Computer Science 336
Fall 2023
Homework 2

- Use the following notation as pseudocode for standard 3D affine transformation matrices. You can refer to these by the names below. There is no need to write out the entries unless you need to actually perform multiplication.

RotateZ(θ) $= \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (Rotate about z-axis, i.e., in the x-y plane)

Translate(tx, ty, tz) $= \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Scale(sx, sy, sz) $= \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

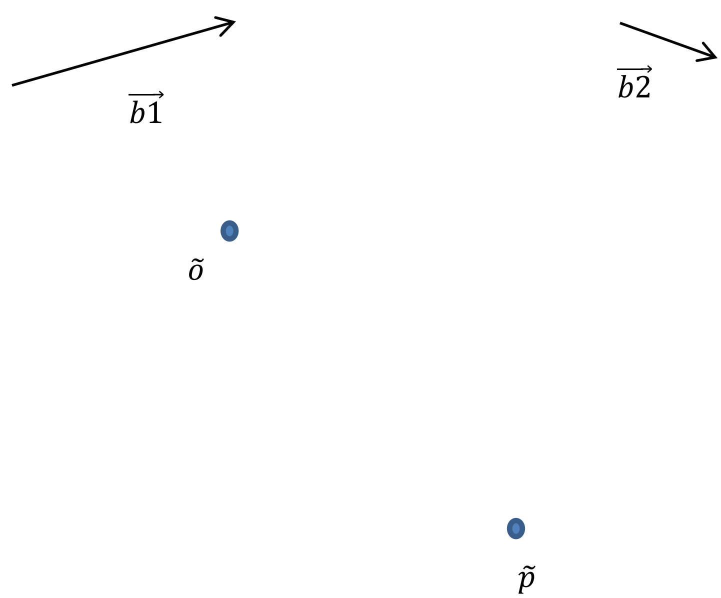
- Please show your work clearly and legibly; remember, we are going to have to read these papers.
- Due Friday, September 22 by 3pm. Bring a hard copy to class on Thursday, or drop it off during my office hours on Thursday, or slide it under my door sometime Friday morning, or drop it off in at my office in person between 2 and 3pm on Friday.

For part 3b, upload a zip file containing your modified version of Transformations1a.js to the Canvas submission link.

1. The illustration below shows two vectors and a point $\tilde{o}$. Define a frame $\mathcal{F} = [\overrightarrow{b1}, \overrightarrow{b2}, \overrightarrow{b3}, \tilde{o}]$, where we can assume that $\overrightarrow{b3}$ is irrelevant here.

a) Sketch, circle and clearly label a point $\tilde{q}$ that has the coordinates $[-1\ 2\ 0\ 1]^T$ with respect to $\mathcal{F}$.

b) The illustration also shows a point $\tilde{p}$. Determine (geometrically) the approximate coordinates of the point $\tilde{p}$ below with respect to the frame $\mathcal{F}$. Justify your answer using a sketch and/or brief explanation. Use a ruler and make a careful sketch and try to estimate the numbers reasonably accurately.

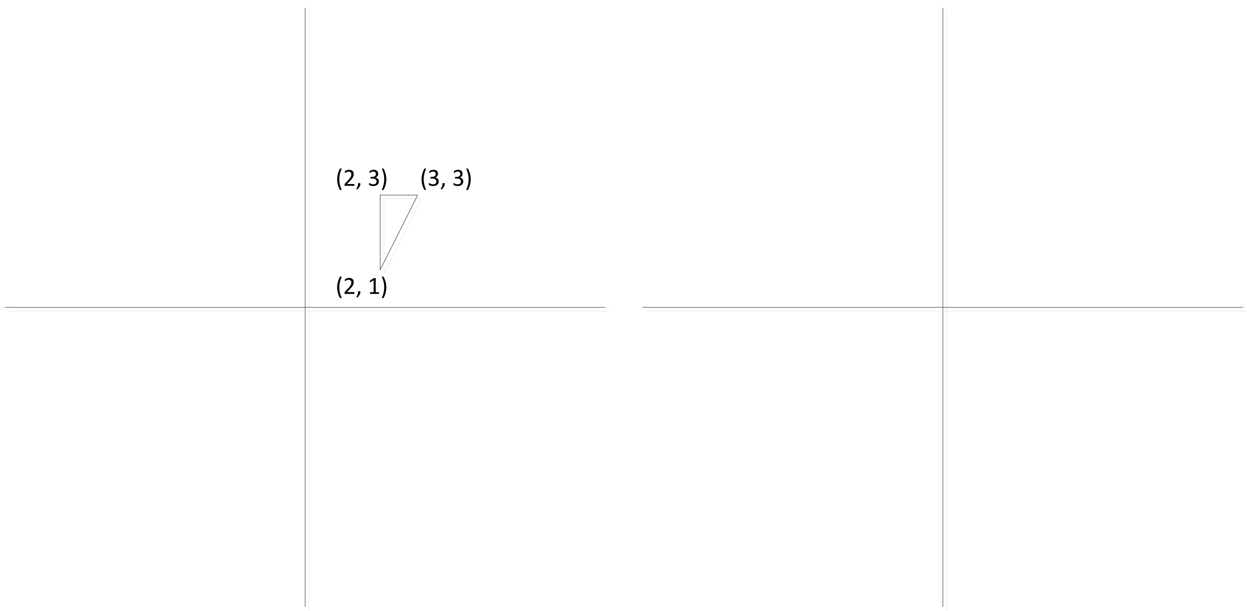


2.

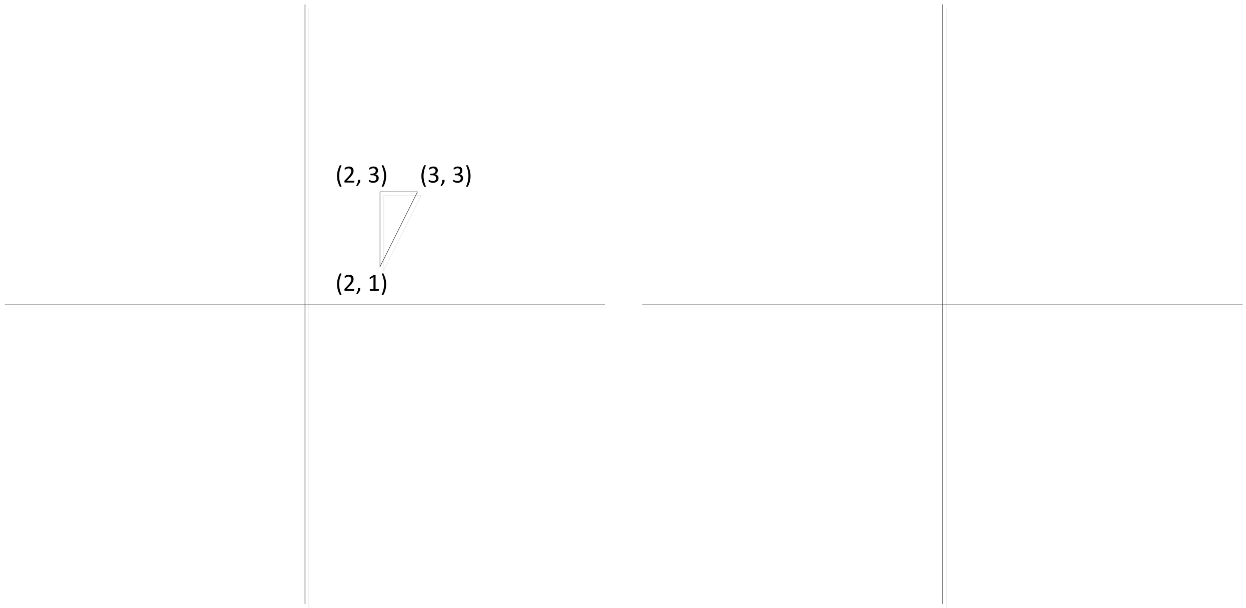
$$\texttt{T = Translate(1,-3,0)} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \texttt{R = Rotate(90)} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) Multiply out the two products TR and RT.

b) In the axes on the right, **sketch** the result applying the transformation RT (T, then R) to the triangle. Use the matrix RT from the previous page to **calculate** the new coordinates of the three vertices, **label** them on your sketch, and **verify** that your result makes sense geometrically. (Does it *look* like you've translated over 1 and down 3, and then rotated 90 degrees about the origin?) Remember that a 2D point such as (2, 3) is represented in homogeneous coordinates as $[2\ 3\ 0\ 1]^T$.



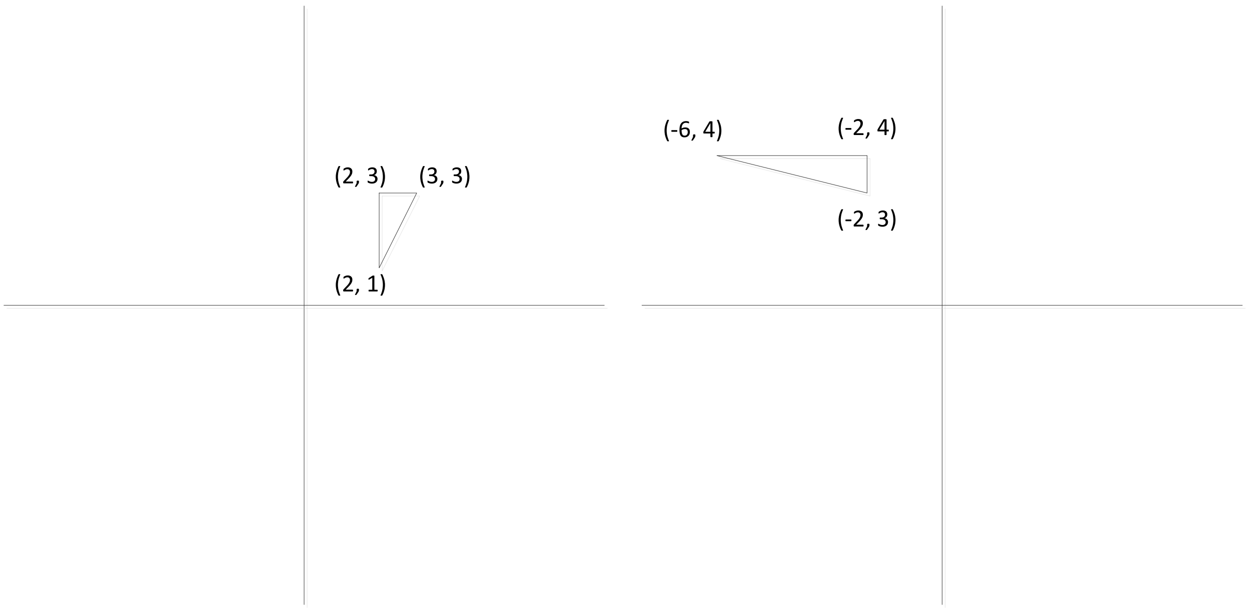
c) Do the same for the transformation TR.



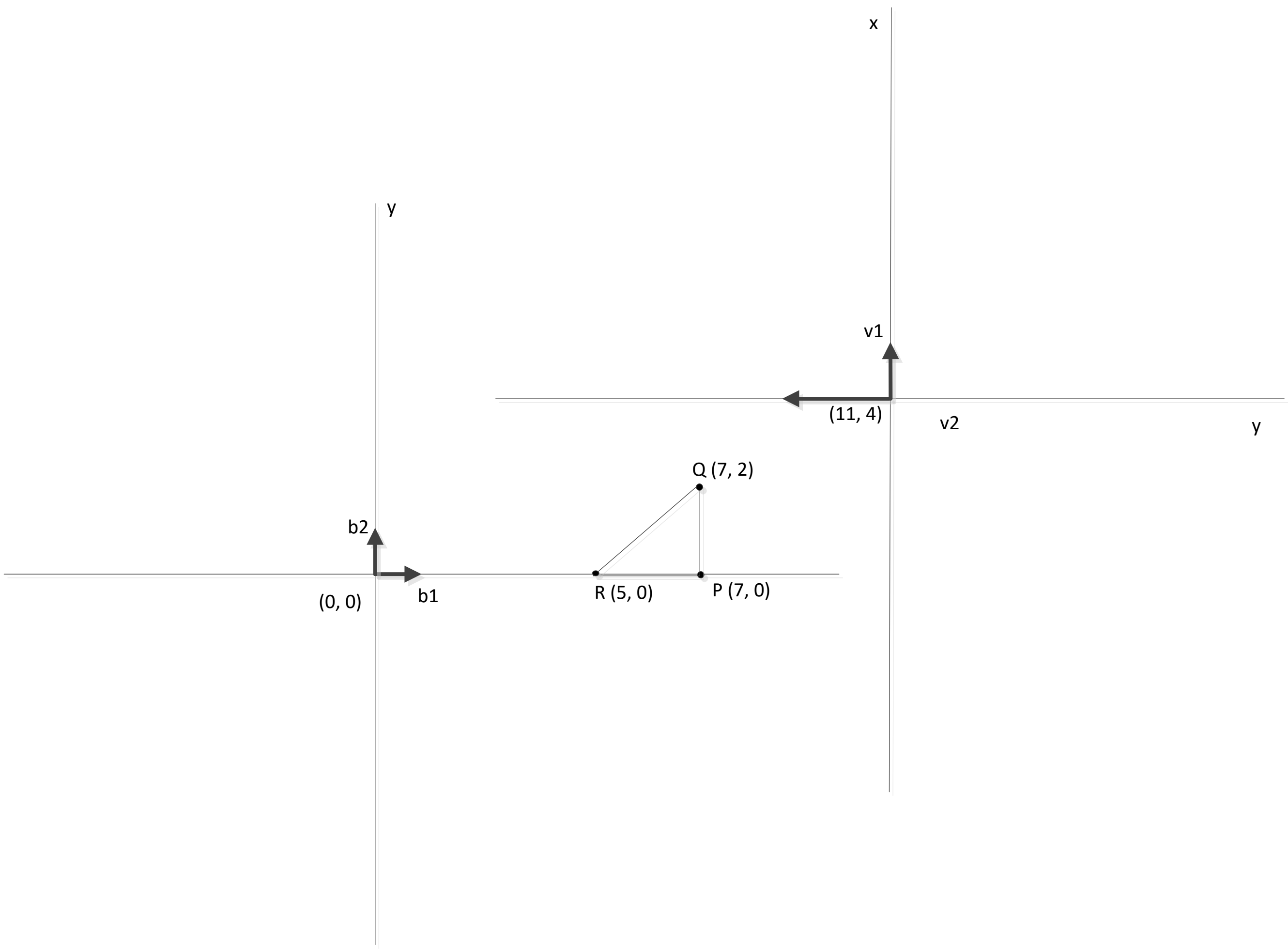
c) T and R are affine transformation matrices, since they have [0 0 0 1] in the bottom row. Any product of affine transformations is also affine. Therefore RT (given to us as a translation followed by a rotation) must be affine. An affine transformation is defined as a linear transformation followed by a translation. So there must be a way to write RT as a product of matrices T'M such that M is linear and T' is a translation. Find T' and M. (A 4x4 matrix represents a *linear* transformation if the bottom row is [0 0 0 1] *and* the rightmost column is also $[0\ 0\ 0\ 1]^T$.)

3. a) A certain affine transformation transforms the triangle on the left into the triangle on the right. Write down a matrix for this transformation as a **product of the standard transformation matrices** defined on the first page. (There are many possible correct answers!) *Use the symbol "`*`" for matrix multiplication and be sure you clearly show the order in which the matrices are multiplied.* You can check your work by multiplying everything out, or better yet, see part b.

b) The file **`examples/transformations/Transformations1a.js`** (along with the .html file) is a variation of **`Transformations1.js`** in which the triangle vertices are those of the triangle below left. Implement your transformation from (a) in the main method, and check that it works.



4. The figure below shows a frame (coordinate system) $\mathcal{F} = [\overrightarrow{b1}, \overrightarrow{b2}, \overrightarrow{b3}, \tilde{o}]$ along with a second frame $\mathcal{F}' = [\overrightarrow{v1}, \overrightarrow{v2}, \overrightarrow{v3}, \tilde{p}]$, where the new origin $\tilde{p}$ has coordinates $[11\ 4\ 0\ 1]^T$ with respect to $\mathcal{F}$. The new frame is rotated 90 degrees counterclockwise and the y-axis is stretched by 2. Note that v1 and v2 are orthogonal, v1 has unit length, and v2 has length 2. (You can ignore the invisible third basis vectors $\overrightarrow{b3}$ and $\overrightarrow{v3}$ since nothing is changing outside of the x-y plane.) Thus the matrix M = Translate(11, 4, 0) * RotateZ(90) * Scale(1, 2, 1) transforms $\mathcal{F}$ to $\mathcal{F}'$, i.e. $\mathcal{F}' = \mathcal{F}M$. Note that the inverse of M is M^{-1} = Scale(1, 0.5, 1) * RotateZ(-90) * Translate(-11, -4, 0).



a) When the triangle PQR is transformed by M, where does it end up? i) Sketch it and **label** the coordinates of the transformed vertices with respect to the original frame $\mathcal{F}$; for example, the point R has coordinates $[5\ 0\ 0\ 1]^T$, and $M[5\ 0\ 0\ 1]^T$ is $[11\ 9\ 0\ 1]^T$. ii) Multiply out M and use the matrix M to **calculate** the coordinates of the transformed triangle with respect to the original frame. Show your calculations and verify geometrically that your results make sense. (Note that the drawing is to scale).

b) i) Determine (geometrically) the coordinates, *with respect to* $\mathcal{F}'$, of the points labeled P, Q, and R. ii) Then, find the inverse M^{-1} of the matrix M from part (a). Calculate $M^{-1}\underline{c}$ for the coordinate vectors $\underline{c}$ for each point P, Q, and R and show that your answers agree with the geometrically obvious ones (Again using R as an example, $M^{-1}[5\ 0\ 0\ 1]$ has to be $[-4\ 3\ 0\ 1]^T$, since we can see from the picture that $-4\overrightarrow{v_1} + 3\overrightarrow{v_2} + \tilde{p}$ is point R.)