This script applies KF to the measurement data. There is a provision to apply estimate filtering to reject spurrious estimates. This filtering using either innovation covariance or mixture model of the sensor.

In [16]:

```
# -*- coding: utf-8 -*-
"""
Created on Mon Nov 13 15:17:51 2017

@author: Nishant
"""
import math
import csv
import numpy as np
import matplotlib.pyplot as plt

# Calculates stats for given data
def stats(reads):
    mu=sum(reads)/len(reads)
    sq=[x*x for x in reads]
    sig=sum(sq)/len(sq)-mu*mu
    return mu,sig

# Returns value of Gaussian pdf of mixture for a passed measurement xs
def sample_from_gaussian(pdf,x):
    pow=-1/2*(x-pdf[0])*(x-pdf[0])/(pdf[1])
    return (1/np.sqrt(2*math.pi*pdf[1]))*np.exp(pow)

# Returns value of exp decay pdf of mixture for a passed measurement xs
def sample_exp_decay(xs):
    return 0.05*np.exp(-0.05*xs)

# Declaration of some global lists
x_measures=[]
x_measurescm=[]
x_estimates=[]
v_estimates=[]
sigx=[]
sigv=[]
sigxv=[]
T=[0]

# Returns a list corresponding to proportion of xi belonging to different compone
def get_mixture_probs(xp,xs):
    return [0.8*sample_from_gaussian([xp[0][0],0.001],xs),0.2*sample_exp_decay(xs
```

In [17]:

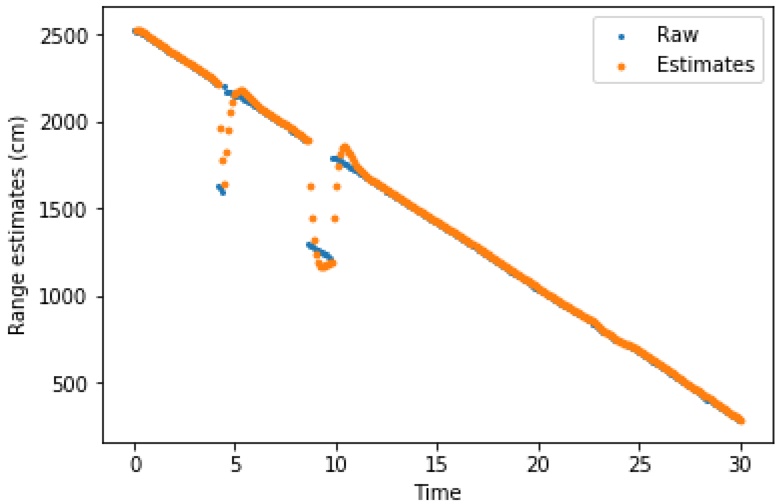
```
# Takes in system Markov model i.e. A,P,R,q, Current measurement xs, Prior xm
# Outputs updated state estimate and updated uncertainty
def KF(xm,xs,P,R,A,q,n,td):

    b=np.array([[+(td**3)*0.01*(0.1**3)-(0.1**3)*0.01*(0.1**3), (td**2)*0.01*(0.1
    c=np.array([[0,(td-1)*0.1],[0,0]])

    A=np.array(A)-np.array(c)
    q=np.array(q)+np.array(b)

    x_predicted=A*xm
    Pn=np.dot(A,P*np.transpose(A))+q
    H=np.asmatrix([1,0])

    a=np.dot(H,(Pn*np.transpose(H))+R)
    K=Pn*np.transpose(H)/a
    #K=np.reshape(np.array([K[0][0],K[1][0]]),(2,1))

    if n==1:
        x_posterior=x_predicted+K*(xs-H*x_predicted)
        P_posterior=(np.identity(2)-K*H)*Pn

        T.append(T[-1]+0.1)
    elif n==2:
        x_temp=x_predicted+K*(xs-H*x_predicted)
        P_temp=(np.identity(2)-K*H)*Pn
        delta=xs-float(x_temp[0])
        inno=np.dot(H,(P_temp*np.transpose(H)))
        var=delta*delta/inno
        if var<9:
            x_posterior=x_predicted+K*(xs-H*x_predicted)
            P_posterior=(np.identity(2)-K*H)*Pn
            T.append(T[-1]+0.1*td)
            td=1

        else:
            td+=1
            return np.array([]),np.array([]),td

    elif n==3:
        x_temp=x_predicted+K*(xs-H*x_predicted)
        p=get_mixture_probs(x_temp,xs)

        if p[0]>p[1]:
            x_posterior=np.array(x_predicted)+np.array(K)*(xs-np.dot(H,x_predicte
            P_posterior=(np.identity(2)-K*H)*Pn
            T.append(T[-1]+0.1*td)
            td=1
        else:
            td+=1
            return np.array([]),np.array([]),td

    return x_posterior,P_posterior,td
```

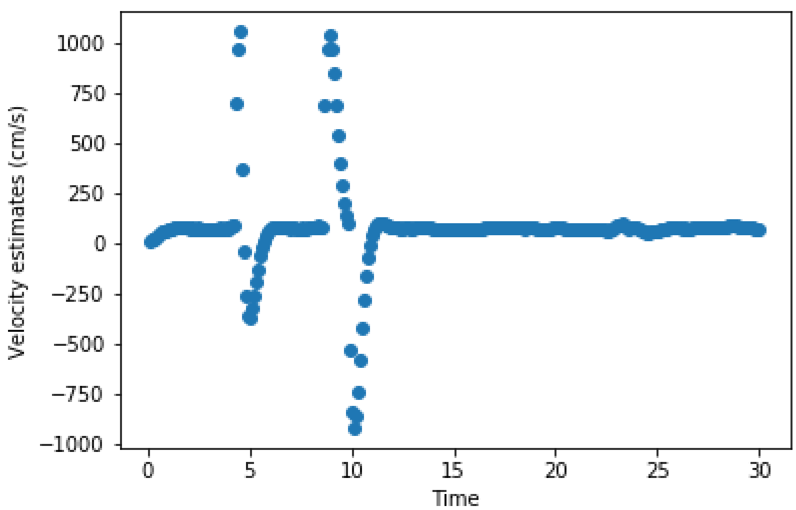
In [18]:
```
# Iterates through measurement data, updates posteriors using KF and stores it to
def estimate(n):
    P0=np.array([[0.01, 0],[0, 0.01]])
    x0=np.asmatrix([25.30,.1]).T
    R=0.001
    Q=[[0.01*0.1*0.1*0.1, 0.01*0.1*0.1],[0.01*0.1*0.1,0.01*0.1]]
    A=[[1,-0.1],[0,1]]
    t=1

    for i in x_measures:
        xm=i
        xt,Pt,t=KF(x0,xm,P0,R,A,Q,n,t)
        if xt.tolist()!=[] and Pt.tolist()!=[]:
            x_estimates.append(float(xt[0])*100)
            v_estimates.append(float(xt[1])*100)
            sigx.append(np.sqrt(Pt.tolist()[0][0]*10000))
            sigv.append(np.sqrt(Pt.tolist()[1][1]*10000))
            sigxv.append(Pt.tolist()[0][1]*10000/np.sqrt(Pt.tolist()[0][0]*10000)
            x0=np.asmatrix(xt)
            P0=np.asmatrix(Pt)
```

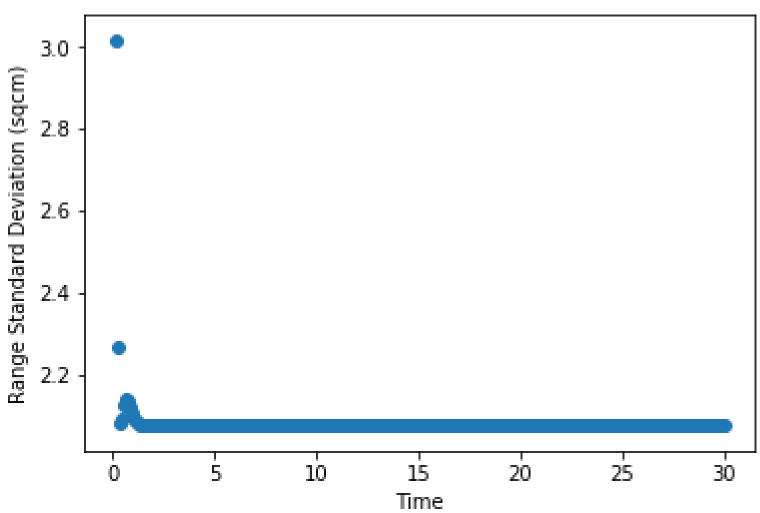
KF without object rejection

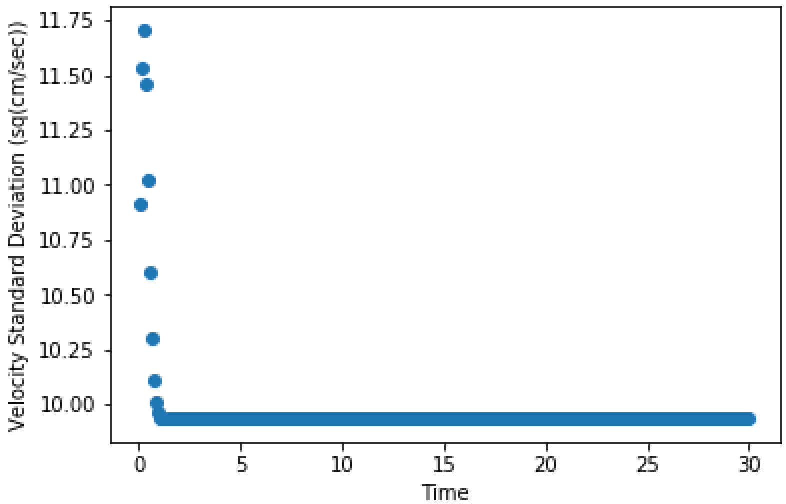
In [19]:

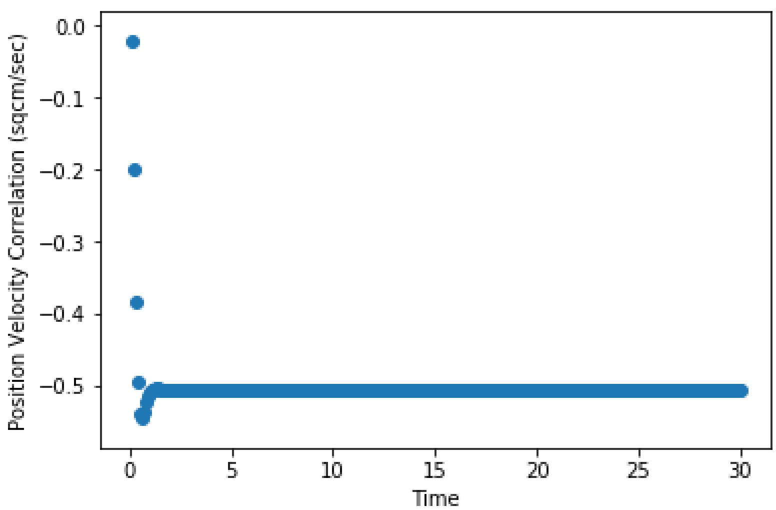
```
#Script starts from here
#Reading data file and queuing data
with open('RBE500-F17-100ms-Constant-Vel.csv', newline='') as csvfile:
 spamreader = csv.reader(csvfile, delimiter=' ', quotechar='|')
 spamreader=list(spamreader)
 for row in spamreader:
    x_measures.append(float(row[0])/100)
    x_measurescm.append(float(row[0]))


# Input the following numbers as arguments to estimate() for the corresponding po
# 1: Conventional KF   2: KF with object rejection (innovation)  3: KF with objec
estimate(1)


# Plot data
T.remove(0)
Ta=np.arange(0.0, 30.0, 0.1)

plt.scatter(Ta,x_measurescm,s=4,label='Raw')
plt.scatter(T,x_estimates,s=8,label='Estimates')
plt.xlabel('Time')
plt.legend()
plt.ylabel('Range estimates (cm)')
plt.show()

plt.scatter(T,v_estimates)
plt.xlabel('Time')
plt.ylabel('Velocity estimates (cm/s)')
plt.show()

plt.scatter(T,sigx)
plt.xlabel('Time')
plt.ylabel('Range Standard Deviation (sqcm)')
plt.show()

plt.scatter(T,sigv)
plt.xlabel('Time')
plt.ylabel('Velocity Standard Deviation (sq(cm/sec))')
plt.show()

plt.scatter(T,sigxv)
plt.xlabel('Time')
plt.ylabel('Position Velocity Correlation (sqcm/sec)')
plt.show()
```


Raw
Estimates
Range estimates (cm)
Time


Velocity estimates (cm/s)
Time


Range Standard Deviation (sqcm)
Time





Interpretation: It can be seen that KF doesn't handle the sudden variations in measurements due to external sources very well. If the datais not sufficient it may output wrong estimates. Also the velocity estimate can be seen to spike during external disturbance which may also affect the estimates in the case of insufficient data.

This script applies KF to the measurement data. There is a provision to apply estimate filtering to reject spurrious estimates. This filtering using either innovation covariance or mixture model of the sensor.

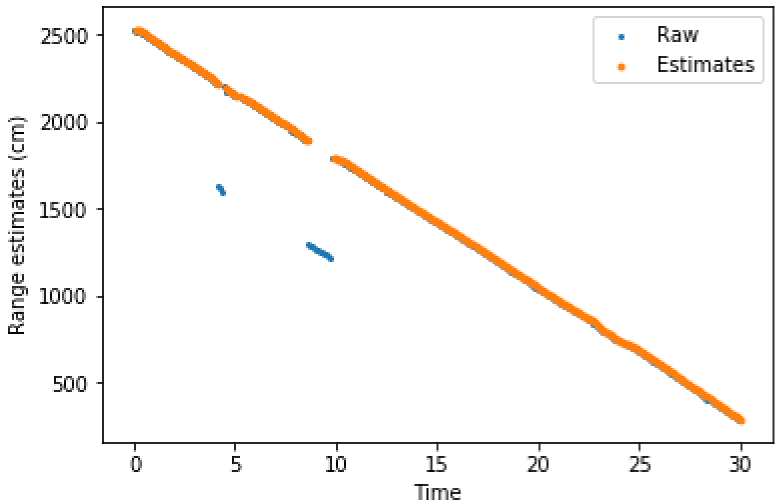
In [5]:
```
# -*- coding: utf-8 -*-
"""
Created on Mon Nov 13 15:17:51 2017

@author: Nishant
"""
import math
import csv
import numpy as np
import matplotlib.pyplot as plt

# Calculates stats for given data
def stats(reads):
    mu=sum(reads)/len(reads)
    sq=[x*x for x in reads]
    sig=sum(sq)/len(sq)-mu*mu
    return mu,sig

# Returns value of Gaussian pdf of mixture for a passed measurement xs
def sample_from_gaussian(pdf,x):
    pow=-1/2*(x-pdf[0])*(x-pdf[0])/(pdf[1])
    return (1/np.sqrt(2*math.pi*pdf[1]))*np.exp(pow)

# Returns value of exp decay pdf of mixture for a passed measurement xs
def sample_exp_decay(xs):
    return 0.05*np.exp(-0.05*xs)

# Declaration of some global lists
x_measures=[]
x_measurescm=[]
x_estimates=[]
v_estimates=[]
sigx=[]
sigv=[]
sigxv=[]
T=[0]

# Returns a list corresponding to proportion of xi belonging to different compone
def get_mixture_probs(xp,xs):
    return [0.8*sample_from_gaussian([xp[0][0],0.001],xs),0.2*sample_exp_decay(xs
```

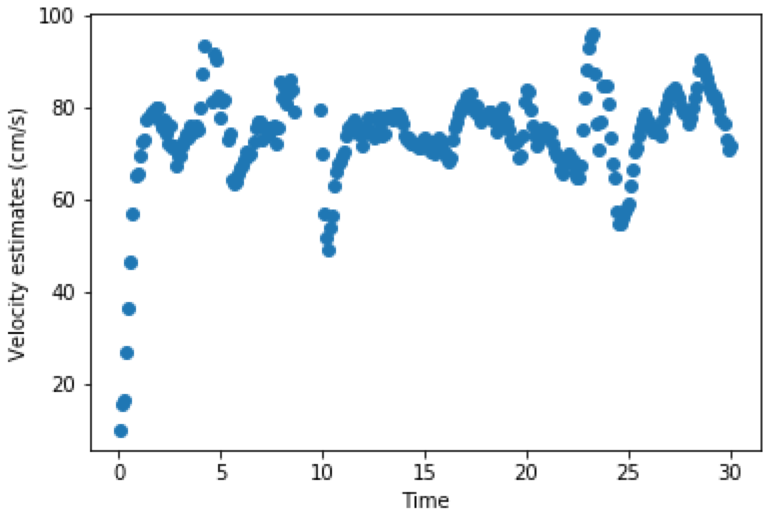
In [6]:
```
# Takes in system Markov model i.e. A,P,R,q, Current measurement xs, Prior xm
# Outputs updated state estimate and updated uncertainty
def KF(xm,xs,P,R,A,q,n,td):

    b=np.array([[+(td**3)*0.01*(0.1**3)-(0.1**3)*0.01*(0.1**3), (td**2)*0.01*(0.1
    c=np.array([[0,(td-1)*0.1],[0,0]])

    A=np.array(A)-np.array(c)
    q=np.array(q)+np.array(b)

    x_predicted=A*xm
    Pn=np.dot(A,P*np.transpose(A))+q
    H=np.asmatrix([1,0])

    a=np.dot(H,(Pn*np.transpose(H))+R)
    K=Pn*np.transpose(H)/a
    #K=np.reshape(np.array([K[0][0],K[1][0]]),(2,1))

    if n==1:
        x_posterior=x_predicted+K*(xs-H*x_predicted)
        P_posterior=(np.identity(2)-K*H)*Pn

        T.append(T[-1]+0.1)
    elif n==2:
        x_temp=x_predicted+K*(xs-H*x_predicted)
        P_temp=(np.identity(2)-K*H)*Pn
        delta=xs-float(x_temp[0])
        inno=np.dot(H,(P_temp*np.transpose(H)))
        var=delta*delta/inno
        if var<9:
            x_posterior=x_predicted+K*(xs-H*x_predicted)
            P_posterior=(np.identity(2)-K*H)*Pn
            T.append(T[-1]+0.1*td)
            td=1

        else:
            td+=1
            return np.array([]),np.array([]),td

    elif n==3:
        x_temp=x_predicted+K*(xs-H*x_predicted)
        p=get_mixture_probs(x_temp,xs)

        if p[0]>p[1]:
            x_posterior=np.array(x_predicted)+np.array(K)*(xs-np.dot(H,x_predicte
            P_posterior=(np.identity(2)-K*H)*Pn
            T.append(T[-1]+0.1*td)
            td=1
        else:
            td+=1
            return np.array([]),np.array([]),td

    return x_posterior,P_posterior,td
```

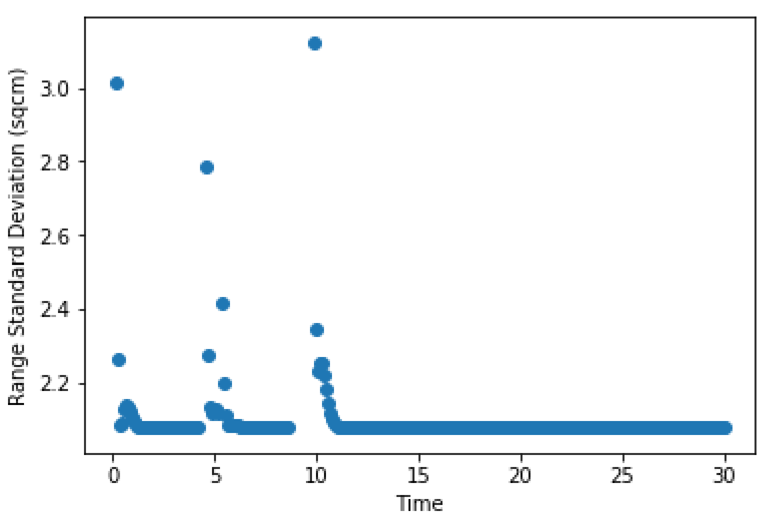
In [7]:
```
# Iterates through measurement data, updates posteriors using KF and stores it to
def estimate(n):
    P0=np.array([[0.01, 0],[0, 0.01]])
    x0=np.asmatrix([25.30,.1]).T
    R=0.001
    Q=[[0.01*0.1*0.1*0.1, 0.01*0.1*0.1],[0.01*0.1*0.1,0.01*0.1]]
    A=[[1,-0.1],[0,1]]
    t=1

    for i in x_measures:
        xm=i
        xt,Pt,t=KF(x0,xm,P0,R,A,Q,n,t)
        if xt.tolist()!=[] and Pt.tolist()!=[]:
            x_estimates.append(float(xt[0])*100)
            v_estimates.append(float(xt[1])*100)
            sigx.append(np.sqrt(Pt.tolist()[0][0]*10000))
            sigv.append(np.sqrt(Pt.tolist()[1][1]*10000))
            sigxv.append(Pt.tolist()[0][1]*10000/np.sqrt(Pt.tolist()[0][0]*10000)
            x0=np.asmatrix(xt)
            P0=np.asmatrix(Pt)
```

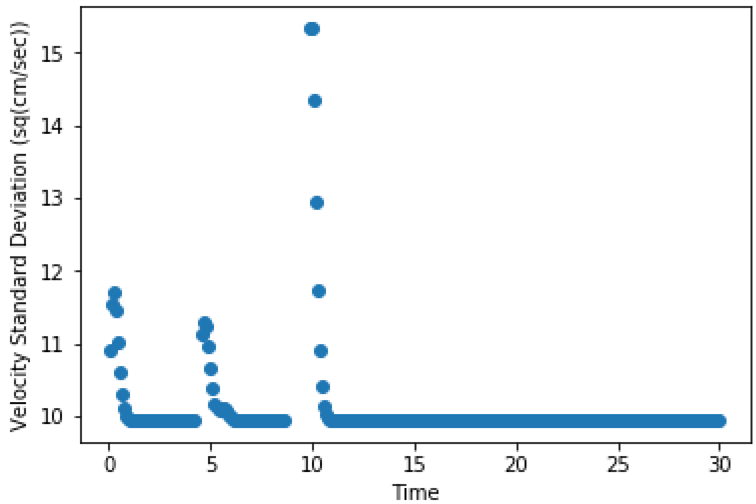
KF with innovation covariance based rejection

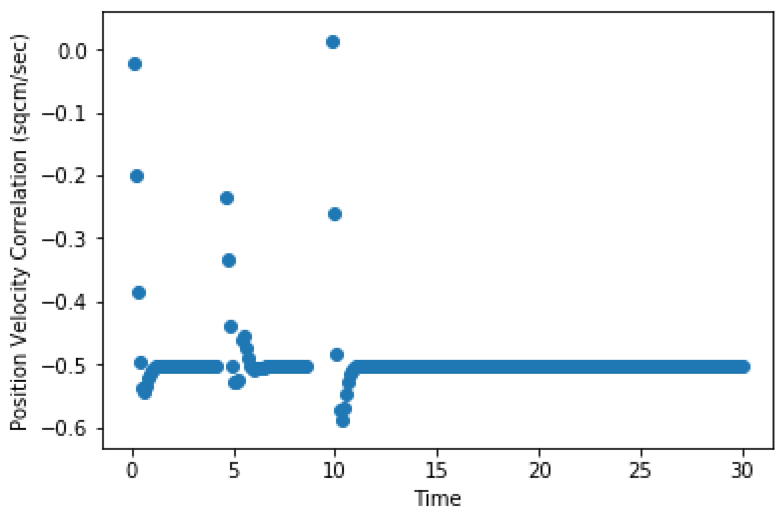
In [8]:
```
#Script starts from here
#Reading data file and queuing data
with open('RBE500-F17-100ms-Constant-Vel.csv', newline='') as csvfile:
 spamreader = csv.reader(csvfile, delimiter=' ', quotechar='|')
 spamreader=list(spamreader)
 for row in spamreader:
    x_measures.append(float(row[0])/100)
    x_measurescm.append(float(row[0]))

# Input the following numbers as arguments to estimate() for the corresponding po
# 1: Conventional KF   2: KF with object rejection (innovation)  3: KF with objec
estimate(2)


# Plot data
T.remove(0)
Ta=np.arange(0.0, 30.0, 0.1)

plt.scatter(Ta,x_measurescm,s=4,label='Raw')
plt.scatter(T,x_estimates,s=8,label='Estimates')
plt.xlabel('Time')
plt.legend()
plt.ylabel('Range estimates (cm)')
plt.show()

plt.scatter(T,v_estimates)
plt.xlabel('Time')
plt.ylabel('Velocity estimates (cm/s)')
plt.show()

plt.scatter(T,sigx)
plt.xlabel('Time')
plt.ylabel('Range Standard Deviation (sqcm)')
plt.show()

plt.scatter(T,sigv)
plt.xlabel('Time')
plt.ylabel('Velocity Standard Deviation (sq(cm/sec))')
plt.show()

plt.scatter(T,sigxv)
plt.xlabel('Time')
plt.ylabel('Position Velocity Correlation (sqcm/sec)')
plt.show()
```


Raw
Estimates
Range estimates (cm)
Time


Velocity estimates (cm/s)
Time


Range Standard Deviation (sqcm)
Time





Interpretation: The object rejection works by trading off precision for accuracy of the estimate. As seen in the plots the range estimate has gaps in areas of high uncertainty. This type of rejection also maintains a good estimate of velocity. Hence a bit of extra processing of estimates can improve the estimate accuracy.

This script applies KF to the measurement data. There is a provision to apply estimate filtering to reject spurrious estimates. This filtering using either innovation covariance or mixture model of the sensor.

In [5]:
```
# -*- coding: utf-8 -*-
"""
Created on Mon Nov 13 15:17:51 2017

@author: Nishant
"""
import math
import csv
import numpy as np
import matplotlib.pyplot as plt

# Calculates stats for given data
def stats(reads):
    mu=sum(reads)/len(reads)
    sq=[x*x for x in reads]
    sig=sum(sq)/len(sq)-mu*mu
    return mu,sig

# Returns value of Gaussian pdf of mixture for a passed measurement xs
def sample_from_gaussian(pdf,x):
    pow=-1/2*(x-pdf[0])*(x-pdf[0])/(pdf[1])
    return (1/np.sqrt(2*math.pi*pdf[1]))*np.exp(pow)

# Returns value of exp decay pdf of mixture for a passed measurement xs
def sample_exp_decay(xs):
    return 0.05*np.exp(-0.05*xs)

# Declaration of some global lists
x_measures=[]
x_measurescm=[]
x_estimates=[]
v_estimates=[]
sigx=[]
sigv=[]
sigxv=[]
T=[0]

# Returns a list corresponding to proportion of xi belonging to different compone
def get_mixture_probs(xp,xs):
    return [0.8*sample_from_gaussian([xp[0][0],0.001],xs),0.2*sample_exp_decay(xs
```

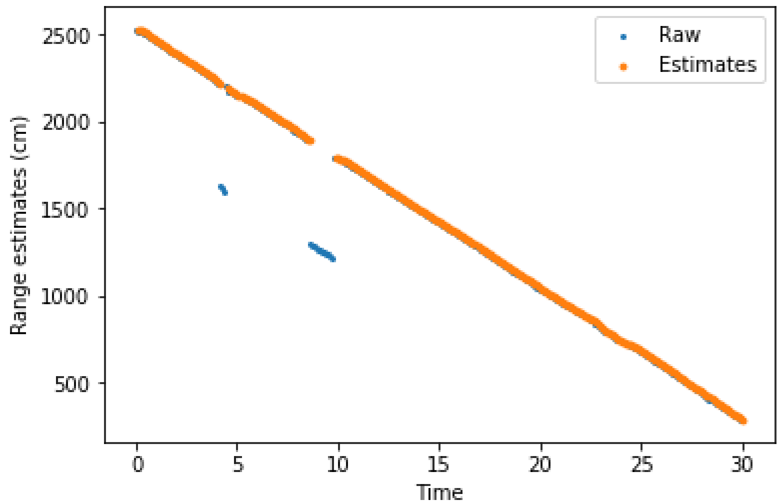
In [6]:
```
# Takes in system Markov model i.e. A,P,R,q, Current measurement xs, Prior xm
# Outputs updated state estimate and updated uncertainty
def KF(xm,xs,P,R,A,q,n,td):

    b=np.array([[+(td**3)*0.01*(0.1**3)-(0.1**3)*0.01*(0.1**3), (td**2)*0.01*(0.1
    c=np.array([[0,(td-1)*0.1],[0,0]])

    A=np.array(A)-np.array(c)
    q=np.array(q)+np.array(b)

    x_predicted=A*xm
    Pn=np.dot(A,P*np.transpose(A))+q
    H=np.asmatrix([1,0])

    a=np.dot(H,(Pn*np.transpose(H))+R)
    K=Pn*np.transpose(H)/a
    #K=np.reshape(np.array([K[0][0],K[1][0]]),(2,1))

    if n==1:
        x_posterior=x_predicted+K*(xs-H*x_predicted)
        P_posterior=(np.identity(2)-K*H)*Pn

        T.append(T[-1]+0.1)
    elif n==2:
        x_temp=x_predicted+K*(xs-H*x_predicted)
        P_temp=(np.identity(2)-K*H)*Pn
        delta=xs-float(x_temp[0])
        inno=np.dot(H,(P_temp*np.transpose(H)))
        var=delta*delta/inno
        if var<9:
            x_posterior=x_predicted+K*(xs-H*x_predicted)
            P_posterior=(np.identity(2)-K*H)*Pn
            T.append(T[-1]+0.1*td)
            td=1

        else:
            td+=1
            return np.array([]),np.array([]),td

    elif n==3:
        x_temp=x_predicted+K*(xs-H*x_predicted)
        p=get_mixture_probs(x_temp,xs)

        if p[0]>p[1]:
            x_posterior=np.array(x_predicted)+np.array(K)*(xs-np.dot(H,x_predicte
            P_posterior=(np.identity(2)-K*H)*Pn
            T.append(T[-1]+0.1*td)
            td=1
        else:
            td+=1
            return np.array([]),np.array([]),td

    return x_posterior,P_posterior,td
```

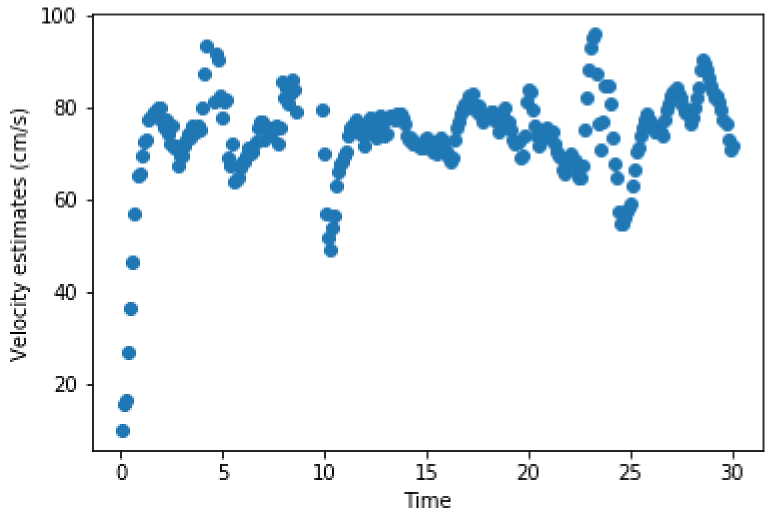
In [7]:
```
# Iterates through measurement data, updates posteriors using KF and stores it to
def estimate(n):
    P0=np.array([[0.01, 0],[0, 0.01]])
    x0=np.asmatrix([25.30,.1]).T
    R=0.001
    Q=[[0.01*0.1*0.1*0.1, 0.01*0.1*0.1],[0.01*0.1*0.1,0.01*0.1]]
    A=[[1,-0.1],[0,1]]
    t=1

    for i in x_measures:
        xm=i
        xt,Pt,t=KF(x0,xm,P0,R,A,Q,n,t)
        if xt.tolist()!=[] and Pt.tolist()!=[]:
            x_estimates.append(float(xt[0])*100)
            v_estimates.append(float(xt[1])*100)
            sigx.append(np.sqrt(Pt.tolist()[0][0]*10000))
            sigv.append(np.sqrt(Pt.tolist()[1][1]*10000))
            sigxv.append(Pt.tolist()[0][1]*10000/np.sqrt(Pt.tolist()[0][0]*10000)
            x0=np.asmatrix(xt)
            P0=np.asmatrix(Pt)
```

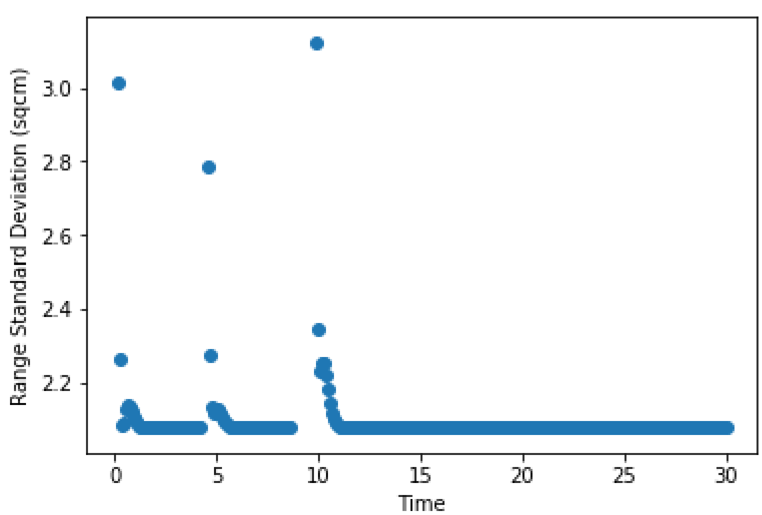
KF with mixture model based rejection

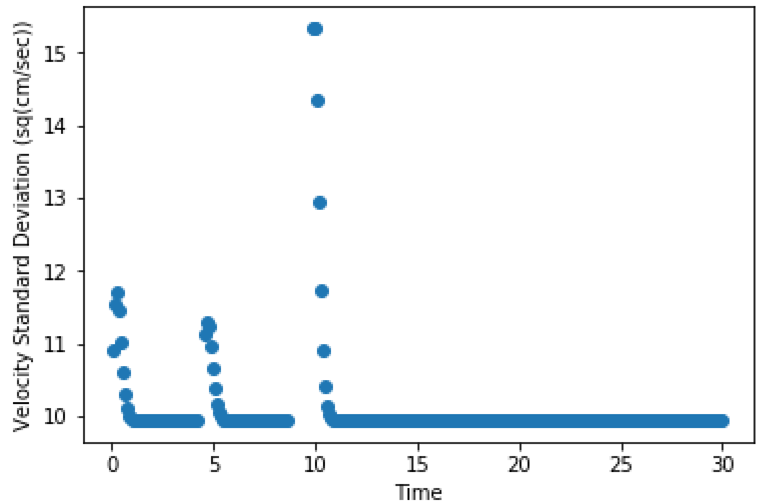
In [8]:

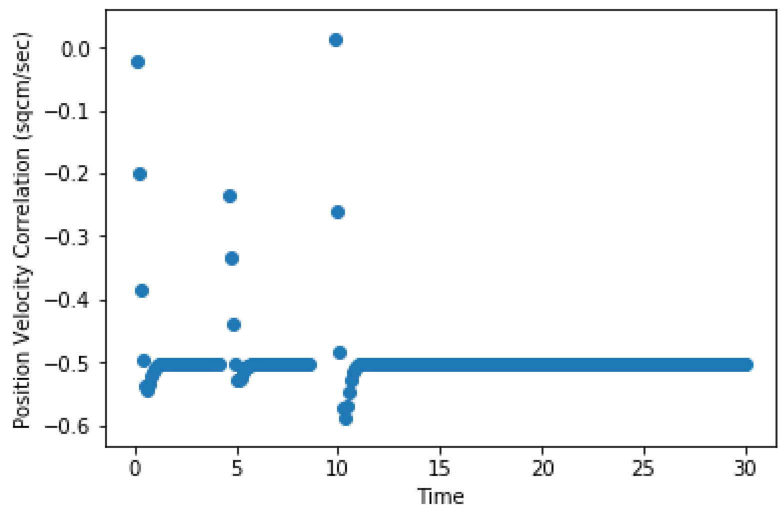
```
#Script starts from here
#Reading data file and queuing data
with open('RBE500-F17-100ms-Constant-Vel.csv', newline='') as csvfile:
 spamreader = csv.reader(csvfile, delimiter=' ', quotechar='|')
 spamreader=list(spamreader)
 for row in spamreader:
    x_measures.append(float(row[0])/100)
    x_measurescm.append(float(row[0]))


# Input the following numbers as arguments to estimate() for the corresponding po
# 1: Conventional KF   2: KF with object rejection (innovation)  3: KF with objec
estimate(3)


# Plot data
T.remove(0)
Ta=np.arange(0.0, 30.0, 0.1)
ptypes=['Raw', 'Estimates']

plt.scatter(Ta,x_measurescm,s=4,label='Raw')
plt.scatter(T,x_estimates,s=8,label='Estimates')
plt.xlabel('Time')
plt.legend()
plt.ylabel('Range estimates (cm)')

plt.show()

plt.scatter(T,v_estimates)
plt.xlabel('Time')
plt.ylabel('Velocity estimates (cm/s)')
plt.show()

plt.scatter(T,sigx)
plt.xlabel('Time')
plt.ylabel('Range Standard Deviation (sqcm)')
plt.show()

plt.scatter(T,sigv)
plt.xlabel('Time')
plt.ylabel('Velocity Standard Deviation (sq(cm/sec))')
plt.show()

plt.scatter(T,sigxv)
plt.xlabel('Time')
plt.ylabel('Position Velocity Correlation (sqcm/sec)')
plt.show()
```


Raw
Estimates
Range estimates (cm)
2500
2000
1500
1000
500
0
5
10
15
20
25
30
Time


Velocity estimates (cm/s)
100
80
60
40
20
0
5
10
15
20
25
30
Time


Range Standard Deviation (sqcm)
3.0
2.8
2.6
2.4
2.2
0
5
10
15
20
25
30
Time





Interpretation: The mixture model based object rejection performs similar to innovation covariance based rejection in this case but its rejection criterian may vary based on the soundness of the mixture model. As seen in the plots the range estimate has gaps in areas of high uncertainty. This type of rejection also maintains a good estimate of velocity. The only downside being that it requires a bit more preprocessing of selecting and evaluating the appropriate mixture model by running Expectation Maximization on the collected data.

Homework 010

c) Compare the performance of the three approaches. Were any anomalous points processed by mistake? Were any valid points rejected? How can each of these methods fail? **[1 point]**

KF doesn't handle the sudden variations in measurements due to external sources very well. If the data is not sufficient it may output wrong estimates. Also the velocity estimate can be seen to spike during external disturbance which may also affect the estimates in the case of insufficient data. KF usually fails in case of higher order systems or sudden external disturbances where the uncertainty shoots up to give unreliable estimates

KF with innovation covariance based rejection performs much better than conventional KF by neglecting the measurements exceeding the considered covariance band. The uncertainty increases as the number of consecutive points rejected increases. A limitation of this approach may be that deciding the acceptable covariance band adjusts the sensitivity of the rejection to the disturbances in the measurements. A narrower band my sometimes perform worse and lead to many unnecessary rejections. Whereas a broader band may fail to reject many disturbances.

KF with mixture model based rejection performs equivalent to innovation covariance based rejection in this case. But its performance highly depends on the integrity of the mixture model being considered. A well chosen and evaluated mixture model would lead to lesser false rejections and inclusions and hence may perform even better than innovation covariance based rejection. But the challenging bit is selection of appropriate components for the mixture and weighing these components using Expectation Maximization.