ME 504 Numerical Methods for Engineering Computation 2019
Assignment 2 (Root Finding)

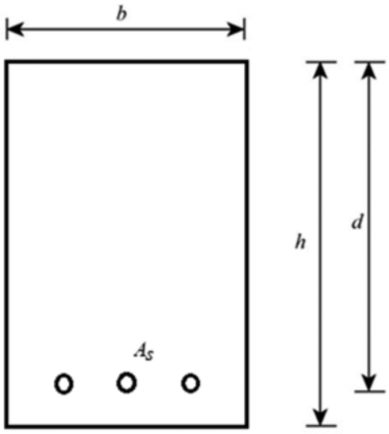
Marks : 30

(1) The following equation can be used as a simple model in Chandrayaan Mission (rocket dynamics)

$$m_0\left[1 - e^{-\left\{\frac{(v+gt)}{v_r}\right\}}\right] = u_f t$$

where m_0 is the mass of the rocket at time t = 0, v is its upward velocity at time t seconds, v_r is the relative velocity at which the fuel is ejected, u_f is the fuel consumption rate and g is the acceleration due to gravity (9.81 m/s^2). Determine time instant t, when v = 1500 m/s, m_0 = 200,000 kg, v_r = 2500 m/s and u_f = 3000 kg/s. Using MATLAB/Octave code using method of false position. [5 marks]

(2) In the design of a singly reinforced concrete beam section, the area of the reinforcement is given by the equation 1. Report the minimum and maximum possible depths *d (mm)* when the upper and lower limits of provided reinforcement area (A_{st}) are 0.8% and 4% of the effective area (Effective Area = *bd*) of the beam, respectively. Solve the problem by coding it using secant method



$$\frac{A_{st}}{bd} = \frac{f_c}{2f_y}\left[1 - \sqrt{1 - 4.598\,R/f_c}\right] \qquad (1)$$

where $R = M_u/bd^2$

Other input parameters are as follows :

b (width of Beam) = 250mm

M_u (Factored moment) = 126KNm

f_c (Characteristic strength of concrete) = 25MPa

f_y (Yield strength of steel) = 415MPa [5 marks]

(3) In control systems analysis, transfer functions are developed that mathematically relate the dynamics of a system's input to its output. A transfer function for a robotic positioning system is given by

$$G(s) = \frac{C(s)}{N(s)} = \frac{s^3 + 9s^2 + 26s + 24}{s^4 + 15s^3 + 77s^2 + 153s + 90}$$

where G(s) = system gain, C(s) = system output, N(s) = system input, and s = Laplace transform complex frequency. Use MATLAB/Octave to find the roots of the numerator and denominator and factor these into the form

$$G(s) = \frac{(s + a1)(s + a2)(s + a3)}{(s + b1)(s + b2)(s + b3)(s + b4)}$$

where a_i and b_i = the roots of the numerator and denominator, respectively. [5 marks]

(4) The natural frequencies of a uniform cantilever beam are related to the roots βi of the frequency equation *cosh β cos β + 1 = 0*, where

$$\beta i^4 = (2\pi f i)^2 \frac{mL^3}{EI}$$

f_i = ith natural frequency (cps)

m = mass of the beam

L = length of the beam

E = modulus of elasticity

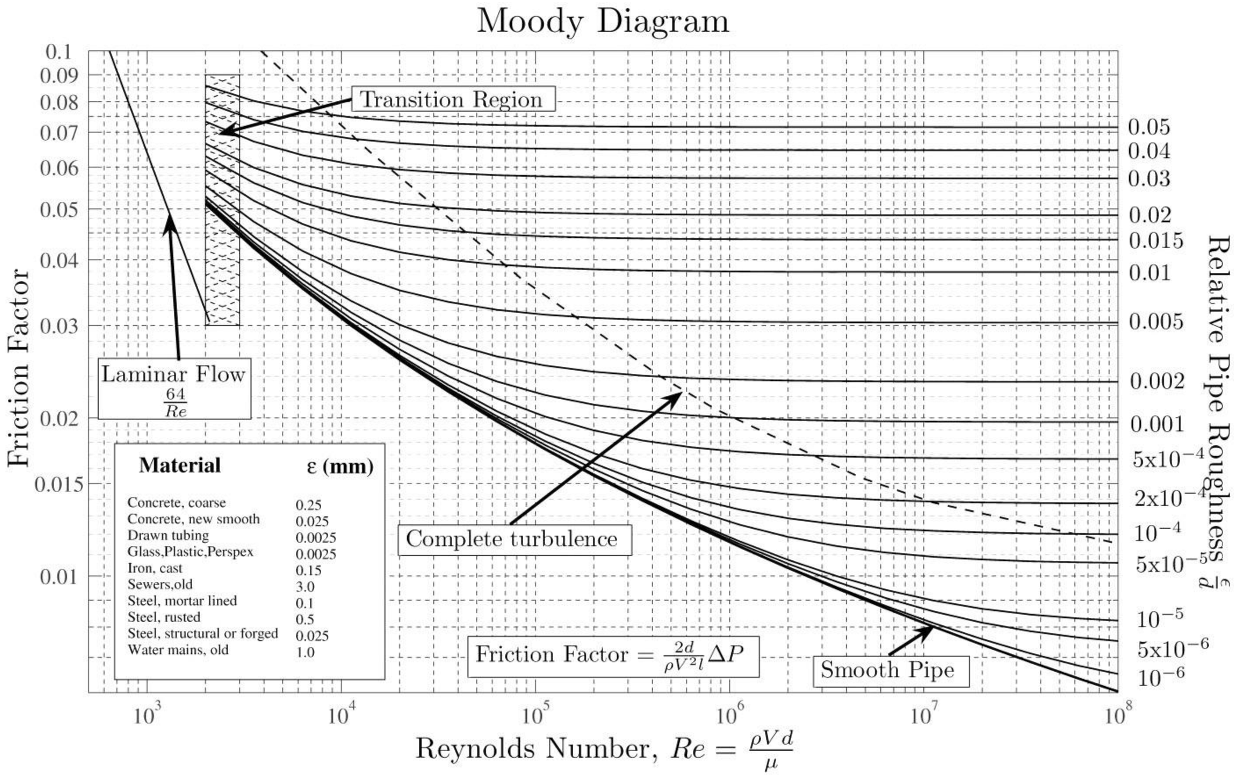
I = moment of inertia of the cross section

Using *fzero* function, determine the lowest two frequencies of a steel beam 0.9 m long, with a rectangular cross section 25 mm wide and 2.5 mm high. The mass density of steel is 7850 kg/m^3 and E = 200 GPa. [5 marks]

(5) The Colebrook–White equation is used to express Darcy friction factor ***f*** as a function of Reynolds number ***Re*** and pipe relative roughness ***ε / D***, fitting the data of experimental studies of turbulent flow in smooth and rough pipes. The equation is given as

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{\epsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right]$$

Generally, for such nonlinear equations, the solutions are provided either as tables or as charts in books. The chart for such nonlinear phenomenon from text book is shown below.



Hope that you can read the approximate value of *f* from chart for the lab exam problem condition -- Re (Reynolds Number) = 5000, ***ε*** ***(***surface roughness) for rusted steel pipe is 0.5 mm, D (diameter of pipe) is 10 cm.

Part of equation has already been solved for given Re, ***ε*** and D in lab exam and class. Using *fzero* or your own solver, can you try to come up with a plot as shown above? (Hint : have to vary over two parameters Re and ***ε/D*** and solve for ***f*.** Also to plot xaxis on log scale – help semilogx)

[10 marks]