Threshold public good games with model uncertainty

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Abstract

Various collective action problems can be described as discrete public good games with a threshold. In such games, a public good is provided when total contributions exceed a contribution threshold. Yet, the latter is often not known with certainty because multiple models predict its true value. We derive equilibria when players are ambiguity averse about which model to pick. We show that the players' ambiguity aversion affects the number of contributors. This contrasts with what Kishishita and Ozaki (2020) shows when ambiguity arises in place of risk over a unique model. Our paper also claims that a vague model established with the aim of discouraging contributions may actually increase contributions, provided the public good has great value.

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1 Introduction

Discrete public good games are simplified representations of many collective action scenarios. For examples, neighbors may decide to petition to build a public project; fishers may restrict fishing to enable fish reproduction; and countries may engage to reduce their carbon emissions. These games posit that a public good is provided if the number of contributors equals or exceeds a threshold. Interestingly, this threshold is often uncertain. One reason is the existence of a diversity of models predicting the true threshold. In our previous examples, the neighbors may not know the public official who will notice their petition and its subjective threshold number of petitioners; fishing organizations may propose various fish reproduction models[1]; and scientists may propose various climate change models (see Figure 1 below).

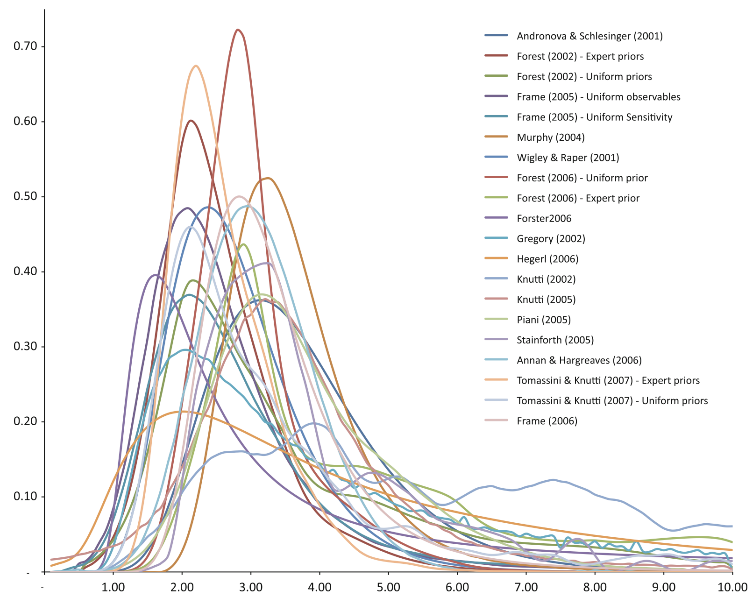


Figure 1: Estimated probability density functions for climate sensitivity from a variety of published studies (copy-past of Fig. 1 in Millner *et al.* (2013))

The uncertainty triggered by the presence of multiple models is known as *model uncertainty* (Berger et al. ...). Model uncertainty arises when risk arises as a first layer of uncertainty (predictions are known probability distributions about the thresholds) while ambiguity appears as a

[1]Refer to the following site by the Food and Agricultural Organization to know more about the diversity of fishing restrictions in the world `http://www.fao.org/3/AC865E/AC865E05.htm`.

second layer (players are unable to discriminate between the distributions). Numerous experimental evidence show that players exhibit ambiguity aversion in the lab, i.e. players seem to dislike uncertainty (Eichberger Kelsey, ...). When players dislike uncertainty, they consider the worst possible outcomes when taking a decision. Yet, it is difficult to guess which model the players will consider and thus how they decide whether to contribute.

The present study precisely investigates this issue. We use the framework by Çelen and Özgür (2018) to frame *model uncertainty* and derive the model used by the players when they take their decisions. We then show that ambiguity averse players tend to contribute more when the cost-benefit ratio of the public good is sufficiently low. At the opposite, they contribute less when the cost-benefit ratio of the public good is sufficiently high. So we recover the result by McBride (2006).

Recent papers explore the effect of threshold uncertainty on the players' contributions both theoretically (McBride, 2006; Kishishita and Ozaki, 2020) or experimentally (Dannenberg *et al.*, 2015; McBride, 2010). The closest papers to ours are the theoretical works by McBride (2006) and by Kishishita and Ozaki (2020). The two studies assume a unique threshold distribution. The former shows that higher risk (mean-preserving spread) increases contributions when the cost-benefit ratio of the public good is sufficiently low. The latter shows that higher ambiguity (less confidence in the threshold distribution) decreases contributions, irrespective of the players' attitude towards uncertainty. Our analysis shows that players' attitude does affect contributions when ambiguity arises as a second layer of uncertainty. Specifically, pessimists (players who dislike ambiguity) tend to contribute more whenever the cost-benefit ratio of the public good is sufficiently low.

The remainder of the paper is as follows. Section 2 introduces the model. Section 3 and Section 4 run the analyses respectively with and without ambiguity. Finally, Section 5 concludes. All proof are in the appendix.

2 The model

Players and actions. Following McBride (2006), we consider a discrete public good game with n players where $2 < n < \infty$. We denote the set of all players by $\mathcal{I} = \{1, 2, ..., n\}$. Player $i \in \mathcal{I}$ takes action $a_i \in \mathcal{A}_i = \{0,1\}$ where $a_i = 1$ means he contributes while $a_i = 0$ means he does not. The cost of contribution is $c \in (0, \infty)$, and the benefit of the public good provision is $v \in (0, \infty)$. The public good is successfully provided only when the sum of the contributions exceeds a threshold $s^* \in S = \{0, 1, ..., n+1\}$ i.e. formally $\sum_i a_i \geq s^*$. When $s^* = n+1$, even if all the players contribute the public good is not provided whereas when $s^* = 0$ players do not even have to contribute.

Multiple threshold theories. The contribution threshold s^* is chosen from a *theory* which is a normal cumulative distribution F with mean s and variance σ^2. We denote the associated probability density function by f. There is a set of conceivable theories about s^*, which is commonly known by the players. We denote such set by $\mathcal{F} := \{F(.|s,\sigma) : \underline{s} \leq s \leq \overline{s}\,,\ \underline{\sigma} \leq \sigma \leq \overline{\sigma}\}$ and call it a *media*.[2] Both s and σ have natural interpretations. The parameter s is the theory's predicted threshold while σ gives information about the theory's degree of precision. Therefore, the set $[\underline{s}, \overline{s}]$ captures the extent of different theoretical predictions while $[\underline{\sigma}, \overline{\sigma}]$ captures variety of theories' precisions. Given a theory that is represented by $F(.|s,\sigma)$, and the beliefs about the contributions of the other players $x - 1 = \sum_{j \neq i} a_j$, the expected utility of player $i \in \mathcal{I}$ is

$$\mathcal{U}_i^{F(|s,\sigma)}(a_i, x-1) = \begin{cases} vF(s^* \leq x|s,\sigma) - c & \text{if } a_i = 1 \\ vF(s^* \leq x-1|s,\sigma) & \text{otherwise} \end{cases} \tag{1}$$

The game. The present public good problem is a n-player normal form game $<\mathcal{I}, (\mathcal{U}_i^{\mathcal{F}})>$, where each player i has the strategy space $A_i = \{0,1\}$, set of prior beliefs $\mathcal{F}$, and an expected utility function $\mathcal{U}_i^{\mathcal{F}}$, which represents preferences of player i given its expectations about the theories in the media. We solve the game using the Nash equilibrium concept and focus on pure strategy equilibria.

[2] Figure 1 in appendix gives an empirical illustration of such a set of theories (though distributions are not all Gaussian, most of them are unimodal)

3 Complete awareness

Players' awareness. In this section, we suppose the players know that a certain theory from the media, say $F(.|\tilde{s},\tilde{\sigma})$ is the correct one.[3] The players could operate such selection provided they have the ability, knowledge or experience to discriminate between theories. Player $i \in \mathcal{I}$ thus uses this theory to take decisions and his expected utility given $\mathcal{F}$ simplifies to:

$$\mathcal{U}_i^{\mathcal{F}}(a_i, x-1) = \mathcal{U}_i^{F(.|\tilde{s},\tilde{\sigma})}(a_i, x-1) \tag{2}$$

Equilibria. Given all the relevant information about the other players' contributions and the threshold level, player i contributes when its expected utility from contributing is greater than the one when he does not: $\mathcal{U}_i^{\mathcal{F}}(1, x-1) \geq \mathcal{U}_i^{\mathcal{F}}(0, x-1) \Leftrightarrow vF(s^* \leq x|\tilde{s},\tilde{\sigma}) - c \geq vF(s^* \leq x-1|\tilde{s},\tilde{\sigma})$. Denote $f(x|\tilde{s},\tilde{\sigma}) = F(s^* \leq x|\tilde{s},\tilde{\sigma}) - F(s^* \leq x-1|\tilde{s},\tilde{\sigma})$ the probability of player i of being pivotal given other players' contributions $x-1$, then player i contributes if and only if this probability is greater than the cost-benefit ratio of contributing:

$$f(x|\tilde{s},\tilde{\sigma}) \geq \frac{c}{v} \tag{3}$$

Let us denote C^* the number of players contributing at equilibrium, then we retrieve the characterization of equilibria by McBride (2006) and Kishishita and Ozaki (2020), and the same applies to the existence property of at least one of these equilibria.

$$C^* = \begin{cases} 0 & \text{if } f(1|\tilde{s},\tilde{\sigma}) < \frac{c}{v} \\ x \in \{1, ..., n-1\} & \text{if } f(x|\tilde{s},\tilde{\sigma}) \geq \frac{c}{v} \text{ and } f(x+1|\tilde{s},\tilde{\sigma}) < \frac{c}{v} \\ n & \text{if } f(n|\tilde{s},\tilde{\sigma}) \geq \frac{c}{v} \end{cases} \tag{4}$$

[3] Note that one could also suppose that the players are rational and use Lebesgue's principle of insufficient reason to form their expectations over the theories. This principle asserts that in face of total ignorance of a probability distribution one must assume uniform probability distribution. This would create a compound lottery $F(.|\hat{s},\hat{\sigma})$. One condition for the results to follow the same pattern as under awareness is that such compound lottery is unimodal.

4 Complete ignorance

Players' ignorance. We now assume players are ambiguous about which theory to pick. The players could not operate such selection provided they lack the ability, knowledge or experience to discriminate between the theories displayed in the media. To decide whether to contribute, we suppose players only consider the best or worst possible outcomes depending on their attitude towards ambiguity. More formally, we suppose players hold a common attitude towards uncertainty that we denote $\alpha \in \{0, 1\}$ and have α-maxmin expected utility (Hurwicz, 1951). For simplicity, we focus on the extreme ambiguity attitudes such that $\alpha = 0$ means player i is pessimistic and thinks Nature minimizes its expected utility whereas $\alpha = 1$ means player is optimistic and thinks Nature maximizes its utility. The preferences of player $i \in \mathcal{I}$ with attitude $\alpha \in \{0, 1\}$ is thus represented by expected utility:

$$\mathcal{U}_i^{\mathcal{F}}(a_i, x-1) = \alpha \max_{F \in \mathcal{F}} \mathcal{U}_i^F(a_i, x-1) + (1-\alpha) \min_{F \in \mathcal{F}} \mathcal{U}_i^F(a_i, x-1) \tag{5}$$

Lemma 1. *Given belief about the contribution of the other players $x-1$, Nature's best response to player i's choice is as follows. (i) For any $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, the unique minimizer to (1) is $\arg\min_{F \in \mathcal{F}} U_i^F(x) = F(.|\overline{s}, \sigma)$ and the unique maximizer to (1) is $\arg\max_{F \in \mathcal{F}} U_i^F(x) = F(.|\underline{s}, \sigma)$. (ii) For any $s \in [\underline{s}, \overline{s}]$, the unique minimizer to (1) is $\arg\min_{F \in \mathcal{F}} U_i^F(x) = F(.|s, \overline{\sigma})$ when $s \leq x'$ and the unique maximizer to (1) is $\arg\max_{F \in \mathcal{F}} U_i^F(x) = F(.|s, \underline{\sigma})$ when $s \leq x'$. The reverse applies when $s > x'$.*

Intuitively, the expected utility of contributing or not, $\mathcal{U}_i^F(a_i, x-1)$, is increasing in the probability that the threshold is reached, $F(s^* \leq x'|s, \sigma)$ with $x' \in \{x-1, x\}$. Therefore, if Nature aims to minimize the utility, it boils down to minimize this probability. The reverse reasoning applies for maximization. Appendix details the reasoning for probability minimization and maximization. By taking nature's behavior into account, player i's expected utility simplifies to:

$$\mathcal{U}_i^{\mathcal{F}}(a_i, x-1) = \begin{cases} vF_\alpha(s^* \leq x|s, \sigma) - c & \text{if } a_i = 1 \\ vF_\alpha(s^* \leq x-1|s, \sigma) & \text{otherwise} \end{cases} \tag{6}$$

where $F_\alpha(s^* \leq x'|s,\sigma) = \alpha F(s^* \leq x'|\underline{s},\underline{\sigma}) + (1-\alpha)F(s^* \leq x'|\overline{s},\overline{\sigma})$ if $s \leq x'$, and $F_\alpha(s^* \leq x'|s,\sigma) = \alpha F(s^* \leq x'|\underline{s},\overline{\sigma}) + (1-\alpha)F(s^* \leq x'|\overline{s},\underline{\sigma})$, otherwise.

Equilibria. By the same reasoning as in the previous section, player i contributes whenever $f_\alpha(x|s,\sigma) \geq \frac{c}{v}$ where $f_\alpha(x|s,\sigma) = F_\alpha(s^* \leq x|s,\sigma) - F_\alpha(s^* \leq x-1|s,\sigma)$ denotes player i's probability of being pivotal under ambiguous expectations. Let us denote C^*_α the number of players contributing at equilibrium, we then retrieve our previous characterization of equilibria except that $f(x)$ now is $f_\alpha(x)$.[4]

$$C^*_\alpha = \begin{cases} 0 & \text{if } f_\alpha(1|s,\sigma) < \frac{c}{v} \\ x \in \{1,...,n-1\} & \text{if } f_\alpha(x|s,\sigma) \geq \frac{c}{v} \text{ and } f_\alpha(x+1|s,\sigma) < \frac{c}{v} \\ n & \text{if } f_\alpha(n|s,\sigma) \geq \frac{c}{v} \end{cases} \tag{7}$$

Effect of ambiguity. Let us now compare C^* to C^*_α with a focus on non-unanimous equilibria where $0 < C^* < n$ and $0 < C^*_\alpha < n$.

Proposition 1. *Suppose non-unanimous equilibria exist under both awareness and ignorance, then pessimistic (respestively optimistic) players weakly contributes more (respestively less) than aware players provided the cost-benefit ratio is sufficiently low.*

Proposition 1 contrasts with Kishishita and Ozaki (2020)'s findings that higher ambiguity, in the sense of less confidence in a unique threshold distribution, decreases contributions, irrespective of the players' attitude towards ambiguity. We provide evidence that when ambiguity appears as a second layer of uncertainty, i.e. when threshold distribution is not unique and players are unable to pick one distribution, then ambiguity attitude plays a major role in contribution decision.

Intuitively, pessimism leads the players to think, on the one hand, that the correct threshold is the highest one which incites more players to contribute. On the other hand, players tend to think that the prediction is highly uncertain which increases risk and incites even more players to contribute when the ratio cost-benefit is sufficiently low (this effect links to McBride (2006)). The reverse occurs with optimism.

[4] Note that there is a discontinuity around s, that does not affect our proposition.

Corollary 1. *Provided the cost-benefit ratio is sufficiently low, the presence of a vague theory incites pessimistic players to contribute more than aware players whereas a precise theory leads optimistic players to contribute less than aware players.*

Corollary 1 underlines the following situations, provided the cost-benefit ratio is sufficiently small. Imagine an agent creates a fake and vague theory into the media with a low predicted threshold $s' \leq \overline{s}$ in order to incite players not to contribute. If the theory has the lowest precision degree $\sigma' = \overline{\sigma} > \tilde{\sigma}$, then, counterintuitively, it has the effect of inciting pessimistic players to contribute. In the same vein, let us now suppose an agent engages research to provide a very precise theory about a high predicted threshold $s' > \underline{s}$ to encourage players to contribute. If the theory has the highest precision degree $\sigma' = \underline{\sigma} < \tilde{\sigma}$, then it discourages players to contribute if they are optimistic.

5 Conclusion

Various collective action problems can be described as discrete public good games with a contribution threshold. The latter is often not known with certainty because multiple theories about its true value appear. We show that the players' attitudes towards ambiguity affects contributions. This contrasts with what Kishishita and Ozaki (2020) shows when ambiguity arises in place of risk over a unique theory about the true threshold value.

References

Çelen, B. and Özgür, O. (2018). "Final-offer arbitration with uncertainty averse parties". *Games and Economic Behavior*, **109**:484–500.

Dannenberg, A., Löschel, A., Paolacci, G., Reif, C., and Tavoni, A. (2015). "On the provision of public goods with probabilistic and ambiguous thresholds". *Environmental and Resource economics*, **61**(3):365–383.

Hurwicz, L. (1951). "Optimality criteria for decision making under ignorance". *Cowles Commission Discussion Paper no. 370.*

Kishishita, D. and Ozaki, H. (2020). "Public goods game with ambiguous threshold". *Economics Letters*, **191**:109165.

McBride, M. (2006). "Discrete public goods under threshold uncertainty". *Journal of Public Economics*, **90**(6-7):1181–1199.

McBride, M. (2010). "Threshold uncertainty in discrete public good games: an experimental study". *Economics of governance*, **11**(1):77–99.

Millner, A., Dietz, S., and Heal, G. (2013). "Scientific ambiguity and climate policy". *Environmental and Resource Economics*, **55**(1):21–46.

Appendices

Appendix uses continuous distributions to simplify the graphical understanding, yet, the reader must remind that the analysis is discrete and only considers discrete transformations.

Proof Equation (4): NE with complete awareness

The Nash Equilibrium is such that $\mathcal{U}_i^{\mathcal{F}}(1, \sum_{j\neq i} a_j^*) \geq \mathcal{U}_i^{\mathcal{F}}(0, \sum_{j\neq i} a_j^*)$ for any player i who choose $a_i^* = 1$, while $\mathcal{U}_i^{\mathcal{F}}(0, \sum_{j\neq i} a_j^* - 1) \geq \mathcal{U}_i^{\mathcal{F}}(1, \sum_{j\neq i} a_j^* - 1)$ for any player i who choose $a_i^* = 0$.

• $C^* = 0$. Suppose there is no contributor, $C^* = 0$, then it must be that each player thinks the other players do not contribute $x - 1 = 0$ and each player i decides not to contribute. This happens as long as the player's utility from contributing is strictly lower than his utility from non contributing $vF(1|\tilde{s}, \tilde{\sigma}) - c < vF(0|\tilde{s}, \tilde{\sigma})$ which implies $f(1) < \frac{c}{v}$. No player with belief $x - 1 = 0$ wishes to deviate and contribute.

• $C^* = n$. Suppose there is no contributor, $C^* = n$, then it must be that each player thinks the other players contribute $x - 1 = n - 1$ and each player i decides to contribute. This happens as long as the player's utility from contributing is higher than his utility from non contributing $vF(n|\tilde{s}, \tilde{\sigma}) - c \geq vF(n-1|\tilde{s}, \tilde{\sigma})$ which implies $f(n) \geq \frac{c}{v}$. No player with belief $x - 1 = n - 1$ wishes to deviate and not to contribute.

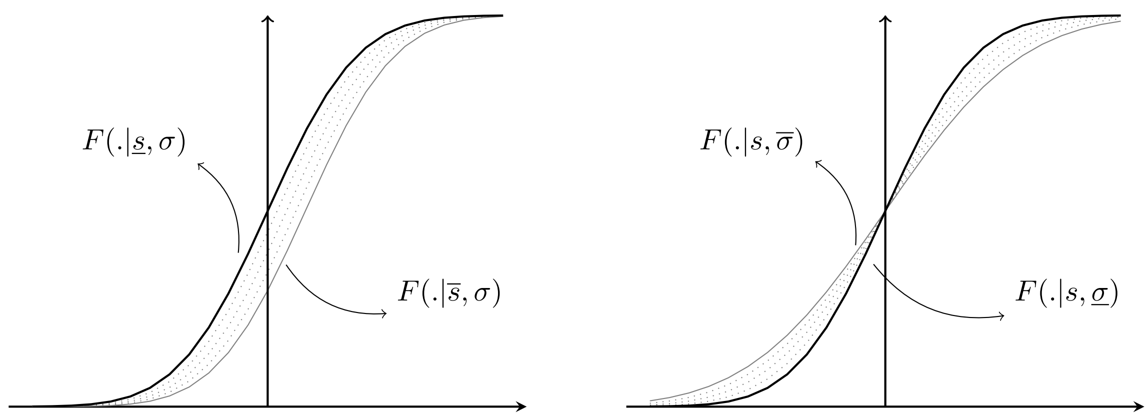
• $C^* = x \in \{1, ..., n-1\}$. Suppose $C^* = x \in \{1, ..., n-1\}$, then it must be that each contributing player thinks that there is $x - 1$ other contributors and given this beliefs that their utility from contributing is greater than that from non contributing $vF(x|\tilde{s}, \tilde{\sigma}) - c \geq vF(x-1|\tilde{s}, \tilde{\sigma})$ which implies $f(x) \geq \frac{c}{v}$. No contributing player with such beliefs wishes to deviate and not contribute. On the other hand, each not-contributing player must think there is x other contributors and that given this beliefs their utility from contributing is strictly lower than that of not contributing: $vF(x+1|\tilde{s}, \tilde{\sigma}) - c < vF(x|\tilde{s}, \tilde{\sigma})$ which implies $f(x+1) < \frac{c}{v}$. No not contributing player with such beliefs wishes to deviate and contribute.

• Existence. We assume that there does not exist equilibria and show that this leads to a contradiction. First suppose that $f(n) \geq \frac{c}{v}$. Then, we are done since by Equation 4 we have

$C^* = n$. Therefore, it must be that $f(n) < \frac{c}{v}$. Next, let $x \in \{1, 2, ..., n-1\}$ be the largest integer such that $f(x) \geq c/v$, if any. Then, both $f(x+1) < c/v$ and $f(x) \geq c/v$ hold, and hence, by Equation 4 we have $C^* = x$. Therefore, it must be that $f(1) < c/v$. But again, by Equation 4 we have $C^* = 0$. We find a contradiction and it must be that there exist at least one of the above equilibria.

□

Proof of Lemma 1



(a) Given σ, $F(.|\overline{s}, \sigma)$ minimizes player i's belief of reaching the threshold

(b) Given s, $F(.|s, \overline{\sigma})$ minimizes player i's belief of reaching the threshold i.e. $s \leq x'$

Figure 2: Minimizing player i's belief $F(s^* \leq x')$

Consider a continuous cumulative distribution function (cdf) $F(s^* \leq x|s, \sigma)$ as Figure 2 displays in black. For a given $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, we observe that a rise of s makes it is less likely that $F(s^* \leq x')$ because it pushes the cdf to the right of the graph - see Figure 2a. On the other hand, for a given $s \in [\underline{s}, \overline{s}]$, we observe that a rise of σ makes it is less likely that $F(s^* \leq x')$ if $s \leq x'$ but it makes it more likely that $F(s^* \leq x')$ if $s > x'$. The rise of σ distorts the cdf around s - see Figure 2b. It follows that if Nature wishes to minimize $F(s^* \leq x'|s, \sigma)$, it would set $s = \overline{s}$, and $\sigma = \overline{\sigma}$ when $s > x'$, while $\sigma = \underline{\sigma}$ when $s \leq x'$.

Consider now player i's expected utility given a cumulative density function $F(.|s, \sigma)$. Formally, we remind that we have $\mathcal{U}_i^{F(.|s,\sigma)} = vF(s^* \leq x|s, \sigma) - c$ if $a_i = 1$ and $\mathcal{U}_i^{F(.|s,\sigma)} = vF(s^* \leq x-1|s, \sigma)$

otherwise. The expected utility is strictly increasing in $F(.)$. Given our reasoning above, Nature thus minimizes the utility at $s = \overline{s}$, and $\sigma = \overline{\sigma}$ when $s > x'$, while $\sigma = \underline{\sigma}$ when $s \leq x'$.

The reverse reasoning applies for maximization of player i's expected utility $\mathcal{U}_i^{F(.|s,\sigma)}$. This proves Lemma 1.

$\square$

Proof Equation (7): NE under complete ignorance

The proof uses the same reasoning as the proof Equation (4), just replace $F(.|\tilde{s},\tilde{\sigma})$ by $F_\alpha(.|s,\sigma)$.

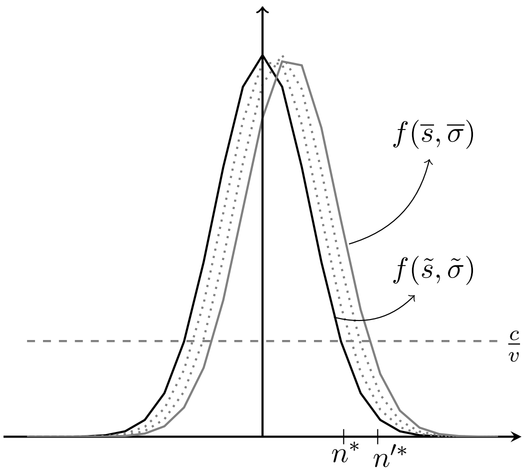
$\square$

Proof of Proposition 1

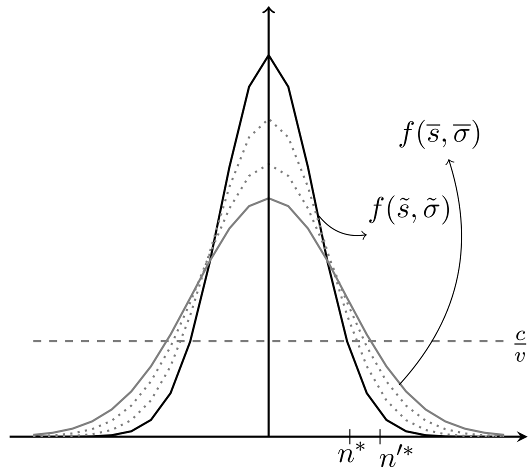
We want to compare the non-unanimous equilibria C^* obtained with complete awareness to C^*_α obtained with complete ignorance, provided they both exist. Note if they both exist, they are situated on the right tail of the aware and ignorant density functions (that is where the mode of the density function is always on the left of existing equilibrium number of players $s < C^*$ and $s < C^*_\alpha$). We thus focus on the right-tail of the density functions.

• Pessimistic players $\alpha = 0$. Suppose players are pessimistic, then following Lemma 1 pessimistic players take decision according to $f(.|\overline{s},\overline{\sigma})$ when $s < x'$ (which is the case with non-unanimous equilibria), while aware players take decision taking $f(.|\tilde{s},\tilde{\sigma})$. Consider k such that $k = f(x|\overline{s},\overline{\sigma}) = f(x|\tilde{s},\tilde{\sigma})$, we observe that $f(.|\overline{s},\overline{\sigma})$ has a fatter interior-right tail $I_R = \{x,...,n\}$ than $f(.|\tilde{s},\tilde{\sigma})$ (see Figure 3). Pick any $\frac{c}{v} \leq k$, then because $f(.|\tilde{s},\tilde{\sigma})$ is strictly unimodal (by assumption) and downward slopping over I_R the highest contribution level is $y \in I_R$ such that $f(y|\tilde{s},\tilde{\sigma}) \geq \frac{c}{v}$ and $f(y+1|\overline{s},\overline{\sigma}) < \frac{c}{v}$ which implies $C^* = y$ by Equation (4). Since $f(.|\overline{s},\overline{\sigma})$ has a fatter interior-right tail than $f(.|\tilde{s},\tilde{\sigma})$ for each contribution level in I_R by definition of the interior-right tail we find that $f(C^*_0|\overline{s},\overline{\sigma}) \geq f(C^*|\tilde{s},\tilde{\sigma})$. Using Equation (7) we have $C^*_0 \geq C^*$. Pessimistic players contribute more than aware players when $\frac{c}{v}$ is lower than k where k is such that $k = f(x|\overline{s},\overline{\sigma}) = f(x|\tilde{s},\tilde{\sigma})$.

• Optimistic players $\alpha = 1$. Suppose players are pessimistic, then following Lemma 1 optimistic players take decision according to $f(.|\underline{s},\underline{\sigma})$ when $s < x'$ (which is the case with non-unanimous equilibria), while aware players take decision taking $f(.|\tilde{s},\tilde{\sigma})$. The reverse reasoning with a thiner interior-right tail applies for optimistic players and we find $C_1^* \leq C^*$. $\square$



(a) $\tilde{s} < \overline{s}$ and $\tilde{\sigma} = \overline{\sigma}$



(b) $\tilde{s} = \overline{s}$ and $\tilde{\sigma} < \overline{\sigma}$

Figure 3: Impact of ambiguity with pessimists