

Problem 1:

Assuming  $\nabla D = 0$

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n \vec{v}) + \nabla \cdot (D \nabla n) + R = -\nabla \cdot (n \vec{v}) + D \nabla^2 n + R$$

$$\nabla^2 n \equiv \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial f}{\partial \rho} \right) + \frac{\partial^2 f}{\partial z^2} \quad (\text{in 2D, assuming } f=n)$$

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial f}{\partial \rho} \right) = \frac{1}{\rho} \left[ \left( \frac{\partial}{\partial \rho} \rho \right) \frac{\partial f}{\partial \rho} + \rho \frac{\partial^2 f}{\partial \rho^2} \right] = \frac{1}{\rho} \frac{\partial f}{\partial \rho} + \frac{\partial^2 f}{\partial \rho^2}$$

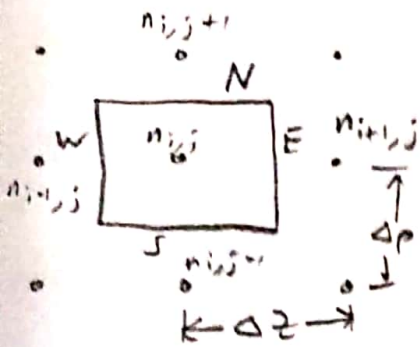
$$\Rightarrow \nabla^2 n \approx \frac{1}{\rho} \left( \frac{n_{\rho+1,z} - n_{\rho,z}}{\Delta \rho} \right) + \frac{n_{\rho-1,z} - 2n_{\rho,z} + n_{\rho+1,z}}{\Delta^2 \rho} + \frac{n_{\rho,z+1} - 2n_{\rho,z} + n_{\rho,z+1}}{\Delta^2 z}$$

or, using central difference:

$$\nabla^2 n \approx \frac{1}{\rho} \left( \frac{n_{\rho+1,z} - n_{\rho-1,z}}{2 \Delta \rho} \right) + \frac{n_{\rho-1,z} - 2n_{\rho,z} + n_{\rho+1,z}}{\Delta^2 \rho} + \frac{n_{\rho,z+1} - 2n_{\rho,z} + n_{\rho,z-1}}{\Delta^2 z}$$

Problem 2:

$$\nabla^2 n = ? ; \int_V \nabla^2 n \, dV = \oint_S (\nabla n) \cdot \hat{n} \, dA$$



$$(\hat{n} A)_N = \Delta z (2\pi(\rho + \Delta\rho))$$

$$(\hat{n} A)_S = -\Delta z (2\pi\rho)$$

$$(\hat{n} A)_E = \pi(\rho + \Delta\rho)^2 - \pi\rho^2 = \pi(\rho^2 + 2\rho\Delta\rho + \Delta\rho^2) - \pi\rho^2 = 2\pi\rho\Delta\rho$$

$$(\hat{n} A)_W = -2\pi\rho\Delta\rho$$

$$\oint_S (\nabla n) \cdot \hat{n} \, dA = \underbrace{\frac{n_{i,j+1} - n_{i,j}}{\Delta\rho} [\Delta z (2\pi\rho + 2\pi\Delta\rho)]}_{(1)} + \underbrace{\frac{n_{i+1,j} - n_{i,j}}{\Delta z} (2\pi\rho\Delta\rho)}_{(2)} + \underbrace{\frac{n_{i,j} - n_{i,j-1}}{\Delta\rho} [-\Delta z (2\pi\rho)]}_{(3)} + \underbrace{\frac{n_{i,j} - n_{i-1,j}}{\Delta z} (-2\pi\rho\Delta\rho)}_{(4)}$$

$$\text{Term (1)} = \nabla n_N \Delta z (2\pi\rho) + \frac{n_{i,j+1} - n_{i,j}}{\Delta\rho} 2\pi\Delta\rho \Delta z$$

$$\Rightarrow \oint_S (\nabla n) \cdot \hat{n} \, dA = \underbrace{\left( \frac{n_{i,j+1} - 2n_{i,j} + n_{i,j-1}}{\Delta\rho} \right) [\Delta z (2\pi\rho)]}_{(4)} + \underbrace{\left( \frac{n_{i+1,j} - 2n_{i,j} + n_{i-1,j}}{\Delta z} \right) [\Delta\rho (2\pi\rho)]}_{(5)} + \underbrace{(n_{i,j+1} - n_{i,j}) (2\pi\Delta z)}_{(1)}$$

$$\Delta V = 2\pi\rho\Delta\rho\Delta z$$

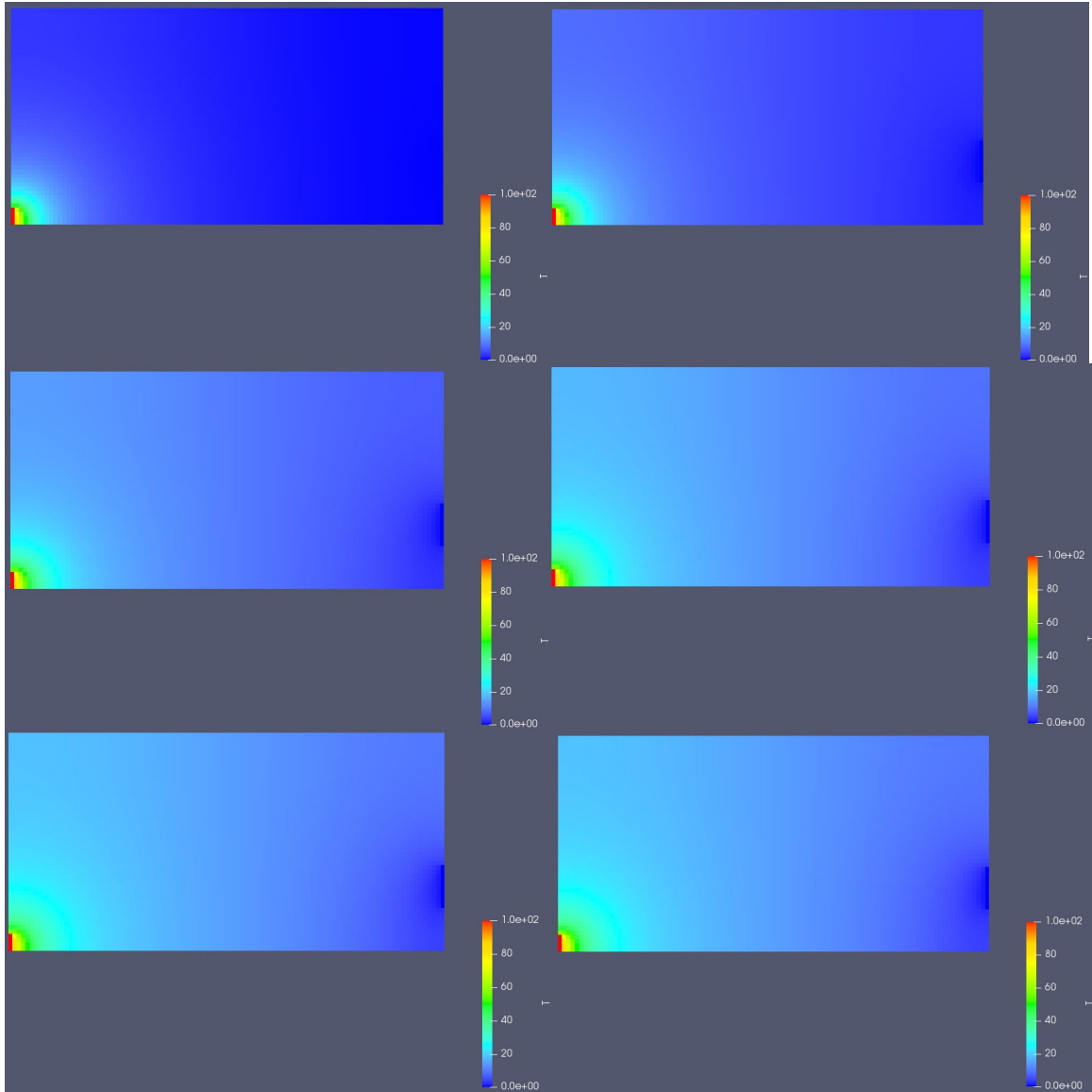
$$\nabla^2 n = \frac{1}{\Delta V} \oint_S (\nabla n) \cdot \hat{n} \, dA = \underbrace{\frac{\Delta z 2\pi\rho}{2\pi\rho\Delta\rho\Delta z}}_{(4)} + \underbrace{\frac{\Delta\rho 2\pi\rho}{2\pi\rho\Delta\rho\Delta z}}_{(5)} + \underbrace{(n_{i,j+1} - n_{i,j}) \frac{2\pi\Delta z}{2\pi\rho\Delta\rho\Delta z}}_{(1)}$$

$$\Rightarrow \boxed{\nabla^2 n = \frac{1}{\rho} \left( \frac{n_{i,j+1} - n_{i,j-1}}{\Delta\rho} \right) + \left( \frac{n_{i,j+1} - 2n_{i,j} + n_{i,j-1}}{\Delta^2\rho} \right) + \left( \frac{n_{i+1,j} - 2n_{i,j} + n_{i-1,j}}{\Delta^2 z} \right)}$$

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**Problem 3:**

**Part a:** For this, I used 0.04 for the inlet and 0.1-0.2 (open) for the outlet. The timesteps are 100, 400, 1000, 2000, 5000, 50000 (left to right, top down). The mesh size is 101 by 51.



**Part b:** For this, I used 0.04 for the inlet and 0.1-0.1 (closed) for the outlet. Timesteps are: 1000, 4000, 10000, 20000, 37000. Mesh size is 101 by 51.

