Winners and Losers of Gatekeeper-Induced Consumer Preference Distortion in Promoting Personalized Pricing by Asymmetric Firms *

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Abstract

We present a model of duopoly competition in a marketplace with a Hotelling segment of consumers, where two business users (firms) have access to raw consumer data. The firms can choose between personalized prices (PP), using a costly personalized program device provided by the marketplace, or uniform prices at no additional cost. One firm has a higher level of experience in utilizing consumer data, resulting in a lower cost of price personalization (PP device cost). In order to promote its personalized program device, the marketplace may have an incentive to distort consumer preferences from a uniform to a triangular distribution. Our findings indicate that the marketplace is more likely to distort consumer preferences under specific conditions. This occurs when there is moderate asymmetry in experience between the firms and a high tariff for the program, or when there is weak asymmetry and a moderate program tariff. In these parameter regions, the distortion of consumer preferences negatively impact the profits of the sellers while benefiting the consumers. These insights contribute to a better understanding of the dynamics of digital marketplaces and have implications for policymakers and competition authorities.

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1 Introduction

The emergence of digital markets has presented policymakers, academics, and managers with unprecedented challenges, particularly concerning the access to vast amounts of customer data and the role of gatekeepers. Firstly, customer data provides valuable insights into consumer behavior and preferences, which can be leveraged to employ personalized prices, a form of price discrimination that involves charging different prices to consumers with different valuations. Secondly, gatekeepers are increasingly important in certain markets, exerting significant influence over both businesses and consumers. Gatekeepers refer to digital platforms that play a crucial role as intermediaries between business users and consumers, and their dominant position can give them significant power to act as private rule makers.

As the largest online marketplace in the world, Amazon's services, policies, and actions can have a profound impact on the success or failure of businesses that rely on its platform for sales. For example, Amazon Web Services (AWS) provides machine learning services, such as AWS Personalize, which uses historical data and customer behavior to help sellers create personalized strategies for their customers, including recommendations, prices, and advertising.[1] Moreover, Amazon's advertising strategies can potentially affect the distribution of consumer preferences for brands, for instance, from a uniform to a triangular distribution, and thereby influence the pricing strategies of third-party sellers. This could have significant implications for sellers' profits and consumer surplus.

The practice of personalized pricing has garnered attention from various reports aiming to explore its implications for consumers (EC, 2018; OECD, 2018). Policymakers, such as Ofcom in the UK (e.g. Ofcom (2020)), have expressed concerns regarding the increasing prevalence of personalized pricing in digital markets. In the United States, the Federal Trade Commission (FTC) has taken

[1]Amazon operates through its subsidiary AWS, which is considered as the overall market leader in cloud. Amazon categorises its overall operations into three segments: AWS, North America and International. While AWS is Amazon's smallest operating segment by revenue, representing about 16 percent of Amazon's revenue in 2022, it was Amazon's only profitable segment in the same year.

steps to revitalize the Robinson-Patman Act, an older legislation focused on price discrimination.[2] In light of these developments, it becomes crucial to examine the implications of using consumer data for personalized pricing and identify the potential winners and losers in this scenario.

The economics literature has shown that when consumer preferences are uniformly distributed, and firms are symmetric, price personalization becomes a dominant strategy for all firms (Thisse and Vives, 1988; Matsumura and Matsushima, 2015). Price personalization intensifies competition, leading to reduced profits for firms but benefiting consumers (Thisse and Vives, 1988). A similar outcome can be observed under behavior-based price discrimination models, where different prices are charged to consumers based on their purchase history (e.g., distinguishing between old and new customers, Fudenberg and Tirole (2000)).

However, when firms are symmetric, and consumer preferences follow a triangular distribution, Esteves *et al.* (2022) demonstrate that behavior-based price discrimination can lead to increased profits for firms but at the expense of consumer welfare. It is important to note that while the assumption of uniform consumer preferences is convenient for analytical purposes, it may not hold true in many real-world markets. In practice, markets often consist of a mix of consumers with varying preferences, including some with average preferences and others with more extreme preferences.

To our knowledge, the literature on PP has not explored the implications of this business practice in markets where consumer preferences follow a triangular distribution. Therefore, this paper aims to assess the profit and welfare effects of PP in markets where the density of consumer preferences is better represented by a triangular distribution. Some relevant questions are: What happens to the sellers' price decisions and profits when consumer tastes change from a uniform to a triangular distribution?

Managers face the important task of determining whether adopting personalized pricing (PP) as a strategy will enhance profitability, especially when relying on tools provided by gatekeepers. The case of Amazon exemplifies this scenario. Amazon offers the "AWS Personalize" tool for personalized pricing to its business users, which encompasses various components, including a training component that may not be necessary for experienced data analytics users. As a result, there may

[2] The FTC recently initiated a preliminary investigation into potential price discrimination by PepsiCo and Coca-Cola Co., alleging a violation of the Robinson-Patman Act. This investigation signals the FTC's renewed focus on enforcing the RPA, which has seen limited action in recent decades. Since 2021, the FTC has been emphasizing the revival of RPA enforcement as a top priority.

be an asymmetry in the efficiency of employing PP among different sellers based on their experience levels. Additionally, Amazon utilizes its "A10 algorithm," a secret ranking algorithm, to provide recommendations to consumers and list products from business users on its website. These recommendations and rankings have the potential to influence and affect (distort) consumer preferences, transitioning them, from instance, from a uniform distribution to a triangular one, similar to the effects of persuasive advertising (Bloch and Manceau, 1999).

The potential distortion of consumer preferences by gatekeepers like Amazon raises crucial questions: Under what market conditions would Amazon be incentivized to manipulate consumer preferences? How does the sellers' asymmetric data experience impact their price decisions and profits, and Amazon's motivation to do so? What are the potential risks for consumers and sellers' profits? Answering these questions is vital for competition agencies to identify instances where Amazon's strategies could potentially harm consumers or business users. By understanding the dynamics and risks involved, appropriate enforcement measures can be implemented to safeguard consumer welfare and ensure fair competition in the marketplace.

In the European Union, the legislative framework for digital markets has experienced significant expansion and refinement, with ongoing efforts to further fine-tune it. This can be observed in the legislative package introduced as part of the European Data Strategy, particularly the proposed Digital Markets Act (DMA), which entered into force in May, 2023. The proposed DMA seeks to establish a set of rules and obligations for large online platforms considered as gatekeepers, such as Amazon. The DMA is aimed at addressing unfair practices by companies acting as gatekeepers in the online platform economy. It prohibits gatekeepers from engaging in certain behaviors that could harm consumers (e.g. treating services and products offered by the gatekeeper itself more favourably in ranking than similar services or products offered by third parties on the gatekeeper's platform). While the DMA introduces important measures to promote fair competition in the digital marketplace (e.g. data sharing) there are still some limitations that should be noted. Firstly, the mandatory data sharing provisions (Delbono *et al.*, 2022) may not fully address data asymmetry issues among business users, as not all users may have the same ability to process data (Belleflamme *et al.*, 2020). This could potentially limit the benefits of data for smaller businesses, creating a competitive disadvantage for them. Secondly, while the DMA prohibits gatekeepers from unfairly favoring their own products and services in rankings, there are other avenues that gatekeepers can take that can affect competition and consumers. In a statement published on April 5, 2023, Ofcom, the Britain's media and communications regulator said it has "significant concerns" that Amazon and Microsoft could be harming competition in the market for cloud services.[3]

We investigate these questions by employing a Hotelling framework to model duopoly competition with personalized pricing. Initially, consumers are uniformly distributed, but we introduce the assumption that the gatekeeper has the ability to distort consumer preferences. This is achieved by recommending a greater mix of brands, creating a state where consumers become more indifferent between the two firms and leading to a triangular distribution of consumer preferences (Esteves *et al.*, 2022; Tabuchi and Thisse, 1995).[4] In our model, the gatekeeper provides raw data to the firms, in accordance with the DMA regulations. This data allows the firms to have insights into the consumer distribution. However, to extract more detailed information from the raw data and offer personalized prices, the firms are required to purchase an optional device provided by the gatekeeper. Importantly, we consider that the cost of this device is lower for one of the firms (referred to as the experienced firm) compared to the other (referred to as the inexperienced firm). After obtaining the raw data and considering the cost of the device, the firms make simultaneous decisions regarding whether to quote personalized prices or uniform prices.

Our findings suggest that in a symmetric scenario, the marketplace cannot effectively incentivize firms to adopt personalized pricing when the device tariff is either sufficiently high or sufficiently low. When the tariff is high, firms never choose to use personalized pricing, while when it is low, they always opt for personalized pricing. The marketplace only distorts consumer preferences when the device tariff is intermediate. This occurs because the increased competition for the indifferent consumer, under a triangular distribution of preferences, prompts firms that were not using personalized pricing (under a uniform distribution) to adopt it (under a triangular distribution).

In contrast, our results demonstrate that when firms have asymmetric experience, the experienced firm is more likely to utilize personalized pricing when the cost of PP is higher. This, in turn, compels the inexperienced firm to adopt personalized pricing to remain competitive. As a result, in the asymmetric setting, the marketplace can distort consumer preferences for higher device tariffs

[3]For more details see https://www.ofcom.org.uk/news-centre/2023/ofcom-proposes-to-refer-uk-cloud-market-for-investigation.

[4]We do not study asymmetric distribution of consumers preferences (such as the ones proposed by Belleflamme and Peitz (2015, IO book) because it would clearly favor one business user at the detriment of the other and would clearly be seen as unfair by the DMA. The analysis would also become too complex.

compared to the symmetric case, due to increased competition. However, as the level of asymmetry becomes too pronounced, the experienced firm becomes so efficient that it becomes the sole user of PP. In such cases, the gatekeeper can only distort consumer preferences if it encourages the experienced firm to utilize PP for a larger fraction of consumers, which occurs when the degree of asymmetry is not too large.

Our study reveals that in certain parameter regions where the gatekeeper is incentivized to distort consumer preferences, it can have a detrimental effect on the profits of sellers operating on its platform. However, these actions ultimately lead to benefits for consumers. This finding should reassure competition authorities that despite the potential for preference distortion, the gatekeeper's actions can promote increased competition among firms and encourage the adoption of personalized pricing, resulting in even greater advantages for consumers. However, it is crucial for competing agencies to carefully assess the potential risks and implications of these practices for sellers, especially when the fixed costs of operating in the market are high. If the gatekeeper's actions disproportionately impact smaller or less experience sellers, it could result in market consolidation and reduced diversity of offerings. This concentration of power in the hands of a few dominant sellers could limit consumer choice and hinder competition, ultimately leading to higher prices and lower quality products or services. Competition authorities must assess these risks to maintain a fair marketplace that encourages competition, innovation, and benefits consumers.

The remainder of the paper is organized as follows. Section 2 explains how our results contribute to the existing literature. In Section 4, we analyze the equilibria under the uniform distribution of consumer preferences. Section 5 examines the results under the triangular distribution and compares them to those obtained under the uniform distribution. Section 6 investigates whether the marketplace decides to distort consumer preferences from uniform to triangular. The implications for sellers' profits and consumer welfare are discussed in Section 7. Finally, Section 8 provides the conclusion. All proofs not included in the main text can be found in the Appendix.

2 Related Literature

Endogenous pricing policy. The ability of firms to use consumer data to price discriminate is not a new topic in economics. The pioneering work on personalized pricing in imperfectly competitive markets is the one by Thisse and Vives (1988) which is based on the Hotelling model. In this literature, some papers investigate why asymmetry about the employment of personalized pricing occurs and have presented several plausible answers (Shaffer and Zhang, 2002; Choudhary *et al.*, 2005; Ghose and Huang, 2009; Matsumura and Matsushima, 2015). Only Shaffer and Zhang (2002) and Matsumura and Matsushima (2015) discuss price discrimination as a costly activity.

Shaffer and Zhang (2002) share with our paper the assumption that the cost of price personalization - under the form of individual promotions - takes the form of a per-consumer cost. Nevertheless, their study remains on symmetric price personalization costs and they do not explicitly discuss whether each firm commits to employing personalized pricing over its whole segment of consumers. Matsumura and Matsushima (2015) model a duopoly with asymmetric production costs and allow firms to choose uniform or personalized prices. Their model closely relates to Choudhary *et al.* (2005) but they additionally elaborate on what occurs when price personalization yields a fixed cost (e.g. payment of a device, or access to data set). In that sense, their framework is close to ours. However, we get rid of production costs while suppose personalization costs are asymmetric and per consumer.

Similar to the findings of Matsumura and Matsushima (2015), our study reveals that three equilibria can emerge regardless of consumer preferences: (i) both firms employ personalized prices, (ii) only the efficient firm employs personalized prices, and (iii) neither firm employs personalized prices. However, while Matsumura and Matsushima (2015) focus on the production cost difference and fixed cost of price personalization as determinants, we highlight the significance of the cost difference of price personalization and the degree of product differentiation. Interestingly, we discover that a minimum level of asymmetry is required to achieve the equilibrium where only the efficient firm adopts personalized prices. This implies that even with unrestricted access to data, the inefficient firm may refrain from employing personalized prices due to the asymmetry in price personalization efficiency.

Triangular consumer preferences. There is a limited number of studies that explore the effects of the distribution shape of consumer preferences in a duopoly competition setting. The seminal work by Tabuchi and Thisse (1995) focuses on a spatial competition model, while the recent study by Esteves *et al.* (2022) examines behavior-based price competition. To the best of our knowledge, our paper is the first to investigate the impact of non-uniform distribution of consumer preferences on duopoly firms' decisions to personalize prices in an endogenous setting. Our study fills an important gap in the literature by shedding light on this aspect of personalized pricing in a competitive market environment.

We contribute to this literature by examining the influence of a triangular distribution on firms' decisions to employ personalized pricing. We know from the literature that While symmetric firms are generally inclined to adopt personalized prices (Thisse and Vives, 1988), this relationship becomes less definitive when firms exhibit asymmetry (Matsumura and Matsushima, 2015). Our analysis shows that the triangular distribution reduces the minimum level of asymmetry required for the equilibrium where only the efficient firm personalizes prices. This implies that the asymmetric efficiency of price personalization is more likely to discourage the inefficient firm from employing personalized prices when consumers have greater indifference between the firms. In terms of welfare implications, we demonstrate that when the marketplace distorts consumer preferences towards a triangular distribution, particularly in regions where it has an incentive to do so, it negatively affects the profits of the firms while benefiting the consumers.

3 The Model

Two firms A and B sell competing differentiated brands to consumers through an online marketplace M. The marginal production cost is assumed equal to zero, with no loss of generality.[5] There is a mass of consumers normalized to one. Following the Hotelling framework, consumers are continuously distributed in the segment of length one. The firms are located at the opposite ends of this segment $[0, 1]$ with firm A (respectively B) located at 0 (1). Each consumer demands at most one unit of the product, either from A or B. Formally, a consumer located at $x \in [0, 1]$ receives instantaneous utility of $u_A(x) = v - p_A - tx$ if she buys from firm A at price p_A. If she buys from firm B, her utility will be $u_B(x) = v - p_B - t(1 - x)$. The parameter x therefore stands for a consumer relative brand preference such that consumers with $x < 1/2$ are more loyal to brand A whereas those with $x > 1/2$ are more loyal to brand B. We assume that v is sufficiently large so that all consumers buy in equilibrium (covered market).

The distribution of consumer preferences can follow two patterns: uniform or triangular. If there is no activity from the marketplace to influence consumer preferences, we assume they are uniformly distributed over the interval [0,1], i.e. $f(x) = 1$. This is the standard assumption in the literature. In contrast, if the marketplace attempts to distort the distribution of consumer preferences from

[5] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

uniform to triangular, there will be more consumers near the center and fewer consumers near the ends. This can be achieved, for instance, by displaying ads about firm j's product, $j \neq i$, to loyal-consumers of firm i (in the spirit of persuasive advertising (Bloch and Manceau, 1999)).[6] Formally, we will use the triangular distribution by (Esteves et al. 2022): $f(x) = 4x$ if $x \in \left[0, \frac{1}{2}\right]$, and $4(1-x)+$ if $x \in \left(\frac{1}{2}, 1\right]$. Note that we do not study intermediate distributions nor assume an advertising cost. This is mainly due to computational complexities at the optimal pricing policy choice. Nevertheless, bear in mind that the focus of the paper is to exhibit a marketplace's incentive to distort consumers' preferences, for instance through advertising, to increase its profits. In other words, like Bloch and Manceau (1999) we do not to look for the optimal advertising level.

Once the marketplace makes its decision, firms can access raw data on consumer preferences at no extra cost, giving them knowledge of the distribution. For example, Amazon Marketplace provides a tool called Amazon Marketplace Web Services that allows sellers to extract consumer characteristics and obtain descriptive statistics. [7] However, the descriptive statistics alone cannot be used to personalize prices. Firms need a refined device that can extract behavioral patterns from the raw data. In our setting, the marketplace offers such a refined device that reveals the locations of the consumers of interest, enabling price personalization. In real life, machine learning devices such as AWS Personalize, offered by Amazon Web Services, are powerful tools for price personalization.

It is assumed that firm A and firm B respectively incur costs $c_A > 0$ and $c_B > 0$ per consumer location revealed. For example, in the case of AWS Personalize, the cost of the device varies with the size of the dataset, as shown in the tariff table on the AWS Personalize website. This reinforces our assumption that the cost is per-consumer location, as the cost increases as the number of consumers to analyze increases. Nevertheless, one firm might have a previous knowledge of the device or just have more experience so that it is able to reduce its use of the device and therefore its cost. To account for this heterogeneity, we suppose $c_A \neq c_B$ and especially that $c_A = \gamma c_B$ where $\gamma \in [0, 1]$. This means that firm A can reduce the cost of the device thanks to its data experience at rate γ. To simplify notations, we will suppose that $c_A = \gamma c$ and $c_B = c$ with $c > 0$. In other words, c

[6]In contrast to Bloch and Manceau (1999) where advertising is widely made over the whole segment of consumers, we assume the marketplace targets specific segments of customers - which is possible because she has access to the customers history (e.g. Prime, Standard Registered users or via users' Cookies) and can know their relative brand loyalty.

[7]Note that, since 2019, Amazon also provides its sellers Amazon Brand Analytics (ABA), free-of-charge, to complement that

can be seen as the total price of the device while $c\gamma$ would stand for the price of using only part of the device. For example, the device could consists of three components: data ingestion, formation and segmentation such that an inexperienced firm would pay for the three components while an experienced firm would only pay for data ingestion and segmentation, thus avoiding to pay for the formation. The parameter γ has therefore a wide interpretation: it can be the length of experience of firm A, or the length in optional formation by the marketplace.

The cost of the firms turns out to be revenues for the marketplace. Note that we suppose the marketplace does not set the price. This is because we want to assess whether the marketplace has an incentive to use a non-market strategy to promote its device. However, we will also consider the potential trade-offs the marketplace would face if it were to modify the price. This will be discussed in more detail at the end of our paper.

As a result, the firms have two pricing options: either quote a uniform price (henceforth, *up*) or subscribe to the marketplace's program to quote personalized prices (henceforth, *pp*). When firm $i \in \{A, B\}$ uses M's program and employs *pp*, it formally quotes $p_i(x)$ to each consumer located at x. Otherwise firm $i \in \{A, B\}$ quotes uniform price p_i.

The game then runs as follows. At stage 1, the marketplace decides whether to use advertisement and curb consumers' preferences or not. At stage 2, the firms get raw data from the marketplace and observe the distribution consumers preferences. The firms then simultaneously determine their pricing policy: uniform vs. personalized prices, knowing that upon choosing personalized prices they will have to bear additional cost for the device. At stage 3, a firm that employs uniform pricing offers a uniform price that is observable. After that, a firm that employs personalized pricing offers personalized prices that depend on the locations of the consumers. If the two firms adopt the same pricing scheme, they simultaneously determine their prices. Consumers buy and pay-offs are realized. The solution concept is the subgame perfect Nash equilibrium (henceforth SPNE). We therefore solve the game using backward induction. The timing structure of our model follows that in the related papers (Thisse and Vives, 1988; Shaffer and Zhang, 2002; Liu and Serfes, 2004; Matsumura and Matsushima, 2015).

4 The benchmark pricing policy with uniform distribution

4.1 The four sub-game equilibria

Depending on firms' price decisions in the previous stage of the game there are four possible subgames: (up, up) where both firms quote a uniform price, (pp, pp) where both firms quote personalized prices, (up, pp) where the efficient firm quotes uniform price whereas the inefficient firm quotes personalized price, and (pp, up) where the efficient firm quotes a personalized price whereas the inefficient firm quotes uniform price.

• **Both firms quote a uniform price (up, up).** Here the setup is analogous to a standard symmetric Hotelling model. If firms cannot price discriminate in the symmetric equilibrium, they will set the non-discrimination price equals to the transportation cost: $p_{i,\mathcal{U}}^{up,up} = t$. This is because we assume no production cost and only a cost of engaging in personalized pricing. With non discrimination, equilibrium profit per firm is $\pi_{i,\mathcal{U}}^{up,up} = \frac{t}{2}$, and each firms serves half of the market, $\tilde{x}_{\mathcal{U}}^{up,up} = \frac{1}{2}$. The consumer surplus is $CS_{\mathcal{U}}^{up,up} = v - \frac{5}{4}t$, and total welfare is $W_{\mathcal{U}}^{up,up} = v - \frac{t}{4}$. The subscript $\mathcal{U}$ is used for Uniform distribution (henceforth $\mathcal{U}$).

• **Only the efficient firm discriminates (pp, up).** Suppose that firm A discriminates, while B does not. Given firm B's uniform price p_B the indifferent consumer between buying from A and B is located at $p_A(x) = p_B + t(1 - 2x)$. The lowest price firm A is willing to charge to a more distant consumer is equal to its personalization cost $c\gamma$. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x}_{\mathcal{U}}^{pp,up}$ such that $c\gamma = p_B + t(1 - 2\tilde{x}_{\mathcal{U}}^{pp,up})$ which leads to $\tilde{x}_{\mathcal{U}}^{pp,up} = \frac{1}{2t}\left(t + p_B - c\gamma\right)$.

With uniform distribution firm A demand is $\tilde{x}_{\mathcal{U}}^{pp,up}$ and firm B is $1 - \tilde{x}_{\mathcal{U}}^{pp,up}$. As firm B quotes a uniform price it incurs no personalization cost. Its profit is $\pi_{B,\mathcal{U}}^{pp,up} = p_B\left(1 - \frac{1}{2t}\left(t - c\gamma + p_B\right)\right)$. From the FOC for the profit maximization with respect to p_B we obtain that firm B quotes $p_{B,\mathcal{U}}^{pp,up} = \frac{t+c\gamma}{2}$. This then gives $\tilde{x}_{\mathcal{U}}^{pp,up} = \frac{1}{4t}\left(3t - c\gamma\right)$. As $t > c$, we have $t > c\gamma$ and thus firm A serves more than half of the consumers ($\frac{1}{4t}\left(3t - c\gamma\right) = \frac{1}{2} + \frac{t-c\gamma}{4t} > \frac{1}{2}$). Also, $\frac{3t-c\gamma}{4t}$ is always inferior to 1 (because $t > 0 > -c\gamma$) so we always have an interior solution. Firm B serves all consumers in the interval $\left[\frac{1}{4t}\left(3t - c\gamma\right), 1\right]$ while firm A serves all consumers in the remaining interval, i.e., those consumers who belong to the interval $\left[0, \frac{1}{4t}\left(3t - c\gamma\right)\right]$. Substituting p_B in $p_A(x)$ we find that $p_{A,\mathcal{U}}^{pp,up}(x) = \frac{t(3-4x)+c\gamma}{2}$

if $x \leq \frac{1}{4t}(3t - c\gamma)$, and $c\gamma$ otherwise.

With the equilibrium prices, we get firm B and A's profits which write respectively $\pi_{B,\mathcal{U}}^{pp,up} = \frac{1}{8t}(t + c\gamma)^2$ and $\pi_{A,\mathcal{U}}^{pp,up} = \frac{1}{16t}(3t - c\gamma)^2$. In addition, we have $CS_{\mathcal{U}}^{pp,up} = v - \frac{c\gamma}{2} - t$ and $W_{\mathcal{U}}^{pp,up} = v + \frac{3c^2\gamma^2}{16t} - \frac{5c\gamma}{8} - \frac{5t}{16}$.

- **Only the inefficient firm discriminates (up, pp):** The case where B discriminates while A does not is the symmetric of the above case except that $\gamma = 1$. Therefore, firm A's profit is $\pi_{A,\mathcal{U}}^{up,pp} = \frac{1}{8t}(t + c)^2$ while firm B's profit is $\pi_{B,\mathcal{U}}^{up,pp} = \frac{1}{16t}(3t - c)^2$. Also, we have $CS_{\mathcal{U}}^{up,pp} = v - \frac{c}{2} - t$ and $W_{\mathcal{U}}^{up,pp} = \frac{1}{16}\left(\frac{3c^2}{t} - 10c - 5t\right) + v$

Lemma 1 *Under uniform distribution, firm A and B respectively hold the following best responses to uniform pricing denoted $BR_{A,\mathcal{U}}(up)$ and $BR_{B,\mathcal{U}}(up)$:*

$$BR_{A,\mathcal{U}}(up) = \begin{cases} pp & if\, t/c \geq (3 + 2\sqrt{2})\gamma \\ up & otherwise \end{cases} \qquad \& \quad BR_{B,\mathcal{U}}(up) = \begin{cases} pp & if\, t/c \geq (3 + 2\sqrt{2}) \\ up & otherwise \end{cases} \tag{1}$$

It is common knowledge that costless price discrimination unilaterally enables a firm to earn greater revenues than uniform price (Thisse and Vives, 1988; Matsumura and Matsushima, 2015). This essentially happens because personalized prices enables firms to offer lower prices to less loyal consumers and thus provides a competitive advantage over a rival which sets a uniform price. However, a rise of the personalization cost negatively affects these gains and at some level reverses them in losses. This second effect is the meaning of the inequalities. In addition, an asymmetry arises between the efficient firm A and the inefficient firm B. Firm A's threshold turning point becomes lower than that of firm B as firm A becomes more efficient than firm B. Note that firm B's threshold does not depend on γ as firm A does not use pp in this situation.

- **Both firms offer personalized prices (pp, pp):** When both firms analyze data to employ PP, at the cost c_i, $i = A, B$, the best price the more distant firm may set in equilibrium is the marginal cost of personalization. Then, the closest firm needs to provide that consumer the same utility level in order to make a sale. Consider a consumer located near A with $x < \frac{1}{2}$. Given the price B offers to a consumer located at x, $p_B(x)$, in order to make a sale firm A should offer a price that gives this consumer just as good a deal defined by $p_A(x) + tx = p_B(x) + t(1 - x)$.

Taking into account that the best price firm B offers to a consumer located near the rival is its personalization cost c, we have that $p_A(x) + tx = c + t(1-x)$, which yields $p_A(x) = c + t(1-2x)$. Additionally we need to impose that $p_A(x) \geq c\gamma$, from which we get $p_A(x) = c + t(1-2x)$ as long as $x \leq \frac{1}{2} + \frac{(1-\gamma)c}{2t} \equiv \tilde{x}^{pp,pp}$. Since $1-\gamma > 0$, firm A serves more than half of the market. Also as $\frac{t}{c} \geq 2 > 1-\gamma$, firm A serves less than the whole market, and firm B thus serves a positive segment of the market (interior solution). Before proceeding, we may establish the following result. Proposition 1 establishes that regardless the distribution of consumer preferences, firms set the same price to each consumer.

Proposition 1 *(Personalized Prices) When the two firms quote personalized prices, then firm A and B price schedule is given by*

$$p_A^{pp,pp}(x) = \begin{cases} c + t(1-2x) & if \quad x \leq \tilde{x}^{pp,pp} \\ c\gamma & if \quad x > \tilde{x}^{pp,pp} \end{cases},$$

$$p_B^{pp,pp}(x) = \begin{cases} c\gamma + t\,(2x-1) & if \quad x \geq \tilde{x}^{pp,pp} \\ c & if \quad x < \tilde{x}^{pp,pp} \end{cases}.$$

irrespective of consumer distribution.

The firms' profits then are $\pi_{A,\mathcal{U}}^{pp,pp} = \frac{1}{4t}\left(t - c\gamma + c\right)^2$ and $\pi_{B,\mathcal{U}}^{pp,pp} = \frac{1}{4t}\left(t + c\gamma - c\right)^2$. In addition, we have $CS_{\mathcal{U}}^{pp,pp} = \frac{1}{4t}[t(4v-3t) - c^2(1-\gamma)^2 - 2c(\gamma+1)t]$ and $W_{\mathcal{U}}^{pp,pp} = \frac{1}{4t}[c^2(\gamma-1)^2 - 2c(\gamma+1)t - t(t-4v)]$.

Lemma 2 *Under uniform distribution, firm A and B respectively hold the following best responses to personalized pricing denoted $BR_{A,\mathcal{U}}(pp)$ and $BR_{B,\mathcal{U}}(pp)$:*

$$BR_{A,\mathcal{U}}(pp) = \begin{cases} pp & if\, t/c \geq (2+\sqrt{2})\gamma - 1 \\ up & otherwise \end{cases} \quad \& \quad BR_{B,\mathcal{U}}(pp) = \begin{cases} pp & if\, t/c \geq 2+\sqrt{2} - \gamma \\ up & otherwise \end{cases} \tag{2}$$

In contrast to above, firm B's threshold now depends on γ because firm A - its rival - now quotes *pp* in the studied situation. Moreover, note that the threshold for firm A is now increasing in γ whereas the one of firm B is decreasing in γ. This makes sense as the greater efficiency of firm A provides it a greater competitive advantage when the two firms employ *pp*. Therefore, *pp* is again a best response as long as its associated cost does not exceed a threshold but greater efficiency from firm A will relax its threshold while makes the one by firm B more stringent.

4.2 The partition of equilibria

Figure 1 summarizes our findings building on Lemma 1 and 2 and also plots the equilibrium regions. Proposition 2 summarizes the results in terms of possible equilibria.

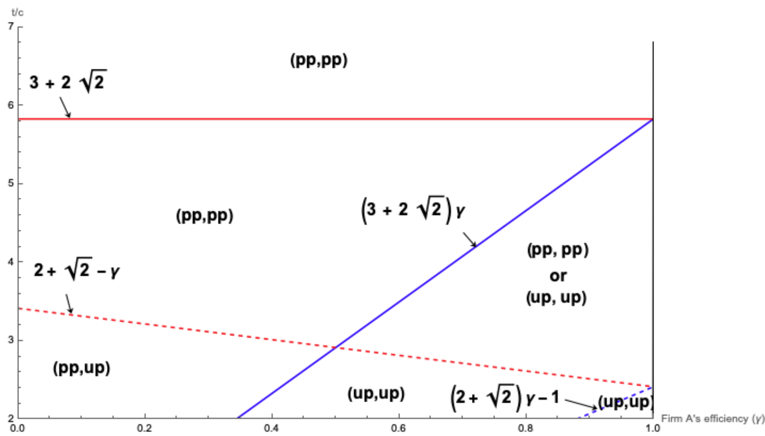


Figure 1: Partition of equilibria (uniform distribution)

Proposition 2 *Under uniform distribution of consumer preferences, three Nash Equilibria can appear: (up,up) when* $\frac{t}{c} \leq (3+2\sqrt{2})\gamma$*, (pp,up) when* $(3+2\sqrt{2})\gamma < \frac{t}{c} < 2+\sqrt{2}-\gamma$*, (pp,pp) when* $\frac{t}{c} \geq 2+\sqrt{2}-\gamma$*.*

When asymmetry is low (γ low) and the personalization cost (c) is high, the two firms are likely to quote uniform prices. In contrast, when the asymmetry is large and the personalization cost is high, then efficient firm A will prefer to quote personalized prices while the inefficient firm will remain on uniform prices. When the personalization cost is intermediate or low, then the two firms will personalize prices.

Note that Matsumura and Matsushima (2015) also shows the presence of a region with two SPNE when firms hold asymmetric production costs and a symmetric fixed cost for *pp*. Our result thus extends Matsumura and Matsushima (2015) to a context where firms have symmetric production cost but asymmetric per-consumer personalization costs. Furthermore, next section will depart from the assumption of uniform distribution of consumers preferences.

5 Pricing policy with triangular distribution

We now assume that the distribution of consumer preferences is triangular. Again, depending on firms' price decisions, in the beginning of the game there are four possible sub-games.

5.1 The four sub-game equilibria

• **Both firms quote a uniform price (up, up).** Given the uniform prices p_A and p_B, the marginal consumer who is indifferent between buying from the two firms is determined by $v - t\tilde{x}_{\mathcal{T}}^{up,up} - p_A = v - p_B - t(1 - \tilde{x}_{\mathcal{T}}^{up,up})$, which yields, $\tilde{x}_{\mathcal{T}}^{up,up} = \frac{1}{2} + \frac{p_B - p_A}{2t}$. Firm A and B's profits are now respectively $\Pi_{A,\mathcal{T}}^{up,up} = p_A F(\tilde{x}_{\mathcal{T}}^{up,up})$ and $\Pi_{B,\mathcal{T}}^{up,up} = p_B \left[1 - F(\tilde{x}_{\mathcal{T}}^{up,up})\right]$, where $F(x) = 2x^2$ if $x \leq 1/2$ and $F(x) = 4x - 2x^2 - 1$ otherwise. Suppose $\tilde{x}_{\mathcal{T}}^{up,up} \leq 1/2$, then $F(x) = 2x^2$ and under uniform pricing each firm i quotes price $p_{i,\mathcal{T}}^{up,up} = \frac{t}{2}$. This gives $\tilde{x}_{\mathcal{T}}^{up,up} = \frac{1}{2}$ which is indeed lower or equal than one half, and each firm's overall profit is $\pi_{i,\mathcal{T}}^{up,up} = \frac{t}{4}$. In addition, we have $CS_{\mathcal{T}}^{up,up} = v - \frac{5t}{6}$ and $W_{\mathcal{T}}^{up,up} = v - \frac{t}{3}$. The subscript $\mathcal{T}$ is used for Triangular distribution (henceforth $\mathcal{T}$).

• **Only the efficient firm discriminates (pp, up):** Suppose that firm A discriminates, while B does not. The method is the same as with uniform distribution. Given firm B's uniform price p_B the indifferent consumer between buying from A and B is located at $p_A(x) = p_B + t(1 - 2x)$. The lowest price firm A is willing to charge to a more distant consumer is equal to its personalization cost $c\gamma$. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x}_{\mathcal{T}}^{pp,up} = \frac{1}{2t}(t + p_B - c\gamma)$. Note that $\tilde{x}_{\mathcal{T}}^{pp,up} = \frac{1}{2} + \frac{p_B - c\gamma}{2t} > \frac{1}{2}$ as long as $p_B > c\gamma$. Otherwise if $p_B < c\gamma$ then $\tilde{x} < \frac{1}{2}$. Let's remind remind that firm B only bears the constraint that its price p_B is positive (it does not personalize its price), and therefore $p_B < c\gamma$ is feasible.

Compared with the framework where consumers are uniformly distributed, the location of the indifferent consumer under triangular distribution is now of great importance as it affects the computations of firms' demand functions. Two cases appear: (i) $p_B > c\gamma$ which implies $\tilde{x} > 1/2$, or (ii) $c\gamma > p_B > 0$ which implies $\tilde{x} < 1/2$. The appendix shows that the assumption $t/c > 2$ impedes the candidate equilibrium price p_B to be lower than $c\gamma$, and that only $p_B > c\gamma$ holds in equilibrium.[8]

[8]The appendix also provides an analysis of what happens when we relax this assumption.

In what follows, we thus assume that $p_B > c\gamma$, which then leads to $\tilde{x} > 1/2$. With triangular distribution firm B's demand is

$$q_B = \int_{\frac{1}{2t}(t+p_B-c\gamma)}^{1} 4(1-x)dx = \frac{(t-p_B+c\gamma)^2}{2t^2}$$

As firm B quotes a uniform price it incurs no personalization cost. Its profit is $\pi_B = p_B\left(\frac{(t-p_B+c\gamma)^2}{2t^2}\right)$. From the FOC (and SOC) for the profit maximization with respect to p_B we obtain that, in equilibrium, firm B quotes $p^{pp,up}_{B,\mathcal{T}} = \frac{t+c\gamma}{3}$. The indifferent consumer is located at $\tilde{x}^{pp,up}_{\mathcal{T}} = \frac{2t-c\gamma}{3t}$.

Note that $p^{pp,up}_{B,\mathcal{T}}$ is indeed superior to $c\gamma$ whenever $\frac{t}{c} > 2\gamma$ which holds true as by assumption $\frac{t}{c} > 2$. Also, $\frac{t}{c} > 2$ implies that $\frac{2t-c\gamma}{3t}$ is indeed greater than one half, and, in addition, it triggers an interior solution ($\frac{2t-c\gamma}{3t} \leq 1$). Firm B thus serves all consumers in the interval $\left[\frac{2t-c\gamma}{3t}, 1\right]$ while Firm A serves all consumers in the remaining interval $\left[0, \frac{2t-c\gamma}{3t}\right]$.

Substituting p_B in $p_A(x)$ we find that $p^{pp,up}_{A,\mathcal{T}}(x) = \frac{t(4-6x)+c\gamma}{3}$ if $x \leq \frac{2t-c\gamma}{3t}$ and $c\gamma$ otherwise. Firm B and A's profits are respectively (remind that A serves more than half the market): $\pi^{pp,up}_{B,\mathcal{T}} = \frac{2}{27}\frac{(t+c\gamma)^3}{t^2}$,and $\pi^{pp,up}_{A,\mathcal{T}} = \frac{4c^3\gamma^3+12c^2\gamma^2t-42c\gamma t^2+31t^3}{81t^2}$. Last, we find: $CS^{pp,up}_{\mathcal{T}} = v - \frac{c\gamma}{3} - \frac{5t}{6}$ and, $W^{pp,up}_{\mathcal{T}} = \frac{4c^3\gamma^3+12c^2\gamma^2t-42c\gamma t^2+31t^3}{81t^2} - \frac{c\gamma}{3} + \frac{2(c\gamma+t)^3}{27t^2} - \frac{5t}{6} + v$.

• **Only the inefficient firm discriminates (up, pp):** Suppose that firm B discriminates, while A does not. Then the result is again symmetric to the above situation except that $\gamma = 1$. Thus, firm A and B's profits are respectively $\pi^{up,pp}_{A,\mathcal{T}} = \frac{2(c+t)^3}{27t^2}$ and $\pi^{up,pp}_{B,\mathcal{T}} = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2}$. Last, we have: $CS^{up,pp}_{\mathcal{T}} = -\frac{c}{3} - \frac{5t}{6} + v$ and $W^{up,pp}_{\mathcal{T}} = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2} + \frac{2(c+t)^3}{27t^2} - \frac{c}{3} - \frac{5t}{6} + v$.

Lemma 3 *Under triangular distribution, firm A and B respectively hold the following best responses to uniform pricing denoted $BR_{A,\mathcal{T}}(up)$ and $BR_{B,\mathcal{T}}(up)$:*

$$BR_{A,\mathcal{T}}(up) = \begin{cases} pp & if\, t/c \geq 3.56\gamma \\ up & otherwise \end{cases} \quad \& \quad BR_{B,\mathcal{T}}(up) = \begin{cases} pp & if\, t/c \geq 3.56 \\ up & otherwise \end{cases} \tag{3}$$

We observe the same pattern as under uniform distribution. Yet, the thresholds are both lower meaning that triangular distribution encourages the use of pp when the rival quotes a up. The intuition is as follows.

Consider symmetric firms. Suppose firm i's rival quotes up, then if firm i quotes up, the two firms equally share the market. The only effect of moving from a uniform to a triangular distribution is that it boosts competition for consumers in the middle, which incites both firms to decrease their *up* (*price effect*), $\Delta p_i^{up,up} = -\frac{t}{2}$. In contrast, if the firm quotes *pp*, then it gets more demand than the rival. Triangular distribution now has three effects on firm i. Firstly, there is a *demand effect*. As more consumers are in the middle of the segment, the demand for firm i automatically increases ($F(\tilde{x}_{i,\mathcal{U}}^{pp,up}) > \tilde{x}_{i,\mathcal{U}}^{pp,up}$). Secondly, this triggers a *price effect*. As the rival decreases its *up* in order to compensate for lost demand, because prices are strategic complements, there is a reduction in the *pp* set by firm i ($\Delta p_i = -\frac{t+c}{6}$). The new price structure makes the indifferent consumer closer to firm i ($\Delta\tilde{x}_i^{pp,up} = -\frac{t+c}{12t}$) which mitigates the boosted demand for firm i. Finally, firm i bears a *margin effect* (re-allocation of consumers). Because under triangular distribution firm i's obtains fewer close consumers providing high margins and more distant consumers providing low margins, the re-allocation of consumers limits firm i's maximum margin through *pp*.

Note that profits from uniform to triangular distribution decrease, in every sub-game. This is clear for the (up, up) equilibrium. But for the (pp, up) or (up, pp) equilibria, it means that the negative price and margin effects dominate the positive demand effect. In addition, it can be shown that $|\Delta\pi^{pp,up}| < |\Delta\pi^{up,up}|$. This means that, overall, the negative effects of triangular distribution on firm i's profits is lower under *pp* than *up*. This is clear for the price effect which decreases less firm i's price than the price effect under *up* ($|\Delta p_i^{up,up}| = \frac{t}{2} > |\Delta p_i^{pp,up}| = \frac{t+c}{6}$). Nonetheless, the result suggests that the presence of the positive demand effect helps mitigating the additional negative margin effect.

Asymmetry then favors firm A in quoting *pp* when the rival sets *up*. This translates for example in the negative price effect under *pp* being lower for firm A $|\Delta p_i| = \frac{t+c\gamma}{6} < \frac{t+c}{6}$. As a result, we recover the similar pattern as under uniform distribution: firm A's threshold decreases while firm B's threshold remains the same.

• **Both firms offer personalized prices (pp, pp).** From our previous analysis the price schedule when both firms quote personalized prices is independent of consumer distribution (Proposition 1). However, profits, consumer surplus and total welfare will be different.

Lemma 4 *Under triangular distribution, firm A and B respectively hold the following best responses* *to personalized pricing denoted $BR_{A,\mathcal{T}}(pp)$ and $BR_{B,\mathcal{T}}(pp)$:*

$$BR_{A,\mathcal{T}}(pp) = \begin{cases} pp & if\, t/c \geq m(\gamma) \\ up & otherwise \end{cases} \qquad \& \quad BR_{B,\mathcal{T}}(pp) = \begin{cases} pp & if\, t/c \geq 4.22 - \gamma \\ up & otherwise \end{cases} \tag{4}$$

where $m(\gamma)$ is totally defined in the Appendix.

We observe the same pattern as under uniform distribution. Yet, it can be shown that the thresholds have increased meaning that it is less interesting for the firms to quote *pp* when the rival quotes *pp*. The intuition is as follows.

Consider symmetric firms. Suppose that firm i's rival quotes *pp*, then if firm i quotes *pp*, it only bears the *margin effect* (re-allocation of consumers) since by symmetry of the demand and from proposition 1, the *pp* remains the same. At the opposite, if the firm quotes *up*, it bears the negative *demand effect* (demand is reduced) and the *price effect* (prices decrease). Bear in mind that the price effect mitigates the demand effect.

Overall, we find that the impact of the negative demand and price effects on firm i's profits under *up* is lower than the negative margin effect on firm i's profits under *pp*. Formally, we have $|\Delta\pi^{up,pp}| < |\Delta\pi^{pp,pp}|$. As a result, the firm is relatively less worse-off choosing *up* when consumer distribution becomes triangular and the rival quotes *pp*.

Asymmetry then disadvantages firm B in quoting *pp* when the rival sets *pp*. Triangular distribution creates a negative demand effect on firm B in equilibrium (pp, pp) due to the demand asymmetry triggered by cost asymmetry $(F(\tilde{x}^{(pp,pp)}) > \tilde{x}^{(pp,pp)} > 1/2)$. Therefore, firm B's threshold becomes more stringent while the reverse occurs for firm A.

5.2 The partition of equilibria

Figure 2 summarizes the results building on Lemma 3 and 4 and also plots the equilibrium regions. Note that, again, both *pp* and *up* can appear as the dominant strategy for each firm. Proposition 3 summarizes the results about the possible equilibria.

Proposition 3 *Under triangular distribution of consumer preferences, three Nash Equilibria can appear: (u,u) when $\frac{t}{c} \leq 3.56\gamma$, (pp,u) when $3.56\gamma < \frac{t}{c} < 4.22 - \gamma$, (pp,pp) when $\frac{t}{c} \geq 4.22 - \gamma$.*

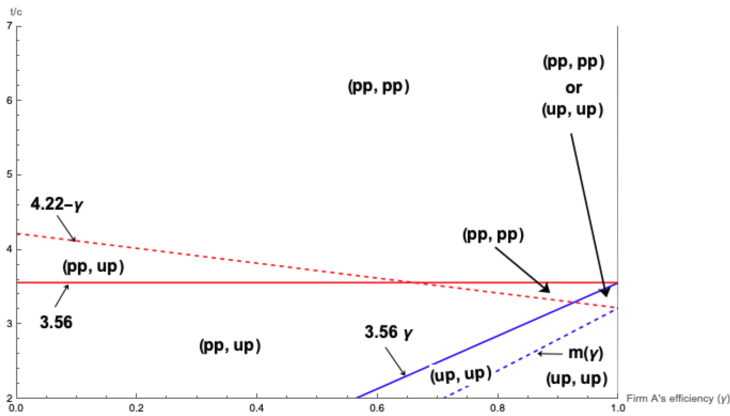
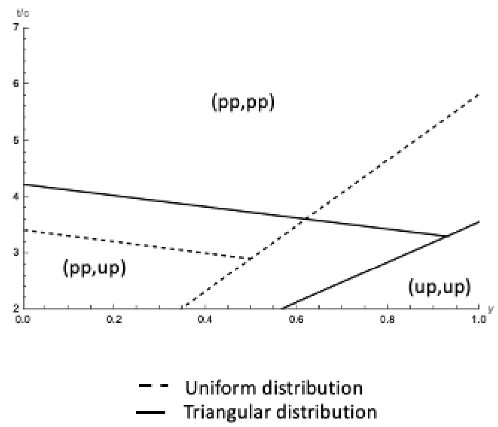


Figure 2: Partition of equilibria (triangular distribution)

Comparison with uniform distribution. To have neat comparisons, we suppose in what follows, and without loss of generality, that $t/c \leq 7$. This enables us to compute the portion of parameter regions for each consumer distribution and compare them.

Note that a concern arises in the region where the two equilibria (pp, pp) and (up, up) can occur. We thus have to determine how firms settle this indeterminacy of equilibrium. To do that, we will use Schelling (1960)'s focal point. Schelling (1960) suggested that two players facing a coordination problem might be able to converge their behavior by finding a focal point of the game, i.e., a point of convergence of expectations and beliefs without communication but by the mean of a salient contextual aspect of the game. Schelling's hypothesis has been examined experimentally in several studies which show that people are able to identify focal points in 'pure' coordination games - games where players get the same payoff in any equilibrium (Crawford *et al.*, 2008; Isoni *et al.*, 2013; Parravano and Poulsen, 2015). Applied to our setting, the salient feature would be the profits of the firms, and the Schelling criterion yields that firms converge their expectations and thus choices on the equilibrium (up, up). We take this view in the following discussion. Therefore, in what follows we assume that the firms coordinate on the more profitable equilibrium, i.e. (up, up). The same argument is used by Jeitschko *et al.* (2017).

Figure 3 summarizes the results about equilibrium pricing policies for the two distributions (i.e. Proposition 2 & 3). We observe that the region where equilibrium (pp, up) occurs increases with triangular distribution by 17.6 percentage points (henceforth p.p.). This means that triangular distribution encourages personalization of prices by the most efficient firm (firm A). However, this region only appears if $\gamma < 0.5$ under uniform distribution, and if $\gamma < 0.925$ under triangular distribution.



(a) Parameter regions

Region	Proportion		
	Uniform	Triangular	Difference
(pp, pp)	64.6%	65.4%	0.8%
(up, up)	25.2%	6.8%	-18.4%
(pp, up)	10.2%	27.8%	17.6%
Total	100%	100%	

(b) Proportion

Figure 3: Effect of triangular distribution on pricing policies

This suggest that under the uniform distribution (pp, up) arises if firms are sufficiently asymmetric. Concerning the two other equilibria (pp, pp) and (up, up), the overall portion of regions yielding equilibrium (pp, pp) increases by 0.8 p.p. while the overall region with equilibrium (up, up) diminishes by 18.4 p.p.

Proposition 4 *Compared to uniform distribution of consumer preferences, a triangular distribution increases the parameter region with personalized prices.*

Proposition 4 suggests that the marketplace has incentives to distort consumers preferences. In the subsequent section, we will delve into specific cases where the gatekeeper has a clear incentive to distort consumers' preferences.

6 The Marketplace decision

Bear in mind that the marketplace obtains profits from selling its personalization program to the firms who decide to personalize prices. Formally, the marketplace profit writes

$$\pi_M^k = c\gamma \int_0^{\tilde{x}^k} f(x)dx \; \mathbb{1}_{k=(pp,pp),(pp,up)} + c \int_{\tilde{x}^k}^1 f(x)dx \; \mathbb{1}_{k=(pp,pp),(up,pp)}$$

where $\tilde{x}^k$ denotes the indifferent consumer under equilibrium type $k = (pp, pp), (pp, up), (up, pp), (up, up)$ and $\mathbb{1}$ is a dummy variable which equals one if the condition is true and zero otherwise. In other words, the marketplace profit boils down to the average personalization cost.

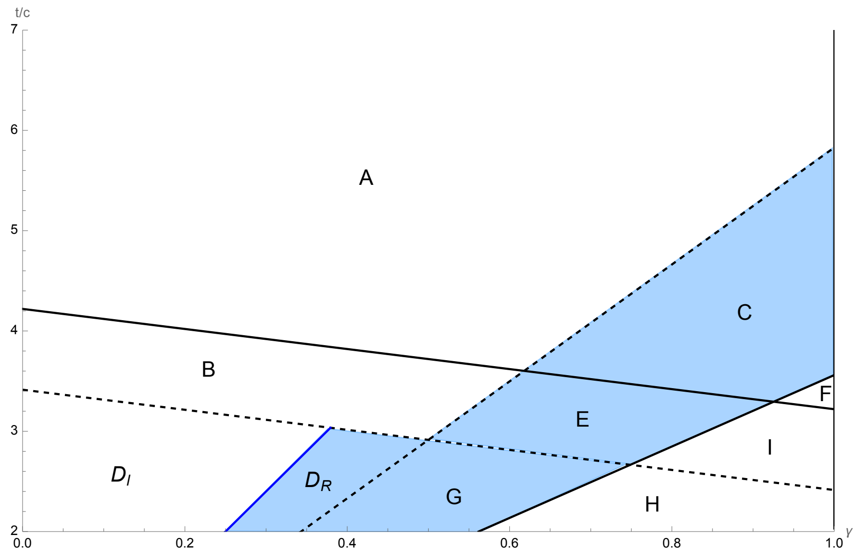


Figure 4: Regions where the marketplace's profits increases

Note that the term $\int_0^{\tilde{x}^k} f(x)dx$ and $\int_{\tilde{x}^k}^1 f(x)dx$ are simply the demand of firm A and firm B, respectively. We can therefore rewrite the marketplace profits as $\pi_M^k = c\gamma D_A^k \, \mathbb{1}_{k=(pp,pp),(pp,up)} + c(1-D_A^k) \, \mathbb{1}_{k=(pp,pp),(up,pp)}$. Then, intuitively, the profit of the marketplace depends on three main factors (i) the demand of firm A, (ii) sellers' asymmetry and (iii) the firms' decision to quote *pp* or *up*. Figure 4 plots the regions where the marketplace's profits increase when moving from a uniform to a triangular distribution (in blue). Conversely, outside of these regions, the profits of the marketplace do not increase.

Proposition 5 *(Blue region) The marketplace has a clear incentive to distort consumer preferences when (i) asymmetry is intermediate ($0.25 < \gamma < 0.75$) and the ratio t/c is sufficiently low (regions D_R and G); or (ii) when asymmetry is weak ($\gamma \geq 0.5$) and t/c is medium (regions E and C).*

In region A, the two firms quote *pp*. The decrease of M's profits therefore comes from the increase of firm A's demand and thus the decrease of firm B's demand ($\Delta D_A = \frac{c(1-\gamma)(t-c(1-\gamma))}{2t^2} > 0$). In region B, M makes profits solely from firm A under triangular distribution, while under the uniform distribution both firms would quote PP. Therefore, despite the increase in sales for firm A ($\Delta D_A = \frac{-4c^2\gamma^2 - c(9-\gamma)t + 5t^2}{18t^2} > 0$), the marketplace is actually worse off under the triangular distribution due to the loss of sales from firm B. In region D, M makes profits only from firm A irrespective of the distribution of consumer preferences. Thus, in this region its profits increase whenever the demand of firm A increases ($\Delta D_A = \frac{(t-8c\gamma)(c\gamma+t)}{36t^2}) > 0$) i.e. whenever $t/c > 8\gamma$ (the blue line on the Figure), region D_R. In regions C, E and G, the marketplace was not generating any profit under uniform distribution because no firm was offering personalized prices. However, if the distribution shifts from uniform to triangular, firm A begins to offer personalized prices, resulting in positive profits for the marketplace. Furthermore, M's profits increase in this region. Finally, in region H, both firms continue to offer uniform prices irrespective of consumer preferences, resulting in no profits for the marketplace.

It is worth noting that if the firms were symmetric, the marketplace would be unable to incentivize them to offer personalized prices (PP) in two cases: when the device tariff is too high (as firms would never choose to offer PP), or when the device tariff is too low (as firms would already offer PP). In this case, the marketplace can only distort consumer preferences when the device tariff is intermediate, because the increased competition for the indifferent consumer encourages firms that previously did not offer PP to start doing so.

Asymmetric experience then encourages the experienced firm to offer PP in situations where it did not previously, due to the high cost involved. This then prompts the inexperienced firm to also offer PP for higher device tariffs in order to remain competitive. In comparison to the symmetric setting, increased competition resulting from asymmetric experience only causes the gatekeeper to distort consumer preferences for higher device tariffs. At a certain degree of asymmetry, the experienced firm may become so efficient that it is the only one offering PP. In such a scenario, the marketplace can distort consumer preferences only by encouraging the experienced firm to offer PP to a larger fraction of consumers (which occurs when the asymmetry is not too large).

7 The impact of the Marketplace's decision on firms and consumers

The marketplace's decisions can significantly affect the pricing behavior of firms, their profits and consumer welfare, and it is crucial to carefully consider these potential impacts, particularly in markets with features such as firms' asymmetry in data use and the personalized pricing device costs.

This section analyzes the changes in firms' profits, consumer surplus, and total welfare in the regions where the marketplace has an incentive to distort consumer preferences. Specifically, the analysis focuses on regions D_R, G, E and C such that: in region D_R, the equilibrium remains (pp, up); in regions G and E, the equilibrium changes from (up, up) to (pp, up); and in region C, it changes from (up, up) to (pp, pp).

7.1 The firms' profits.

Based on our analysis, we find that firms' profits consistently decrease when the marketplace distorts consumer preferences in regions where it has an incentive to do so (regions D_R, G, E, C and F). This result holds for both experienced and inexperienced firms and for various levels of device cost and data asymmetry. However, the magnitude of the profit reduction differs across regions and depends on the specific market conditions.

Proposition 6 *If the marketplace distorts consumer preferences in the regions where it has an incentive to do so, both firms experience a decrease in profits.*

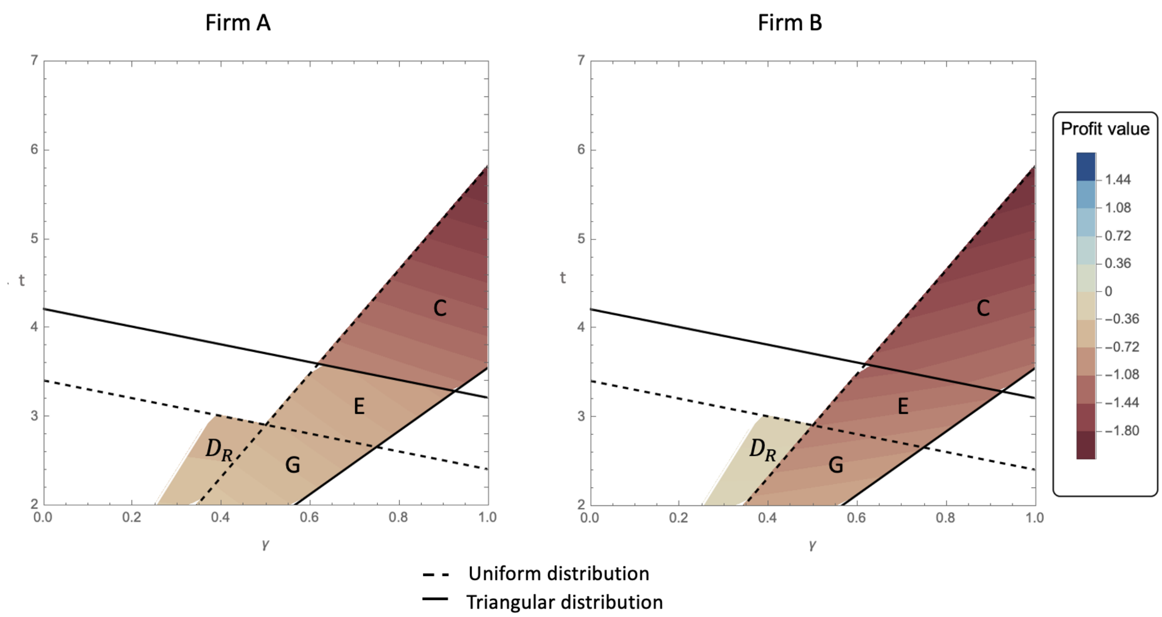


Figure 5: Variations of firms' profits (illustration for $c = 1$)

In region D_R, where the equilibrium pricing policy remains the same, the marketplace's distortion to a triangular distribution leads to increased competition for consumers in the middle, resulting in a decrease in profits for both firms. Notably, firm B experiences a smaller decrease in profits than firm A. Bear in mind that under uniform distribution, firm A's efficiency increases its demand and price competition. Therefore, strong asymmetry implies that triangular distribution has limited demand and price effects for both firms. The difference in profits thus mainly resides with firm A additionally incurring the negative margin effect (more consumers buying at lower prices).

Interestingly, in regions G and E where the equilibrium changes from (up, up) to (pp, up), the ranking of losses reverses. The novelty in these regions is the *switching of price policy effect* whereby firm A changes from quoting uniform to personalized prices. Under uniform distribution, this *switching policy effect* yields a boost of competition $(p_{B,\mathcal{U}}^{(pp,up)} - p_{B,\mathcal{U}}^{(up,up)} = -\frac{t-c\gamma}{2} < 0)$ in favor of firm A. Then, as previously, triangular distribution (the change of preference distribution from $(pp, up)_{\mathcal{U}}$ to $(pp, up)_{\mathcal{T}}$) will dampen the two firms' profits, albeit affecting more firm B. Overall, the two firms' profits decrease but the switching price policy effect yields that firm B's profits now decrease more than those of firm A.

Finally, the result of the change in profits in region C is more obvious and relates to the prisoner dilemma result documented in most papers on personalized pricing. However, PP further decreases profits when moving from uniform to triangular distribution.

The impact of online marketplaces on competition and business success has been a widely debated topic. Our study sheds light on the potential effects of marketplaces' strategies to increase profits from their web and cloud services. We have already noted that although Amazon Web Services (AWS) is its smallest operating segment by revenue, it was Amazon's only profitable segment in 2022. This suggests that this platform may have incentives to strategically promote these services.

While our study focused on the Amazon Personalize cloud service example, it could have equally examined the cloud storage or computing services, also important for business users to personalized their strategies. As long as Amazon profits from these services increase with business users' personalized pricing decisions, and this pricing practice is more likely to be chosen under a triangular distribution, then the marketplace may have stronger incentives to manipulate the distribution of consumer preferences in order to increase profits from its cloud services. Our study indicates that a policy like this has the potential to harm the profits of business users on the platform, especially when there is significant asymmetry among them. Furthermore, it is worth noting that if there is a fixed cost involved in remaining in the market, the market place behavior could potentially lead to the exclusion of certain business users, particularly those who are smaller or less experienced. These findings highlight the importance of policymakers taking into account the potential impact of marketplace policies, when developing regulations for online marketplaces under the Digital Markets Act.

7.2 Consumer surplus

Let's analyze the impact of marketplace behavior on consumer surplus. Our findings indicate that while the marketplace distortion may lead to falling profits for businesses, consumer surplus always increases when the marketplace distorts consumer preferences in the regions where it has an incentive to do so (regions D_R, G, E and C).

Proposition 7 *If the marketplace distorts consumer preferences in the regions where it has an incentive to do so, then consumer surplus increases.*

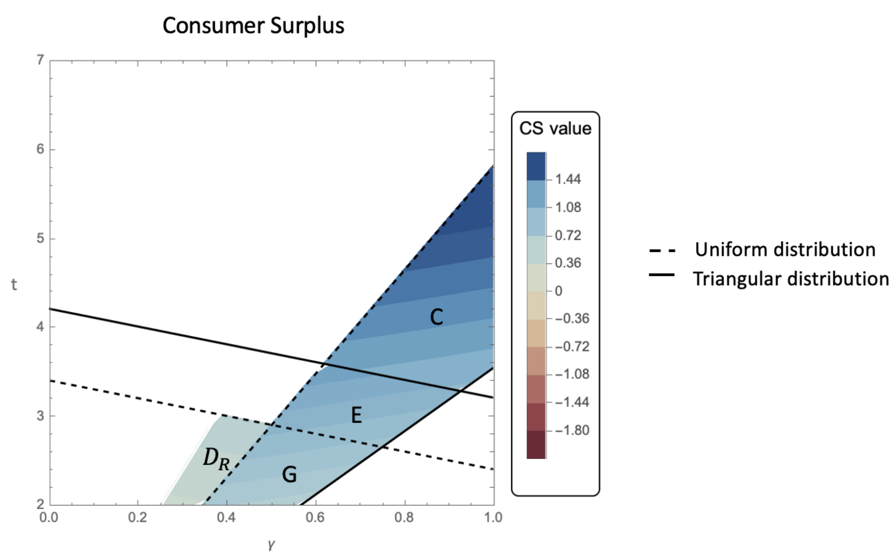


Figure 6: Variations of consumer surplus (illustration for $c = 1$)

Intuitively, consumer surplus is affected by the firms' price decisions, which may switch some consumers' purchase decisions, and the distance consumers have to travel to buy from the firms.

Consider the hypothetic case where the equilibrium remains (up, up). We saw that triangular distribution triggers a *price effect* ($\Delta p = -\frac{t}{2} < 0$) that equivalently benefits all consumers as they all pay the same price. Therefore, the *average price* paid by the consumers decreases ($\Delta AP = -\frac{t}{2} < 0$). At the opposite, under the triangular distribution, more consumers have to travel a greater distance to buy from their firm of interest. This increases the *average transportation cost* ($\Delta ATC = \frac{t}{4} > 0$).

Overall, we find that the positive AP effect overcomes the negative ATC effect and consumers are better off.

Interestingly, in the opposite hypothetic case where the equilibrium remains (pp, pp), triangular distribution triggers no price effect (price is the same under both distributions). However, consumers still witness a decrease of the average price as triangular distribution diverts consumers towards the middle where the prices are lower ($\Delta AP = -\frac{4c^3(1-\gamma)^3+3c^2(1-\gamma)^2t+t^3}{6t^2} < 0$). On the other hand, consumers again have to travel a greater distance to buy from their preferred firm, and the ATC increases ($\Delta ATC = \frac{c^2(1-\gamma)^2(3t-4c(1-\gamma))+t^3}{12t^2} > 0$). Overall, we find that the AP effect again overcomes the ATC effect, thus consumer surplus increases.

In region D_R, where the equilibrium remains (pp, up), the price effect, triggered by triangular distribution, has the additional effect of modifying the location of the indifferent consumer so that it is closer to firm A. This mitigates the negative ATC effect. Overall, we still find that the AP effect overcomes the ATC effect and consumers are better off. In regions E and G, the equilibrium changes from (up, up) to (pp, up), which will have an additional impact on the AP and ATC effects. This shift in equilibrium leads to increased competition, which, combined with the AP and ATC effects, results in a greater benefit for consumers. Finally, in region C, the shift to the triangular distribution enhances competition under (pp, pp), leading to a further increase in consumer surplus. This finding is consistent with previous research in the field, highlighting the positive effects of personalized pricing on consumer welfare in terms of improved market competitiveness and greater benefits for consumers (higher under the triangular distribution). Indeed, while the marketplace's behavior can have positive implications for consumer welfare, it is essential to consider the potential negative effects on sellers' profits and the risk of exclusion of smaller and less experienced sellers. The goals of the Digital Markets Act (DMA) emphasize the need for fair and competitive digital markets. Policymakers must align their considerations with these goals, aiming to create an environment that not only enhances consumer welfare but also promotes a level playing field for all market participants. Striking a balance between consumer welfare and seller profitability, and ensuring the inclusion of smaller and less experienced sellers, is crucial for the effective implementation of the DMA. By addressing these concerns, policymakers can foster a digital marketplace that upholds competition and sustainable growth for all stakeholders involved.

8 Conclusion and discussion

In this study, we have modeled duopoly duopoly competition on a marketplace represented by a Hotelling segment of consumers. Initially, consumers are uniformly distributed, but the marketplace has the ability to manipulate consumer preferences by altering the mix of brand recommendations, making consumers more indifferent between the competing firms and resulting in triangular preferences (Esteves *et al.*, 2022; Tabuchi and Thisse, 1995). The marketplace provides raw data to the firms, allowing them to gain insights into the consumer distribution. Importantly, the firms have the option to purchase a marketplace device that provides additional information from the raw data, enabling them to personalize prices. Following our previous reasoning on data experience, we further suppose that the cost of the device is lower for one firm (henceforth the experienced firm) than the other (henceforth the inexperienced firm). After getting raw data, the firms simultaneously decide whether to quote personalized or uniform prices.

Our findings indicate that the marketplace has an incentive to distort consumer preferences when there is intermediate asymmetry between the firms and a high device tariff. In the absence of preference distortion, the efficient firm is either the sole provider of personalized prices or both firms opt for uniform prices. However, when consumer indifference increases (triangular distribution), competition intensifies, prompting the efficient firm to extend personalized pricing to a larger consumer base. This benefits the marketplace. Interestingly, even when asymmetry and the subscription tariff decrease, the marketplace still retains an incentive to distort preferences. In such cases, each firm initially decides to offer personalized prices only if the other firm does so. The literature suggests that they would eventually converge on uniform prices, prompting the marketplace to distort consumer preferences to encourage the efficient firm to employ personalized pricing. This also encourages the other firm to adopt personalized prices when the levels of asymmetry and cost are low.

Overall, when the marketplace distorts consumer preferences within the identified parameter regions, it negatively affects the sellers' profits but benefits consumers. While the marketplace's behavior can have positive implications for consumer welfare, it is essential to consider the potential negative effects on sellers' profits and the risk of exclusion of smaller and less experienced sellers. These findings highlight the importance of considering the implications of marketplace behavior and its impact on business users profits and consumer welfare. They further underscore the significance of regulatory frameworks such as the Digital Markets Act in ensuring competitive digital markets.

Policymakers should carefully evaluate the dynamics between marketplace platforms, firms, and consumers to formulate effective regulations that strike a balance between promoting competition, protecting sellers' interests, and safeguarding consumer welfare in the digital marketplace ecosystem.

It is worth mentioning that the model can be extended to incorporate a scenario where consumers experience a psychological cost from targeted or disliked ads. Interestingly, our findings reveal that this cost associated with viewing targeted advertising can mitigate the increase in consumer surplus. Technically, it can be interpreted as consumers perceiving a lower reservation value for products under the triangular distribution (although it remains sufficiently high to maintain market coverage at equilibrium). Consequently, there is a possibility of a decrease in consumer surplus, particularly in the region D_R, characterized by high personalized costs and low firm asymmetry.

Finally, the model presented in this study has certain limitations that should be acknowledged. Firstly, it does not lend itself to tractability when considering intermediate consumer preferences, where the distribution of consumers falls between the uniform and pure triangular shapes. While this limitation exists, it is important to emphasize that our primary focus is on examining the marketplace's incentive to distort consumer preferences rather than determining the optimal distortion strategy. Further research is necessary to explore this aspect in more detail, and Appendix F provides preliminary insights by highlighting the non-monotonic impact of intermediate consumer preferences on firms' decisions regarding personalized pricing. Additionally, our model does not consider the ability of the marketplace to modify tariff choices. This exclusion is due to the challenges associated with adapting tariffs to specific situations, as the tariff remains the same for all firms. However, it is worth noting that our model highlights the pricing trade-off faced by the marketplace. Setting a high price runs the risk of supplying only the efficient firm or no firm at all, while setting a low price increases the likelihood of supplying both firms.

These limitations point to avenues for future research to enhance our understanding of marketplace dynamics, consumer preferences, and pricing strategies. By addressing these limitations, we can gain further insights into the complex interactions between marketplaces, firms, and consumers, and inform the development of effective regulatory frameworks to promote fair and competitive digital markets.

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APPENDIX

A Figures

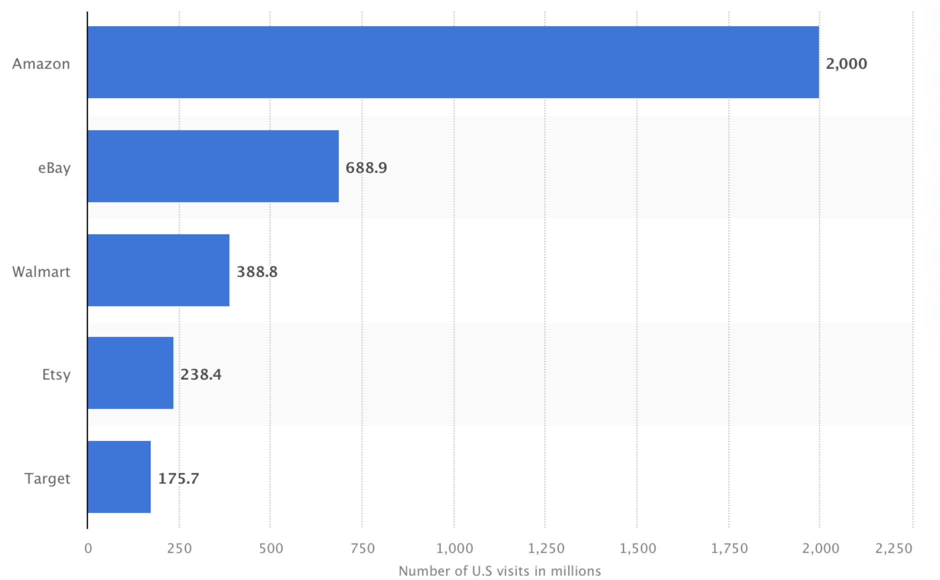

Figure 7: Leading online marketplaces in the United States as of April 2021, based on number of monthly visits (Statista, 2022)

B Proof of Lemma 1 & 2 and Proposition 2

◇ The four sub-game equilibria with uniform distribution

In what follows, we omit the subcript $\mathcal{U}$ and the superscript of equilibria. It alleviates notations and facilitates the reading. The reader just has to refer to the subsection of interest to get the associated equilibrium values.

• **Equilibrium** (up, up)**.**

The indifferent customer, $\tilde{x}$, is indifferent between buying form firm A or B. Its utilities satisfy $u_A(\tilde{x}) = u_B(\tilde{x})$, which writes $v - p_A - t\tilde{x} = v - p_B - t(1 - \tilde{x})$ and leads to $\tilde{x} = \frac{1}{2} + \frac{p_B - p_A}{2t}$. Given uniform distribution, the demand for A is $q_A = \tilde{x}$ while the demand for B is $q_B = 1 - \tilde{x}$. Each firm $i \in \{A, B\}$ maximizes profit $\pi_i = p_i q_i$ with respect to price p_i (remind there is no production cost). The first order condition (FOC) of A gives $\tilde{x} + \frac{d\tilde{x}}{dp_A} p_A = 0$ which boils down to $2p_A - p_B = t$. Similarly for firm B, we find $2p_B - p_A = t$. Because firms are symmetric, we get at equilibrium that $p_A = p_B = t$. The SOC are satisfied: we have $\frac{d^2\pi_A}{d(p_A)^2} = \frac{d^2\pi_B}{d(p_B)^2} = -1/t < 0$. Since prices are the same, the indifferent customer is situated at $\tilde{x} = \frac{1}{2}$. Therefore, the equilibrium profits of the firms are $\pi_A = \pi_B = \frac{t}{2}$. And the consumer surplus is $CS = \tilde{x}(v-p_A-t\tilde{x})+(1-\tilde{x})(v-p_B-t(1-\tilde{x})) = \frac{1}{2}(v-t-t\frac{1}{2})+\frac{1}{2}(v-t-t\frac{1}{2}) = v-\frac{5}{4}t$. Overall, the welfare is $W = \pi_A + \pi_B + CS = v - \frac{t}{4}$. $\square$

- **Equilibrium (pp, pp).**

Consider a consumer located near A with $x < \frac{1}{2}$. Given the price B offers to a consumer located at x, $p_B(x)$, in order to make a sale firm A should offer a price that gives this consumer just as good a deal defined by $v - p_A(x) - tx = v - p_B(x) - t(1-x)$ that is $p_A(x) + tx = p_B(x) + t(1-x)$. Taking into account that the best price firm B offers to a consumer located near the rival is its marginal cost of personalization c, we have that $p_A(x) + tx = c + t(1-x)$, which yields $p_A(x) = c + t(1-2x)$. Additionally we need to impose that the price of A is superior to its own personalization marginal cost $p_A(x) \geq c\gamma$, from which we get $p_A(x) = c+t(1-2x)$ as long as $x \leq \frac{1}{2} + \frac{(1-\gamma)}{2t} \equiv \tilde{x}$ and $p_A(x) = c\gamma$ otherwise. Note that as $1-\gamma > 0$ then firm A serves always more than half of the market. The profit of firm A is $\pi_A = \int_0^{\tilde{x}} (p_A(x) - c\gamma)dx = [(c - c\gamma + t)x - tx^2]_0^{\tilde{x}} = \frac{(c-c\gamma+t)^2}{4t}$.

By symmetry, consider now a consumer located near B with $x > \frac{1}{2}$. Similarly to above, firm B should offer a price that gives this consumer just as good a deal defined by $p_A(x) + tx = p_B(x) + t(1-x)$. Taking into account that the best price firm A offers to a consumer located near the rival is its marginal cost of personalization $c\gamma$, we have that $p_B(x) = c\gamma + t(2x-1)$. Again, we need to impose that the price of B is superior to its own personalization marginal cost $p_B(x) \geq c$, from which we get $p_B(x) = c\gamma + t(2x-1)$ as long as $x \geq \frac{1}{2} + \frac{(1-\gamma)}{2t} = \tilde{x}$ and $p_B(x) = c$ otherwise. Firm B serves always less than half of the market. The profit of firm B is $\pi_B = \int_{\tilde{x}}^1 (p_B(x) - c)dx = \frac{(c\gamma-c+t)^2}{4t}$, which is symmetric to the profit of firm A.

The consumer surplus is $CS = \int_0^{\frac{1}{2}+\frac{c(1-\gamma)}{2t}} (v - p_A(x) - tx)dx + \int_{\frac{1}{2}+\frac{c(1-\gamma)}{2t}}^1 (v - p_B(x) - t(1-x))dx = \frac{1}{4t}[t(4v-3t) - c^2(1-\gamma)^2 - 2c(\gamma+1)t]$ and welfare is $W = \pi_A + \pi_B + CS = \frac{1}{4t}[c^2(1-\gamma)^2 - 2c(1+\gamma)t + t(4v-t)]$. $\square$

- **Equilibrium (pp, up).**

Suppose that firm A discriminates, while B does not. Given firm B's uniform price p_B the indifferent consumer between buying from A and B is located at x such that $v - p_A(x) - tx = v - p_B - t(1-x)$ which leads to $p_A(x) = p_B + t(1-2x)$. The lowest price firm A is willing to charge to a more distant consumer is equal to its personalization cost $c\gamma$. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x}$ such that $c\gamma = p_B + t(1-2\tilde{x})$ which leads to $\tilde{x} = \frac{1}{2t}(t + p_B - c\gamma)$. With uniform distribution firm A demand is $q_A = \tilde{x}$ and firm B is $q_B = 1 - \tilde{x}$. As firm B quotes a uniform price it incurs no personalization cost. Its profit is $\pi_B = p_B q_B = p_B\left(1 - \frac{1}{2t}(t - c\gamma + p_B)\right)$. The FOC for the profit maximization with respect to p_B gives $-2p_B + t + c\gamma = 0$ and we obtain that firm B quotes $p_B = \frac{t+c\gamma}{2}$. The SOC is satisfied: we have $\frac{d^2\pi_B}{d(p_B)^2} = -1/t < 0$. This then gives $\tilde{x} = \frac{1}{4t}(3t - c\gamma)$.

As $t > c$, we have $t > c\gamma$ and thus firm A serves more than half of the consumers ($\frac{1}{4t}(3t - c\gamma) = \frac{1}{2} + \frac{t-c\gamma}{4t} > \frac{1}{2}$). Also, $\frac{3t-c\gamma}{4t}$ is always inferior to 1 (because $t > 0 > -c\gamma$) so we always have an interior solution. Firm B serves all consumers in the interval $\left[\frac{1}{4t}(3t - c\gamma), 1\right]$ while firm A serves all consumers in the remaining interval, i.e., those consumers who belong to the interval $\left[0, \frac{1}{4t}(3t - c\gamma)\right]$.

Substituting p_B in $p_A(x)$ we find that $p_A(x) = \frac{t(3-4x)+c\gamma}{2}$ if $x \leq \frac{1}{4t}(3t - c\gamma)$, and $c\gamma$ otherwise. With these two equilibrium prices, we get firm B and A's profits which are respectively $\pi_B = p_B q_B = \frac{1}{8t}(t + c\gamma)^2$ and $\pi_A = \int_0^{\frac{1}{2}+\frac{t-c\gamma}{4t}} (p_A(x) - c\gamma)dx = \frac{1}{16t}(3t - c\gamma)^2$. In addition, we have $CS = \int_0^{\frac{1}{2}+\frac{t-c\gamma}{4t}} (v - p_A(x) - tx)\, dx + \int_{\frac{1}{2}+\frac{t-c\gamma}{4t}}^1 (v - p_B - t(1-x))\, dx = v - \frac{c\gamma}{2} - t$ and $W = v + \frac{3c^2\gamma^2}{16t} - \frac{5c\gamma}{8} - \frac{5t}{16}$.$\square$

• **Equilibrium (up, pp).** Consider now the case where B discriminates while A does not. Given firm A's uniform price p_A the indifferent consumer between buying from A and B is located at x such that $v - p_A - tx = v - p_B(x) - t(1-x)$ which leads to $p_B(x) = p_A + t(2x-1)$. The lowest price firm B is willing to charge to a more distant consumer is equal to its personalization cost c. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x}$ such that $c = p_A + t(2\tilde{x} - 1)$ which gives $\tilde{x} = \frac{1}{2t}(t - p_A + c)$.

With uniform distribution firm A demand is $q_A = \tilde{x}$, and firm B is $q_B = 1 - \tilde{x}$. Firm A's profit is $\pi_A = p_A.q_A = p_A \frac{1}{2t}(t - P_A + c)$. From the FOC for the profit maximization with respect to p_A we obtain $c - 2p_A + t = 0$ which gives $p_A = \frac{1}{2}(t + c)$. The SOC is satisfied: we have $\frac{d^2\pi_A}{d(p_A)^2} = -1/t < 0$. Thus, in equilibrium, firm A quotes $p_A = \frac{1}{2}(t + c)$. This implies that $\tilde{x} = \frac{t+c}{4t}$, which is inferior to 1 (because $0 < c < t$) so we always have an interior solution. Firm A serves all consumers in the interval $\left[0, \frac{1}{4t}(c + t)\right]$ while firm B serves all consumers in $\left[\frac{1}{4t}(c + t), 1\right]$. Actually, $\frac{t+c}{4t}$ rewrites $\frac{1}{2} - \frac{t-c}{4t}$ and firm A thus serves less than half of the market.

Substituting p_A in $p_B(x)$, we find that $p_B(x) = \frac{c+t(4x-1)}{2}$ if $x \geq \frac{1}{4t}(c + t)$ and c otherwise. Firm A's profit is $\pi_A = p_A q_A = \frac{1}{8t}(t + c)^2$ while firm B's profit is $\pi_B = \int_{\frac{t+c}{4t}}^1 (p_B(x) - c)dx = \frac{1}{16t}(3t - c)^2$.

Also, we have $CS = \int_0^{\frac{1}{2}-\frac{t-c}{4t}} (v - p_A - tx)\,dx + \int_{\frac{1}{2}-\frac{t-c}{4t}}^{1} (v - p_B(x) - t(1-x))\,dx = v - \frac{c}{2} - t$ and $W = \frac{1}{16}\left(\frac{3c^2}{t} - 10c - 5t\right) + v$. $\square$

$\diamond$ Partition of equilibrium regions under uniform distribution

In same purpose as above, we omit the subscript $\mathcal{U}$.

- Look first at firm A.

$$\begin{aligned}
\pi_A^{pp,up} - \pi_A^{up,up} &= \frac{1}{16t}(3t - c\gamma)^2 - \frac{t}{2} > 0 \\
\gamma &< \frac{t}{c}\left(3 - 2\sqrt{2}\right) \approx 0.171\,57\frac{t}{c} \\
\gamma &< \underline{\gamma} = \frac{t}{c}\left(3 - 2\sqrt{2}\right) \\
or\ \frac{t}{c} &> \frac{\gamma}{(3 - 2\sqrt{2})} = (3 + 2\sqrt{2})\gamma \approx 5.83\gamma \equiv \underline{t}_c(\gamma)
\end{aligned}$$

$$\begin{aligned}
\pi_A^{pp,pp} - \pi_A^{up,pp} &= \frac{1}{4t}(t - c\gamma + c)^2 - \frac{1}{8t}(t + c)^2 > 0 \\
\gamma &< \frac{c+t}{c}\left(\frac{2 - \sqrt{2}}{2}\right) \approx 0.292\,89\left(1 + \frac{t}{c}\right) \\
\gamma &< \overline{\gamma} = \left(1 + \frac{t}{c}\right)\left(\frac{2 - \sqrt{2}}{2}\right) \\
\text{or } \frac{t}{c} &> \gamma\left(2 + \sqrt{2}\right) - 1 \equiv \overline{t}_c(\gamma)
\end{aligned}$$

Finally, we find $\gamma(2 + \sqrt{2}) - 1 \leq \frac{\gamma}{3 - 2\sqrt{2}}$, i.e. $\overline{t}_c(\gamma) \leq \underline{t}_c(\gamma)$. Note that $\overline{t}_c(\gamma)$ can be positive or negative depending on γ.

Proof. Suppose $\gamma(3 + 2\sqrt{2}) \geq \gamma(2 + \sqrt{2}) - 1$, it is equivalent to $\gamma(1 + \sqrt{2}) + 1 \geq 0$ which is always true. $\blacksquare$

Summary: If $\frac{t}{c} \geq \underline{t}_c(\gamma)$ then PP is a strictly dominant strategy for firm A. If $\underline{t}_c(\gamma) > \frac{t}{c} \geq \overline{t}_c(\gamma)$, firm A best-response is to choose PP when B chooses PP and U when B chooses U. Otherwise, when $\frac{t}{c} < \overline{t}_c(\gamma)$ U is a strictly dominant strategy for firm A.

• Look next at firm B.

$$
\begin{aligned}
\pi_B^{up,pp} \geq \pi_B^{up,up} \quad &\Leftrightarrow \quad \frac{(3t-c)^2}{16t} - \frac{t}{2} \\
&\Leftrightarrow \quad (3t-c)^2 \geq 8t^2 \\
&\Leftrightarrow \quad 3t - c \geq 2\sqrt{2}t \\
&\Leftrightarrow \quad \frac{t}{c} \geq \frac{1}{3-2\sqrt{2}} = 3 + 2\sqrt{2} \equiv \underline{t}_c^B
\end{aligned}
$$

Note that $\underline{t}_c^B = \frac{1}{3-2\sqrt{2}} \geq \frac{\gamma}{3-2\sqrt{2}} = \underline{t}_c^A(\gamma)$.

$$
\begin{aligned}
\pi_B^{pp,pp} \geq \pi_B^{pp,up} \quad &\Leftrightarrow \quad \frac{(t+c\gamma-c)^2}{4t} - \frac{(t+c\gamma)^2}{8t} \\
&\Leftrightarrow \quad \sqrt{2}(t+c\gamma-c) \geq t + c\gamma \\
&\Leftrightarrow \quad (t+c\gamma)(\sqrt{2}-1) \geq \sqrt{2}c \\
&\Leftrightarrow \quad \frac{t}{c} \geq \frac{\sqrt{2}}{\sqrt{2}-1} - \gamma \\
&\Leftrightarrow \quad \frac{t}{c} \geq \sqrt{2} + 2 - \gamma \equiv \bar{t}_c^B(\gamma)
\end{aligned}
$$

Again, we find $\bar{t}_c^B(\gamma) < \underline{t}_c^B$.

Summary: If $\frac{t}{c} \geq \underline{t}_c^B(\gamma)$ then *pp* is a strictly dominant strategy for firm B. If $\underline{t}_c^B(\gamma) > \frac{t}{c} \geq \bar{t}_c^B(\gamma)$, firm B best-response is to choose *pp* when A chooses *pp* and *up* when A chooses *up*. Otherwise, when $\frac{t}{c} < \bar{t}_c^B(\gamma)$ *up* is a strictly dominant strategy for firm B. □

⋄ Numerical point examples

We now display numerical point examples in each equilibrium region to provide the reader another way to observe how the best responses work in each region. The best outcomes associated with each best responses are underlined, so that two underlined outcomes constitute an equilibrium.

A \ B	*up*	*pp*
up	$\underline{2}$, $\underline{2}$	0.78, 1.89
pp	1.93, 0.75	$\underline{1.05}$, $\underline{0.95}$

Table 1: Numerical example when $t = 4, \gamma = 0.9, c = 1$ (region 2 NE)

A \ B	*up*	*pp*
up	3.25, 3.25	1.08, $\underline{3.29}$
pp	$\underline{3.47}$, 0.94	$\underline{1.88}$, $\underline{1.39}$

Table 2: Numerical example when $t = 6.5, \gamma = 0.5, c = 1$ (region $((pp,^{p}p))$ top)

A \ B	*up*	*pp*
up	2.5, $\underline{2.5}$	0.9, 2.45
pp	$\underline{2.63}$, 0.76	$\underline{1.51}$, $\underline{1.01}$

Table 3: Numerical example when $t = 5, \gamma = 0.5, c = 1$ (region (pp, pp) bottom)

A \ B	*up*	*pp*
up	$\underline{1}$, $\underline{1}$	0.56, 0.78
pp	0.95, $\underline{0.39}$	$\underline{0.78}$, 0.28

Table 4: Numerical example when $t = 2, \gamma = 0.5, c = 1$ (region (up, up) top)

A \ B	*up*	*pp*
up	$\underline{1}$, $\underline{1}$	$\underline{0.56}$, 0.78
pp	0.80, $\underline{0.54}$	0.53, 0.48

Table 5: Numerical example when $t = 2, \gamma = 0.5, c = 1$ (region (up, up) bottom)

A \ B	*up*	*pp*
up	1, $\underline{1}$	0.56, 0.78
pp	$\underline{1.09}$, $\underline{0.28}$	$\underline{1.05}$, 0.15

Table 6: Numerical example when $t = 2, \gamma = 0.1, c = 1$ (region (pp, up))

◇ (Optional) The mixed equilibrium in the region with 2 NE

In this subsection, we derive the mixed equilibrium strategies in the case where $\frac{t}{c} \geq 2 + \sqrt{2} - \gamma$ and $\frac{t}{c} \leq (3 + 2\sqrt{2})\gamma$. In this case, each firm follows what the other would choose. Suppose Firm B decides to use personalize price with probability v, $1 > v > 0$. Then Firm A quotes personalized prices whenever $v > \bar{v}$, and uniform prices otherwise. The same applies to Firm B. Suppose Firm A decides to use personalize price with probability w, $1 > w > 0$. Then Firm B quotes personalized prices whenever $w > \bar{w}$, and uniform price otherwise. The Mathematica file details how we find the thresholds $\bar{v} = -\frac{c^2\gamma^2-6c\gamma t+t^2}{(\gamma(3\gamma-8)+2)c^2-2(\gamma-2)ct+t^2} > 0$ and $\bar{w} = -\frac{c^2-6ct+t^2}{(2(\gamma-4)\gamma+3)c^2+2(2\gamma-1)ct+t^2} > 0$. $\square$

C Proof of Lemma 3 & 4 and Proposition 3

⋄ The four sub-game equilibria under triangular distribution

In what follows, we omit the subcript $\mathcal{T}$ and the superscript of equilibria. It alleviates notations and facilitates the reading. The reader just has to refer to the subsection of interest to get the associated equilibrium values.

- **Equilibrium (up, up).**

Given the uniform prices p_A and p_B, the marginal consumer $\tilde{x}$ who is indifferent between buying from the two firms is determined by $v - p_A - t\tilde{x} = v - p_B - t(1 - \tilde{x})$, which yields $\tilde{x} = \frac{1}{2} + \frac{p_B - p_A}{2t}$. Because all consumers pay the same price, the demand of firm A is $q_A = F(\tilde{x})$ and the demand of firm B is $q_B = 1 - F(\tilde{x})$, where $F(x) = 2x^2$ if $x < 1/2$ and $F(x) = 4x - 2x^2 - 1$. Firms' profits are given by $\pi_A = p_A q_A$, and $\pi_B = p_B q_B$.

Suppose $\tilde{x} \leq 1/2$, then $F(x) = 2x^2$. The First Order Conditions gives for firm A: $(p_A - p_B - t)(3p_A - p_B - t) = 0$; and for firm B: $2t^2 + 2p_B(p_A - p_B - t) - (p_A - p_B - t)^2 = 0$. Given that a price cannot be negative, this system yields to $p_A = p_B = \frac{t}{2}$ at equilibrium. Also, at these equilibrium prices the SOC are satisfied (we have $\frac{d^2\pi_A}{d(p_A)^2} = -3/2t < 0$ and $\frac{d^2\pi_B}{d(p_B)^2} = -5/2t < 0$). Because prices are equivalent, firms share the market by half (hence at equilibrium we indeed have that $\tilde{x} \leq 1/2$), and each firm's overall profit is $\pi_A^U = \pi_B^U = \frac{t}{4}$.

The consumer surplus is $CS = \int_0^{1/2}(v - p_A - tx)4xdx + \int_{1/2}^1(v - p_B - t(1-x))4(1-x)dx = v - \frac{5t}{6}$, and the welfare is $W = \pi_A + \pi_B + CS = v - \frac{t}{3}$. $\square$

- **Equilibrium (pp, pp).**

From the proof for the equilibrium (pp,pp) under uniform distribution, we prove that the price schedule when both firms quote personalized prices is independent of consumer distribution (this is Proposition 1 is the paper). However, profits, consumer surplus and total welfare differ as follows.

For reminder, firm A and B's personalized prices are respectively $p_A(x) = c + t(1 - 2x)$ if $x \leq \tilde{x}$

and $c\gamma$ otherwise, and $p_B(x) = c\gamma + t\,(2x-1)$ if $x \geq \tilde{x}$ and c otherwise, where $\tilde{x} = \frac{1}{2} + \frac{c(1-\gamma)}{2t} > \frac{1}{2}$.

With triangular distribution, more consumers are situated in the middle and less consumers are situated at the extreme: the distribution changes at $x = \frac{1}{2}$. The firms' profits become $\pi_A = \int_0^{\frac{1}{2}} (p_A(x) - c\gamma) 4x dx + \int_{1/2}^{\frac{1}{2}+\frac{c(1-\gamma)}{2t}} (p_A(x) - c\gamma) 4(1-x) dx = \frac{(t+c-c\gamma)^3}{6t^2} - \frac{2c^3(1-\gamma)^3}{6t^2}$ and $\pi_B = \int_{\frac{1}{2}+\frac{c(1-\gamma)}{2t}}^1 (p_B(x) - c) 4(1-x) dx = \frac{(t-c+c\gamma)^3}{6t^2}$.

In addition, we have

$$\begin{aligned} CS = & \int_0^{\frac{1}{2}} (v - p_A(x) - tx) 4x\, dx + \int_{\frac{1}{2}}^{\frac{1}{2}+\frac{c(1-\gamma)}{2t}} (v - p_A(x) - tx) 4(1-x)\, dx \\ & + \int_{\frac{1}{2}+\frac{c(1-\gamma)}{2t}}^1 (v - p_B(x) - t(1-x)) 4(1-x)\, dx \\ = & \frac{1}{6}\left(-\frac{c^3(\gamma-1)^3}{t^2} - \frac{3c^2(\gamma-1)^2}{t} - 3c(\gamma+1) - 4t + 6v \right) \end{aligned}$$

and $W = \frac{(c(\gamma-1)+t)^3}{6t^2} + \frac{c^3(\gamma-1)^3 + 3c^2(\gamma-1)^2 t - 3c(\gamma-1)t^2 + t^3}{6t^2} + \frac{1}{6}\left(-\frac{c^3(\gamma-1)^3}{t^2} - \frac{3c^2(\gamma-1)^2}{t} - 3c(\gamma+1) - 4t + 6v \right)$.
$\square$

- **Equilibrium (pp, up).**

Suppose that firm A discriminates, while B does not. The method is the same as with uniform distribution. Given firm B's uniform price p_B the indifferent consumer between buying from A and B is located at $p_A(x) = p_B + t(1-2x)$. The lowest price firm A is willing to charge to a more distant consumer is equal to its personalization cost $c\gamma$. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x} = \frac{1}{2t}\,(t + p_B - c\gamma)$. Note that $\tilde{x} = \frac{1}{2} + \frac{p_B - c\gamma}{2t} > \frac{1}{2}$ as long as $p_B > c\gamma$. Otherwise if $p_B < c\gamma$ then $\tilde{x} < \frac{1}{2}$. Let's remind remind that firm B only bears the constraint that its price p_B is positive (it does not personalize its price), and therefore $p_B < c\gamma$ is feasible. Two cases appear: (i) $p_B > c\gamma$ which implies $\tilde{x} > 1/2$, or (ii) $c\gamma > p_B > 0$ which implies $\tilde{x} < 1/2$.

(i) Assume first that $p_B > c\gamma$, which then leads to $\tilde{x} > 1/2$.

With triangular distribution firm B's demand is

$$q_B = \int_{\frac{1}{2t}(t+p_B-c\gamma)}^1 4(1-x) dx = \frac{(t - p_B + c\gamma)^2}{2t^2}$$

As firm B quotes a uniform price it incurs no personalization cost. Its profit is $\pi_B = p_B \left(\frac{(t-p_B+c\gamma)^2}{2t^2} \right)$. The FOC gives $(-3p_B + t + c\gamma)(-p_B + t + c\gamma) = 0$ and there are two potential solutions $p_B^1 = t + c\gamma$

and $p_B^2 = \frac{t+c\gamma}{3}$. The SOC at p_B^1 is not satisfied as $\frac{d^2\pi_B}{d(p_B)^2}(p_B^1) = \frac{t+c\gamma}{t^2} > 0$. At the opposite, the SOC at p_B^2 is satisfied as $\frac{d^2\pi_B}{d(p_B)^2}(p_B^2) = -\frac{t+c\gamma}{t^2} < 0$. Hence, at equilibrium firm B quotes $p_B = \frac{t+c\gamma}{3}$. Note that p_B is indeed superior to $c\gamma$ as long as $\frac{t}{c} > 2\gamma$. Otherwise, the constraint binds and we have $p_B = c\gamma$.

The indifferent consumer is thus located at $\tilde{x} = \frac{2t-c\gamma}{3t}$ which rewrites $\frac{1}{2} + \frac{t-2c\gamma}{6t}$.

Suppose $\frac{t}{c} > 2$, then it implies that $\frac{2t-c\gamma}{3t}$ is indeed greater than one half, and, in addition, it triggers an interior solution ($\frac{2t-c\gamma}{3t} \leq 1$). Firm B thus serves all consumers in the interval $\left[\frac{2t-c\gamma}{3t}, 1\right]$ while Firm A serves all consumers in the remaining interval $\left[0, \frac{2t-c\gamma}{3t}\right]$. Substituting p_B in $p_A(x)$ we find that $p_A(x) = \frac{t(4-6x)+c\gamma}{3}$ if $x \leq \frac{2t-c\gamma}{3t}$ and $c\gamma$ otherwise.

Firm B and A's profits are respectively $\pi_B = p_B q_B = \frac{2}{27}\frac{(t+c\gamma)^3}{t^2}$,and $\pi_A = \int_0^{1/2}(p_A(x) - c\gamma)4x dx + \int_{1/2}^{\frac{1}{2}+\frac{t-2c\gamma}{6t}}(p_A(x) - c\gamma)4(1-x)dx = \frac{4c^3\gamma^3+12c^2\gamma^2 t-42c\gamma t^2+31t^3}{81t^2}$.

Suppose $\frac{t}{c} < 2$, then $p_B = c\gamma$ and $\tilde{x} = 1/2$. Firm A quotes $p_A(x) = c\gamma + t(1-2x)$ if $x \leq 1/2$ and $c\gamma$ otherwise. We then get that $\pi_A = t/6$ and $\pi_B = c\gamma/2$.

(ii) Assume now that $p_B < c\gamma$, which then leads to $\tilde{x} < 1/2$. The demand of firm B is

$$q_B = \int_{\frac{1}{2t}(t+P_B-c\gamma)}^{\frac{1}{2}} 4x dx + \int_{\frac{1}{2}}^{1} 4(1-x)dx = 1 - \frac{(p_B + t - c\gamma)^2}{2t^2}$$

As firm B quotes a uniform price it incurs no personalization cost. Its profit i $\pi_B = p_B\left(1 - \frac{(p_B+t-c\gamma)^2}{2t^2}\right)$.

From the FOC we obtain that $p_B = \frac{2}{3}(c\gamma - t) + \frac{1}{3}\sqrt{c^2\gamma^2 + 7t^2 - 2ct\gamma}$, where $c^2\gamma^2 + 7t^2 - 2ct\gamma = (t - c\gamma)^2 + 6t^2$ is positive. Note that $0 < p_B < c\gamma$ as long as $\frac{t}{c} < 2\gamma$. Otherwise, the constraint binds and $p_B = c\gamma$. Therefore as by hypothesis $t/c > 2$, we have $p_B = c\gamma$ in this case. This leads to $\tilde{x} = 1/2$. Firm A then quotes $p_A(x) = c\gamma + t(1-2x)$ if $x \leq 1/2$ and $c\gamma$ otherwise. We then get that $\pi_A = t/6$ and $\pi_B = c\gamma/2$

Provided $\frac{t}{c} > 2\gamma$ then firm B prefers to quote $p_B > c\gamma$ (case (i)), rather than $p_B = c\gamma$ (case (ii)). Last, given the equilibrium prices, we have:

$$\begin{aligned}
CS = &\int_0^{\frac{1}{2}} (v - p_A(x) - tx)4x\,dx + \int_{\frac{1}{2}}^{\frac{1}{2}+\frac{t-2c\gamma}{6t}} (v - p_A(x) - tx)4(1-x)\,dx \\
&+ \int_{\frac{1}{2}+\frac{t-2c\gamma}{6t}}^{1} (v - p_{B,} - t(1-x))4(1-x)\,dx \\
=&v - \frac{c\gamma}{3} - \frac{5t}{6}
\end{aligned}$$

and $W = \frac{4c^3\gamma^3+12c^2\gamma^2 t-42c\gamma t^2+31t^3}{81t^2} - \frac{c\gamma}{3} + \frac{2(c\gamma+t)^3}{27t^2} - \frac{5t}{6} + v.\square$

- **Equilibrium (up, pp).**

Suppose that firm B discriminates, while A does not. Given firm A's uniform price p_A the indifferent consumer between buying from A and B is located at $p_B(x) = p_A + t(2x-1)$. The lowest price firm B is willing to charge to a more distant consumer is equal to its personalization cost c. Therefore, the consumer who is indifferent between buying from B at the lowest price and from A at price p_A is located at $\tilde{x} = \frac{1}{2} - \frac{p_A - c}{2t}$.

Note that $\tilde{x} < \frac{1}{2}$ whenever $p_A > c$. Otherwise if $p_A < c$ we have $\tilde{x} > \frac{1}{2}$.

(i) Assume first that $p_A > c$, which implies $\tilde{x} < 1/2$. With triangular distribution firm A's demand is

$$q_A = \int_0^{\frac{1}{2t}(t-P_A+c)} 4x dx = \frac{(c-p_A+t)^2}{2t^2}$$

As firm A quotes a uniform price it incurs no personalization cost. Its profit is $\pi_A = p_A \frac{(c-p_A+t)^2}{2t^2}$. From the FOC for the profit maximization with respect to p_A we obtain that $(c-3p_A+t)(c-p_A+t) = 0$ which leads to two candidate prices $p_A^1 = t+c$ and $p_A^2 = \frac{t+c}{3}$. Only the SOC at p_A^2 is satisfied as we have $\frac{d^2\pi_A}{d(p_A)^2}(p_A^1) = \frac{t+c}{t^2} > 0$ and $\frac{d^2\pi_A}{d(p_A)^2}(p_A^2) = -\frac{t+c}{t^2} < 0$. At equilibrium, firm A thus quotes $p_A = \frac{t+c}{3}$. Note that $p_A > c$ whevener $\frac{t}{c} > 2$, otherwise the constraint binds and $p_A = c$.

The indifferent consumer is located at $\tilde{x} = \frac{t+c}{3t}$ which rewrites $\tilde{x} = \frac{1}{2} - \frac{t-2c}{6t}$.

Suppose $\frac{t}{c} > 2$, we recover that $\tilde{x} < 1/2$ and also have an interior solution as $\frac{t+c}{3t} > 0$. Therefore firm A serves all consumers in the interval $\left[0, \frac{t+c}{3t}\right]$. Firm B serves all consumers in the remaining interval, i.e., those consumers who belong to the interval $\left[\frac{t+c}{3t}, 1\right]$. Substituting p_A in $p_B(x)$ we find that $p_B(x) = \frac{1}{3}(c+6tx-2t)$ if $x \geq \frac{t+c}{3t}$ and c otherwise. Firm A and B's profits are respectively $\pi_A = \frac{2(c+t)^3}{27t^2}$ and $\pi_B = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2}$.

Suppose $\frac{t}{c} < 2$, then $p_A = c$ and $\tilde{x} = 1/2$. Firm B quotes $p_B(x) = c + t(2x-1)$ if $x \leq 1/2$ and $c\gamma$ otherwise. We then get that $\pi_A = c/2$ and $\pi_B = t/6$.

(ii) Assume now that $p_A < c$, then $\tilde{x} > 1/2$. The demand of firm A is:

$$q_A = \int_0^{\frac{1}{2}} 4x dx + \int_{\frac{1}{2}}^{\frac{1}{2t}(t-p_A+c)} 4(1-x) dx = \frac{2t(c-p_A)-(c-p_A)^2+t^2}{2t^2}$$

As firm A quotes a uniform price it incurs no personalization cost. Its profit is $\pi_A = p_A q_A$.

From the FOC we obtain that $p_A = \frac{1}{3}\left(\sqrt{c^2-2ct+7t^2}+2c-2t\right)$, where $c^2-2ct+7t^2 = (t-c)^2+6t^2$ is positive. Note that $0 < p_A < c$ as long as $\frac{t}{c} < 2$, otherwise $p_A = c$. Since by assumption $\frac{t}{c} > 2$, we have $p_A = c$ and therefore Firm B quotes $p_B(x) = c + t(2x-1)$ if $x \leq 1/2$ and $c\gamma$ otherwise. We then get that $\pi_A = c/2$ and $\pi_B = t/6$.

Provided $\frac{t}{c} > 2\gamma$ then firm A prefers to quote $p_A > c$ (case (i)), rather than $p_A = c$ (case (ii)).

Last, we have:

$$\begin{aligned}
CS = &\int_0^{\frac{1}{2}-\frac{t-2c}{6t}} (v - p_A - tx)4x\,dx + \int_{\frac{1}{2}-\frac{t-2c}{6t}}^{\frac{1}{2}} (v - p_B(x) - t(1-x))4x\,dx \\
&+ \int_{\frac{1}{2}}^{1} (v - p_B(x) - t(1-x))4(1-x)\,dx \\
= &-\frac{c}{3} - \frac{5t}{6} + v
\end{aligned}$$

and $W = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2} + \frac{2(c+t)^3}{27t^2} - \frac{c}{3} - \frac{5t}{6} + v.\square$

$\diamond$ The equilibrium regions under triangular distribution

In same purpose as above, we omit the subscript $\mathcal{U}$.

A \ B	*up*	*pp*
up	$\pi_A^{up,up}, \pi_B^{up,up}$	$\pi_A^{up,pp}, \pi_B^{up,pp}$
pp	$\pi_A^{pp,up}, \pi_B^{pp,up}$	$\pi_A^{pp,pp}, \pi_B^{pp,pp}$

Firm A

- $\pi_A^{pp,up} > \pi_A^{up,up}$ whenever $\frac{t}{c} \gtrapprox 3.56\gamma$.

Proof. Reminder: $\pi_A^{up,up} = \frac{t}{4}$ and $\pi_A^{pp,up} = \frac{31t^3-42ct^2\gamma+12c^2t\gamma^2+4c^3\gamma^3}{81t^2}$.
Alternatively, we have $\pi_A^{pp,up} = \left(\frac{c^3\gamma^3}{81t^2}\right)\left(31(\frac{t}{c\gamma})^3 - 42(\frac{t}{c\gamma})^2 + 12(\frac{t}{c\gamma}) + 4\right)$. Now suppose $b = \frac{t}{c\gamma} > 2$, then the difference of profits writes:

$$\pi_A^{pp,up} - \pi_A^{up,up} = \left(31b^3 - 42b^2 + 12b + 4\right)\left(\frac{c\gamma}{81b^2}\right) - \frac{t}{4}$$

From the writing of b, we get $t = bc\gamma$ which leads to

$$\pi_A^{pp,up} - \pi_A^{up,up} = \left(31b^3 - 42b^2 + 12b + 4\right)\left(\frac{c\gamma}{81b^2}\right) - \frac{bc\gamma}{4}$$

$$\pi_A^{pp,up} - \pi_A^{up,up} = c\gamma \left((31b^3 - 42b^2 + 12b + 4)(\frac{1}{81b^2}) - \frac{b}{4} \right)$$

Therefore, we find $\pi_A^{pp,up} - \pi_A^{up,up} \geq 0$ whenever $(31b^3 - 42b^2 + 12b + 4)(\frac{1}{81b^2}) - \frac{b}{4} \geq 0 \Leftrightarrow 43b^3 - 168b^2 + 48b + 16 \geq 0$ which is true as long as $b \gtrapprox 3.56$ (see Mathematica file). Put together, we find that $\pi_A^{pp,up} - \pi_A^{up,up} \geq 0$ whenever $\frac{t}{c} \gtrapprox 3.56\gamma$. ■

- $\pi_A^{pp,pp} > \pi_A^{up,pp}$ whenever $\frac{t}{c} > m(\gamma)$, with

$$\begin{aligned} m(\gamma) =& \frac{108\sqrt[3]{2}\gamma^2}{5\sqrt[3]{486\gamma^3 + 36450\gamma^2 + \sqrt{(486\gamma^3 + 36450\gamma^2 - 36450\gamma + 12150)^2 - 136048896\gamma^6} - 36450\gamma + 12150}} \\ &+ \frac{\sqrt[3]{486\gamma^3 + 36450\gamma^2 + \sqrt{(486\gamma^3 + 36450\gamma^2 - 36450\gamma + 12150)^2 - 136048896\gamma^6} - 36450\gamma + 12150}}{15\sqrt[3]{2}} \\ &+ \frac{1}{5}(9\gamma - 5) \end{aligned}$$

which can be interpolated by the polynomial $m(\gamma) \approx 3.206 + 4.11604(-1 + \gamma)$ on the domain $\gamma \in D = [0.707, 1]$.

Proof. Reminder: $\pi_A^{up,pp} = \frac{2(c+t)^3}{27t^2}$ and $\pi_A^{pp,pp} = \frac{t^3 + 3ct^2(1-\gamma) + 3c^2t(1-\gamma)^2 - c^3(1-\gamma)^3}{6t^2}$. Alternatively, we have $\pi_A^{pp,pp} = \left(\frac{c^3}{6t^2}\right)\left((\frac{t}{c})^3 + 3(\frac{t}{c})^2(1-\gamma) + 3(\frac{t}{c})(1-\gamma)^2 - (1-\gamma)^3\right)$ and $\pi_A^{up,pp} = \frac{2c^3}{27t^2}(1 + \frac{t}{c})^3$ Now suppose $b = \frac{t}{c} > 2$, then the difference of profits writes:

$$\pi_A^{pp,pp} - \pi_A^{up,pp} = \frac{c}{3b^2}\left[\frac{1}{2}\left((b)^3 + 3(b)^2(1-\gamma) + 3(b)(1-\gamma)^2 - (1-\gamma)^3\right) - \frac{2}{9}(1+b)^3\right]$$

We find $\frac{1}{2}\left((b)^3 + 3(b)^2(1-\gamma) + 3(b)(1-\gamma)^2 - (1-\gamma)^3\right) - \frac{2}{9}(1+b)^3 \geq 0$ whenever $b > m(\gamma)$ (see Mathematica file). ■

Remarks: we find that $3.56\gamma > 2$ whenever $0.561 \lessapprox \gamma$ and $m(\gamma) > 2$ whenever $0.707 \lessapprox \gamma$. In addition, $3.56\gamma > m(\gamma)$ whenever $0.173 \lessapprox \gamma$. Therefore, we always have that $3.56\gamma \geq m(\gamma)$ whenever both functions are defined and otherwise the constraint $t/c > 2$ prevails.

- We find :

(i) $\frac{t}{c} \gtrapprox 3.56\gamma \Rightarrow pp \succ up$ irrespective of rival's choice ;

(ii) $m(\gamma) < \frac{t}{c} \lessapprox 3.56\gamma \Rightarrow \begin{cases} pp \succ up \text{ when rival uses } pp \\ up \succ pp \text{ when rival uses } up \end{cases}$;

(iii) $\frac{t}{c} < m(\gamma) \Rightarrow up \succ pp$ irrespective of rival's choice.

Firm B

• $\pi_B^{up,pp} > \pi_B^{up,up}$ whenever $\frac{t}{c} \gtrapprox 3.56$.

Proof. Reminder: $\pi_B^{up,up} = \frac{t}{4}$ and $\pi_B^{up,pp} = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2}$.
Alternatively, we have $\pi_B^{pp,up} = \left(\frac{c^3}{81t^2}\right)\left(4+12(\frac{t}{c})-42(\frac{t}{c})^2+31(\frac{t}{c})^3\right)$. Now suppose $b = \frac{t}{c} > 2$, then the difference of profits writes:

$$\pi_B^{up,pp} - \pi_B^{up,up} = \left(31b^3 - 42b^2 + 12b + 4\right)((\frac{c}{81b^2})) - \frac{t}{4}$$

From the writing of b, we get $t = bc$ which leads to

$$\pi_B^{up,pp} - \pi_B^{up,up} = \left(31b^3 - 42b^2 + 12b + 4\right)(\frac{c}{81b^2}) - \frac{bc}{4}$$

$$\pi_B^{up,pp} - \pi_B^{up,up} = c\left((31b^3 - 42b^2 + 12b + 4)(\frac{1}{81b^2}) - \frac{b}{4}\right)$$

Therefore, we find $\pi_B^{pp,up} - \pi_B^{up,up} \geq 0$ whenever $(31b^3 - 42b^2 + 12b + 4)(\frac{1}{81b^2}) - \frac{b}{4} \geq 0 \Leftrightarrow 43b^3 - 168b^2 + 48b + 16 \geq 0$ which is true as long as $b \gtrapprox 3.56$ (see Mathematica file). Put together, we find that $\pi_B^{up,pp} - \pi_B^{up,up} \geq 0$ whenever $\frac{t}{c} \gtrapprox 3.56 > 2$. ■

• $\pi_B^{pp,pp} > \pi_B^{pp,up}$ whenever $\frac{t}{c} \gtrapprox 4.22 - \gamma$.

Proof. Reminder: $\pi_B^{pp,pp} = \frac{(t-c(1-\gamma))^3}{6t^2}$ and $\pi_B^{pp,up} = \frac{2(t+c\gamma)^3}{27t^2}$.
Alternatively, we have $\pi_B^{pp,pp} = \left(\frac{c^3}{6t^2}\right)\left(\frac{t}{c} - (1-\gamma)\right)^3$ and $\pi_B^{pp,up} = \left(\frac{2c^3}{27t^2}\right)\left(\frac{t}{c} + \gamma\right)^3$. Now suppose $b = \frac{t}{c} > 2$, then the difference of profits writes:

$$\pi_B^{pp,pp} - \pi_B^{pp,up} = \left(\frac{c}{3b^2}\right)\left(\frac{1}{2}(b+\gamma-1)^3 - \frac{2}{9}(b+\gamma)^3\right)$$

We have $\pi_B^{pp,pp} \geq \pi_B^{pp,up}$ whenever $5(b+\gamma)^3 - 27(b+\gamma)^2 + 27(b+\gamma) - 9 \geq 0$, which occurs upon $b + \gamma \gtrapprox 4.22$. Put together, we find that $\pi_B^{pp,pp} - \pi_B^{pp,up} \geq 0$ whenever $\frac{t}{c} \gtrapprox 4.22 - \gamma > 2$. ■ □

⋄ Numerical point examples

We now display numerical point examples in each equilibrium region to provide the reader another way to observe how the best responses work in each region. The best outcomes associated with each best responses are underlined, so that two underlined outcomes constitute an equilibrium.

A \ B	*up*	*pp*
up	1.25, 1.25	0.64, $\underline{1.43}$
pp	$\underline{1.76}$, 0.44	$\underline{1.23}$, $\underline{0.53}$

Table 7: Numerical example when $t = 5, \gamma = 0.3, c = 1$ (region (pp, pp) top)

A \ B	*up*	*pp*
up	0.93, 0.93	0.56, $\underline{0.94}$
pp	$\underline{1.26}$, $\underline{0.34}$	$\underline{1.03}$, 0.33

Table 8: Numerical example when $t = 3.7, \gamma = 0.3, c = 1$ (region (pp, up) top)

A \ B	*up*	*pp*
up	0.75, $\underline{0.75}$	0.53, 0.69
pp	$\underline{1}$, $\underline{0.30}$	$\underline{0.93}$, 0.23

Table 9: Numerical example when $t = 3, \gamma = 0.3, c = 1$ (region (pp, up) bottom)

A \ B	*up*	*pp*
up	$\underline{0.75}$, $\underline{0.75}$	0.53, 0.68
pp	0.73,$\underline{0.49}$	$\underline{0.55}$, 0.45

Table 10: Numerical example when $t = 3, \gamma = 0.9, c = 1$ (region (up, up) top)

A \ B	*up*	*pp*
up	$\underline{0.53}$, $\underline{0.53}$	$\underline{0.50}$, 0.37
pp	0.40, $\underline{0.45}$	0.40, 0.30

Table 11: Numerical example when $t = 2.1, \gamma = 0.9, c = 1$ (region (up, up) bottom)

A \ B	*up*	*pp*
up	0.85, $\underline{0.85}$	0.55, 0.83
pp	$\underline{0.87}$, 0.51	$\underline{0.62}$, $\underline{0.52}$

Table 12: Numerical example when $t = 3.4, \gamma = 0.9, c = 1$ (region (pp, pp) bottom)

A \ B	up	pp
up	$\underline{0.85}$, $\underline{0.85}$	0.55, 0.83
pp	0.83, 0.54	$\underline{0.57}$, $\underline{0.56}$

Table 13: Numerical example when $t = 3.4, \gamma = 0.99, c = 1$ (region 2 NE)

C.1 Proof of Proposition 4

■ **Region** (pp, pp)**.**

- *Triangular:*

$$\begin{aligned} AreaPPTri =& (\int_0^1 7\ dx - \int_0^{0.925} (4.22 - x)\ dx - \int_{0.925}^1 (3.56x)\ dx) * (100/5) \\ \approx& 65.34 \end{aligned}$$

- *Uniform:*

$$\begin{aligned} AreaPPUni =& (\int_0^1 7\ dx - \int_0^{0.5} (\sqrt{2} + 2 - x)\ dx - \int_{0.5}^1 (3 + 2\sqrt{2})x\ dx) * (100/5) \\ \approx& 64.64 \end{aligned}$$

■ **Region** (up, up)**.**

- *Triangular:*

$$\begin{aligned} AreaUUTri =& (\int_{0.561}^1 3.56x\ dx - \int_{0.561}^1 2\ dx) * (100/5) \\ \approx& 6.84 \end{aligned}$$

- *Uniform:*

$$\begin{aligned} AreaUUUni =& (\int_{0.34}^1 (3 + 2\sqrt{2})x\ dx - \int_{0.34}^1 2\ dx) * (100/5) \\ \approx& 25.15 \end{aligned}$$

■ **Region** (pp, up)**.**

- *Triangular:*

$$AreaPUTri = (\int_0^{0.925} 4.22 - x\ dx - \int_0^{0.561} 2\ dx - \int_{0.561}^{0.925} 3.56x\ dx) * (100/5) \approx 27.82$$

- *Uniform:*

$$AreaPUUni = (\int_{0}^{0.5} \sqrt{2} - 2 - x\ dx - \int_{0}^{0.34} 2\ dx - \int_{0.34}^{0.5} (3 + 2\sqrt{2})x\ dx) * (100/5) \approx 10.21$$

D Proof of Proposition 6

Mathematica file available upon request.

E Proof of Proposition 7

Mathematica file available upon request.

F Discussion: intermediate values of consumer preferences.

In real-world settings, the consumers are likely heterogenous in their reactions to ads, or the marketplace might advertize in a lesser propension. These reasons would diminish the propensity of consumers gathering at the center of the Hotelling segment. Formally, we will assume that the distribution of consumer preferences generalizes to $f(x) = 4\beta x + 1 - \beta$ if $x < 1/2$, and $f(x) = 4\beta(1 - x) + 1 - \beta$ otherwise, where $\beta \in [0, 1]$. The parameter β denotes the propension of consumers to become brand indifferent, i.e. gather towards the middle of the segment. We retrieve the our cases where when they remain loyal ($\beta = 0$) or indifferent ($\beta = 1$).

The presence of this new parameter drastically complexifies the analysis, we thus counter-balance by focusing the computations on the case where $t = 1$ and $\gamma = 1$. Figure 8 summarizes the partition of equilibria where $l_1(\beta) = 1/c_1(\beta)$ is such that $\pi_A^{pp,pp}(c_1(\beta)) - \pi_A^{up,pp}(c_1(\beta)) = 0$, and $l_2(\beta) = 1/c_2(\beta)$ is such that $\pi_A^{pp,up}(c_2(\beta)) - \pi_A^{up,up}(c_2(\beta)) = 0$.

We are then able to show that $l_1(\beta)$ increases wrt β, whereas the result is more ambiguous for $l_2(\beta)$ (it decreases until $\beta \approx 0.472$ and then increases). In other words, we find that our previous results are likely non monotonic to intermediate values of brand indifference.

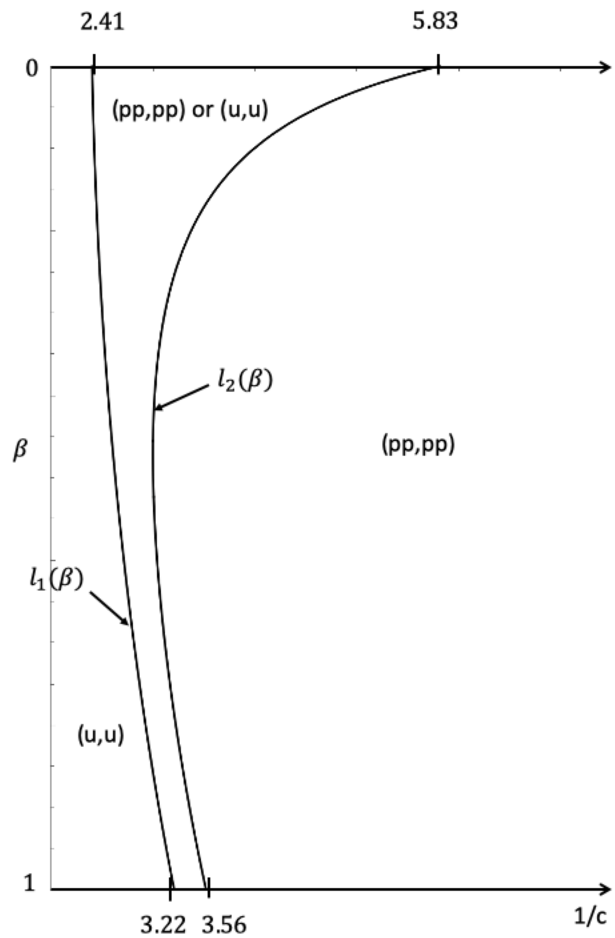

Figure 8: Partition of SPNE with symmetric firms and $t = 1$