s},\underline{\sigma}) < \frac{c}{v} \\ x \in \{1,...,\overline{s}-1\} & \text{if } f(x|\overline{s},\underline{\sigma}) \geq \frac{c}{v} \text{ and } f(x+1|\overline{s},\underline{\sigma}) < \frac{c}{v} \\ x \in \{\overline{s}+1,...,n-1\} & \text{if } f(x|\overline{s},\overline{\sigma}) \geq \frac{c}{v} \text{ and } f(x+1|\overline{s},\overline{\sigma}) < \frac{c}{v} \\ x = \overline{s} & \text{if } f(x|\overline{s},\underline{\sigma}) \geq \frac{c}{v} \text{ and } f(x|\overline{s},\overline{\sigma}) < \frac{c}{v} \\ n & \text{if } f(n|\overline{s},\overline{\sigma}) \geq \frac{c}{v} \end{cases} \tag{7}$$

Lemma 3. *Under total ignorance, the following characterization of pure equilibria arises: (i)* ***(No contribution equilibrium)****:* $C_U^* = 0$ *constitutes an equilibrium if and only if* $f(1|\overline{s},\underline{\sigma}) < \frac{c}{v}$*; (ii)* ***(Non-unanimous equilibrium)****: (a)* $C_U^* = x \in \{\overline{s}+1,...,n-1\}$ *constitutes an equilibrium if and only if* $f(x|\overline{s},\overline{\sigma}) \geq \frac{c}{v}$ *and* $f(x+1|\overline{s},\overline{\sigma}) < \frac{c}{v}$*; (b)* $C_U^* = \overline{s}$ *constitutes an equilibrium if and only if* $f(x|\overline{s},\underline{\sigma}) \geq \frac{c}{v}$ *and* $f(x+1|\overline{s},\overline{\sigma}) < \frac{c}{v}$*; (iii)* ***(Unanimous equilibrium)****:* $C_U^* = n$ *constitutes an equilibrium if and only if* $f(n|\overline{s},\overline{\sigma}) \geq \frac{c}{v}$*; (iv)* ***(Existence)****: There always exists at least one equilibrium.*

4 Comparison

Let us now compare C^* to C_U^* with a focus on non-unanimous equilibria where $0 < C^* < n$ and $0 < C_U^* < n$. We find the following proposition.

Proposition 1. *Suppose non-unanimous equilibria exist under both model certainty and model uncertainty, then model uncertainty yields more contributions provided the cost-benefit ratio is sufficiently low. Otherwise, the effect is ambiguous.*

Intuitively, model uncertainty leads the players to consider a greater expected threshold than under model certainty. For a given degree of precision σ, this effect favors contributions. Graphically, the probability distribution functions moves to the right of the graph. On the other hand, model uncertainty also leads the players to consider a lesser degree of precision which flattens the probability density function. This triggers two effects whose sign depend on the cost-benefit ra-

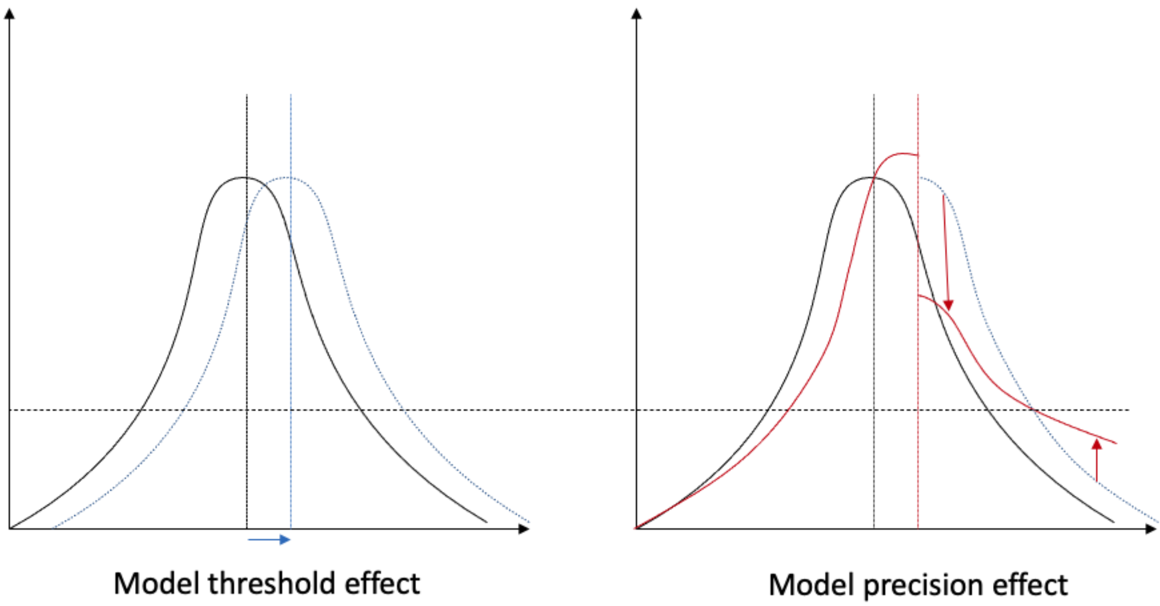

Figure 3: Illustration of the model uncertainty effects

tio: (i) if the cost-benefit ratio is sufficiently high, then the effect might decrease contributions, in contrast (ii) if the cost benefit ratio is sufficiently low, then the effect increases even more the contributions. As a result, if the cost benefit ratio is sufficiently low, model uncertainty increases contributions. Otherwise, the effect is ambiguous as it depends on the length of the positive and negative effects.

Proposition 1 contrasts with Kishishita and Ozaki (2020)'s findings. These authors show that greater ambiguity over a unique model, i.e. players have less confidence in a unique threshold distribution, decreases contributions irrespective of the players' attitude towards this ambiguity. We demonstrate that when ambiguity appears instead as a second layer of uncertainty - i.e. when threshold distribution is not unique and players are unable to pick one distribution - then ambiguity can increase contributions.

Proposition 2. *Suppose non-unanimous equilibria exist under model certainty and model uncertainty, then model uncertainty displays a "ratchet effect" which greatly impedes contributions to fall below the maximum expected true threshold $\overline{s}$.*

The best way to understand this ratchet effect is to consider that the certain model is the one

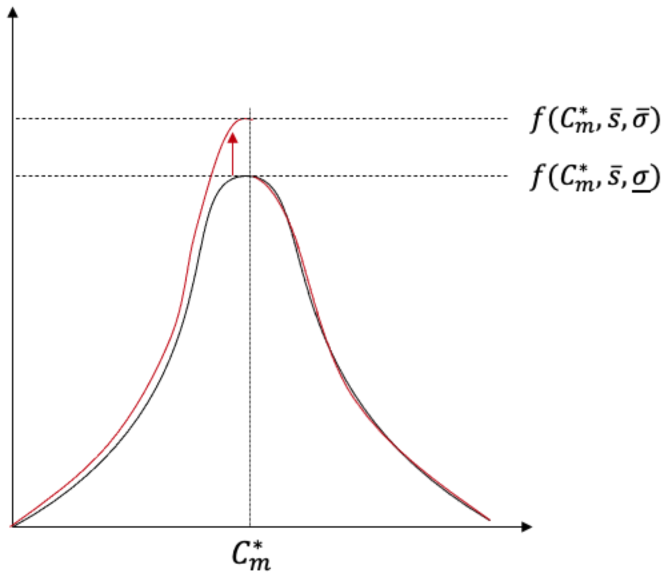

Figure 4: Illustration of ratchet effect

with the maximum average true threshold $\overline{s}$ and the lowest precision $\underline{\sigma}$. In that case, the unanimous equilibria are the same when the cost-benefit ratio is sufficiently low i.e. $c/v \leq f(C_m^*|\overline{s}, \underline{\sigma})$ where C_m^* is the minimum number of contributions with model certainty. Our model show that model uncertainty yields contributions when model certainty does not. Bear in mind that the left-side downward slope is set using the greatest degree of precision $\overline{\sigma}$ which creates non-monotonicity. As a result, contributions at level $\overline{s}$ will stick for greater levels of cost-benefit ratio, even if it exceeds the value of $f(C_m^*|\overline{s}, \underline{\sigma})$. This happens until new value $f(C_m^*|\overline{s}, \overline{\sigma})$.

5 Conclusion

We build on the framework with arbitral uncertainty by Çelen and Özgür (2018) to frame *model uncertainty* in a threshold public good game. Our framework yields the players to build their own predictive model picking the models characteristics that would trigger the worst-case outcomes.

As a result, we first find that provided the contribution cost is sufficiently low, model uncertainty increases contributions. Intuitively, the players facing model uncertainty believe to have to reach a greater expected threshold level of contributions than those facing model certainty. This boosts contributions. On the other hand, these players also believe that the (non-additive) probabilities of having thresholds substantially greater than the expected one increase whereas the (non-additive) probabilities of having thresholds slightly greater than the expected one decrease. This second effect boosts contributions when the contribution cost is sufficiently low. Model uncertainty thus increases contributions with respect to model certainty provided the contribution cost is sufficiently low.

Interestingly, model uncertainty also triggers a "ratchet effect" that could mitigate the fall of contributions when the contribution cost is higher. Indeed, suppose that the certain model is the one with the worst expected threshold and the lowest precision. In that case, model uncertainty still yields the players to believe the (non-additive) probabilities of having thresholds slightly lower than this one increase whereas the (non-additive) probabilities of having thresholds substantially lower decrease. As a result, contributions stick to greater levels of contribution cost.

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Appendices

Appendix uses continuous distributions to simplify the graphical understanding, yet, the reader must remind that the analysis is discrete and only considers discrete transformations.

Illustration of the problem raised by the multiplicity of models

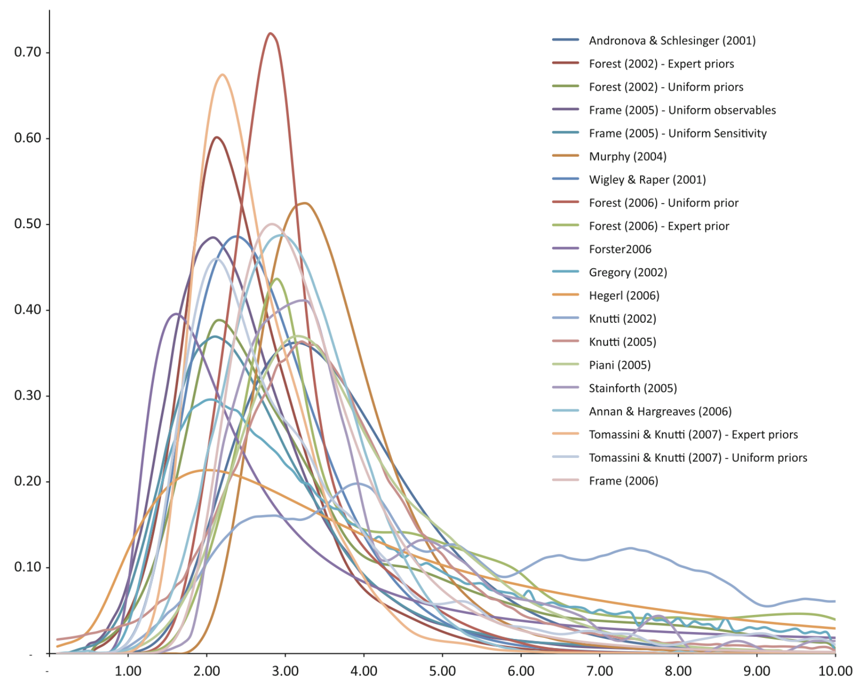


Figure 5: Estimated probability density functions for climate sensitivity from a variety of published studies (copy-past of Fig. 1 in Millner *et al.* (2013))

Proof Equation (3): NE with model certainty

The Nash Equilibrium is such that $\mathcal{U}_i^{\mathcal{F}}(1, \sum_{j\neq i} a_j^*) \geq \mathcal{U}_i^{\mathcal{F}}(0, \sum_{j\neq i} a_j^*)$ for any player i who choose $a_i^* = 1$, while $\mathcal{U}_i^{\mathcal{F}}(0, \sum_{j\neq i} a_j^* - 1) \geq \mathcal{U}_i^{\mathcal{F}}(1, \sum_{j\neq i} a_j^* - 1)$ for any player i who choose $a_i^* = 0$.

• $C^* = 0$. Suppose there is no contributor, $C^* = 0$, then it must be that each player thinks the other players do not contribute $x - 1 = 0$ and each player i decides not to contribute. This happens as long as the player's utility from contributing is strictly lower than his utility from non contributing $vF(1|\tilde{s}, \tilde{\sigma}) - c < vF(0|\tilde{s}, \tilde{\sigma})$ which implies $f(1) < \frac{c}{v}$. No player with belief $x - 1 = 0$

wishes to deviate and contribute.

• $C^* = n$. Suppose there is n contributor, $C^* = n$, then it must be that each player thinks the other players contribute $x-1 = n-1$ and each player i decides to contribute. This happens as long as the player's utility from contributing is higher than his utility from non contributing $vF(n|\tilde{s},\tilde{\sigma}) - c \geq vF(n-1|\tilde{s},\tilde{\sigma})$ which implies $f(n) \geq \frac{c}{v}$. No player with belief $x-1 = n-1$ wishes to deviate and not to contribute.

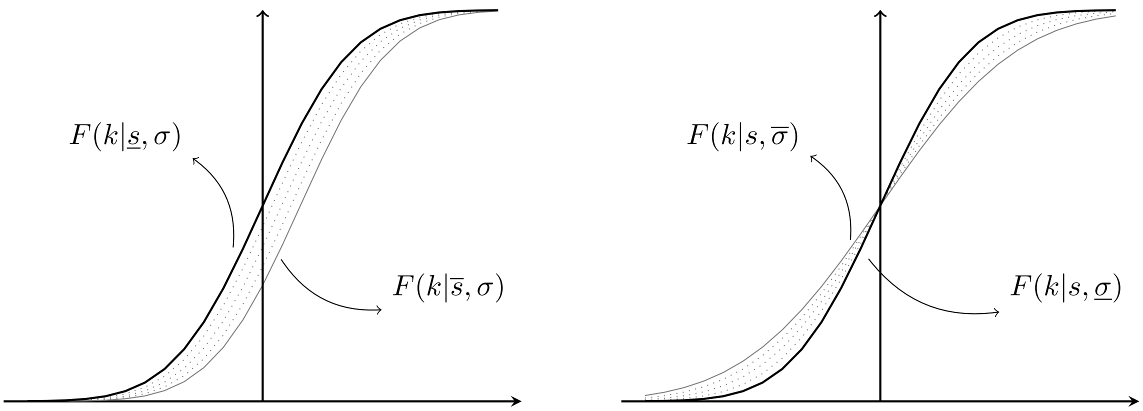
• $C^* = x \in \{1,...,n-1\}$. Suppose $C^* = x \in \{1,...,n-1\}$, then it must be that each contributing player thinks that there is $x-1$ other contributors and given this beliefs that their utility from contributing is greater than that from non contributing $vF(x|\tilde{s},\tilde{\sigma}) - c \geq vF(x-1|\tilde{s},\tilde{\sigma})$ which implies $f(x) \geq \frac{c}{v}$. No contributing player with such beliefs wishes to deviate and not contribute. On the other hand, each not-contributing player must think there is x other contributors and that given this beliefs their utility from contributing is strictly lower than that of not contributing: $vF(x+1|\tilde{s},\tilde{\sigma}) - c < vF(x|\tilde{s},\tilde{\sigma})$ which implies $f(x+1) < \frac{c}{v}$. No not contributing player with such beliefs wishes to deviate and contribute.

• Existence. We assume that there does not exist equilibria and show that this leads to a contradiction. First suppose that $f(n) \geq \frac{c}{v}$. Then, we are done since by Equation 3 we have $C^* = n$. Therefore, it must be that $f(n) < \frac{c}{v}$. Next, let $x \in \{1, 2, ..., n-1\}$ be the largest integer such that $f(x) \geq c/v$, if any. Then, both $f(x+1) < c/v$ and $f(x) \geq c/v$ hold, and hence, by Equation 3 we have $C^* = x$. Therefore, it must be that $f(1) < c/v$. But again, by Equation 3 we have $C^* = 0$. We find a contradiction and it must be that there exist at least one of the above equilibria.

□

Proof of Lemma 2

Consider a continuous cumulative distribution function (cdf) $F(s^* \leq k|s,\sigma)$ as Figure 6 displays in black. For a given $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, we observe that a rise of s makes it is less likely that $F(s^* \leq k)$ because it pushes the cdf to the right of the graph - see Figure 6a. On the other hand, for a given $s \in [\underline{s}, \overline{s}]$, we observe that a rise of σ makes it is less likely that $F(s^* \leq k)$ if $s \leq k$ but it makes



(a) Given σ, $F(k|\overline{s},\sigma)$ minimizes player i's belief of reaching the threshold

(b) Given s, $F(k|s,\overline{\sigma})$ minimizes player i's belief of reaching the threshold i.e. $s \leq k$

Figure 6: Minimizing player i's belief

it more likely that $F(s^* \leq k)$ if $s > k$. The rise of σ distorts the cdf around s - see Figure 6b. It follows that if Nature wishes to minimize $F(s^* \leq k|s,\sigma)$, it would set $s = \overline{s}$, and $\sigma = \overline{\sigma}$ when $s > k$, while $\sigma = \underline{\sigma}$ when $s \leq k$.

Consider now player i's expected utility given a cumulative density function $F(.|s,\sigma)$. Formally, we remind that we have $\mathcal{U}_i^{F(.|s,\sigma)} = vF(s^* \leq x|s,\sigma) - c$ if $a_i = 1$ and $\mathcal{U}_i^{F(.|s,\sigma)} = vF(s^* \leq x-1|s,\sigma)$ otherwise. The expected utility is strictly increasing in $F(.)$. Given our reasoning above, Nature thus minimizes the utility at $s = \overline{s}$, and $\sigma = \overline{\sigma}$ when $s > k$, while $\sigma = \underline{\sigma}$ when $s \leq k$.

This proves Lemma 2.

$\square$

Proof Equation (6)

We find that player i contributes whenever $f_U(x|\overline{s},\sigma) \geq \frac{c}{v}$ where

$$f_U(x|\overline{s},\sigma) = \begin{cases} F(s^* \leq x|\overline{s},\underline{\sigma}) - F(s^* \leq x-1|\overline{s},\underline{\sigma}) & \text{when } x-1 < x < s \\ F(s^* \leq x|\overline{s},\overline{\sigma}) - F(s^* \leq x-1|\overline{s},\overline{\sigma}) & \text{when } s \leq x-1 < x \\ F(s^* \leq x|\overline{s},\overline{\sigma}) - F(s^* \leq x-1|\overline{s},\underline{\sigma}) & \text{when } x-1 < s = x \end{cases}$$

But note that as F is symmetric at exactly $x = \overline{s}$, it implies that $F(s^* \leq x|\overline{s}, \underline{\sigma}) = F(s^* \leq x|\overline{s}, \overline{\sigma})$. Therefore,

$$f_U(x|\overline{s}, \sigma) = \begin{cases} F(s^* \leq x|\overline{s}, \underline{\sigma}) - F(s^* \leq x-1|\overline{s}, \underline{\sigma}) & \text{when } x-1 < x < s \\ F(s^* \leq x|\overline{s}, \overline{\sigma}) - F(s^* \leq x-1|\overline{s}, \overline{\sigma}) & \text{when } s \leq x-1 < x \\ F(s^* \leq x|\overline{s}, \underline{\sigma}) - F(s^* \leq x-1|\overline{s}, \underline{\sigma}) & \text{when } x-1 < s = x \end{cases}$$

leading to

$$f_U(x|\overline{s}, \sigma) = \begin{cases} f_U(x|\overline{s}, \underline{\sigma}) & \text{when } x-1 < x \leq s \\ f_U(x|\overline{s}, \overline{\sigma}) & \text{when } s \leq x-1 < x \end{cases}$$

□

Proof Equation (7): NE under model uncertainty

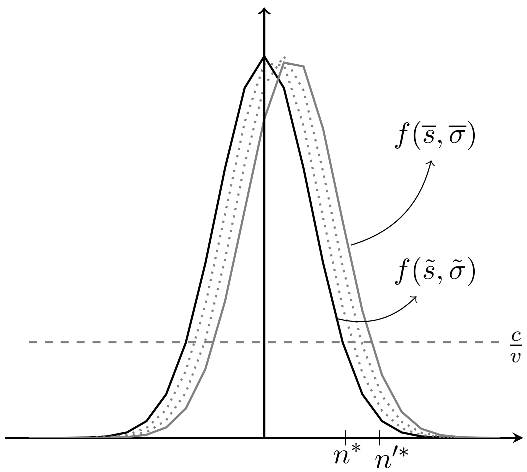
The proof uses the same reasoning as the proof Equation (3).

□

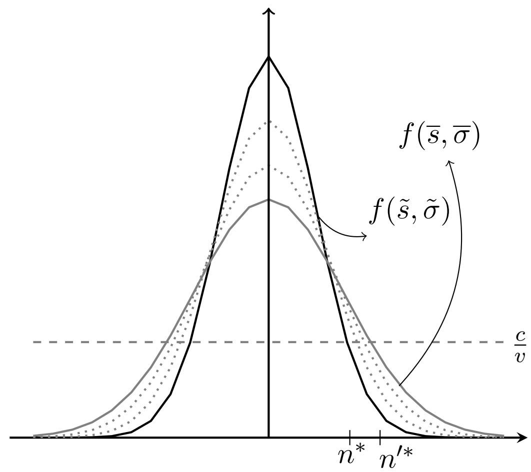
Proof of Proposition 1

We want to compare the non-unanimous equilibria C^* obtained with complete awareness to C^*_U obtained with complete ignorance, provided they both exist. Note if they both exist, they are situated on the right tail of the aware and ignorant density functions (that is where the mode of the density function is always on the left of existing equilibrium number of players $s < C^*$ and $s < C^*_U$). We thus focus on the right-tail of the density functions.

According to Lemma 2 pessimistic players take decision according to $f(.|\overline{s}, \overline{\sigma})$ when $s < k$ (which is the case with non-unanimous equilibria), while aware players take decision taking $f(.|s, \sigma)$. Consider k such that $k = f(x|\overline{s}, \overline{\sigma}) = f(x|s, \sigma)$, we observe that $f(.|\overline{s}, \overline{\sigma})$ has a fatter interior-right tail $I_R = \{x, ..., n\}$ than $f(.|s, \sigma)$ (see Figure 7). Pick any $\frac{c}{v} \leq k$, then because $f(.|s, \sigma)$ is strictly unimodal (by assumption) and downward slopping over I_R the highest contribution level is $y \in I_R$ such that $f(y|s, \sigma) \geq \frac{c}{v}$ and $f(y+1|\overline{s}, \overline{\sigma}) < \frac{c}{v}$ which implies $C^* = y$ by Equation (3). Since $f(.|\overline{s}, \overline{\sigma})$ has a fatter interior-right tail than $f(.|s, \sigma)$ for each contribution level in I_R by definition of the interior-right tail we find that $f(C_U^*|\overline{s}, \overline{\sigma}) \geq f(C^*|s, \sigma)$. Using Equation (7) we have $C_U^* \geq C^*$. Pessimistic players contribute more than aware players when $\frac{c}{v}$ is lower than k where k is such that $k = f(x|\overline{s}, \overline{\sigma}) = f(x|s, \sigma)$.



(a) $\tilde{s} < \overline{s}$ and $\tilde{\sigma} = \overline{\sigma}$



(b) $\tilde{s} = \overline{s}$ and $\tilde{\sigma} < \overline{\sigma}$

Figure 7: Impact of ambiguity with pessimists