Management turnover, strategic ambiguity and supply incentives

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Abstract

When a firm changes its manager, it reopens room for contractual frictions with its partners. We explore *strategic ambiguity* as a potential friction with a supplier. The firm's new manager likely holds fuzzy expectations about the supplier's strategy. An optimistic manager weights favorable strategies more than detrimental ones whereas a pessimistic manager does the reverse. We show that the manager's degree of optimism is critical: above a threshold it can incite the supplier to change its timing of contracting and increase its profits. We also find that this threshold degree of optimism depends on the length of product substitution.

Keywords: vertical contracting; strategic ambiguity; ambiguity attitude

JEL classification: L14; L22; D8

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1 Introduction

An upstream monopolist contracting with two downstream firms may build on repeated relationships to design better contracts and increase its profits (Ryall and Sampson, 2009; Gilo and Yehezkel, 2020). After years of contracting, it is likely that a downstream firm incurs a management turnover implying that the monopolist now has to deal with a new manager. This new relationship reopens room for contractual frictions and potentially threatens the monopolist's profits.

In this paper, we actually show that the monopolist can benefit from contractual friction when the latter takes the form of *strategic ambiguity*. Strategic ambiguity arises when the new manager holds fuzzy expectations about the monopolist's strategy. With such expectations, the new manager holds an objective function which differs from one with rational expectations. In particular, an optimistic manager weights favorable monopolist's strategies more than detrimental ones whereas a pessimistic manager does the reverse.

Experimental evidence shows that individuals exhibit strategic ambiguity and optimism (Eichberger *et al.*, 2007; Ivanov, 2011). Moreover, Armstrong and Huck (2010) points out that entrepreneurs are more prone to optimism, about one's own ability or about the probability of favorable outcomes, than other individuals. Our paper theoretically demonstrates that when the new manager is sufficiently optimistic, the monopolist can earn greater profits than without management turnover. Our paper also shows that product differentiation mitigates this result.

To give room to strategic ambiguity and optimism, the monopolist just has to approach first the new manager and only then the remaining one. When products are perfect substitutes, competition is fierce and the new manager expects the market price to be nil in its worst scenario. Provided the manager is sufficiently optimistic, the monopolist sells it some quantities and then lets the remaining firm serve the residual demand. With unrelated products, competition is inexistant. The new manager expects the same profit irrespective of its optimism and the monopolist extracts the same profits as without turnover. With imperfect substitutes, competition is soft and the manager can expect positive profits even in the worst scenario. The monopolist can still sell it some quantities and then lets the remaining firm serve the residual demand. However, the new manager must be more optimistic than without product differentiation.

Our paper mainly contributes to the flourishing literature that revisits industrial organization settings in light with ambiguity concepts (Eichberger *et al.*, 2009; Król, 2012; Kauffeldt and Wiesenfarth, 2018). To the best of our knowledge this literature is silent regarding vertical relationships. We are thus the first to study a vertical relationship in light with strategic ambiguity. We link management turnover to strategic ambiguity and show that it can benefit an upstream monopolist. In addition, we find that the effect of optimism genuinely depends on the degree of product substitution.

The paper is organized as follows. Section 2 presents the benchmark situation, which is the absence of management turnover Section 3 states the new equilibrium strategies consecutive to management turnover, i.e. to the introduction of strategic ambiguity. Section 4 compares the two situations. Section 5 concludes. All proofs are relegated to the Appendix.

2 The benchmark situation

We assume an upstream monopolist, denoted by U, producing inputs at zero marginal cost. This monopolist supplies two undifferentiated downstream firms, denoted D_1 and D_2. To this end, U proposes a contract $c_i = (q_i, f_i)$ to each D_i, where $i \in 1, 2$ and $q_i \in [0, 1]$ stands for the input quantity delivered to D_i while $f_i \in [0, 1]$ is the fixed tariff paid by D_i to U in exchange for such a quantity.[1] Let $\mathcal{C}_i = [0, 1]^2$ be the set of the contracts U can propose to D_i. Each D_i decides whether to accept, $a_i = 1$, or reject, $a_i = 0$, its contract offer and pays f_i upon acceptance.

A strategy for U is thus to propose two bilateral contracts. Formally, it denotes $c = (c_1, c_2)$, or equivalently $c = (q_1, f_1, q_2, f_2)$. The set of U's strategies is therefore the set of these bilateral contracts, which denotes $\mathcal{C} = \mathcal{C}_1 \times \mathcal{C}_2 = [0, 1]^4$. The monopolist's profit is:

$$\pi_U(c, a_1(c), a_2(c)) = a_1(c) \cdot f_1 + a_2(c) \cdot f_2, \tag{1}$$

[1] We suppose such a contract set to avoid slotting allowances (case where $f < 0$) and unfeasible contracts given our inverse demand (case where $f > 1$).

where $a_i(c)$ is the response by firm i to the monopolist contract strategy c.

Once a contract is accepted or rejected, it is transparently disclosed to the firms. The firms transform the inputs into homogeneous outputs on a one-to-one basis and compete downstream *à la Cournot* with the inverse demand $P(q_i + q_j) = 1 - q_i - \gamma q_j$ with $j \neq i$ and where γ refers to the level of product substitution. When $\gamma = 0$, goods are unrelated whereas as γ tends towards 1, goods become more en more substitutes until they reach perfect substitution $\gamma = 1$.

We assume for simplicity that if the firms have purchased quantities q_1 and q_2, they find it optimal to transform all units of inputs into final goods. Structural reasons such as a sufficiently high cost for stocking or destroying the inputs can sustain this behaviour. Firm i's revenues can thus be summarized by Cournot total revenue functions and its profit is:

$$\pi_i(a_i(c), a_j(c), c) = a_i[(1 - q_i - \gamma a_j(c).q_j)q_i - f_i] \qquad j = 1, 2 \;\&\; j \neq i \tag{2}$$

From the linear demand function and given final outputs, we get the following consumer surplus, denoted by CS:

$$CS = \frac{(q_1)^2 + 2\gamma q_1 q_2 + (q_2)^2}{2} \tag{3}$$

Consider the case where the monopolist makes simultaneous offers and can commit not to renegotiate contracts. That is the best situation in the literature about vertical contracting. We solve the game using the Subgame Perfect Nash Equilibrium (SPNE) concept. We focus on the symmetric equilibrium because firms are undifferentiated at this point. We find the standard result that the monopolist supplies half the monopoly quantity to each firm and the monopolist earns the monopoly profit (Rey and Tirole, 2007). Lemma 1 summarizes our findings:

Lemma 1. *In the benchmark situation, the symmetric SPNE in pure strategy implies the following equilibrium outcomes:*

$$q_1^B(\gamma) = q_2^B(\gamma) = \frac{1}{2(1+\gamma)}\,, \quad f_1^B(\gamma) = f_2^B(\gamma) = \frac{1}{4(1+\gamma)} \tag{4}$$

$$\pi_U^B(\gamma) = \frac{1}{2(1+\gamma)}\,, \quad CS^B(\gamma) = \frac{1}{4+\gamma} \tag{5}$$

In addition, Lemma 1 shows that the monopolist is better off as product substitution decreases ($d\pi_U^B/d\gamma = -1/(2(1+\gamma^2) < 0)$. This happens because consumers disentangle more and more the products and as a consequence competition softens. At some point, i.e. when products are unrelated ($\gamma = 0$), firms even become local monopolists.

3 Management turnover

In this section, we suppose without loss of generality that downstream firm D_1 changes its manager and it raises *strategic ambiguity*. If the supplier continues to make simultaneous offers then observability and completeness mute strategic ambiguity: the new manager is able to observe all the contract when it has to choose whether to accept its own. Consequently, the result would be the same as in the benchmark situation. If we instead suppose the monopolist sequences its timing of contracting, in particular if it enters first into contracts with the new manager and then approaches the other firm, we enable strategic ambiguity to play a major role.

3.1 Strategic ambiguity and sequential contracting

We focus on the situation where the new manager holds strategic ambiguity towards the monopolist's strategy. In contrast, we suppose the new manager has time to monitor the rival (contract histories, etc.) and thus does not feel any strategic ambiguity towards the rival's decision.

In addition, we assume the monopolist knows the attitude towards ambiguity of the new manager: the monopolist has sufficient time to monitor the new manager before making its offer, whereas the new manager likely lacks the time to monitor the monopolist, additively to the rival, when it receives its offer.

Finally, we further suppose that due to the previous repeated relationship with the second firm, the monopolist can still credibly commit to the latter not to renegotiate contracts afterwards (complete contracts).

We model the new manager's strategic ambiguity using an $\alpha-$ MaxMin Expected Utility ($\alpha-$MEU).[2] Specifically, D_1 takes a decision weighting the best and worst profits it can get given the action

[2] In our case, this utility function boils down to the Hurwicz criterion (Hurwicz, 1951; Arrow and Hurwicz, 1972).

of the monopolist, c_1, its expectation about the future strategies available to the monopolist, c_2 - which D_1 feels ambiguous about - and given the expectation about the strategy of the rival, $a_2(c)$ - which D_1 does not feel ambiguous about. The expected profit of D_1 for choosing action $a_1(c_1)$ thus is:

$$E\pi_1(a_1(c_1), a_2(c), c) = a_1(c_1)[(1-\alpha)\max_{c_2 \in \mathcal{C}_2} \pi_1(a_1(c_1), a_2(c), c) + \alpha \min_{c_2 \in \mathcal{C}_2} \pi_1(a_1(c_1), a_2(c), c)] \quad (6)$$

Criterion (6) has been axiomatically characterized by Chateauneuf *et al.* (2020). The parameter $\alpha \in [0, 1]$ represents the level of pessimism of D_1, i.e. its attitude towards strategic ambiguity.

3.2 The optimal choice and final outcomes

We look for the SPNE in pure strategies of the game and thus solve the game by using backward induction. We omit to write the history of strategies as functions of history of past strategies to alleviate notation. When D_2 observes its offer, it also observes the previous contract offer and D_1's decision. D_2 accepts whenever its profit is positive. Formally, it means:

$$a_2^*(c_2, a_1, c_1) = \begin{cases} 1 & \text{if } \pi_2(a_2 = 1|c, a_1) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\pi_2(a_2 = 1|c, a_1) = (1 - \gamma a_1 q_1 - q_2)q_2 - f_2$. We focus on equilibria where the firms accept their offers, this means that the monopolist's offer to D_2 must satisfy $f_2 \leq (1 - \gamma a_1 q_1 - q_2)q_2$.

When the monopolist makes the offer to D_2, it anticipates D_2's decision, given its own previous decision and D_1's one. The monopolist thus maximizes $\pi_U = f_1 + f_2$ where $f_2 \leq (1 - \gamma a_1 q_1 - q_2)q_2$ and f_1 is sunk (because already paid by D_1 at this stage). The profit is increasing in f_2 so the monopolist extracts all the rent and the profit rewrites $\pi_U = (1 - \gamma a_1 q_1 - q_2)q_2$. Obviously, the monopolist maximizes this profit for any contract offer $(q_2^*(c_1, a_1), f_2^*(a_1, c_1))$ such that

$$q_2^*(c_1, a_1) = \frac{1 - \gamma a_1 q_1}{2} \quad \text{and} \quad f_2^*(c_1, a_1) = \frac{(1 - \gamma a_1 q_1)^2}{4}$$

where $q_2^*(c_1, a_1)$ is simply the Cournot best response to $a_1 q_1$.

When D_1 gets its offer c_1, it has to anticipate the other firms' future decisions. This anticipation is critical. D_1 perfectly anticipates D_2's decision, a_2^*. However, since D_1 is ambiguous towards the monopolist's decision, it weighs the best and worst outcome induced by all the strategies available to the monopolist at the next stage, $c_2 \in \mathcal{C}_2$.

More formally, and by applying eq.(6), we get that D_1's expected profit from accepting the offer is $E_{c_2}\pi_1(a_1 = 1, c_2, a_2^*|c_1) = (1-\alpha)\max_{c_2 \in \mathcal{C}_2} \pi_1(a_1 = 1, c_2, a_2^*|c_1) + \alpha \min_{c_2 \in \mathcal{C}_2} \pi_1(a_1 = 1, c_2, a_2^*|c_1)$. D_1 accepts whenever its expected profit is positive and we now formally get:

$$a_1^*(c_1) = \begin{cases} 1 & \text{if } E_{c_2}\pi_1(a_1 = 1, c_2, a_2^*|c_1) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where the expected profit simplifies to $E_{c_2}\pi_1(a_1 = 1, c_2, a_2^*|c_1) = (1-\alpha)(1-q_1)q_1 + \alpha(1-\gamma-q_1)q_1 - f_1$ if $q_1 \leq 1-\gamma$ and $E_{c_2}\pi_1(a_1 = 1, c_2, a_2^*|c_1) = (1-\alpha)(1-q_1)q_1 + 0 - f_1$ otherwise.

Intuitively, given contract offer $c_1 = (q_1, f_1)$, the best outcome appears in the event where the monopolist offers nothing to D_2, $q_2 = 0$, and requests nothing to the latter (so that D_2 accepts this contract). The worst outcome appears in the scenario where the monopolist offers the maximum quantity to D_2, i.e. $q_2 = 1$, which potentially drives the market price to zero, and requests nothing in exchange of such a quantity (so that again D_2 accepts).

We focus on equilibria where the firms accept, so the monopolist's offer to D_1 must satisfy $f_1 \leq (1-\alpha)(1-q_1)q_1 + \alpha \text{Max}\{(1-\gamma-q_1)q_1, 0\}$.

When the monopolist decides the offer for D_1, it anticipates the other firms' strategies. The monopolist thus maximizes $\pi_U = f_1 + f_2$ where $f_2 \leq \frac{(1-\gamma q_1)^2}{4}$ and $f_1 \leq (1-\alpha)(1-q_1)q_1 + \alpha \text{Max}\{(1-\gamma-q_1)q_1, 0\}$. The profit is increasing in the fees so, for a given level of optimism α, the monopolist extracts all the rent.

We find that the monopolist's simplified program depends on the product substitution. We develop first the two extreme cases (perfect substitutes and unrelated products) and then study the intermediate case. Last, we summarize the three cases in a graph.

■ Perfect substitutes ($\gamma = 1$)

With perfect substitutes, downstream competition is fierce. The new manager expects a nil market price - and hence no profit - in the worst scenario because the monopolist would give too much quantity to the rival ($P(q_1 + 1) = 0\,,\ \forall q_1 \in [0, 1]$). Given level of optimism α, the monopolist program is

$$\underset{q_1 \in [0,1]}{\text{Max}}\ \pi_U(q_1|\alpha) = \frac{(1 - \gamma q_1)^2}{4} + (1 - \alpha)(1 - q_1)q_1 \tag{7}$$

We find the following SPNE strategy which depends on the new manager's level of optimism α.

$$q_1^T(\alpha, 1) = \begin{cases} \frac{1-2\alpha}{3-4\alpha} & \text{if } \alpha < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}, \qquad q_2^T(\alpha, 1) = \begin{cases} \frac{1-\alpha}{3-4\alpha} & \text{if } \alpha < \frac{1}{2} \\ \frac{1}{2} & \text{otherwise} \end{cases}, \tag{8}$$

$$f_1^T(\alpha, 1) = \begin{cases} \frac{2(1-\alpha)^2(1-2\alpha)}{(3-4\alpha)^2} & \text{if } \alpha < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}, \qquad f_2^T(\alpha, 1) = \begin{cases} \frac{(1-\alpha)^2}{(3-4\alpha)^2} & \text{if } \alpha < \frac{1}{2} \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

Because the new manager believes that the market price in the worst scenario is nil and so are its profits, it accepts the monopolist's contract only when the best scenario is sufficiently attractive. This happens when optimism and expected profits are jointly sufficiently high. Otherwise, it rejects the offer. In both circumstances, the monopolist then designs the last contract so that the last firm serves the residual demand.

In the end, there is at least the monopoly quantity in the final market ($q_1^T + q_2^T = 1/2 + (1 - 2\alpha)/(6 - 8\alpha) > 1/2\,,\ \forall \alpha < 1/2$ and $q_1^T + q_2^T = 1/2$, otherwise) and the monopolist extracts at least the monopoly profit ($f_1^T + f_2^T = (1 - \alpha)^2)/(3 - 4\alpha) > 1/4\,,\ \forall \alpha < 1/2$ and $f_1^T + f_2^T = 1/4$, otherwise).

Proposition 1. *With perfect substitutes, management turnover enables the monopolist to earn at least the monopoly profit irrespective of the new manager's level of optimism.*

■ Unrelated products ($\gamma = 0$)

In contrast, with unrelated products, firms become local monopolists. The new manager expects the same profit in the worst scenario as in the best scenario because it does not consider the rival.

Irrespective of the manager's optimism, the monopolist program is

$$\underset{q_1\in[0,1]}{\text{Max}}\ \pi_U(q_1|\alpha) = \frac{1}{4} + (1-q_1)q_1 \tag{9}$$

We find that the SPNE strategy this time does not depend on the new manager's level of optimism α. The monopolist offers $q_1^T(\alpha,0) = 1/2$, $q_2^T(\alpha,0) = 1/2$ in exchange for $f_1^T(\alpha,0) = 1/4$ and $f_2^T(\alpha,0) = 1/4$.

With unrelated products, the firms does not compete anymore. The monopolist thus enables each firm to act as local monopolists on their respective markets.

In the end, there is the monopoly quantity in each final market ($q_1^T = q_2^T = 1/2$) and the monopolist extracts the monopoly profits in each market ($f_1^T = f_2^T = 1/4$).

Proposition 2. *With unrelated products, management turnover enables the monopolist to earn the monopoly profit on each market irrespective of the new manager's level of optimism.*

Proposition 2 suggests that as product substitution softens the monopolist seems to be better off. This seems to follow the benchmark pattern, however, as we will see the relation is not that simple with management turnover.

■ Imperfect substitutes ($0 < \gamma < 1$)

With imperfect substitutes, the new manager' expectation about the market price - and hence the profit - in the worst scenario depends on the monopolist offer. In particular, even if the monopolist gives many quantities to the rival but provides a sufficiently low quantity to D_1, the latter can expect a positive market price - and hence positive profits- in the worst case. Given level of optimism α, the monopolist program is

$$\underset{q_1\in[0,1]}{\text{Max}}\ \pi_U(q_1|\alpha) = \frac{(1-\gamma q_1)^2}{4} + (1-\alpha)(1-q_1)q_1 + \alpha\,\text{Max}\{(1-\gamma-q_1)q_1, 0\} \tag{10}$$

We find that the optimal choice depends on the values of parameters α and γ. To ease understanding, let area A' denote the subset $\{(\alpha,\gamma)\in[0,1]\times(0,1):\ \alpha<\bar{\alpha}(\gamma)\,\text{and}\ 1>\gamma>\frac{1}{2}\left(\sqrt{5}-1\right)\}$ where $\bar{\alpha}(\gamma) = (1/8)\left(-\sqrt{(\gamma-2)^2(\gamma-1)^2(\gamma+2)(\gamma(\gamma(\gamma+4)-3)+2)}/\sqrt{\gamma^4}-\gamma^2-4/\gamma^2+(\gamma+8)/\gamma\right)$, area B' denote the subset $\{(\alpha, \gamma) \in [0,1] \times (0,1) : \bar{\bar{\alpha}}(\gamma) < \alpha < 1 \text{ and } 1 > \gamma > 2/3\}$ where $\bar{\bar{\alpha}}(\gamma) = 1/\gamma - 1/2$ and area C' denote the rest of the set of parameters, i.e. $\{(\alpha, \gamma) \in [0,1] \times (0,1) \setminus (A' \cup B')\}$.

Also, let $q_i^{T_k}$ and $f_i^{T_k}$ denote the equilibrium quantity delivered by the monopolist to D_i and the fixed fee paid by D_i to the monopolist, in area $k = A', B', C'$.

When goods are close substitutes then competition is fierce. If the monopolist offers a sufficiently high quantity ($q_1 > 1 - \gamma$), the new manager likely believes that the market price in the worst scenario is nil and so are its profits. It therefore accepts this contract only when the best scenario is sufficiently attractive. This happens when optimism is sufficiently high $\alpha < \bar{\alpha}(\gamma)$. This is the upper bound of area A'.

Note that the substitution degree indirectly affects expected profits and thus also acts on the acceptance decision and threshold $\bar{\alpha}(\gamma)$. Specifically, a lower substitution degree softens competition and increases the fee paid by the second firm ($\partial f_2 / \partial \gamma < 0$). This bolds the monopolist to increase the quantity offer to the first firm ($\partial q_1^{T_{A'}} / \partial \gamma < 0$) and the manager has to hold a higher degree of optimism to accept the offer ($d\bar{\alpha}/d\gamma > 0$).

Suppose now that goods are still close substitutes but $\alpha > \bar{\alpha}(\gamma)$ so that D_1 would reject the high quantity offer. The monopolist then offers a sufficiently low quantity ($q_1 < 1 - \gamma$) so that the new manager likely believes that the market price in the worst scenario remains positive and so are the profits. This releases the pressure on the best scenario and the optimism threshold of acceptance is higher $\bar{\bar{\alpha}}(\gamma) > \bar{\alpha}(\gamma)$. This is the upper-bound of area C' when goods are close substitutes.

Again note that substitution degree affects this acceptance threshold $\bar{\bar{\alpha}}(\gamma)$. Specifically, a lower substitution degree, by softening competition, increases the fee paid by the second firm ($\partial f_2 / \partial \gamma < 0$). Similarly to above, it bolds the monopolist to increase the quantity offer to the first firm ($\partial q_1^{T_{C'}} / \partial \gamma < 0$) however this time the manager can hold a lower degree of optimism to accept the offer ($d\bar{\bar{\alpha}}/d\gamma < 0$) because higher quantities also increase its expected profits in the worst scenario.

Finally, observe that when the new manager prefers to reject the offers then the monopolist is obviously up to offer nothing to the former and we end up in area B'. $\bar{\bar{\alpha}}(\gamma)$ is therefore also the lower bound of area B'.

When goods are soft substitutes then competition is soft, the firm accepts any small quantity

($q_1 < 1-\gamma$) because it expects low competition and thus optimism to not matter for acceptance anymore. This is the rest of area C'. Note that the monopolist still increases its quantity offer as substitution degree decreases ($\partial q_1^{T_{C'}}/\partial\gamma < 0$).

By computing the equilibrium fixed fees in each case, we then find the following proposition.

Proposition 3. *With imperfect substitutes, management turnover can make the monopolist earn strictly less than the monopoly profit.*

■ Summary ($0 \leq \gamma \leq 1$)

Finally, we find that the general solutions with imperfect substitutes in respectively A', B' and C' apply to the cases of perfect substitutes and unrelated goods so that we can generalise these areas to areas A, B and C where the latter include the parameter values $\gamma = 0$ and $\gamma = 1$. Lemma 2 details these solutions in the appendix, and Figure 1 provides a graphical illustration of which solution to consider as function of the substitution degree and the firm's level of optimism. Note that $\bar{\alpha}(\gamma) \geq 1/2$ and $\bar{\bar{\alpha}}(\gamma) \leq 1/2$.

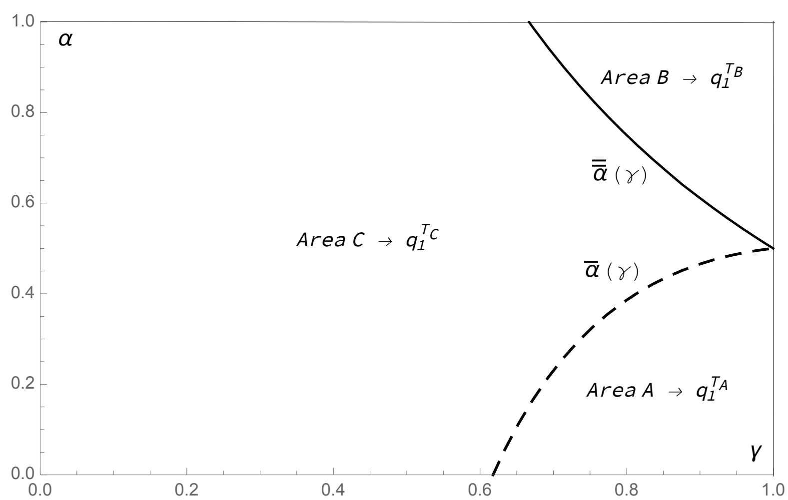

Figure 1: Graph of solution partition

4 Comparison

In this section, we compare the new equilibria obtained under management turnover and sequential contracting with the benchmark situation. From proposition 1 and 2 joint with lemma 1, we first obtain the following statement:

Proposition 4. *With perfect substitutes or unrelated goods, the monopolist is better off contracting first with the new manager irrespective of the manager's optimism level. In addition, with perfect substitutes, it is strictly better off when the manager is sufficiently optimistic ($0 \leq \alpha < 1/2$).*

Imperfect substitution mitigates this result. We find the following pattern.

Proposition 5. *With imperfect substitutes, the monopolist is still better off contracting first with the new manager only when the latter is sufficiently optimistic ($0 \leq \alpha < \tilde{\alpha}(\gamma) < 1/2$). However, the monopolist is worse off otherwise.*

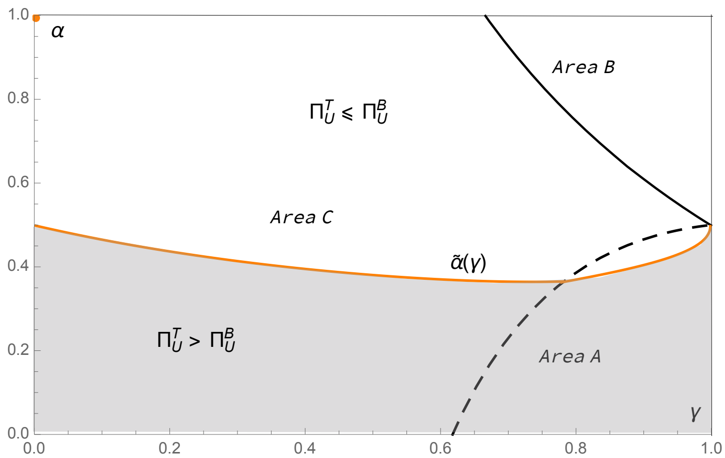

Figure 2: Comparison of the monopolist's profits with imperfect substitutes

The area where the monopolist can take advantage of management turnover is represented in gray on Figure 2. In this part of the graph, the optimism of the new manager lies below the threshold level $\tilde{\alpha}(\gamma)$ which depends on product substitution. Notice that $\tilde{\alpha}(1) = 1/2$ and $\tilde{\alpha}(0) = 1$ so that Proposition 5 also points out that the sufficient level of optimism $\tilde{\alpha}(\gamma)$, which makes management turnover profitable for the monopolist, is greater under imperfect substitutes than with perfect substitutes or unrelated goods. The intuition is as follows.

Consider first perfect product substitution ($\gamma = 1$), we have $q_1^T = q_1^{T_A} = (1 - 2\alpha)/(3 - \alpha)$, providing the manager's optimism is sufficiently high and $q_1^T = 0$ otherwise. At the threshold optimism level $\alpha = \tilde{\alpha}(1) = 1/2$, the equilibrium quantity is nil and the monopolist is indifferent whether management turnover occurs or not, $\pi_U^T = \pi_U^B$. This is because perfect substitutes make the monopolist indifferent between letting both firms being downstream monopolists or letting D_2 being the downstream monopolist. In contrast, when the manager is sufficiently optimistic ($\alpha < \tilde{\alpha}(1) = 1/2$), the monopolist is strictly better off offering strictly positive quantities to the new manager and then letting the remaining firm serve the residual demand.

We now tackle the change of the indifference threshold, $\tilde{\alpha}(\gamma)$, as product substitution decreases ($\gamma < 1$). From $\pi_U^T(\tilde{\alpha}(\gamma), \gamma) - \pi_U^B(\gamma) = 0$, and using the implicit function theorem, we find that the sign of the change is pinned down by the following expression:

$$\text{sign}\left[\frac{d\tilde{\alpha}}{d\gamma}\right] = \text{sign}\left[\frac{\partial f_2}{\partial \gamma}(q_1^T) - \mathbb{1}_{[q_1^T < 1-\gamma]}\frac{\partial f_1}{\partial \gamma}(q_1^T) - \frac{\partial \pi_U^B}{\partial \gamma}\right] \tag{11}$$

As products become less substitutes, the benchmark profit increases ($\partial \pi_U^B/\partial \gamma < 0$) implying that the benchmark situation is more profitable for larger intervals of optimism levels. Nonetheless, a lower product substitution also increases the fixed fee paid by the second firm in the turnover situation ($\partial f_2^T/\partial \gamma < 0$) which increases turnover profit and mitigates the first effect. Henceforth, the positive - but not that steep - slope of $d\tilde{\alpha}/d\gamma$ in area A.

When product substitution falls below a certain level then the monopolist's quantity offer changes in the turnover situation. This induces a decrease of the latter effect (as $q_1^{T_A} > q_1^{T_C}$ and $\partial^2 f_2^T/\partial\gamma\partial q_1 < 0$) but also introduces a new effect linked to the pessimistic term that the new manager now account for ($\mathbb{1}_{[q_1^* < 1-\gamma]} = 1$ in the above equation). This term implies that a decrease in product substitution increases the fixed fee paid by the new manager as even pessimistic manager now expects positive profits ($\partial f_1^T/\partial \gamma < 0$). This additional effect enables turnover profit to overcome the benchmark profit for larger level of optimism, henceforth the negative slope that appears at sufficiently low degree of product substitution.

Finally, when goods are unrelated ($\gamma = 0$), then each firm is a local monopolist and the manager's optimism does not matter anymore. The monopolist is indifferent between the two situations and $\tilde{\alpha}(0) = 1$.

We now turn to the consumer surplus. To do that we replace the equilibrium outputs into the general consumer surplus function in Eq. (3). We find the following result.

Proposition 6. *With respect to the benchmark situation, the consumers are better off when contracting is sequential and the new manager is sufficiently optimistic ($\alpha < \hat{\alpha}(\gamma)$). Otherwise, the consumers are worse off. This sufficient level of optimism also depends on the substitution degree.*

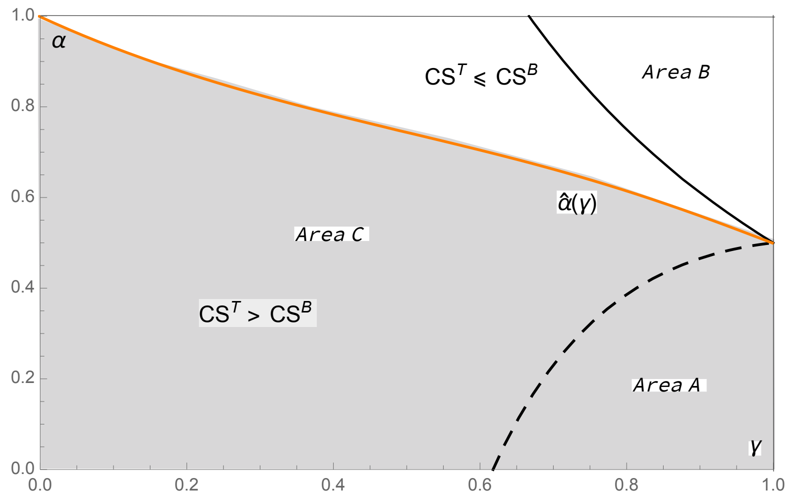

Figure 3: The consumer surplus

We find that $\hat{\alpha}(\gamma) \geq \tilde{\alpha}(\gamma)\ \forall\gamma$, which means that consumers overall benefit from management turnover for a wider range of optimism levels than the monopolist.

This happens because the monopolist's profit depends on the expectations about the fixed fee while the consumer surplus actually directly depends on the quantities released in the final markets. In addition, an increase in q_2 can compensate a decrease in q_1.

To see this, we rewrite the expression of the consumer surplus below. The first term on the right-hand side of the equal sign details the difference between the consumer surplus in market for good 1 and the second term the surplus difference in the market for good 2.

$$CS^T - CS^B = \frac{(q_1^T)^2 - (q^B)^2 + \gamma(q_1^T q_2^T - (q^B)^2)}{2} + \frac{(q_2^T)^2 - (q^B)^2 + \gamma(q_1^T q_2^T - (q^B)^2)}{2}$$

Note that even though the quantities in one market decreases, say $q_1^T < q_1^B$, a sufficient increase in the quantities in the other market, that is $q_2^T > q_2^B$, can lead to an increase of the consumer surplus. In other words, an increase in q_2 can compensate a decrease in q_1. This is because the increase in quantities of good 2 somehow overcomes the scarcity of good 1 through the lower price of good 1, provided there is some product substitution, $0 < \gamma \geq 1$.

5 Conclusion

When a firm changes its manager, it reopens room for contractual frictions with its partners. In this paper, we explore *strategic ambiguity* as a potential friction with a supplier. Compared with its experienced predecessor, the firm's new manager likely holds fuzzy expectations about the supplier's strategy. An optimistic manager weights favorable strategies more than detrimental ones whereas a pessimistic manager does the reverse.

We show that the manager's degree of optimism is critical because above a certain threshold it incites the supplier to change its timing of contracting and increase its profits. We also find that this threshold degree of optimism is greater when products are imperfect substitutes. This happens because competition is softer so that the new manager may expect positive profits even in the worst scenario.

There is a flourishing literature that revisits industrial organization settings in light with ambiguity concepts (Eichberger *et al.*, 2009; Król, 2012; Kauffeldt and Wiesenfarth, 2018). To the best of our knowledge this literature is silent regarding vertical relationships. Our paper contributes to this literature by pointing out that an upstream monopolist can temporarily benefit from a management turnover because of the rise of strategic ambiguity and this result depends on product substitution.

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Conflict of Interest

The authors declare that there is no conflict of interest.

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Appendices

Proofs

Proof of SPNE with perfect substitutes or Eq. (8). At the time the monopolist enters into contract with the first firm, it maximizes

$$\pi_U = (1-\alpha)(1-q_1)q_1 + (\frac{1-q_1}{2})^2.$$

The first order condition and the second order condition gives respectively:

$$\frac{\partial \pi_U}{\partial q_1} = 0 \quad \Leftrightarrow \quad (1-2\alpha) - (3-4\alpha)q_1 = 0 \tag{12}$$

$$\frac{\partial^2 \pi_U}{\partial^2 q_1} \leq 0 \quad \Leftrightarrow \quad (1-\alpha)(-2) + (1/2) \leq 0 \tag{13}$$

The FOC is satisfied when evaluated at $q_1(\alpha) = (1-2\alpha)/(3-4\alpha)$.

When $0 \leq \alpha \leq 1/2$, both eq. (12) and eq. (13) hold. Therefore, $q_1(\alpha) = (1-2\alpha)/(3-4\alpha) \geq 0$ is a maximum. We then obtain that $q_1^T(\alpha) = (1-2\alpha)/(3-4\alpha)$ and $f_1^T = (1-\alpha)(1-q_1^T)q_1^T = 2(1-\alpha)^2(1-2\alpha)/(3-4\alpha)^2$ when $0 \leq \alpha \leq 1/2$.

When $\alpha > 1/2$, the SOC becomes positive (eq. (13)). On the one hand, when $3/4 > \alpha > 1/2$, $(\partial\pi_U/\partial q_1)$ is negative. Therefore, the profit is decreasing on $q_1 \in [0,1]$ and we find that the maximum actually lies at $q_1 = 0$ in that case. On the other hand, when $\alpha > 3/4 > 1/2$, $(\partial\pi_U/\partial q_1)$ is negative until $q_1(\alpha) = (1-2\alpha)/(3-4\alpha) \geq 0$ and positive above. This time $q_1(\alpha) = (1-2\alpha)/(3-4\alpha)$ is thus a minimum. By computing the profit value at the extrema the interval, we find that $\pi_U(1) = 0$ and $\pi_U(0) = 1/4$. Therefore, the maximum profit is again reached at $q_1 = 0$. To sum up, when $\alpha > 1/2$, the maximum is reach at $q_1 = 0$. We then obtain that $q_1^T(\alpha) = 0$ and $f_1^T = 0$ when $\alpha > 1/2$.

Finally, $q_2^T(\alpha)$ is obtained by implementing the value of q_1 into the Cournot best response function of D_2, $q_2^T(\alpha) = [1-q_1(\alpha)]/2 = (1-\alpha)/(3-4\alpha)$ if $\alpha < 1/2$, and $1/2$ otherwise. Similarly, the corresponding fixed fee $f_2^T(\alpha)$ is such that $f_2^T(\alpha) = (1-q_1)^2/4 = (1-\alpha)^2/(3-4\alpha)^2$ when

$\alpha < 1/2$, and 1/4 otherwise. $\square$

Proof of proposition 1 Denote by $\pi_U(\alpha)$ the profit of the monopolist under strategic ambiguity. By the results under perfect substitutes obtained in the above proof and displayed in Eq. (8) (or in the lemma 2 in the proof of Proposition 5), we have:

$$\pi_U(\alpha) = f_1(\alpha) + f_2(\alpha) = \frac{(1-\alpha)^2}{3-4\alpha}, \tag{14}$$

as long as $\alpha \leq 1/2$. For higher values of α, π_U is equal to 1/4. The monopoly profit with perfect substitutes is $\pi_U^M = 1/4$. We then get:

$$\pi_U(\alpha) - \pi_U^M = \frac{(1-2\alpha)^2}{4(3-4\alpha)} \text{ when } \alpha < 1/2 \text{ ,} \quad \text{and } 0 \quad \text{otherwise}$$

which is strictly positive as long as $\alpha < 1/2$ and null otherwise. $\square$

Proof of the monopolist's SPNE with imperfect substitutes. At the time the monopolist enters into contract with the first firm, it maximizes

$$\pi_U = (\frac{1-q_1}{2})^2 + (1-\alpha)(1-q_1)q_1 + \alpha \max\{(1-\gamma-q_1)q_1, 0\}$$

Two cases arise:

(i) *Consider $q_1 < 1-\gamma$,*

We have $\max\{(1-\gamma-q_1)q_1, 0\} = (1-\gamma-q_1)q_1$, and the first order condition and the second order condition respectively are:

$$\frac{\partial \pi_U}{\partial q_1} = 0 \quad \Leftrightarrow \quad \frac{1}{2}\left(-(2\alpha+1)\gamma + \left(\gamma^2-4\right)q_1 + 2\right) = 0 \tag{15}$$

$$\frac{\partial^2 \pi_U}{\partial^2 q_1} \leq 0 \quad \Leftrightarrow \quad \frac{1}{2}\left(\gamma^2-4\right) \leq 0 \tag{16}$$

The FOC is satisfied when evaluated at $q_1(\alpha,\gamma) = \frac{2-\gamma(1+2\alpha)}{4-\gamma^2}$. Let us suppose $q_1^L(\alpha,\gamma) \equiv \frac{2-\gamma(1+2\alpha)}{4-\gamma^2}$.

Eq. (16) holds whenever $(\alpha,\gamma) \in [0,1]^2$. Therefore, $q_1^L(\alpha,\gamma)$ is always a maximum for π_U.

When $\alpha > \bar{\bar{\alpha}}(\gamma) \equiv \frac{1}{\gamma} - \frac{1}{2}$, we find $q_1^L(\alpha,\gamma) \leq 0$ meaning the maximum on this region is $q_1(\alpha,\gamma) = 0$. On the other hand, $q_1(\alpha,\gamma) < 1-\gamma$ is satisfied as long as $\alpha < \frac{1}{2}\left(-\gamma^2 + \gamma - \frac{2}{\gamma} + 3\right)$ and henceforth $q_1(\alpha,\gamma) = 1-\gamma$ is the maximum on the region where $\alpha > \frac{1}{2}\left(-\gamma^2 + \gamma - \frac{2}{\gamma} + 3\right)$.

Figure 4 summarizes our findings. The red line refers to $\bar{\bar{\alpha}}(\gamma)$ and the blue line refers to $\frac{1}{2}\left(-\gamma^2 + \gamma - \frac{2}{\gamma} + 3\right)$.

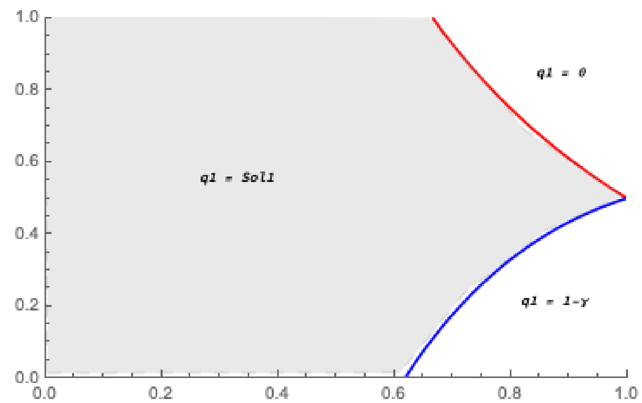

Figure 4: The solution for $q_1 < 1-\gamma$

(ii) *For $q_1 \geq 1-\gamma$,*

We have $\max\{(1-\gamma-q_1)q_1, 0\} = 0$, and the first order condition and the second order condition respectively are:

$$\frac{\partial \pi_U}{\partial q_1} = 0 \quad \Leftrightarrow \quad \frac{1}{2}\left(-2\alpha - \gamma + \text{q1}\left(4\alpha + \gamma^2 - 4\right) + 2\right) = 0 \tag{17}$$

$$\frac{\partial^2 \pi_U}{\partial^2 q_1} \leq 0 \quad \Leftrightarrow \quad \frac{1}{2}\left(4\alpha + \gamma^2 - 4\right) \leq 0 \tag{18}$$

The FOC is satisfied when evaluated at $q_1(\alpha,\gamma) = \frac{2(1-\alpha)-\gamma}{4(1-\alpha)-\gamma^2}$. Let us suppose $q_1^H(\alpha,\gamma) \equiv \frac{2(1-\alpha)-\gamma}{4(1-\alpha)-\gamma^2}$.

Eq. (18) holds whenever $\gamma \in [0,1]$ and $0 < \alpha < \frac{1}{4}\left(4-\gamma^2\right)$. Therefore, $q_1^H(\alpha,\gamma)$ is a maximum for π_U when $0 < \gamma < 1$ and $0 < \alpha < \frac{1}{4}\left(4-\gamma^2\right)$, and a minimum otherwise.

When $0 < \alpha < \frac{1}{4}\left(4-\gamma^2\right)$, we find $q_1^H(\alpha,\gamma) > 1-\gamma$ only if $\alpha < \frac{(\gamma-2)\left(\gamma^2+\gamma-1\right)}{2-4\gamma}$. It implies that (a) $q_1^H(\alpha,\gamma)$ is the maximum in the area where $\alpha < \frac{(\gamma-2)\left(\gamma^2+\gamma-1\right)}{2-4\gamma}$ and (b) $q_1(\alpha,\gamma) = 1-\gamma \geq 0$ is the maximum in the area where $\frac{(\gamma-2)\left(\gamma^2+\gamma-1\right)}{2-4\gamma} < \alpha < \frac{1}{4}\left(4-\gamma^2\right)$.

When $\alpha > \frac{1}{4}\left(4-\gamma^2\right)$, $q_1^H(\alpha,\gamma)$ is a minimum, we find that the value is higher than $1-\gamma$

meaning that the maximum on this part is either in $q_1(\alpha,\gamma) = 1$ or $q_1(\alpha,\gamma) = 1-\gamma$. It can be shown that the profit at $q_1(\alpha,\gamma) = 1-\gamma$ is higher than that at $q_1(\alpha,\gamma) = 1$ so that $q_1(\alpha,\gamma) = 1-\gamma$ is the maximum on this area.

Figure 5 summarizes our findings. The black line refers to $(1/4)(4-\gamma^2)$ and the blue line refers to $\frac{(\gamma-2)(\gamma^2+\gamma-1)}{2-4\gamma}$.

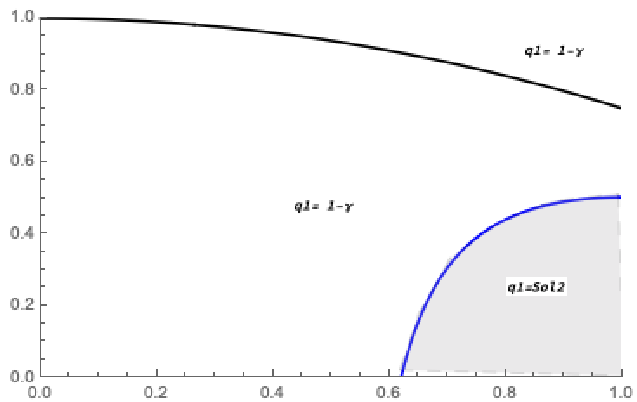

Figure 5: The solution for $q_1 < 1-\gamma$

We now derive which solution is the best for the monopolist in each region.

◇ *Maximum profit at optimum solutions.*

Let's denote by π_U^L the profit function when $q_1 < 1-\gamma$ and π_U^H the profit function otherwise. Several cases arise:

(i) In the area where $\alpha < \frac{1}{2}\left(-\gamma^2+\gamma-\frac{2}{\gamma}+3\right)$, i.e. below the blue line of Fig 4, we find that

$$\pi_U^L(1-\gamma) - \pi_U^H(1-\gamma) = 0 \geq \pi_U^L(1-\gamma) - \pi_U^H(q_1^H)$$

Therefore, q_1^H is solution in this region.

(ii) In the area where $\frac{1}{2}\left(-\gamma^2+\gamma-\frac{2}{\gamma}+3\right) < \alpha < \frac{(\gamma-2)(\gamma^2+\gamma-1)}{2-4\gamma}$, i.e. between the blue lines of Fig 4 and Fig 5, we find that

$$\pi_U^L(q_1^L) - \pi_U^H(q_1^H) = \frac{2(1-\alpha\gamma)+\alpha(\alpha+1)\gamma^2-\gamma}{4-\gamma^2} - \frac{(1-\alpha)(2-\alpha-\gamma)}{4(1-\alpha)-\gamma^2}$$

This is positive whenever $\alpha > \bar{\alpha}(\gamma) \equiv \frac{1}{8}\left(-\sqrt{\frac{(\gamma-2)^2(\gamma-1)^2(\gamma+2)(\gamma(\gamma(\gamma+4)-3)+2)}{\gamma^4}} - \gamma^2 - \frac{4}{\gamma^2} + \frac{\gamma+8}{\gamma}\right)$ and negative otherwise. The solution in this region is thus $q_1^L(\alpha,\gamma)$ when $\alpha > \bar{\alpha}(\gamma)$ and $q_1^H(\alpha,\gamma)$, otherwise.

(iii) In the area where $\alpha > \bar{\bar{\alpha}}(\gamma)$, above the red line of Fig 4, we find that

$$\pi_U^L(0) - \pi_U^H(1-\gamma) = \frac{1}{4}(1-\gamma)\gamma(4\alpha - (2-\gamma)(\gamma+1)) \geq 0$$

The solution is this region is $q_1(\alpha,\gamma) = 0$.

(iiv) In the last area, we find that

$$\pi_U^L(q_1^L) - \pi_U^H(1-\gamma) = \frac{(\gamma(2\alpha + (\gamma-1)\gamma - 3) + 2)^2}{4(4-\gamma^2)} \geq 0$$

The solution is this region is $q_1^L(\alpha,\gamma)$.

These thresholds are summarized in figure 6 below, where $q_1^{T_{A'}} = q_1^H(\alpha,\gamma)$, $q_1^{T_{B'}} = 0$ and $q_1^{T_{C'}} = q_1^L(\alpha,\gamma)$.

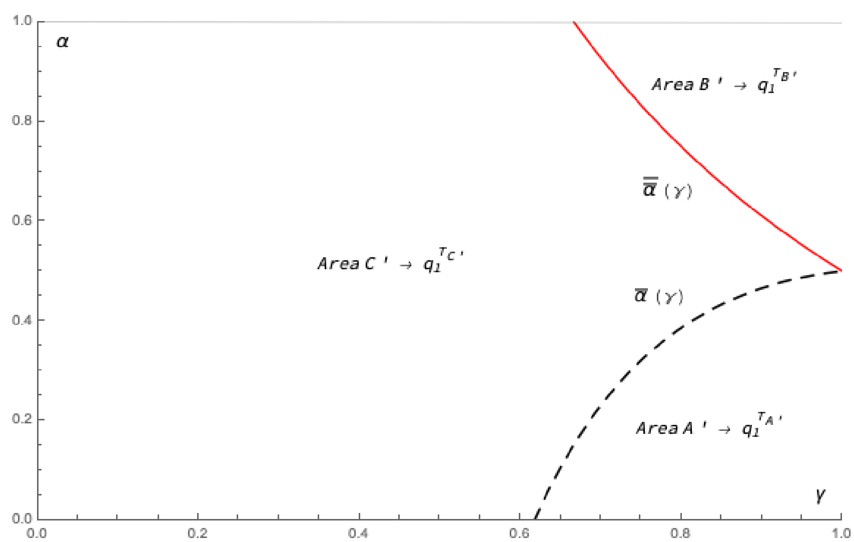

Figure 6: (Equiv. fig 1) Graph of solution partition

With management turnover and imperfect substitutes, the SPNE strategy of U jointly is:

$$q_1^T(\alpha,\gamma)=\begin{cases}\frac{2(1-\alpha)-\gamma}{4(1-\alpha)-\gamma^2}, & \text{if } (\alpha,\gamma)\in AreaA' \\ 0 & \text{if } (\alpha,\gamma)\in AreaB' \\ \frac{2-\gamma(1+2\alpha)}{4-\gamma^2} & \text{otherwise}\end{cases}, \quad q_2^T(\alpha,\gamma)=\begin{cases}\frac{(1-\alpha)(2-\gamma)}{4(1-\alpha)-\gamma^2}, & \text{if } (\alpha,\gamma)\in AreaA' \\ \frac{1}{2} & \text{if } (\alpha,\gamma)\in AreaB' \\ \frac{2-\gamma(1-\alpha\gamma)}{4-\gamma^2} & \text{otherwise}\end{cases},$$

$$f_1^T(\alpha,\gamma)=\begin{cases}\frac{(1-\alpha)(2(1-\alpha)-\gamma)((2-\gamma)(1+\gamma)-2\alpha)}{(4(1-\alpha)-\gamma^2)^2}, & \text{if } (\alpha,\gamma)\in AreaA' \\ 0 & \text{if } (\alpha,\gamma)\in AreaB' \\ \frac{(1-\alpha)(2(1-\alpha\gamma)-\gamma)(2(1+\alpha\gamma)+\gamma-\gamma^2)}{(4-\gamma^2)^2} & \text{otherwise}\end{cases},$$

$$f_2^T(\alpha,\gamma)=\begin{cases}\frac{(1-\alpha)^2(2-\gamma)^2)}{(4(1-\alpha)-\gamma^2)^2}, & \text{if } (\alpha,\gamma)\in AreaA' \\ \frac{1}{4} & \text{if } (\alpha,\gamma)\in AreaB' \\ \frac{(2-(1-\alpha\gamma)\gamma)^2}{(4-\gamma^2)^2} & \text{otherwise}\end{cases}$$

where Area A' denotes the subset $\{(\alpha,\gamma)\in[0,1]\times(0,1):\ \alpha<\bar{\alpha}(\gamma)\,\text{and}\,1>\gamma>\frac{1}{2}\left(\sqrt{5}-1\right)\}$ such that $\bar{\alpha}(\gamma)=(1/8)\left(-\sqrt{(\gamma-2)^2(\gamma-1)^2(\gamma+2)(\gamma(\gamma(\gamma+4)-3)+2)}/\sqrt{\gamma^4}-\gamma^2-4/\gamma^2+(\gamma+8)/\gamma\right)$, Area B' denotes the subset $\{(\alpha,\gamma)\in[0,1]\times(0,1):\ \bar{\bar{\alpha}}(\gamma)<\alpha<1\,\text{and}\,1>\gamma>2/3\}$ such that $\bar{\bar{\alpha}}(\gamma)=1/\gamma-1/2$ and Area C' denotes the rest of the set of parameters. □

Proof of proposition 3 Take for example $\alpha=1$ and a sufficiently high γ, say $\gamma>\gamma'$, so that we are in area B. We get $f_1^T+f_2^T=1/4$ while the monopoly profit is $\pi^M=\pi_U^B=1/(2+2\gamma)$. With imperfect substitutes, we have $\gamma'<\gamma<1$, which implies that $f_1^T+f_2^T=1/4<1/(2+2\gamma)=\pi^M$. □

Proof of proposition 5 Let's remind that $\pi_U^B=1/(2(1+\gamma))$ is the benchmark profit without management turnover. Let's now denote by π_U^T the profit of the monopolist with management turnover. Lemma 2 summarizes the monopolist's SPNE strategies according to the parameter values.

Lemma 2. *With management turnover, the monopolist's SPNE strategy jointly depends on the new* *manager's level of optimism α and the product substitution γ such that:*

$$q_1^T(\alpha,\gamma) = \begin{cases} \frac{2(1-\alpha)-\gamma}{4(1-\alpha)-\gamma^2}, & \textit{if } (\alpha,\gamma) \in AreaA \\ 0 & \textit{if } (\alpha,\gamma) \in AreaB \\ \frac{2-\gamma(1+2\alpha)}{4-\gamma^2} & \textit{otherwise} \end{cases}, \quad q_2^T(\alpha,\gamma) = \begin{cases} \frac{(1-\alpha)(2-\gamma)}{4(1-\alpha)-\gamma^2}, & \textit{if } (\alpha,\gamma) \in AreaA \\ \frac{1}{2} & \textit{if } (\alpha,\gamma) \in AreaB \\ \frac{2-\gamma(1-\alpha\gamma)}{4-\gamma^2} & \textit{otherwise} \end{cases},$$

$$f_1^T(\alpha,\gamma) = \begin{cases} \frac{(1-\alpha)(2(1-\alpha)-\gamma)((2-\gamma)(1+\gamma)-2\alpha)}{(4(1-\alpha)-\gamma^2)^2}, & \textit{if } (\alpha,\gamma) \in AreaA \\ 0 & \textit{if } (\alpha,\gamma) \in AreaB \\ \frac{(1-\alpha)(2(1-\alpha\gamma)-\gamma)(2(1+\alpha\gamma)+\gamma-\gamma^2)}{(4-\gamma^2)^2} & \textit{otherwise} \end{cases},$$

$$f_2^T(\alpha,\gamma) = \begin{cases} \frac{(1-\alpha)^2(2-\gamma)^2)}{(4(1-\alpha)-\gamma^2)^2}, & \textit{if } (\alpha,\gamma) \in AreaA \\ \frac{1}{4} & \textit{if } (\alpha,\gamma) \in AreaB \\ \frac{(2-(1-\alpha\gamma)\gamma)^2}{(4-\gamma^2)^2} & \textit{otherwise} \end{cases}$$

where Area A denotes the subset $\{(\alpha,\gamma) \in [0,1]^2 : \ \alpha < \bar{\alpha}(\gamma) \textit{ and } 1 > \gamma > \frac{1}{2}\left(\sqrt{5}-1\right)\}$ *such that* $\bar{\alpha}(\gamma) = (1/8)\left(-\sqrt{(\gamma-2)^2(\gamma-1)^2(\gamma+2)(\gamma(\gamma(\gamma+4)-3)+2)}/\sqrt{\gamma^4} - \gamma^2 - 4/\gamma^2 + (\gamma+8)/\gamma\right)$, *Area B denotes the subset* $\{(\alpha,\gamma) \in [0,1]^2 : \ \bar{\bar{\alpha}}(\gamma) < \alpha < 1 \textit{ and } 1 > \gamma > 2/3\}$ *such that* $\bar{\bar{\alpha}}(\gamma) = 1/\gamma - 1/2$ *and Area C denotes the rest of the set of parameters.*

Since on these SPNE, firms always accept, we have our next lemma which displays the monopolist's equilibrium profits with respect to the parameter values.

Lemma 3. *With management turnover, the monopolist earns*

$$\pi_U^T(\alpha,\gamma) = \begin{cases} \frac{(1-\alpha)(2-\alpha-\gamma)}{4(1-\alpha)-\gamma^2} & \textit{if } (\alpha,\gamma) \in AreaA \\ 1/4 & \textit{if } (\alpha,\gamma) \in AreaB \\ \frac{2(1-\alpha\gamma)+\alpha(1+\alpha)\gamma^2-\gamma}{4-\gamma^2} & \textit{otherwise} \end{cases} \tag{19}$$

We then get:

$$\pi_U^T(\alpha,\gamma)-\pi_U^B(\gamma)=\begin{cases}\frac{(1-\alpha)(2-\alpha-\gamma)}{4(1-\alpha)-\gamma^2}-\frac{1}{2(1+\gamma)} & \text{if } (\alpha,\gamma)\in AreaA\\ -\frac{1-\gamma}{4(\gamma+1)} & \text{if } (\alpha,\gamma)\in AreaB\\ \frac{\gamma(-2\alpha(\gamma+1)(\alpha\gamma+\gamma-2)+\gamma-2)}{2(\gamma+1)(\gamma^2-4)} & \text{otherwise}\end{cases} \tag{20}$$

which is positive whenever $\alpha<\tilde{\alpha}^a(\gamma)\equiv\frac{1}{2}\left(-\sqrt{\frac{(\gamma-2)\gamma^3+1}{(\gamma+1)^2}}-\gamma-\frac{2}{\gamma+1}+3\right)$ and negative otherwise in area A, always negative in area B and positive whenever $\alpha<\tilde{\alpha}^c(\gamma)\equiv\frac{-\gamma^2-\sqrt{\gamma^4-5\gamma^2+4}+\gamma+2}{2(\gamma^2+\gamma)}$ while negative otherwise in area C. Figure 2 summarizes our findings and is redisplayed below.

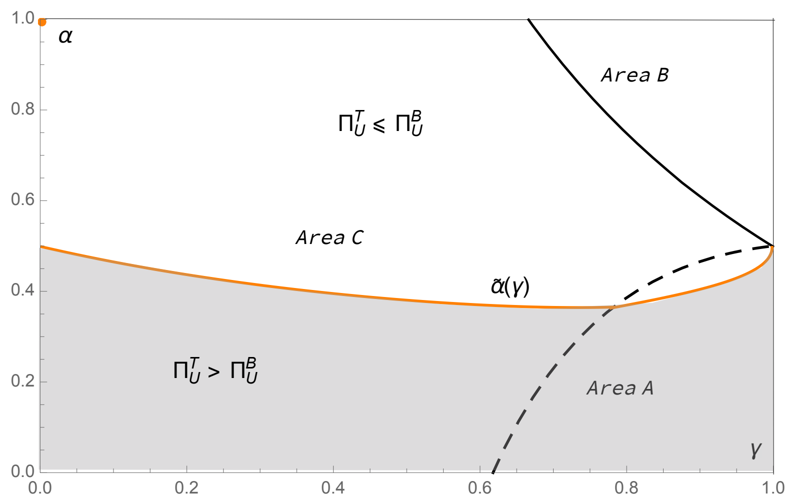

Figure 7: (Equiv. Fig 2) The monopolist's profits

$\diamond$ Let's additionnally prove that the thresholds are inferior to 1/2 for $\gamma\in(0,1)$,

- Consider $\tilde{\alpha}^c(\gamma)=\frac{-\gamma^2-\sqrt{\gamma^4-5\gamma^2+4}+\gamma+2}{2(\gamma^2+\gamma)}\leq 1/2$. It is equivalent to $2+\gamma-\gamma^2-\sqrt{4-5\gamma^2+\gamma^4}\leq(1/2)2(\gamma+\gamma^2)\Rightarrow 2-2\gamma^2\leq\sqrt{4-5\gamma^2+\gamma^4}\Rightarrow 4(1-\gamma^2)^2\leq 4-5\gamma^2+\gamma^4\Rightarrow 4-8\gamma^2-4\gamma^4\leq 4-5\gamma^2+\gamma 4$ which becomes obviously true for $\gamma\in(0,1)$.

- For $\tilde{\alpha}^a(\gamma)=\frac{1}{2}\left(-\sqrt{\frac{(\gamma-2)\gamma^3+1}{(\gamma+1)^2}}-\gamma-\frac{2}{\gamma+1}+3\right)\leq 1/2$, we have $\left(-\sqrt{\frac{(\gamma-2)\gamma^3+1}{(\gamma+1)^2}}-\gamma-\frac{2}{\gamma+1}+3\right)\leq 1\Rightarrow -\gamma-\frac{2}{\gamma+1}+3-1\leq\sqrt{\frac{(\gamma-2)\gamma^3+1}{(\gamma+1)^2}}\Rightarrow\frac{\gamma-\gamma^2}{\gamma+1}\leq\sqrt{\frac{(\gamma-2)\gamma^3+1}{(\gamma+1)^2}}\Rightarrow\gamma-\gamma^2\leq\sqrt{(\gamma-2)\gamma^3+1}\Rightarrow(\gamma-\gamma^2)^2\leq 1+\gamma^4-2\gamma^3\Rightarrow\gamma^2-2\gamma^3+\gamma^4\leq 1-2\gamma^3+\gamma^4$ which becomes obviously true for $\gamma\in(0,1)$.

$\square$

Proof of Equation 11 From $\pi_U^T(\tilde{\alpha}(\gamma),\gamma) - \pi_U^B(\gamma) = 0$, we get:

$$\frac{d\tilde{\alpha}}{d\gamma}(\gamma) = -\frac{\frac{\partial \pi_U^T}{\partial \gamma} - \frac{\partial \pi_U^B}{\partial \gamma}}{\frac{d\pi_U^T}{d\alpha}}$$

We can then decompose the profits. First, using $\pi_U^T(q_2^*(q_1), q_1)$ at $q_1^T(\alpha,\gamma)$, we have

$$\frac{\partial \pi_U^T}{\partial \gamma} = \frac{\partial \pi_U^T}{\partial q_1}\frac{\partial q_1^T}{\partial \gamma} + \frac{\partial \pi_U^T}{\partial \gamma}|_{q_1=q_1^T}$$

At this point $(\partial \pi_U^T/\partial q_1)(q_1^T) = 0$, the expression simplifies to

$$\frac{\partial \pi_U^T}{\partial \gamma} = 0 + \frac{\partial f_1}{\partial \gamma}|_{q_1=q_1^T} + \frac{\partial f_2}{\partial \gamma}|_{q_1=q_1^T}$$

Because $f_1(q_1^T) = (1-\alpha)(1-q_1^T)q_1^T + \mathbb{1}_{[q_1^T<1-\gamma]}\alpha(1-\gamma-q_1^T)q_1^T$ and $f_2(q_1^T) = (1/4)(1-\gamma q_1^T)^2$, we find

$$\frac{\partial \pi_U^T}{\partial \gamma} = -\mathbb{1}_{[q_1^T<1-\gamma]}\alpha . q_1^T - \frac{1}{2}q_1^T(1-\gamma q_1^T) < 0 \tag{21}$$

By the same process, we obtain

$$\frac{\partial \pi_U^T}{\partial \alpha} = -(1-q_1^T)q_1^T + \mathbb{1}_{[q_1^T<1-\gamma]}(1-\gamma-q_1^T)q_1^T < 0 \tag{22}$$

Finally, it is easy to see that $\pi_U^B(q_1^*, q_2^*)$ implies the same derivative irrespective of the area considered. We get

$$\frac{\partial \pi_U^B}{\partial \gamma} = -\frac{1}{2(1+\gamma)^2} < 0 \tag{23}$$

From Eq. (21), Eq. (22) and Eq. (23), we find that:

$$\begin{aligned}\text{sign}[\frac{d\tilde{\alpha}}{d\gamma}] &= \text{sign}[\frac{\partial \pi_U^T}{\partial \gamma} - \frac{\partial \pi_U^B}{\partial \gamma}] \\ &= \text{sign}\left[\frac{\partial f_2}{\partial \gamma}(q_1^T) - \mathbb{1}_{[q_1^T<1-\gamma]}\frac{\partial f_1}{\partial \gamma}(q_1^T) - \frac{\partial \pi_U^B}{\partial \gamma}\right]\end{aligned}$$

This is Equation 11 in our main text.

In area A, where $q_1^T = q_1^{T_A}$ we have:

$$\text{sign}[\frac{d\tilde{\alpha}}{d\gamma}] = \text{sign}[\frac{(1-\alpha)(2-\gamma)(2\alpha+\gamma-2)}{(4\alpha+\gamma^2-4)^2} + \frac{1}{2(\gamma+1)^2}]$$

And the sign is positive.

In area C, where $q_1^T = q_1^{T_C}$ we have:

$$\text{sign}[\frac{d\tilde{\alpha}}{d\gamma}] = \text{sign}[\frac{\alpha^2}{(\gamma-2)^2} - \frac{(\alpha+1)^2}{(\gamma+2)^2} + \frac{1}{2(\gamma+1)^2}]$$

And the sign can be positive or negative. However, evaluating the sign at $\alpha = \tilde{\alpha}^c(\gamma)$ we get that

$$\text{sign}[\frac{d\tilde{\alpha}}{d\gamma}] = \text{sign}[\frac{2\left(\sqrt{\gamma^4-5\gamma^2+4}-2\right)+\gamma\left(\gamma^2+2\sqrt{\gamma^4-5\gamma^2+4}+4\gamma-4\right)}{2\gamma(\gamma+1)^2\left(\gamma^2-4\right)}]$$

which is negative provided $0 < \gamma < -1+\sqrt{3} \approx 0.730$. This threshold is lower than the threshold at which $\tilde{\alpha}$ intersects area A, $\gamma \approx 0.784$. Therefore, the slope is slightly positive above $\gamma = -1+\sqrt{3}$ and negative below. □

Proof of proposition 6 Let CS denote the consumer surplus. With a linear demand, the consumer surplus simplifies to:

$$CS(q_1, q_2) = \frac{(q_1)^2 + (q_2)^2 + 2\gamma q_1 q_2}{2}$$

From the benchmark optimal quantity we find that the consumer surplus without management turnover is $CS^B = 1/(4(1+\gamma))$. Let CS^T denote the consumer surplus with management turnover. By lemma 2, we have:

Lemma 4. *With management turnover, the consumer surplus is:*

$$CS^T(\alpha,\gamma) = \begin{cases} \frac{\alpha^2((4-3\gamma)\gamma+8)-2\alpha(\gamma-4)(\gamma-2)(\gamma+1)+2(\gamma-3)\gamma^2+8}{2(4\alpha+\gamma^2-4)^2} & \textit{if } (\alpha,\gamma) \in AreaA \\ 1/8 & \textit{if } (\alpha,\gamma) \in AreaB \\ \frac{\gamma\left(\alpha^2\gamma\left(4-3\gamma^2\right)-2\alpha(\gamma-2)^2(\gamma+1)+2(\gamma-3)\gamma\right)+8}{2(\gamma^2-4)^2} & \textit{otherwise} \end{cases} \tag{24}$$

We then get (omiting writing CS as functions):

$$CS^T - CS^B = \begin{cases} \frac{\alpha^2((4-3\gamma)\gamma+8)-2\alpha(\gamma-4)(\gamma-2)(\gamma+1)+2(\gamma-3)\gamma^2+8}{2(4\alpha+\gamma^2-4)^2} - \frac{1}{4(1+\gamma)} & \text{if } (\alpha,\gamma) \in AreaA \\ -\frac{1-\gamma}{8(\gamma+1)} & \text{if } (\alpha,\gamma) \in AreaB \\ \frac{\gamma\left(\alpha^2\gamma\left(4-3\gamma^2\right)-2\alpha(\gamma-2)^2(\gamma+1)+2(\gamma-3)\gamma\right)+8}{2(\gamma^2-4)^2} - \frac{1}{4(1+\gamma)} & \text{otherwise} \end{cases} \tag{25}$$

which is always positive in area A, always negative in area B and positive if $\alpha < \frac{(\gamma-2)^2(\gamma+1)}{4\gamma-3\gamma^3} - \frac{\sqrt{\frac{(\gamma^2-4)^2(2\gamma^3-\gamma^2-2\gamma+2)}{\gamma^2(\gamma+1)(3\gamma^2-4)^2}}}{\sqrt{2}} \equiv \hat{\alpha}(\gamma)$ while negative otherwise in area C. Figure 3 summarizes our findings and is redisplayed below.

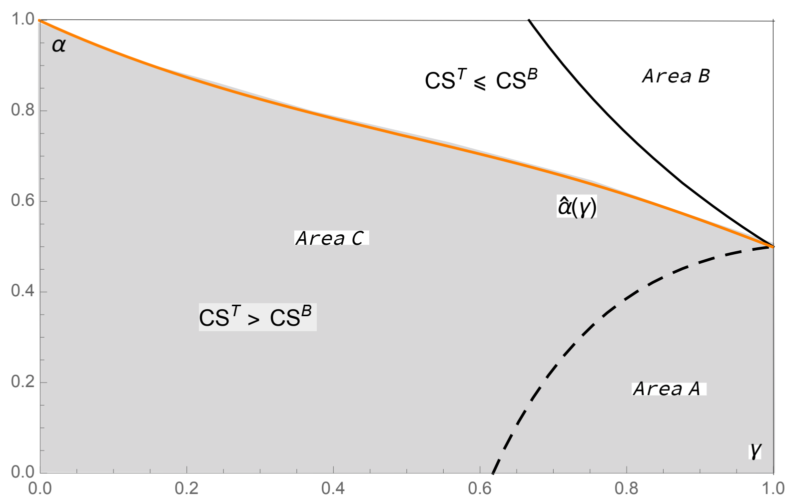

Figure 8: (Equiv. Fig 3) The consumer surplus

□