Tuesday, 5 November 2019

Assignment 4

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Software :- Matlab *Roll:-B17133*

Solution 01.

Matrix generated by $rand(5,5)$ *function is:-*

$$A = \begin{bmatrix} 0.3015 & 0.6665 & 0.0326 & 0.3689 & 0.6448 \\ 0.7011 & 0.1781 & 0.5612 & 0.4607 & 0.3763 \\ 0.6663 & 0.1280 & 0.8819 & 0.9816 & 0.1909 \\ 0.5391 & 0.9991 & 0.6692 & 0.1564 & 0.4283 \\ 0.6981 & 0.1711 & 0.1904 & 0.8555 & 0.4820 \end{bmatrix}$$

Note:- $rand()$ ***function gives different matrixes for each execution of programme.***

Eigen Values calculated for Matrix A *using* $eig()$ *function are :*

$$eigValues = \begin{pmatrix} 2.4572 + 0.0000i \\ 0.5883 + 0.0000i \\ -0.4283 + 0.1432i \\ -0.4283 - 0.1432i \\ -0.1891 + 0.0000i \end{pmatrix}$$

Time taken by $eig()$ *function is :*

$t = 0.0011$ *sec.*

Eigen Values calculated for Matrix A *using* $roots()$ *function are :*

$$eigValues = \begin{pmatrix} 2.4572 + 0.0000i \\ 0.5883 + 0.0000i \\ -0.4283 + 0.1432i \\ -0.4283 - 0.1432i \\ -0.1891 + 0.0000i \end{pmatrix}$$

Time taken by $roots()$ function is :

$$t = 0.0266 \text{ sec.}$$

Absolute Error Calculated using $norm()$ function is:

$$error = 8.8818e - 16$$

Sol. 02)

Given Tridiagonal Matrix is:-

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

Eigen Values calculated for Matrix A using $eig()$ function are :

$$eigValues = \begin{pmatrix} 0.2679 \\ 1.0000 \\ 2.0000 \\ 3.0000 \\ 3.7321 \end{pmatrix}$$

Eigen Values calculated via given expression

$$eigValues = 2 - 2cos(j\frac{\pi}{n+1})$$

$$eigValues = \begin{pmatrix} 0.2679 \\ 1.0000 \\ 2.0000 \\ 3.0000 \\ 3.7321 \end{pmatrix}$$

Eigen values are same in both expressions.

The given expression works for Matrix of Higher order.

Sol. 03)

Given Matrix is:-

$$A = \begin{bmatrix} 0.9901 & 0.002 \\ -0.0001 & 0.9904 \end{bmatrix}$$

Solving for initial value $x^0 = [1,0.9]^T$.

Eigen Values and Eigen-vectors calculated via ***power method*** *are:-*

$$eigValues = \begin{pmatrix} 0.9912 \\ 0.9903 \end{pmatrix}$$

$$eigVectors = \begin{pmatrix} 0.8855 \\ 0.4646 \end{pmatrix}, \begin{pmatrix} 0.7054 \\ 0.7088 \end{pmatrix}$$

Eigen Values and Eigen-vectors calculated via $eig()$ *function are:-*

$$eigValues = \begin{pmatrix} 0.9903 + 0.0004i \\ 0.9903 - 0.0004i \end{pmatrix}$$

$$eigVectors = \begin{pmatrix} 0.9759 \\ 0.0732 + 0.2056i \end{pmatrix}, \begin{pmatrix} 0.9759 \\ 0.0732 + 0.2056i \end{pmatrix}$$

The issue with the power method is that it gives real Eigen values and its corresponding Eigen vectors.

Given another Matrix is:-

$$A = \begin{bmatrix} 8 & -1 & -5 \\ -4 & 4 & -2 \\ 18 & -5 & -7 \end{bmatrix}$$

Solving for initial value $x^0 = [1,0.8,1]^T$.

Eigen Values and Eigen-vectors calculated via $eig()$ function are:-

$$eigValues = \begin{pmatrix} 2.0000 + 4.0000i \\ 2.0000 - 4.0000i \\ 1.0000 \end{pmatrix}$$

$$eigVectors = \begin{pmatrix} 0.3162 - 0.3162i \\ 0.6325 \\ -0.6325i \end{pmatrix}, \begin{pmatrix} 0.3162 + 0.3162i \\ 0.6325 \\ 0.6325i \end{pmatrix}, \begin{pmatrix} 0.4082 \\ 0.8165 \\ 0.4082 \end{pmatrix}$$

Power method diverges while calculating Eigen vector and Eigen values, so it can't be calculated, and shifted power method is calculating wrong Eigen value and Eigen Vector.

***Fortunately,** inverse power method successfully calculated "Real" Eigen Vector and Eigen Value, which is :-*

$$eigValues = (1.0000) \quad eigVectors = \begin{pmatrix} 0.4082 \\ 0.8165 \\ 0.4082 \end{pmatrix}$$

The reason is the other two Eigen-values are imaginary and power method can not solve for imaginary values.

Solution. 04)

A) *Given equations are:-*

$$x^3 - 2xy + y^7 - 4x^3y = 5 \qquad \text{......... (1)}$$

$$y\sin(x) + 3x^2y + \tan(x) = 4 \qquad \text{......... (2)}$$

We can write Eq.2 as

$$y = \frac{4 - tan(x)}{3x^2 + sin(x)} \qquad \text{......... (3)}$$

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By substituting y from Eq.(2) to Eq (1):-

$$x^3 - 2x\frac{4 - tan(x)}{3x^2 + sin(x)} + \left(\frac{4 - tan(x)}{3x^2 + sin(x)}\right)^7 - 4x^3\frac{4 - tan(x)}{3x^2 + sin(x)} = 5$$

If we solve this non-liner equation by newton method with initial value $x_0 = 1$*, we get root of equation x*

$$x = 0.7319$$

if we put x in Eq.3

$$y = 1.3630$$

B) If we solve system of non-linear Eq.1 and Eq2 Directly , we need to calculate Jacobian of given system of non-linear equations which is

$$J = \begin{bmatrix} 3x^2 - 2y - 12x^2y & 7y^6 - 2x - 4x^3 \\ y\cos(x) + 6xy + \tan^2 x + 1 & sin(x) + 3x^2 \end{bmatrix}$$

using this J on newton method, the non-liner equation is solved.

root is :

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.7319 \\ 1.3630 \end{pmatrix}$$