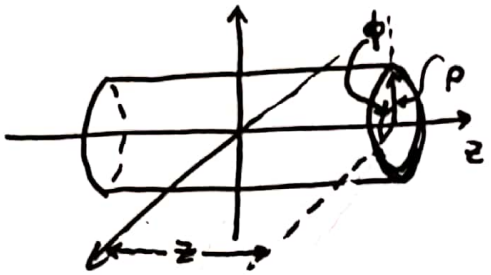
Nikita Persikov ASTE 404 HW 10

Problem 1

Laplacian for cylindrical coordinates: $$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

where:



$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) = \frac{1}{\rho}\left[\left(\frac{\partial}{\partial \rho}\rho\right)\frac{\partial f}{\partial \rho} + \rho \frac{\partial^2 f}{\partial \rho^2}\right]$$

$$= \frac{1}{\rho}\left[\frac{\partial f}{\partial \rho} + \rho \frac{\partial^2 f}{\partial \rho^2}\right]$$

$$\nabla^2 f \approx \frac{1}{\rho}\left(\frac{f_{\rho+1,\phi,z} - f_{\rho,\phi,z}}{\Delta \rho}\right) + \left(\frac{f_{\rho-1,\phi,z} - 2f_{\rho,\phi,z} + f_{\rho+1,\phi,z}}{\Delta^2 \rho}\right) +$$

$$+ \frac{1}{\rho^2}\left(\frac{f_{\rho,\phi-1,z} - 2f_{\rho,\phi,z} + f_{\rho,\phi+1,z}}{\Delta^2 \phi}\right) + \left(\frac{f_{\rho,\phi,z-1} - 2f_{\rho,\phi,z} + f_{\rho,\phi,z+1}}{\Delta^2 z}\right)$$