Friday, 25 October 2019

Assignment 3

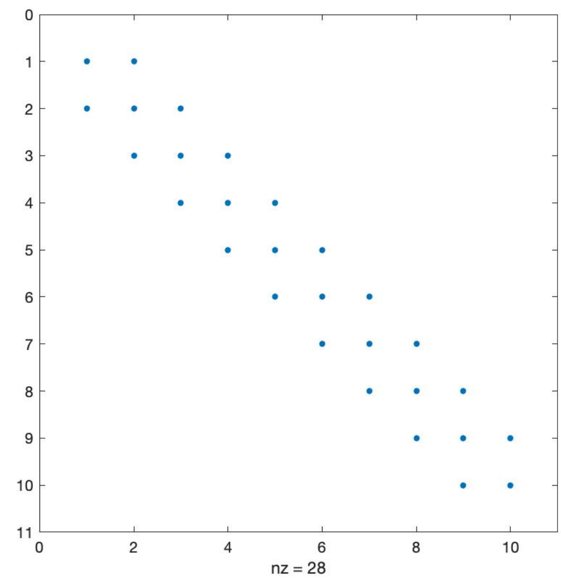
Pradeep Nag

Software :- Matlab *Roll:-B17133*

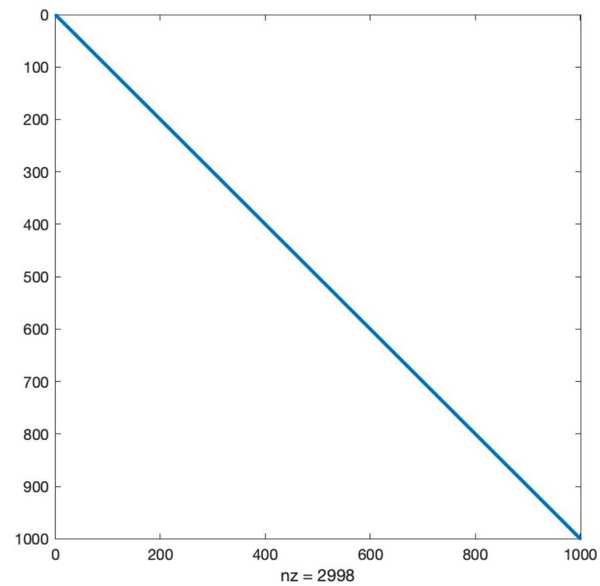
Solution 01.

A) Spy Plots for given Matrixes :-

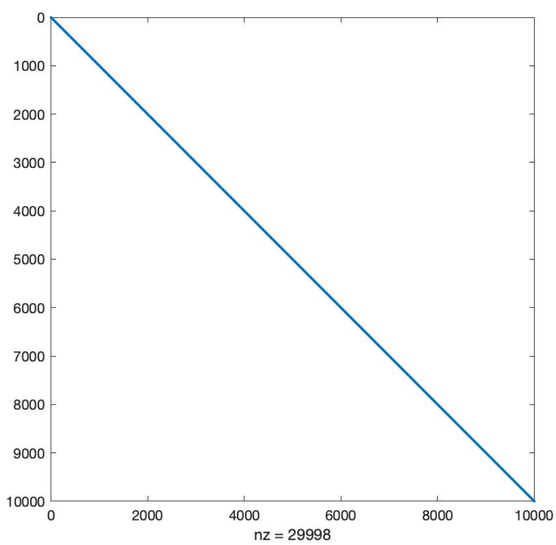
Trigonal 10 x 10 Matrix



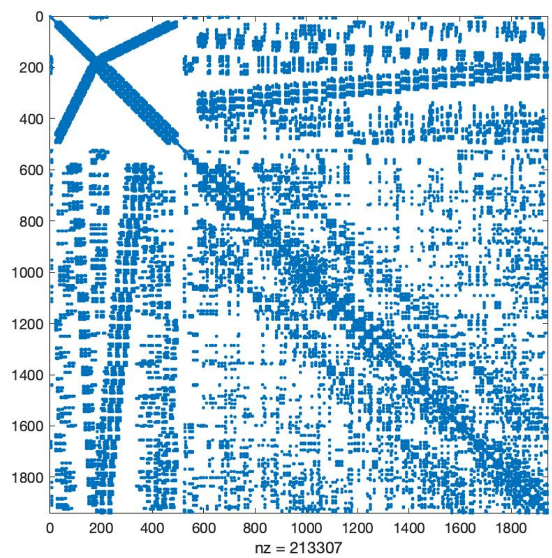
Trigonal 1000 x 1000 Matrix

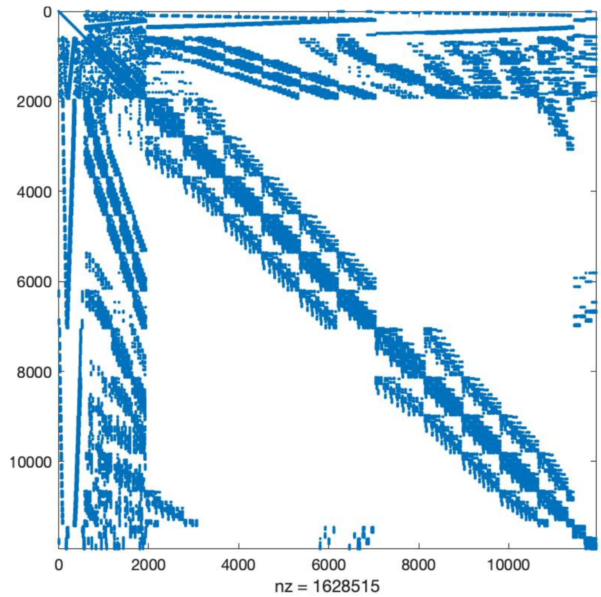


Trigonal 100000 x 10000 Matrix



General 1941 x1941 Matrix





General 11943 x 11943 Matrix

B.)

Here is the Table of time taken by different methods to solve trie-diagonal matrix of having different Size .

	TIME (seconds)		
Methods	10x10 TDM	1000x1000 TDM	10000x10000 TDM
Gauss Elimination	0.005218	5.9372	Couldn't Calculated
Matlab Solver	9.1E-05	0.023371	6.4263
TDMA	0.003865	0.005361	0.017821
Jacobi	0.003928	0.05493	13.9816
Gauss Seedal	0.005523	0.12145	17.3753
SOR	0.006425	0.13319	17.6726

While solving 10,000 x 10,000 tridiagonal matrix , Gauss elimination method was taking too much time , I run the Gauss elimination method and left it , after 4 Hours, Matlab was still solving it i.e. Gauss Elimination method take too Much time among all 6 methods. Exact Gauss elimination Time was not calculated. running time of other 5 methods is plotted in bar chart because running time of Gauss Elimination method is very high among all 6 methods.

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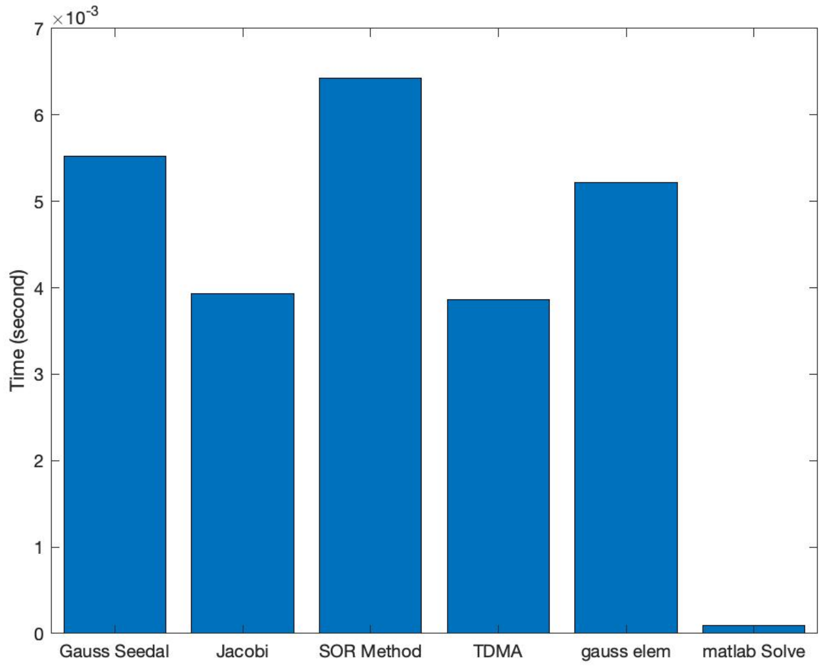
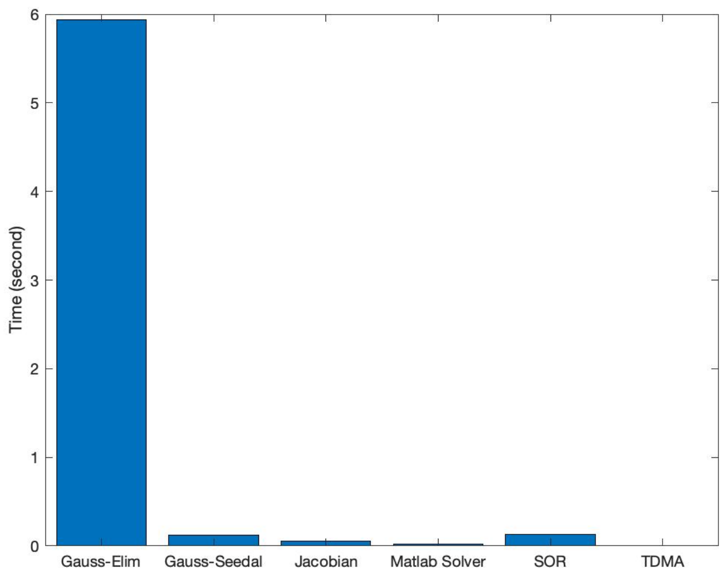
Bar Charts of time for different tridiagonal matrices



10 x10 Tridiagonal Matrix



1000 x1000 Tridiagonal Matrix

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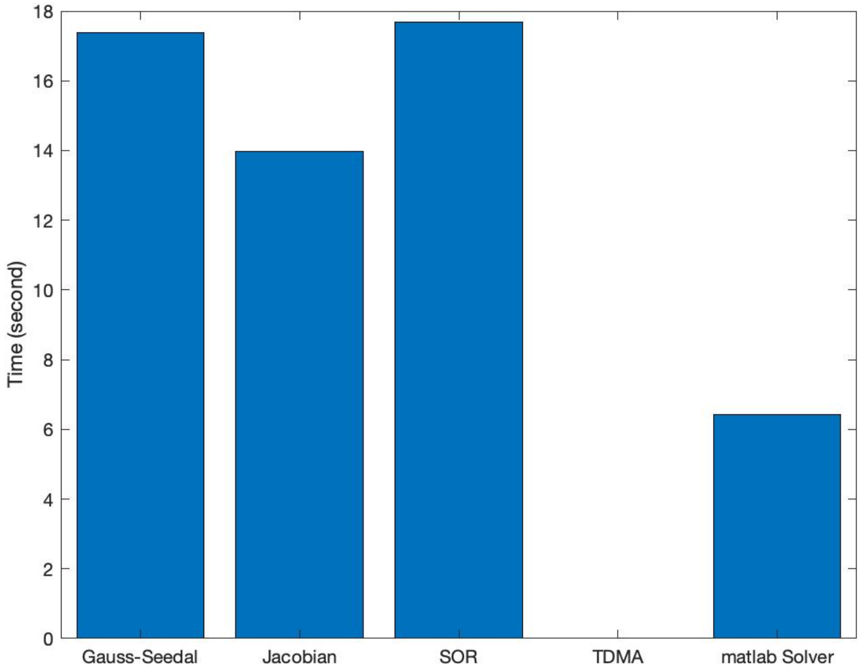
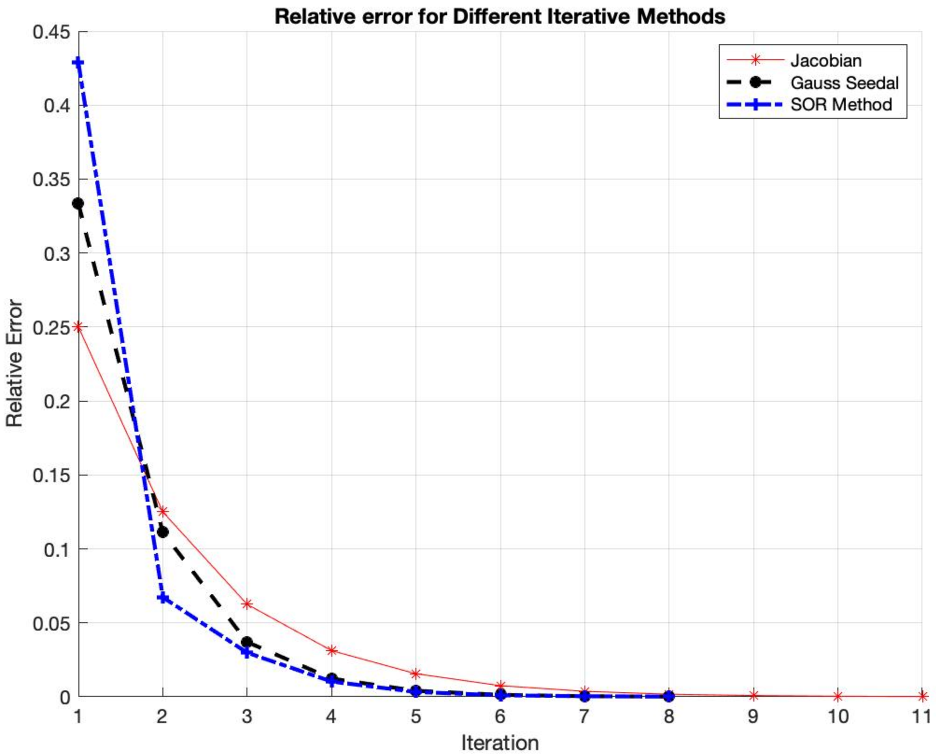
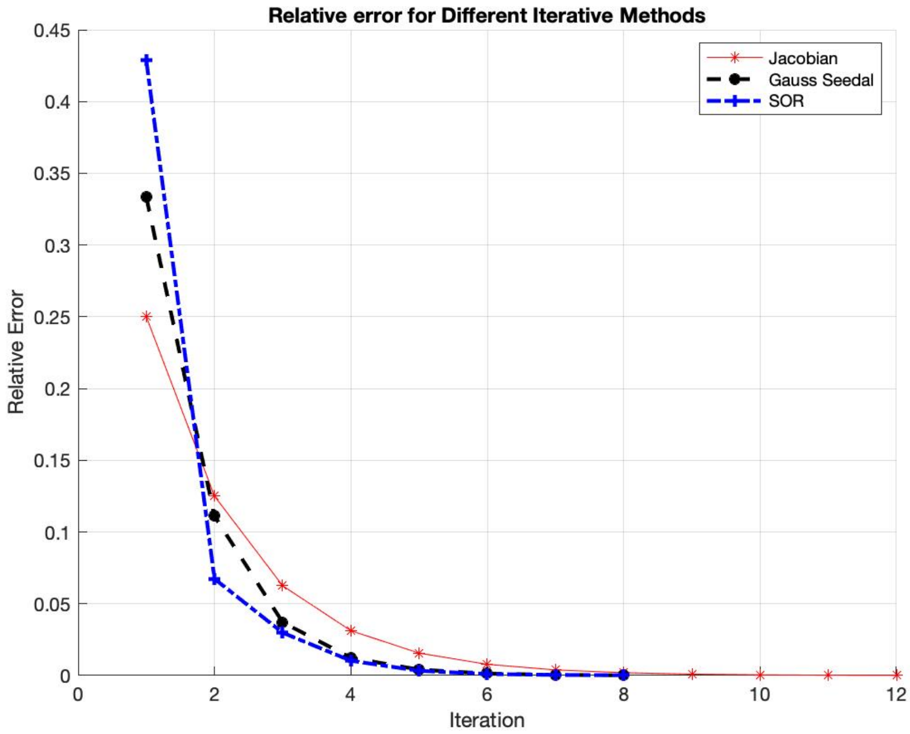


10000x10000 Trigonal Matrix

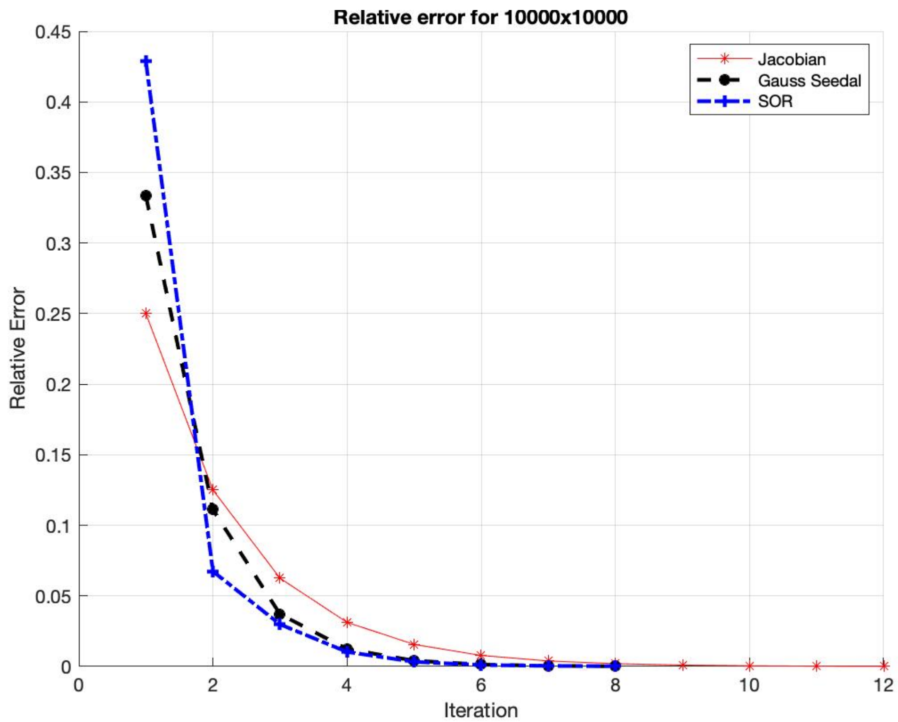
The relative Errors for different Iterative Methods Are Plotted below

10x10 Trigonal Matrix





1000x1000 Trigonal Matrix



10000x10000 Trigonal Matrix

Clearly Relative error is independent of size of the matrix. In All Matrices relative errors are similar.

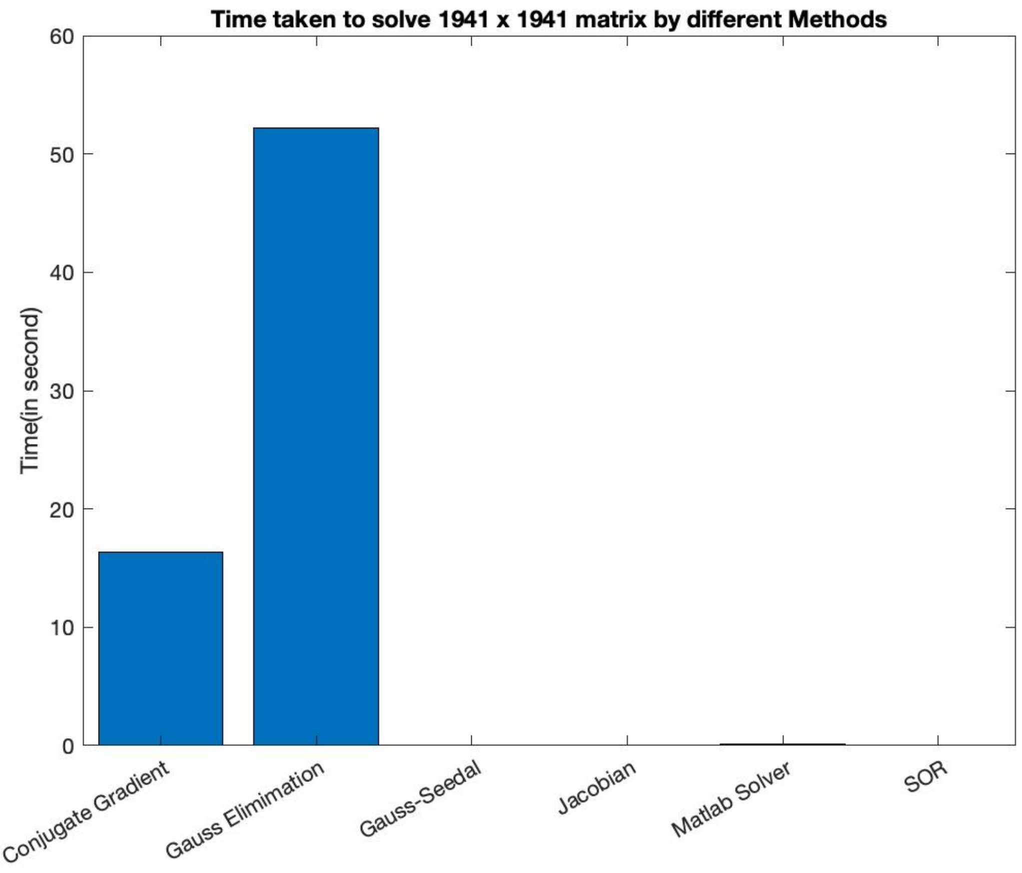
C)

For general Matrices;-

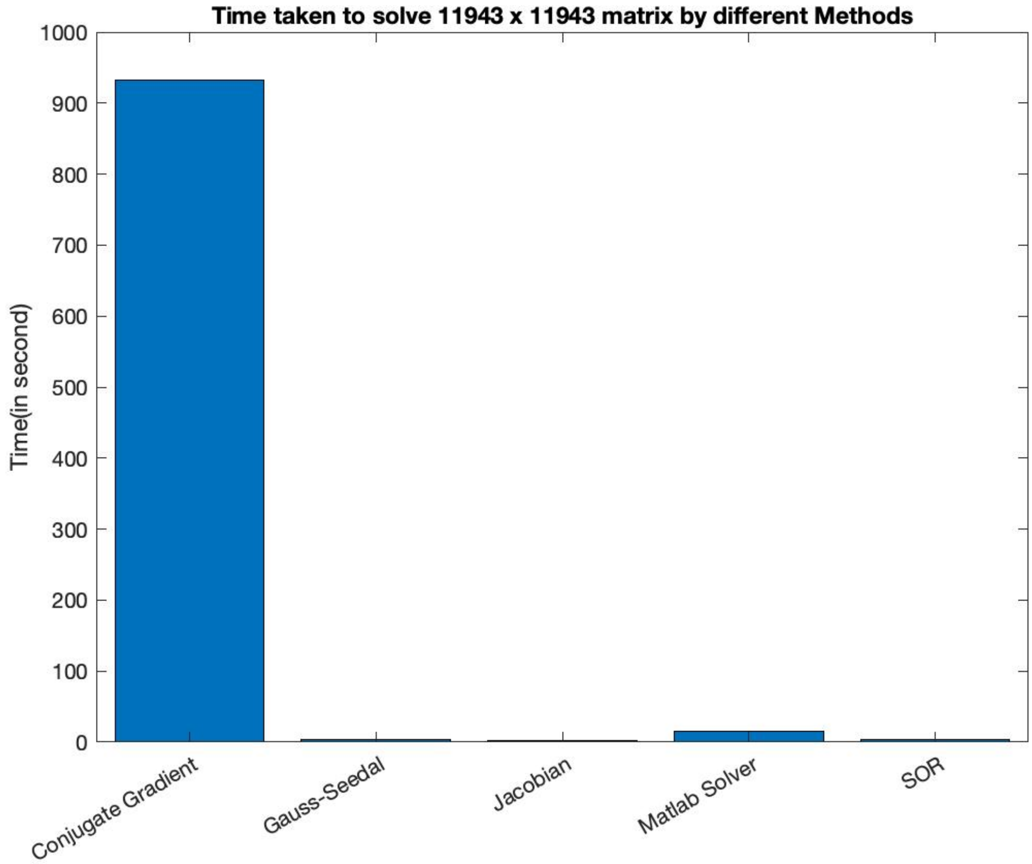
Here is the respective times taken by different methods, ***Times are in second***

Methods	*1941x1951 Matrix*	*11943x11943 Matrix*
Gauss Elimimation	52.2224	Couldn't Calculated
Matlab solver	0.10561	15.4155
Jacobi Method	0.020245	1.9642
Gauss Seedal Method	0.038288	3.8091
SOR Method	0.045106	3.5759
Conjugate Gradient	16.3938	932.363

Generally Gauss Elimination method



Again Gauss elimination method takes more than 4 Hours to solve 11943x11963 matrix, which is not shown in Graph, this time is too long among 6 methods to solve Matrix Liner Equation



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Sol. 02)

Hilbert Matrix is defined as

$$a_{ij} = (i + j - 1)^{-1}$$

where i and j are row and column indices, if dimension N of square Matrix increases, matrix terms having higher indices become smaller and smaller or Most of the terms become too small .

If we solve Matrix equation $Ax = b$ for higher $N * N$ Hilbert matrix

via Gauss Elimination, algebraic operation on very small terms are performed and Matlab approximate some terms and gives Wrong Solution i.e. Error in calculating Solution x Increases.

Clearly if We take Higher Order Hilbert Matrix Error in solving Matrix equation $Ax = b$ increases.

Here is the Absolute error And residue calculated via Solving Matrix equation $Ax = b$ for $3 <= n <= 15$.

N	*Absolute Error*	*Residue *1e-14*
3	0.000000000000014	0
4	0.000000000000691	0
5	0.000000000004426	0.045775667985222
6	0.000000000742406	0.022204460492503
7	0.000000023679008	0.056610488670037
8	0.000000388231498	0.041540741810552
9	0.000026683875153	0.040029660424867
10	0.000763332952281	0.065681679907166
11	0.011177437519037	0.054389598220421
12	0.348532844374694	0.059787339602818
13	25.3437567684893	0.098678780894485
14	4.02687169862121	0.060809419444881
15	15.5764286428996	0.103554709841456

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Sol. 04)

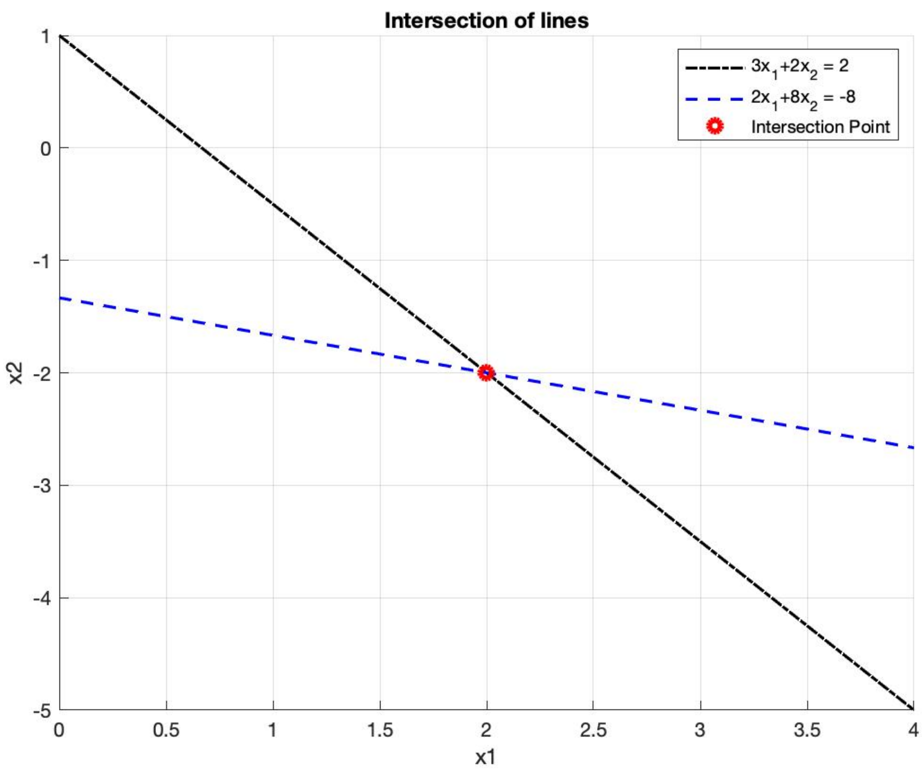
A) *The Matrix Equation $Ax = b$ is :-*

$$\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

Which can be written in form of these Two straight line:

$$\begin{bmatrix} 3x_1 + 2x_2 \\ 2x_1 + 6x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

The solution of Matrix Equation $Ax = b$ lie in the intersection of these line As shown below in Graph.



Clearly solution of Equation is

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

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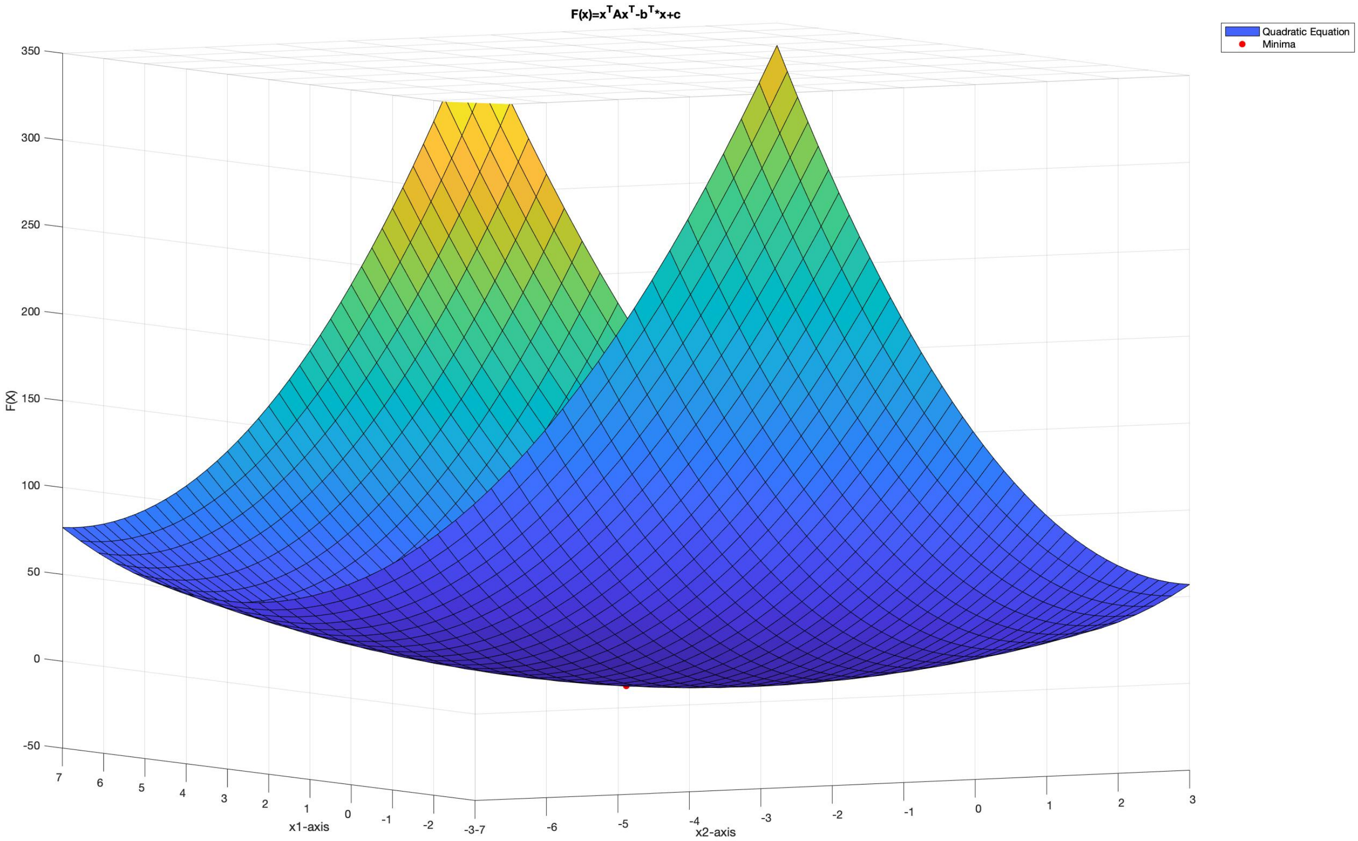
B) *The Quadratic Equation is*

$$F(x) = \frac{1}{2}x^T A x - b^t x + c$$

where $c = 0$ *.*

Here is the Plot of Quadratic Equation. and



if we Differentiate The Quadratic Equation, We get :_

$$F'(x) = Ax - b$$

Minima of this quadratic equation exists at $F'(x) = 0$ *or when* $Ax = b$*.*

i.e. Minima of the Quadratic equation is The Solution of Matrix Equation . The Minima is Plotted as Red dot in Graph shown above.

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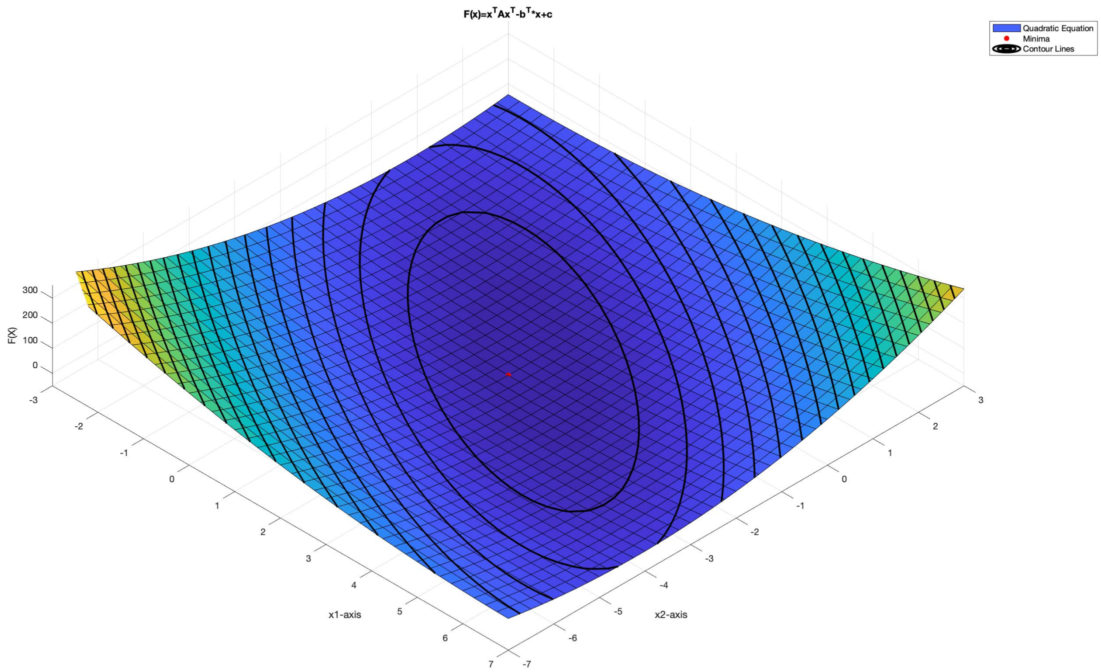
C)

Here is the plot of Contours of This Quadratic form where $F(x)$ has constant value. Which are shown as Black Elliptical shape.



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Sol. 0.5)

The Matrix equation is:

$$\begin{bmatrix} 7 & 3 & -1 & 2 \\ 3 & 8 & 1 & -4 \\ -1 & 1 & 4 & -1 \\ 2 & 4 & -1 & 6 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 3 \end{bmatrix}$$

which is solved by Jacobi, Gauss-seedal and SOR $(\omega = 1.4)$ method .

Number of iteration taken in Jacobi method is 17 and Gauss_seedal is 7 but SOR method diverges and fails to Solve the Matrix equation. it Iterated upto 2799 and exited.

Sol. 06)

SOR Method converges Faster for $\omega = 1$ and successfully solve metric equation. SOR method works like Gauss Seedal Method. i.e No of iterations are same.

Here is the Graph of No of iteration w.r.t relaxation Parameter

such that $\omega = 1 : 0.1 : 2$