1. (10 points) A simple undirected graph is called regular if every vertex has the same degree. How many vertices does a regular graph of degree four with 10 edges have?
V is the number of vertices in a regular graph, and D is the degree of each vertex. The number of edges (E) that we have is 10.
2E = V * D
2*10 = V * 4. 20 = 4V
V = 5 vertices
2. (20 points) Let G be a simple undirected graph. $\overline{G}$ represents the complementary graph of G. This graph has the same vertex set as G and is obtained as follows. If (u, v) is an edge in G, then (u, v) is not an edge in $\overline{G}$. Conversely, if (u, v) is not an edge in G, then (u, v) is an edge in $\overline{G}$.

• Suppose that G has 15 edges and $\overline{G}$ has 13 edges. Then, how many vertices does G have?
V is the number of vertices in G.The number of edges in -G is 13.
2 * 13 = V(V-1) -15 since G has 15 edges
15(edges in G) + 13(edges in -G) = (V*(V-1)) / 2
56 = V * (V-1) 56 = V^2 -V
V^2 - V - 56 = 0 (V-8) (V+7)
V = 8 vertices because the number of vertices cannot be negative.

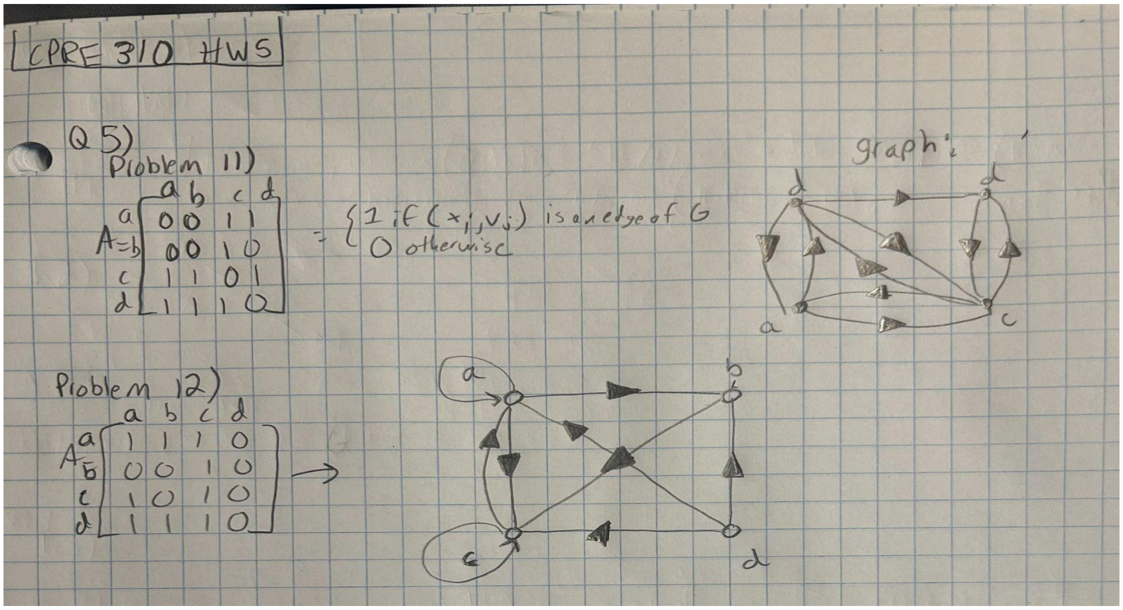
3. (20 points) Recall that the adjacency matrix A of a graph G is such that A(i, j) = 1 if nodes i and j are adjacent and A(i, j) = 0 otherwise. Let αk(i, j) be the number of paths of length k between nodes i and j. For instance, the number of paths of length-1 between nodes i and j in a simple undirected graph is 1 if they are adjacent and zero otherwise. Show that one can determine α2(i, j) by determining the (i, j)-th entry of A2, i.e., the square of A
Eliminate V-1 non-sink vertices and check the only remaining vertex for the sink property.
To eliminate non-sink vertices, we check whether MG[ptr1][ptr2] in the adjacency matrix is 1 or 0.
If it is 0, the vertex corresponding to index 'ptr2' cannot be a sink.
if it is 1, it means the vertex corresponding to index 'ptr1' cannot be a sink.

4. (5+5 points) Problems 7 and 8 of Section 10.3 of the book (pg. 710).
7) The adjacency matrix representing G with respect to the order of the vertices a,b,c,d is
[1 1 1 1
0 0 0 1
1 1 0 0
0 1 1 1]

8)
The adjacency matrix representing G with respect to the order of the vertices a,b,c,d,e is
[0 1 0 1 0
1 0 1 1 1
0 1 1 0 0

1 0 0 0 1
0 0 1 0 1]

5. (5+5 points) Problems 11 and 12 of Section 10.3 of the book (pg. 710).



6. (10 points) What is the sum of the rows of the adjacency matrix for an undirected graph? For a directed graph?
For an undirected graph, the sum of the entries in a row represents the degree of a vertex.
For a directed graph, the sum of the entries in a row represents the out-degree of a vertex.

7. (20 points) Problem 11 of Section 10.4 of the book (pg. 725).

a) The graph is not strongly connected because there is no path from a to b.
But, the graph is weakly connected because of the underlying undirected graph.

b) The graph is not strongly connected because there is no path from c to b
But the graph is weakly connected because the underlying undirected graph

c) The graph is not strongly connected because there is no path from a to b.
The graph is not even weakly connected because of the underlying undirected graph.

8. (10 points) Problem 25 of Section 5.1 of the book (pg. 351).
Let P(n) => "If h > -1, then (1 + nh) <= (1 + h)n for all non-negative integers n" be true.
Putting n = 0 equals to 1
Since, (1 + nh) <= (1 + h)n holds good, therefore P(0) is true.
Now, putting n = 1, we get: 1 + h Since, (1 + nh) <= (1 + h)n holds good, therefore P(1) is also true.
So, we find that if P(n) is true, then P(0), P(1) are true and P(n+1) is also true.
Therefore, we conclude that the induction hypothesis is true.

9. (10 points) Problem 36 of Section 5.1 of the book (pg. 351)

The basic step is true. 21 divides 21. So, the given statement is true for x = 1

Use Mathematical induction to test whether the given statement is true for n = 1

After that, Assume the given statement is true for n = k. Now we have to prove the given statement is true for n = k + 1

Q9) $21\left(4^{(k+1)+1}+5^{2(k+1)-1}\right)$ (Base Case) $n=1 = 21 \mid 4^{1+1}+5^{(2\cdot 1-1)} = 21$ is True

$= 4\cdot 4^{k+1} + 25\cdot 5^{2k-1}$

$4\cdot 4^{k+1} + (4+21)\cdot 5^{2k-1} = 4\cdot 4^{k+1} + (4+21)\cdot 5^{2k-1} = 4\left(4^{k+1}+5^{2k-1}\right) + 21\cdot 5^{2k-1}$

The term. $4\left(4^{k+1}+5^{2k-1}\right)$ is divisible by 21 therefore

$21 \mid \left(4^{(k+1)+1}+5^{2(k+1)-1}\right)$ and by mathematical induction $21 \mid \left(4^{n+1}+5^{2n-1}\right)$

whenever n is a pos integer.