Decentralization and consumer welfare with substitutes or complements

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Abstract

We study a vertically integrated producer (VIP) supplying a downstream firm under price competition. The VIP may decentralize the final price decision to its downstream unit, thereby obliging the latter to ignore the effect of the output price on upstream sales. We find that decentralization benefits the VIP, irrespective of products being substitutes or complements. Decentralization also benefits the consumers when products are substitutes, but it harms them when products are complements. Interestingly, when products are substitutes, decentralization decreases both output prices despite restoring a double margin on the downstream unit's sales.

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1 Introduction

The common wisdom is that a centralized decision maker privy to all pertinent information would generate greater profits than those achieved under a decentralized structure. However, vertically integrated producers (VIPs) that supply and compete with downstream rivals often have their downstream units acting independently of the whole structure. For example, empirical studies of the media market in Israel (Gilo and Spiegel, 2011) and of the television industry in the United States (Crawford *et al.*, 2018) find that the VIP's downstream units take decisions that only partially internalize the effect on the upstream sales.

In particular, it is puzzling why a VIP would let, or even deliberately enable, its downstream unit to set the final price independently, as the latter would fail to consider the price effect on upstream sales. Such decentralization would moreover restore a double margin on the downstream unit's sales, which would mean reduced profits for the VIP. Our paper explores this puzzle by comparing the decentralized equilibrium outcomes with the benchmark centralized equilibrium outcomes where the whole structure quotes the final price.

We model an industry composed of a VIP supplying a downstream rival under price competition. We consider the widely used Singh and Vives (1984)'s linear demand with product substitutes or complements. We find that the decentralized VIP raises the rival's input price irrespective of substitutes or complements, while it quotes a distinctive input price to its downstream unit to boost its revenues on the downstream market. The incentives to divert sales through discriminatory pricing is the main cause of concern in antitrust policy and has recently been emphasized in the FTC and DoJ (2020) Vertical Merger Guidelines. Our results show that decentralized input prices benefit the VIP under price competition irrespective of substitutes or complements. However, they only benefit consumers with substitutes, and harm them with complements.

The intuition is as follows. Consider first the limitation of centralization: once the input price is set, the centralized VIP will consider the effect of its output price decision on the flow of upstream profits issued from the input sales to other firm. This is the Chen effect (Moresi and Schwartz, 2021; Chen, 2001). With substitutes, this effect will prompt the VIP to reduce competition in favor of the other firm (by increasing its output price and diverting demand to the other firm). In contrast, with complements, it incites the VIP to improve collaboration in favor of the other firm (by decreasing its output price and attracting demand for the other firm). Expecting this Chen effect, the other firm increases more its output price upon an increase of the input price (increased pass-through rate), irrespective of substitutes or complements. This reduces the upstream sales and thus the VIP's flow of upstream profits. Consequently, the VIP makes an input price concession to boost competition or collaboration from the other firm which limits this reduced flow of upstream profits.

Decentralization enables the VIP to commit to be less concern about upstream profits (get rid of the Chen effect) and thus avoid the associated input price concession. It also enables to redirect competition or collaboration in favor of the downstream unit through the additional instrument of the input price transfer. This boosts the flow of downstream profits. Overall, the VIP benefits from a more efficient balance of profits. Interestingly, with substitutes, the elimination of the Chen effect induces a pro-competitive effect which dominates the anti-competitive effects of the re-introduction of the double margin (input price transfer between the affiliated units) and of the increased input price to the other firm (avoidance of the price concession). As a result, the output prices decrease which benefits consumers.

The remainder of the paper is as follows. Section 2 briefly explains our contribution to the existing literature. Section 3 introduces the model. Sections 4 and section 5 derive the analysis with demand substitutes and complements. Finally, section 6 discusses some extensions of the model and section 7 concludes. All proofs are in the Appendix.[1]

2 Related literature

The existing literature claims that a centralized VIP, which also supplies a downstream rival, faces a problem of effectively balancing upstream and downstream profits (Arya *et al.*, 2008b; Moresi and Schwartz, 2017). In particular, Arya *et al.* (2008b) shows that decentralization improves the VIP's balance of profits under Cournot competition and thus is more profitable than centralization. However, it harms consumers. Interestingly, Moresi and Schwartz (2017) points out with a specific

[1] At the exception of the proofs of Section 6 which are in the Supplementary Appendix.

numerical example (after having studied the centralized VIP's incentive) that decentralization may benefit consumers under Bertrand competition.

Our paper contributes to this literature by deriving the equilibrium under decentralization and Bertrand competition using a linear demand where products can be either substitutes or complements. We generalize Moresi and Schwartz (2017)'s result with product substitutes. In particular, we find that, even though decentralization restores an additional margin on the downstream unit's sales, it triggers lower final prices and indeed benefits consumers. For demand complements, we establish that decentralization remains more profitable for the VIP but yields a higher price from the downstream unit, which offsets the decrease of the other firm's price, therefore harming consumers.

3 The model

We assume a vertically integrated producer (VIP) and a downstream rival. The VIP consists of two entities, an upstream unit and a downstream unit. The upstream unit is the monopoly producer of a key input to the downstream unit (D_1) and an independent downstream rival (D_2). Each unit of output requires one unit of the input. We assume the firms bear neither production nor transformation costs. We relax this assumption in section 6.3.

D_1 and D_2 respectively sell horizontally-differentiated products to a representative consumer at prices p_1 and p_2. The representative consumer holds the linear demand function *à la Singh and Vives (1984)*. Formally, for product $i \in \{1, 2\}$, the consumer's demand is $q_i(p_i, p_j) = \frac{\alpha}{1+\gamma} - \frac{1}{1-\gamma^2} p_i + \frac{\gamma}{1-\gamma^2} p_j$, where $j \in \{1, 2\}$, $j \neq i$, $\gamma^2 < 1$, and $\alpha > 0$. Products are substitutes when $\gamma > 0$, complements when $\gamma < 0$ and unrelated when $\gamma = 0$. With the inverse demand function $p_i(q_i, q_j) = \alpha - q_i - \gamma q_j$, $\forall i = 1, 2, j \neq i$, we find that the consumer surplus writes $CS = [(q_1)^2 + 2\gamma q_1 q_2 + (q_2)^2]/2$.

The VIP can choose between two organizational schemes: centralization or decentralization. With centralization, the VIP first sets the input price w_2 to the downstream rival, and then quotes the output price p_1 while the rival quotes the output price p_2. Furthermore, the VIP sets the input price and the output price considering the joint profits, that we denote V, which is the sum of its downstream unit's profits (from output sales to the consumers) and its upstream unit's profits (from the input sales to the rival). In contrast, the independent rival only considers its downstream profit π_2 (from the output sales to the consumers and given the input price). Formally, we have:

$$V = p_1.q_1 + w_2.q_2 \tag{1}$$

$$\pi_2 = (p_2 - w_2)q_2 \tag{2}$$

Under decentralization, the VIP continues to set the input price w_2 to the downstream rival, but now also sets an input price w_1 to its downstream unit. In exchange of this input price w_1, the VIP then lets the downstream unit independently set the output price p_1 while the independent rival continues to set p_2. The VIP thus sets the input prices considering the joint profits V displayed in Equation (1). However, once input prices are set, the independent downstream unit sets the output price p_1 considering only its downstream profits π_1 given the input price w_1, thereby abstracting from upstream sales to the rival. Equation (3) below displays the formal expression. The rival continues to maximize its downstream profits as displayed in Equation (2).

$$\pi_1 = (p_1 - w_1).q_1 \tag{3}$$

Under both organizational schemes, we use the Sub-game Perfect Nash Equilibrium (SPNE) concept to solve the game.

4 Results for susbstitutes

The typical view is that a centralized decision maker privy to all pertinent information can generate greater profits than those achieved under a decentralized structure. Decentralization would for example restore an inefficient double margin on the downstream unit's sales, which would mean reduced profits for the VIP. On the other hand, Moresi and Schwartz (2017) and Arya *et al.* (2008b) claim that a centralized VIP, which also supplies a downstream rival, faces a problem of effectively balancing upstream and downstream profits. This section shows that decentralization improves the VIP's balance of profits even though it does restore double marginalization.

4.1 Centralization

Under centralization, the VIP supplies inputs to the downstream rival D_2 at per-unit price w_2. The timing of the game is as follows: (1) the VIP charges the input price w_2; and (2) the VIP and the rival set output prices p_1 and p_2. Profits are made. We use the Sub-game Perfect Nash Equilibrium (SPNE) concept to solve the game.

We use backward induction to find the SPNE. At stage 2, the firms maximize their profits $V(p_1, p_2)$ and $\pi_2(p_2, p_1)$ with respect to their output prices, given input price w_2. Remind that $V(p_1, p_2)$ includes the upstream unit's sales to the rival (refer to Equation (1)). Formally, the firms maximize the following profit functions $V(p_1, p_2) = p_1.q_1(p_1, p_2) + w_2.q_2(p_1, p_2)$ and $\pi_2(p_1, p_2) = (p_2 - w_2)q_2(p_1, p_2)$. In particular, the VIP's optimization program leads to the following First Order Condition (FOC) : $\frac{dV}{dp_1} = 0 \Leftrightarrow \frac{dq_1}{dp_1}p_1 + q_1 + w_2\frac{dq_2}{dp_1} = 0$. We observe that the VIP anticipates that an increase of the output price will increase the upstream sales to the rival ($w_2(\partial q_2/\partial p_1) \geq 0$). We can rewrite this FOC as follows:

$$q_1 + \frac{dq_1}{dp_1}\left(p_1 - \underbrace{\left[w_2\left(-\frac{dq_2/dp_1}{dq_1/dp_1}\right)\right]}_{C_1(w_2)}\right) = 0 \tag{4}$$

The downstream unit considers an opportunity cost from reduced input sales to the rival which equals $w_2\left(-\frac{dq_2/dp_1}{dq_1/dp_1}\right)$ (Moresi and Schwartz, 2017). The VIP anticipates that for each new quantity obtained through a decrease of its output price (dq_1/dp_1), there will be some quantities which are diverted from the rival's demand ($-\frac{dq_2/dp_1}{dq_1/dp_1}$) and will thus not be sold at price w_2. To ease notations, we denote $C_1(w_2) = w_2\left(-\frac{dq_2/dp_1}{dq_1/dp_1}\right)$. The opportunity cost $C_1(w_2)$ acts as any production cost and tends to push upward the downstream unit's price.

The opportunity cost stands for the incentive of a merged firm, supplying a downstream rival, to raise the output price because diversion of output sales to the rival increases its input sales to the rival. The recent literature denotes this downstream temptation by the *Chen effect* (Moresi and Schwartz, 2021; Moresi and Salop, 2021; Chen, 2001). Intuitively, integration implies that the downstream unit somehow considers it has obtained a "financial interest" in the rival and earns a "dividend" on these sales which thereby limits its incentive to compete (Moresi and Salop, 2021).

The rival knows that this Chen effect reduces the VIP's incentive to compete by pushing upward its output price strategy. It thus considers the VIP's opportunity cost into its own price strategy as if it was a production cost. The firms' optimization problems yield the following sub-game equilibrium prices:

$$p_1^C(w_2) = \frac{\alpha(1-\gamma)(2+\gamma) + 2\,C_1(w_2) + \gamma w_2}{4-\gamma^2} \quad ; \quad p_2^C(w_2) = \frac{\alpha(1-\gamma)(2+\gamma) + 2w_2 + \gamma\,C_1(w_2)}{4-\gamma^2} \tag{5}$$

With our linear demand, the diversion ratio $(-(\partial q_2/\partial p_1)/(\partial q_1/\partial p_1))$ simplifies to the level of product substitution γ so the opportunity cost neatly boils down to $C_1(w_2) = \gamma w_2$. Note that, as intuition would suggest, the VIP's opportunity cost also pushes upward the rival's output price (similarly to a setting where the VIP would bear a production cost).

In addition, the rival knows that the opportunity cost is increasing with the input price w_2. Intuitively, it knows that the "dividend" the downstream unit earns on each of the rival's sale depends positively on the input margin w_2. The rival thus expects less intense competition upon an increase of the input price and increases more its output price following an increase of the input price. In other words, the Chen effect increases the rival's pass-through rate of the input price on the output price ($\frac{dp_2^C}{dw_2} = \frac{\partial p_2^C}{\partial w_2} + \frac{\partial p_2^C}{\partial C_1(w_2)}\frac{\partial C_1(w_2)}{\partial w_2} > \frac{\partial p_2^C}{\partial w_2}$).

At stage 1, the VIP anticipates the sub-game prices and thus maximizes its profit $V(p_1^C(w_2), p_2^C(w_2), w_2)$ with respect to w_2. Lemma 1 displays the equilibrium outcomes.

Lemma 1. *Under centralization, the equilibrium input price is:*

$$w_2^C = \frac{\alpha}{2} - \frac{\alpha(1-\gamma)\gamma^2}{2(8+\gamma^2)}$$

and the final prices, firms' profits and consumer surplus, respectively, are

$$p_1^C = \frac{\alpha(4-\gamma)(2+\gamma)}{2(8+\gamma^2)}\,, \quad p_2^C = \frac{\alpha[2(6+\gamma^2)-\gamma(4+\gamma^2)]}{2(8+\gamma^2)}\,, \quad V^C = \frac{\alpha^2(\gamma+2)\left(\gamma^2-\gamma+6\right)}{4(\gamma+1)\left(\gamma^2+8\right)}$$
$$\pi_2^C = \frac{\alpha^2(1-\gamma)\left(\gamma^2+2\right)^2}{(\gamma+1)\left(\gamma^2+8\right)^2}, \text{ and } \quad CS^C = \frac{\alpha^2\left(5\gamma^5+\gamma^4+24\gamma^3+36\gamma^2+16\gamma+80\right)}{8(\gamma+1)\left(\gamma^2+8\right)^2}$$

In line with Lu *et al.* (2007) and Arya *et al.* (2008a), we find that the VIP does not foreclose the rival as long as products remain imperfect substitutes ($\gamma < 1$). Intuitively, imperfect substitution implies that the VIP can only sell to customers outside its downstream unit's reach, through the rival. This means that the VIP may opt not to foreclose the rival but instead seek profits both in the upstream and downstream arenas.

Regarding the outcomes, the general view is that a vertical merger has two opposite effects. One the one hand, it incites the merged entity to increase the input price to the downstream rival so as to increase the rival's output price and drive demand towards the downstream affiliate. This is the Raise Rival's Cost (RRC) effect by Salop and Scheffman (1983)[2] which tends to increase the output prices. On the other hand, the merger eliminates the upstream margin thereby decreasing the downstream affiliate's output price on the downstream market. This is the Elimination of Double Marginalization (EDM) effect by Spengler (1950)[3] which tends to decrease the output prices.

Our computations for the equilibrium outcomes show a third effect at stake: the Chen effect, which is the downstream unit's incentive to raise its output price once the rival's input price is set (Moresi and Schwartz, 2021; Moresi and Salop, 2021; Chen, 2001). It occurs because the VIP considers again the flow of upstream profits when setting the output price which incites it to reduce the downstream competition in favor of the rival. The opportunity cost $C_1(w_2)$ reflects this pattern and tends to increase the VIP's output price thus mitigating EDM and driving demand towards the rival. Expected by the rival, the Chen effect incites the latter to increase more its output price upon an increase of the input price (increased pass-through rate) which mitigates the flow of upstream profits. Consequently, the VIP has to make an input price concession to impede this reduced flow of upstream profits. This reduced input price tends to decrease the rival's output price and mitigate RRC which drives demand to the rival.[4]

The VIP would be better off avoiding this input price concession while increase its downstream

[2]or Indirect Horizontal Consolidation by Moresi and Salop (2021)

[3]This paper is the most cited reference regarding Double Marginalization although it contains only an informal treatment of the question (Linnemer, 2022).

[4]This relates to the result by Arya *et al.* (2008b) under Cournot competition. The authors show that the merged VIP bears an ex-post temptation to encroach excessively (i.e. increase its output quantity) which also induces an input price concession (because the rival would shrink its output quantity due to strategic substitutability).

affiliate's efficiency. In other words, the VIP would be better off reducing the Chen effect on the RRC and the EDM, and rebalance the flow of profits with more downstream profits. However, the VIP cannot do that under centralization as it cannot commit not to reduce competition once having set the input price, i.e. not re-considering the flow of upstream profits in its output price decision. One way to correct for this incentive is to decentralize the output price decision thus suppressing the consideration of the downstream unit for the rival's sales, once the input prices are set. The rival would raise less its output price following a rise of its input price which enables the VIP to avoid the input price concession.

4.2 Decentralization

Under decentralization, the VIP continues to charge the downstream rival a per-unit price w_2 but it now also charges its downstream unit a per-unit price w_1. The downstream unit and the rival observe these input prices (more formally, contracts are *observable*) and then compete in prices, respectively setting prices p_1 and p_2.

Decentralization can be achieved by hiring a manager and writing a contract that would incentivize the downstream unit to maximize only the downstream profits. Another way would be to have minority outside shareholders i.e., the shareholders spin off the control rights of the downstream unit to outside shareholders and become passive majority shareholders (O'Brien and Salop, 1999). Last, decentralization can be viewed as organizational frictions between units (Arya *et al.*, 2008b)

We solve the game using the SPNE concept. By backward induction, at the price competition stage, the downstream unit and the rival respectively set the downstream prices p_1 and p_2 to maximize their profits $\pi_1(p_1, p_2)$ and $\pi_2(p_2, p_1)$. Note that the downstream unit maximizes only the downstream profit π_1 and in particular does not consider the upstream unit's sales to the rival. As a result, it now maximizes $\pi_1 = (p_1 - w_1).q_1(p_1, p_2)$. The firms' optimization problems yield the following new sub-game equilibrium prices:

$$p_1^D(w_1, w_2) = \frac{\alpha(2-\gamma-\gamma^2) + 2w_1 + \gamma w_2}{4-\gamma^2} \;\; ; \;\; p_2^D(w_1, w_2) = \frac{\alpha(2-\gamma-\gamma^2) + 2w_2 + \gamma w_1}{4-\gamma^2} \qquad (6)$$

The downstream unit's input price w_1 has replaced the VIP's opportunity cost $C_1(w_2)$. Decentralization thus has two opposite effects on downstream competition. On the one hand, decentralization enables the VIP to eliminate the *Chen effect*. The VIP commits not to consider upstream sales, once the input prices are set. This boosts competition and also implies that w_2 has a lesser effect on the rival's pass-through rate ($dp_2^D/dw_2 = \partial p_2^C/\partial w_2 < dp_2^C/dw_2$). On the other hand, it restores the VIP's margin on its downstream unit's sales which softens downstream competition.

At the contracting stage, the VIP maximizes its expected profit $V(p_1^D(w_1,w_2), p_2^D(w_1,w_2), w_1, w_2)$ with respect to w_1 and w_2. Note that the decentralized VIP continues to consider the joint profits when quoting the input prices but with the limitation that the downstream unit's decision is based on only the downstream unit's separate maximizing decision. We obtain the equilibrium input prices w_1^D and w_2^D displayed in Lemma 2 below. Proposition 1 then compares the values with those under centralization.

Lemma 2. *Under decentralization, the equilibrium input prices are*

$$w_1^D = \gamma\left(\frac{\alpha}{2} - \frac{\alpha(1-\gamma)}{4}\right) \quad ; \quad w_2^D = \frac{\alpha}{2} \tag{7}$$

Proposition 1. *With substitutes, decentralization increases the rival's input price ($w_2^D \geq w_2^C$) but sets the downstream unit's input price lower than the centralized opportunity cost ($w_1^D \leq \gamma w_2^C$). Inequalities are strict except when the goods are unrelated ($\gamma = 0$).*

We observe that the rival's input price has increased. Intuitively, decentralization has enabled the VIP to abstract from upstream sales once input prices are set which eliminates the Chen effect and thereby has reduced the rival's price responsiveness to an increase of the input price w_2. The VIP can thus avoid the previous price concession and increase the rival's input price. This result confirms Moresi and Schwartz (2017)'s proposition that decentralization would, at the margin, embolden the VIP to increase the rival's input price. Regarding the result on the input price transfer w_1, it will be clearer with the result on the output prices below.

By substituting the input price values in the sub-game price strategies, we find p_1^D and p_2^D and the rest of the equilibrium outcomes q_1^D, q_2^D, V^D and π_2^D.

Lemma 3. *Under decentralization, the equilibrium prices, profits and consumer surplus are:*

$$p_1^D = \frac{\alpha}{2}\ , \quad p_2^D = \frac{\alpha(3-\gamma)}{4}\ , \quad V^D = \frac{\alpha^2(\gamma+3)}{8(\gamma+1)}\ , \quad \pi_2^D = \frac{\alpha^2(1-\gamma)}{16(\gamma+1)}\ , \text{ and } CS^D = \frac{\alpha^2(3\gamma+5)}{32(\gamma+1)}.$$

Proposition 2. *With substitutes, decentralization decreases the final prices ($p_1^D \leq p_1^C$ and $p_2^D \leq p_2^C$). Inequalities are strict except when the goods are unrelated ($\gamma = 0$).*

Despite the increase of the rival's input price and the restoration of the double marginalization issued from the input price transfer w_1, decentralization has overall decreased the output prices. This happens because the downstream unit's input price is set sufficiently below the level of the opportunity cost under centralization ($w_1^D < C_1(w_2^C)$) so that the elimination of the Chen effect dominates the restoration of the double margin.

Intuitively, the VIP sets a low input price to commit its downstream unit will fiercely compete and thus encourage fierce competition from the rival (hence the decrease of the rival's output price despite the rise of the rival's input price). Greater competition from the rival induces an increase of its output sales that will mitigate the reduced upstream sales issued from the increased rival's input price (Moresi and Schwartz, 2017). In addition, it also improves the downstream unit's efficiency which boosts the VIP's downstream profits. Decentralization thus rebalances the VIP's flow of profits in favor of more downstream profits by increasing the RRC effect and EDM effect initially triggered by integration.

Since decentralization has increased RRC (avoidance of the input price concession) and EDM (elimination of the Chen effect), it harms the rival. Also, we find that the boost on EDM dominates the reminiscent RRC which induces greater competition and benefits the consumer surplus.

Figure 1 provides a decomposition of the variation of the VIP's profits. It shows that the increase in downstream revenues ($p_1^D q_1^D > p_1^C q_1^C$) overcomes the decrease in upstream profits from the rival sales ($w_2^D q_2^D < w_2^C q_2^C$). Ultimately, the VIP is better off with this improved balance of upstream-downstream profits.

Proposition 3. *With substitutes, decentralization increases the VIP's profit, decreases the rival's profit and increases consumer surplus.*

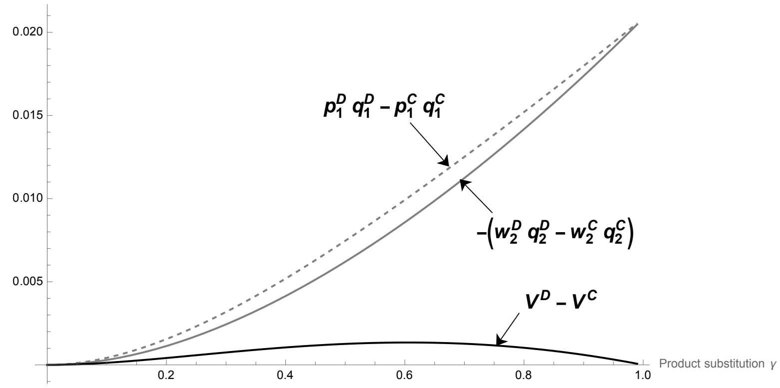

Figure 1: Comparative statics (substitutes)
(Note: we consider $\alpha = 1$ and $0 \leq \gamma \leq 0.99 < 1$)

Our propositions generalize the results of Moresi and Schwartz (2017) to a linear demand function. It addition, they show that the gains for the VIP depend on the length of product substitution. Starting at unrelated products ($\gamma = 0$), the VIP cannot gain from decentralization as products are independent and the Chen effect is already muted. When products are more substitutable ($0 < \gamma < 1$), the VIP can replace the Chen effect and benefit from decentralization even though upstream profits diminish. Above a certain substitution degree, competition is very fierce so downstream margins are short. The effect of the replacement of the Chen effect is softened relative to the reminiscent RRC (avoidance of price concession) which becomes more pronounced. As a result, the VIP's behavior is closer to that under centralization. When $\gamma \to 1$, the VIP forecloses D_2 in both structures and becomes indifferent.

4.3 Private transfer to the downstream unit

In any real-world setting the competition enforcement agencies would likely be suspicious as to why the downstream unit's input price is being made public, and might suspect collusion. On the other hand, the downstream unit would likely know the input price that the VIP was charging the rival, and this would likely appear to be "natural" and not suspicious to the competition agencies. This section studies the informational structure where the rival is unable to observe w_1, but knows that the downstream unit observes w_2. We use the Perfect Bayesian Equilibrium concept to solve this variant of the game where information is imperfect.

Consider the competition stage. The rival is unable to observe the input price transfer between the two affiliated units of the VIP before it makes its output choice. So the rival is unable to (verifiably) calculate the expected reaction function of the downstream unit. Assuming the widely used specification of passive beliefs (Gaudin, 2019), the rival treats the downstream unit's reaction as a constant $\tilde{p}_1$ which sticks to the equilibrium outcome, irrespective of the input price w_2 it obtains from the VIP (Petrakis and Skartados, 2022).[5] Consequently, the rival maximizes its profit $\pi_2(\tilde{p}_1, p_2)$ which gives the following reaction function (and sub-game price strategy at the same time) $p_2^h(w_2) = \frac{1}{2}(A + w_2)$ where $A = \alpha(1-\gamma) + \gamma\tilde{p}_1$. The rival thus acts as a monopolist over its residual demand, and its pass-through rate of the input price decreases more than under decentralization with observable contracts ($dp_2^h/dw_2 \leq dp_2^D/dw_2 \leq dp_2^C/dw_2$). The downstream unit instead knows the two input prices and anticipates the reaction of the rival. Its sub-game price strategy is the reaction function under decentralization with observable contracts $R_1^h(w_1, p_2) = \frac{1}{2}(\alpha(1-\gamma) + \gamma p_2 + w_1)$ to which we implement the above rival's reaction function: $p_1^h(w_1, w_2, \tilde{p}_1) = R_1^h(w_1, p_2^h(w_2))$.

At the contracting stage, the VIP maximizes its profits with respect to w_1 and w_2 taking into account the sub-game equilibrium price strategies. Solving the system composed by the first order conditions with respect to w_1 and w_2 and considering the beliefs hold true at equilibrium ($p_1^h = \tilde{p}_1$), we obtain the following optimal values $w_1^h = \gamma\frac{\alpha(\gamma+2)^2}{8(\gamma+1)}$, $w_2^h = \frac{\alpha(\gamma+2)^2}{8(\gamma+1)}$ and $p_1^h = \frac{\alpha(5\gamma+4)}{8(\gamma+1)}$. These equilibrium values are optimal only when $-\frac{2\sqrt{2}}{3} < \gamma < \frac{2\sqrt{2}}{3}$, i.e. when the Second Order Condition holds (Hessian matrix must be definite semi-negative (Rey and Verge, 2004)). Consequently, the rival's price is $p_2^h = \frac{\alpha((4-\gamma)\gamma+6)}{8(\gamma+1)}$ and the VIP's profit is $V^h = \frac{\alpha^2(\gamma(7\gamma+32)+24)}{64(\gamma+1)^2}$.

Proposition 4. *With substitutes, decentralization with unobservable input price w_1 by the rival: increases the rival's input price above the level of the decentralized observable one ($w_2^h \geq w_2^D$), and sets the downstream unit's input price above the centralized opportunity cost ($w_1^h \geq C_1(w_2^C)$).*

As the rival's pass-through rate has decreased, the VIP can again avoid the price concession of

[5] More formally, $\tilde{p}_1 = (\alpha(1-\gamma) + \tilde{w}_1 + \gamma\tilde{\tilde{p}}_2)/2$, where $\tilde{w}_1$ is the rival's first order belief about the downstream unit's input price, and $\tilde{\tilde{p}}_2$ is the second-order belief that the rival forms about the downstream unit's belief on the rival's equilibrium output (Brandenburger and Dekel (1993)). With passive beliefs, $\tilde{p}_1$ does not depend on w_2.

Centralization. However, the pass-through rate decreases more than under decentralization with observable contracts ($dp_2^h/dw_2 \leq dp_2^D/dw_2 \leq dp_2^C/dw_2$) which implies the VIP sets a higher input price than under decentralization. This happens because the rival does not integrate the potential output price reaction of the downstream unit (it acts as a monopolist on its market). In that sense, the VIP also cannot use the input price w_1 as a commitment device to encourage the rival to compete more and mitigate the decrease of upstream sales. It has to mitigate the decrease by itself by setting a higher input price to its downstream unit which shrinks the latter's sales and profits. Note that we find $w_1^h = \gamma w_2^h$ which is a similar relation as the opportunity cost and the rival's input price under Centralization, and shows that the VIP indeed makes its downstream unit somehow internalize the reduced upstream sales. Because the changes in the input prices do not fit the VIP's marginal incentive under centralization if follows that the VIP is worse off when it decentralizes with an unobservable input price transfer.

Proposition 5. *With substitutes, decentralization with unobservable input price w_1 by the rival is less profitable than centralization ($V^h \leq V^C$).*

Corollary 1. *In order to make decentralization more profitable than centralization, it is crucial that the rival observes the downstream unit's input price w_1.*

Corollary 1 supports the idea that competition authorities should actually be less suspicious about input price revelation in vertical markets when contracts are linear (by contrast to the case where contracts are non-linear – refer to the seminal paper by McAfee and Schwartz (1994)) because decentralization with input price revelation benefits the consumers. This result complements the one by Gaudin (2019) that input price revelation (studied as *information exchange between firms*) benefits the consumers in a setting where firms are separated and compete in prices.

5 Results for complements

The outcomes are formally the same as under substitutes. However, product complementarity yields $\gamma < 0$ which modifies our results and conclusions. In particular, note that firms' price reactions become *strategic substitutes* with *complements* ($\gamma < 0$), irrespective of centralization or decentralization.[6] As the interaction between the downstream firms D_1 and D_2 has changed, such that a decrease of one firm's price yields an increase of both firms' demand, we rename D_2 the *complementor* of D_1 instead of its *rival* (i.e. we use the same terminology as in Hagiu *et al.* (2020)).

5.1 Examples of vertical structures with product complements

The Cloud market provides readily recognizable examples of product complements with a market structure which displays high upstream concentration joint with vertically integrated producers (e.g. Amazon, Microsoft, Google). Consider first an example of imperfect complements with Amazon Web Service. Amazon owns Amazon Cloud which it uses to stream content on PrimeVideo, but also supplies its access to video-streaming competitors like Netflix.[7] Research by "Futuresource" found that there is high overlap between Netflix and Amazon Prime Video in the USA, UK and Germany: nearly half of Netflix subscribers also use Amazon Prime Video in the U.K. and the U.S., and 30% of total respondents to the research group's survey use both Amazon Prime Video and Netflix.[8] Consider now an example of perfect complements with Microsoft. Microsoft owns Azure (Microsoft's Cloud) which it links to its Xbox console but also supplies its access to independent game developers or big video-game companies like Ubisoft.[9] Videogame consoles and video games are obvious perfect complements.

In addition, the relation between two products is often fuzzy and it can happen that substitute products actually turn out to be demand complements. This is the case for network goods (Pouyet and Trégouët, 2021). Consider Google which owns Android OS and produces Pixel smartphones in-house. Google also supplies the smartphone manufacturer Samsung. The firms' smartphones are gross imperfect substitutes, but a greater number of users on one device may increase the valuation of the other device (e.g. network exclusive apps, ...). Accounting for such a relation (network effects) may turn these gross substitutes into demand complements.[10] Intuitively, if the number of

[6] We have that $\partial^2 V/\partial p_2 \partial p_1 = \partial^2 \pi_2/\partial p_1 \partial p_2 = \partial^2 \pi_1/\partial p_2 \partial p_1 = \gamma/(1-\gamma^2)$.

[7] Refer to `https://aws.amazon.com/solutions/case-studies/netflix`

[8] Refer to this article in Forbes: `https://www.forbes.com/sites/greatspeculations/2017/06/01/netflix-and-amazon-competitors-or-complementary/?sh=1504e0cd56cc` ; and this one in Futuresource : `https://www.futuresource-consulting.com/insights/amazon-bucks-the-trend-of-netflix-video-leadership`.

[9] See `https://azure.microsoft.com/en-us/solutions/gaming` and `https://customers.microsoft.com/en-us/story/ubisoft-media-telco-azure`

[10] ***Formal network effect example.*** *By implementing network effects* $(1/2)\mu(q_1+q_2)^2$*, where* μ *denotes the strength*

users on one device increases the valuation of the other device then a decrease in the price of one device increases the demand of the two devices.

5.2 Centralization

Since the computations for the centralized equilibrium outcomes are the same as with substitutes, the VIP still considers the effect of its output price decision on upstream sales after the input price is set (Chen effect). However, in contrast to substitutes, it internalizes that a decrease of its output price will attract new demand for the two firms and thus *increase* the complementor's demand (the diversion ratio is now negative since $\gamma < 0$). In other words, it integrates an opportunity benefit from increased input sales to the complementor $(C_1(w_2) < 0)$: for each new quantity obtained through a decrease of the output price, there will be some quantity that will go to the complementor and will be sold at input price w_2.

Lemma 4. *With complements, the Chen effect increases the VIP's incentive to divert sales towards the complementor and tends to decrease its output price, after the input price is set.*

Similarly to substitutes, the VIP considers again the flow of upstream profits when setting the output price. However, in contrast to substitutes, this incites it to increase the downstream collaboration in favor of the complementor. The opportunity benefit $(C_1(w_2) \leq 0)$ reflects this pattern by acting as a subsidy and tends to decrease the VIP's output price thus attracting new demand for the complementor. Expected by the complementor, the Chen effect still incites the latter to increase more its output price upon an increase of the input price (the pass-through rate still increases, due to strategic substitution) which mitigates the flow of upstream profits. Consequently, as with substitutes, the VIP has to make an input price concession to impede this reduced flow of upstream profits. This input price concession tends to decrease the rival's output price thus attracting new demand to the complementor.

of network effects, in Bowley's utility function (which is the origin of the demand functions used in the paper), we obtain the following gross utility function $U(q_1, q_2) = m + \alpha(q_1 + q_2) - (\beta(q_1^2 + q_2^2) + 2\gamma' q_1 q_2)/2 + (1/2)\mu(q_1 + q_2)^2$ *where* γ' *is the gross degree of substitution between the goods.* m, α *and* β *are positive parameters. Such utility gives the following linear inverse demand* $p_i = \alpha - (\beta - \mu)q_i - (\gamma' - \mu)q_j$, $\forall i, j$.[11] *For a given level of network effects* $\mu \in [0, 1]$, *take* $\beta = 1 + \mu$ *then we get the inverse demand used in the paper that is* $p_i = \alpha - q_i - \gamma q_j$ *where* $\gamma = \gamma' - \mu$. *Goods are substitutes when product substitution is stronger than network effects* $\gamma > 0$, *and complements otherwise* $\gamma < 0$.

5.3 Decentralization

Compared with substitutes, we find the following new Propositions with complements.

Proposition 6. *With complements, decentralization increases the rival's input price ($w_2^D \geq w_2^C$) and sets the downstream unit's subsidy lower than the centralized opportunity benefit ($0 > w_1^D \geq \gamma w_2^C$). Inequalities are strict except when the goods are unrelated ($\gamma = 0$).*

Proposition 7. *With complements, decentralization increases the downstream unit's price ($p_1^D \geq p_1^C$) but decreases the complementor's price ($p_2^D \leq p_2^C$). Inequalities are strict except when the goods are unrelated ($\gamma = 0$).*

With complements, decentralization again enables to increase the flow of downstream profits and limit the loss on upstream sales. To do that, the VIP sets the downstream unit's subsidy below the opportunity benefit (i.e. the input price is greater: $C_1(w_2^C) \leq w_1^D \leq 0$) which induces less collaboration by its downstream unit and forces the complementor to collaborate more, while the avoidance of the complementor's input price concession increases the margin on the latter's sales ($w_2^D \geq w_2^C$). Less collaboration by the downstream unit tends to increase the downstream unit's output price while decrease the complementor's output price. Meanwhile, the avoidance of the price concession tends to do the reverse. Overall, we find that the effect of a lesser collaboration dominates the effect of the avoidance of the input price concession so that the downstream unit's output price increases while the complementor's price decreases.

Proposition 8. *With complements, decentralization increases the VIP's profit, decreases the complementor's profit and decreases consumer surplus.*

Figure 1 provides a decomposition of the variation of the VIP profits. It shows that, as with substitutes, the increase in downstream revenues ($p_1^D q_1^D > p_1^C q_1^C$) overcomes the decrease in upstream profits from the rival sales ($w_2^D q_2^D < w_2^C q_2^C$). Ultimately, the VIP is better off. However, and in contrast to substitutes, the consumers are worse off due to the rise of the downstream unit's price, which overcomes the decrease of the complementor's price. These results extend Moresi and Schwartz (2017)'s analysis which remained at the centralization equilibrium outcome.

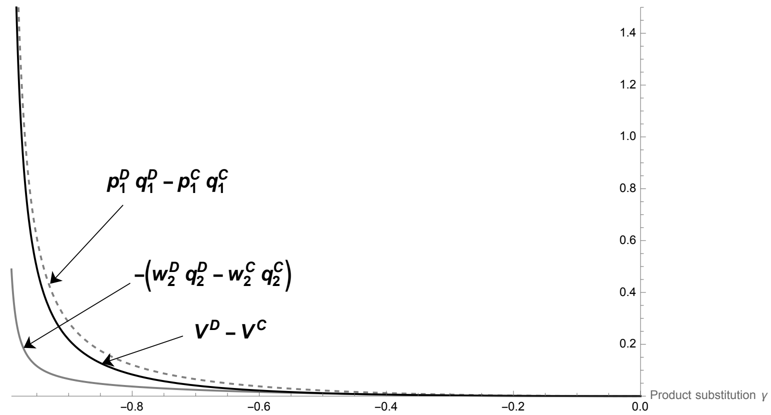

Figure 2: Comparative statics (complements)

The gains for the VIP furthermore depend on the length of product complementarity. In contrast to substitutes, the VIP's gains from decentralization always increases with respect to product complementarity. We need to bear in mind that the objective of the VIP is to boost downstream profits. Starting at unrelated goods ($\gamma = 0$), the VIP cannot gain from decentralization, as products are independent. When products are more complements ($0 > \gamma > 1$), the monopolist can replace the Chen incentive and benefit from decentralization even though upstream profits diminish. When complementarity is strong, the potential for collaboration is great and the replacement of the Chen incentive distorts collaboration even more. As it entails relatively fewer upstream losses than downstream gains, the VIP's behavior is further away to that under centralization.

Finally, it remains necessary that the input price transfer is observable. If the complementor considers the input price as private, then the similar issue as under decentralization with substitutes arises. As we substitutes, the complementor does not integrate the potential output price reaction of the downstream unit, and so acts as if it were a monopolist on its market. As a result, the complementor's pass-through rate decreases more than under decentralization with observable contracts which implies the VIP sets a higher input price than under decentralization (w_2^h remains greater than w_2^C with complements). Also because of that monopolistic pattern by the complementor, the VIP cannot use the input price w_1 as a commitment device to encourage the complementor to collaborate more and mitigate the decrease of upstream sales. It has to do it itself by setting a greater subsidy to its downstream unit ($w_1^h \leq C_1(w_2^C) \leq 0$) which boosts its collaboration for the complementor. Again, we find that $w_1^h = \gamma w_2^h$ which is a similar relation as the opportunity cost and the complementor's input price under Centralization, and shows that the VIP again makes its downstream unit somehow internalize the reduced upstream sales. Because the changes in the input prices do not fit the VIP's marginal incentive under centralization if follows that the VIP is worse off when it decentralizes with an unobservable input price transfer.

6 Extensions

6.1 Opportunism, secret contracting and beliefs

A related issue to the setting with a private transfer to the downstream unit is whether the VIP has an incentive to deviate from the input price transfer in the decentralized setting with observable input prices. Consider substitutes. Intuitively, the VIP would like to set an input price w_1 to encourage the rival to reduce its output price. This enables the VIP to increase the flow of downstream profits while mitigating the reduced upstream sales (from the greater rival's input price). Once the rival output price is set, the VIP would be better off secretly raising the downstream unit's input price to increase the latter's output price. This would increase its margin in the downstream sales, and since the rival does not react it would only slightly harm the downstream unit's sales.[12] A similar opportunistic incentive exists for complements. In that case, the VIP would like to secretly increase the subsidy. The other firm could fear such opportunistic incentive, and the VIP must therefore build commitment on the observable input price transfer.

Another possibility is that the VIP decides to also hide the rival's input price to its downstream unit. But even if the VIP has decided to delegate and thereby decentralize the pricing decision to its downstream unit, it is hard to see why it would find it worthwhile to prevent its downstream unit from knowing the price that it charges the downstream rival. In addition, one can show that

[12]Formally, consider that the VIP has made the two decentralized equilibrium offers to the firms. The VIP then expects the downstream unit to react to a secret change of input price w_1, taking the rival's behavior as given ($p_2 = p_2^D$). A similar argument applies to the rival, which takes $p_1 = p_1^D$ as given. We then implement these reactions into the VIP's profit function, derive the latter with respect to w_1, and evaluate the sign at the decentralized optimal input prices w_1^D. We find that $\partial V/\partial w_1(w_1^D) > 0$ which means that the VIP would like to raise w_1 above w_1^D.

there is no opportunistic incentive on the rival's input price once at the decentralized equilibrium. As total secrecy is therefore unlikely, we therefore relegate the computations of secret contracts to the Appendix. Overall, we find that decentralization under secret contract harms the VIP's profits.

By a similar argument as in sub-section 4.3, the VIP cannot improve the balance of profits due to the unobservability of w_1 and is worse off. This result holds true when firms hold passive beliefs. The results with two other beliefs (*symmetric beliefs* and *wary beliefs* (McAfee and Schwartz, 1994)) are relegated to the Supplementary Appendix as they overall yield to the similar conclusion.

6.2 Non-linear contracts

All our results lie on the assumption that the VIP makes linear contracts under decentralization. We assume so because linear pricing yields more interesting results and is empirically more relevant for our analysis than non-linear contracts.[13] Nonetheless, we discuss below what would occur if we assumed non-linear contracts.

The first thing to notice is that non-linear contracts, in the decentralization scheme and public contracts, enable the VIP to earn the profits of a multi-product monopolist. The VIP simply has to offer the same input prices to both firms so as to induce the latter to set the multi-product monopoly prices. The VIP can then capture the firms' entire profits through the fixed fees. Decentralization with public contracts is therefore preferred to centralization: the monopolist cannot earn more than the multi-product monopolist profit.

Turning to secret contracts, we again find that the opportunism issue remains present and centralization remains more profitable than decentralization. Nonetheless, Gilo and Yehezkel (2020) shows that non-linear contracts genuinely afford firms the possibility to vertically collude when interactions are repeated. Such a pattern may make decentralization more profitable than centralization under secret contracting. However, the study of repeated interactions exceeds the focus of our paper and we thus leave it to future research.

[13] Empirically, Crawford and Yurukoglu (2012) finds that payments between distributors and content providers rarely or negligibly include fixed monetary transfers and Dobson and Waterson (2007) argues that negotiations between vertically-related firms typically occur infrequently (e.g., annually) which makes challenging to set fixed fees.

6.3 Asymmetric transformation costs

We derive our main results restricting the setting to symmetric firms. This restriction has the great advantage of simplifying the exposition and the explanations. However, in real life, firms are likely to differ widely in their ability to transform or use the input product. We briefly provide intuition for the effect of such an alternative hypothesis on the main results.

When the rival becomes less efficient, the VIP's downstream unit is already more competitive with substitutes (or less collaborative with complements). The VIP earns less from being a greater competitor (or softer collaborator) and therefore earns less from decentralization. In contrast, when the rival becomes more efficient, the VIP earns more from decentralization for the opposite reason.

7 Conclusion

A vertically integrated producer (VIP) supplying inputs to downstream rivals often prefers to leave some independence to its integrated units. Decentralization of the final pricing decision is one way to implement such independence. This new scheme enables the downstream unit to be fully responsible for the final pricing decision while the VIP confines itself to offer supply contracts.

Our paper demonstrates that the VIP prefers decentralization over centralization as long as the contract with its downstream unit is disclosed to the rival. Decentralization with public contracts enables the VIP to act on the rival's anticipation about the downstream unit's behavior. We also show that decentralization, though creating an additional margin, reduces the final prices and benefits consumers with substitutes. In contrast, decentralization with complements increases final prices and harms consumers without creating any additional margin.

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Appendices

A Proofs.

Proof of Lemma 1. We look for the SPNE in pure strategies of the benchmark game. We use backward induction to solve this game. At the competition stage, the VIP and the rival maximize respectively $V(p_1, p_2) = p_1 q_1(p_1, p_2) + w_2 q_2(p_1, p_2)$ and $\pi_2(p_2, p_1) = (p_2 - w_2) q_2(p_2, p_1)$ with respect to p_1 and p_2. This gives the following first order conditions $\text{FOC}_{p_1} : \alpha(\gamma - 1) + 2p_1 - \gamma(p_2 + w_2) = 0$ and $\text{FOC}_{p_2} : \alpha\gamma - \alpha - \gamma p_1 + 2p_2 - w_2 = 0$. The linear demand specification implies that the second order conditions are satisfied ($\text{SOC}_{p_i} : \frac{2}{\gamma^2 - 1} < 0, \forall i$). The FOC system delivers the sub-game pricing strategies $p_1^C(w_2)$ and $p_2^C(w_2)$ of equation 5.

At the contracting stage, the VIP accounts for sub-game strategies. We thus substitute the sub-game strategies into the VIP's profit function leading to $V(w_2) = p_1^C(w_2) q_1(p_1^C(w_2), p_2^C(w_2)) + w_2 q_2(p_2^C(w_2), p_1^C(w_2))$ and maximize this expression with respect to the input price w_2. We obtain the following first order condition $\text{FOC}_{w_2} : \alpha\left(\gamma^4 + \gamma^3 + 8\gamma + 8\right) - 2(\gamma + 1)\left(\gamma^2 + 8\right) w_2 = 0$. The second order condition is satisfied $\text{SOC}_{w_2} : -\frac{2\left(\gamma^2 + 8\right)}{\left(\gamma^2 - 4\right)^2} < 0$. By isolating w_2 in the FOC, we obtain the equilibrium input price set by the VIP under centralization w_2^C.

Finally, we substitute w_2 by w_2^C into the sub-game pricing strategies to obtain the equilibrium prices $p_1^C = p_1^C(w_2^C)$ and $p_2^C = p_2^C(w_2^C)$. It remains to substitute these equilibrium prices into the demand functions, profit functions and consumer surplus to get the outcomes of Lemma 1. □

Proof of Lemma 2 and 3. We look for the SPNE in pure strategies of the game with decentralization. We use backward induction to solve the game. At the competition stage, the independent unit and the rival maximize respectively $\pi_1(p_1, p_2) = (p_1 - w_1) q_1(p_1, p_2)$ and $\pi_2(p_2, p_1) = (p_2 - w_2) q_2(p_2, p_1)$ with respect to p_1 and p_2. This gives the following first order conditions $\text{FOC}_{p_1} : \alpha\gamma - \alpha + 2p_1 - \gamma p_2 - w_1 = 0$ and $\text{FOC}_{p_2} : \alpha\gamma - \alpha - \gamma p_1 + 2p_2 - w_2 = 0$. The second order conditions are again satisfied ($\text{SOC}_{p_i} : \frac{2}{\gamma^2 - 1} < 0, \forall i$). The FOC system delivers the sub-game pricing strategies $p_1^D(w_1, w_2)$ and $p_2^D(w_1, w_2)$ of equation 6.

At the contracting stage, the VIP accounts for the sub-game strategies so that the VIP's expected profit function is $V(w_1,w_2) = p_1^D(w_1,w_2)q_1(p_1^D(w_1,w_2),p_2^D(w_1,w_2))+w_2q_2(p_2^D(w_1,w_2),p_1^D(w_1,w_2))$. The VIP maximizes this expression with respect to the input prices w_1 and w_2. We obtain the following first order conditions $\text{FOC}_{w_1}: \gamma\left(\alpha\gamma\left(\gamma^2+\gamma-2\right)-4w_2\right)-4\left(\gamma^2-2\right)w_1=0$, and FOC_{w_2} : $\alpha\left(\gamma^2\left(-\gamma^2+\gamma+8\right)-8\right)-4\gamma w_1+2\left(\gamma^4-7\gamma^2+8\right)w_2=0$. The second order conditions require the Hessian matrix to be definite semi-negative. We find $\partial^2V/\partial w_1^2 = -\frac{4\left(2-\gamma^2\right)}{(1-\gamma^2)(\gamma^2-4)^2} < 0$, $\partial^2V/\partial w_2^2 = -\frac{2\left(\gamma^4-7\gamma^2+8\right)}{(1-\gamma^2)(4-\gamma^2)^2} < 0$, and $(\partial^2V/\partial w_1^2)(\partial^2V/\partial w_2^2)-(\partial^2V/\partial w_1\partial w_2)(\partial^2V/\partial w_2\partial w_1) = \frac{8}{(1-\gamma^2)(4-\gamma^2)^2} > 0$ meaning that the Hessian matrix is definite semi-negative and the SOC is always satisfied. By solving the FOC system, we find the equilibrium input prices w_1^D and w_2^D, displayed in Lemma 2.

Finally, we substitute these input prices by their equilibrium values into the sub-game pricing strategies to obtain the equilibrium prices $p_1^D = p_1^D(w_1^D,w_2^D)$ and $p_2^D = p_2^D(w_1^D,w_2^D)$. It remains to substitute these equilibrium prices into the demand functions, profit functions and consumer surplus to get the equilibrium outcomes of Lemma 3. $\square$

Proof of Proposition 1 and 6. Lemma 1 and 2 provide the equilibrium input prices. By rewriting these values, we find:

$$w_2^D = \frac{\alpha}{2} \geq \frac{\alpha}{2} - \frac{\alpha(1-\gamma)\gamma^2}{2(8+\gamma^2)} = w_2^C \ ; \ \ |w_1^D| = |\gamma\frac{\alpha(1+\gamma)}{4}| \leq |\gamma(\frac{\alpha(1+\gamma)}{4} + \frac{\left(8-\gamma^2\right)\alpha(1-\gamma)}{4\left(\gamma^2+8\right)})| = |\gamma w_2^C| \quad \square$$

Proof of Proposition 3 and 8. Lemma 1 and 3 provide the equilibrium profits and consumer plus. We just have to compute the differences between firms' profits and consumer surplus under the two organizational schemes. They are as follows:

$$V^D - V^C = \frac{\alpha^2(1-\gamma)\gamma^2}{8(\gamma+1)\left(\gamma^2+8\right)} \geq 0 \quad , \quad \pi_2^D - \pi_2^C = \frac{3\alpha^2(\gamma-1)\gamma^2\left(5\gamma^2+16\right)}{16(\gamma+1)\left(\gamma^2+8\right)^2} \leq 0$$

$$CS^D - CS^C = \frac{\alpha^2\gamma\left(\gamma\left(\gamma\left(-17\gamma^2+\gamma-48\right)-64\right)+128\right)}{32(\gamma+1)\left(\gamma^2+8\right)^2} \geq (<)\, 0 \quad \text{when} \quad \gamma \geq (<)\, 0 \quad \square$$

Proof of Explanations below P3 and P8. The decomposition of VIP's profit gives

$$p_1^D q_1^D - p_1^C q_1^C = \frac{3\alpha^2\gamma^2\left(\gamma^3+8\right)}{8(\gamma+1)\left(\gamma^2+8\right)^2} \geq 0 \quad , \quad w_2^D q_2^D - w_2^C q_2^C = \frac{\alpha^2\gamma^2\left(\gamma\left(-4\gamma^2+\gamma-8\right)-16\right)}{8(\gamma+1)\left(\gamma^2+8\right)^2} \leq 0 \quad \square$$