1. For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a){(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
It is not Reflexive as (4,4) is missing
It is not Symmetric as (4,2) is missing
It is not Antisymmetric as (2,3) and (3,2) are present
It is Transitive Relation .{ (2,3),(3,2) so (2,2) is present
b){(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
It is Reflexive
It is Symmetric
It is not Antisymmetric
It is Transitive as all elements are present
c){(2, 4), (4, 2)}
It is not a Reflexive, Anti symmetry, Transitive
It is only a Symmetric function
d){(1, 2), (2, 3), (3, 4)}
The given relation is only Anti-Symmetric
e){(1, 1), (2, 2), (3, 3), (4, 4)}
The given relation is Reflexive and Anti-Symmetric
f){(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}
The given relation has no Symmetry because (4,1) is not present
not transitive as it not contains (2,1) .....{ (2,3),(3,1)
It is not Anti-symmetry as it present (1,3),(3,1)
And no reflexivity

2. Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

a)$x + y = 0$.
Reflexive: No, because for any real number x , x + x = 2x does not = 0 unless x = 0 , which is a particular case.
Symmetric: Yes, because if x + y = 0, then y + x = 0.
Antisymmetric: No, because if x + y = 0 and reverse, it doesn't imply that x = y.
Transitive: Yes, because if x + y = 0 and y + z = 0, then x + z = 0.
b)$x = \pm y$.
Reflexive: Yes, because for any real number x, x = +- x
Symmetric: Yes, because if x = +-y, then y = +-x.
Antisymmetric: No, because if x = ±y and reverse, it doesn't imply that x=y unless x=0 or y=0.
Transitive: Yes, because if x= +-y and y= +-z, then x= +-z.
c)$x - y$ is a rational number.
Reflexive: Yes, because x - x = 0, a rational number.
Symmetric: No, because if x-y is a rational number, it doesn't imply that y-x is also a rational number

Antisymmetric: No, because if x-y is a rational number and y-x is a rational number, it doesn't imply that x=y

Transitive: Yes, because if x-y and y-z are rational numbers, then x-z is also rational.

d)x = 2y.

Reflexive: No, because for any real number x, x does not equal 2x unless x=0, which is a special case.

Symmetric: No, because if x=2y, it doesn't imply that y=2x unless x=y=0.

Antisymmetric: Yes, because if x=2y and y=2x , it implies that x=y=0

Transitive: Yes, because if x=2y and y=2z, then x=2z.

e)xy ≥ 0.

Reflexive: Yes, because for any real number x, x = x^2 ≥ 0 .

Symmetric: No, because if xy ≥ 0, it doesn't imply yx ≥ 0.

Antisymmetric: No, because if xy ≥ 0 and yx ≥ 0, it doesn't imply that x=y.

Transitive: Yes, because if xy ≥ 0 and yz ≥ 0, then xz ≥ 0.

f)xy = 0.

Reflexive: Yes, because for any real number x, x * 0 =0

Symmetric: Yes, because if xy = 0, yx = 0.

Antisymmetric: Yes, because if xy=0 and yx=0, it implies that x=y=0.

Transitive: Yes, because if xy=0 and yz=0, then xz=0. If either x or z is zero, their product will be zero.

g)x = 1.

Reflexive: No, because any real number x ≠ 1 (x,x) is not in R.

Symmetric: No, because if (x,y) is in R, it implies x=1, but it doesn't guarantee that (y,x) is in R since y can be any real number.

Antisymmetric: No, because if (x,y) is in R and (y,x) is in R, it would imply x=1 and y=1. However, this violates the definition of antisymmetry since x does not equal y.

Transitive: Yes, because if (x,y) is in R and (y,z) is in R, it implies x=1 and z=1. Therefore, (x,z) is in R

h)x = 1 or y = 1.

Reflexive: No, because any real number x does not = 1 (x,x) is not in R.

Symmetric: No, because if (x, y) ∈ R, it means either x = 1 or y = 1. It does not guarantee that (y, x) ∈ R since the condition may not hold for the other variable.

Antisymmetric: No, because if (x, y) ∈ R and (y, x) ∈ R, it would imply either x = 1 or y = 1 for both pairs. However, this violates the definition of antisymmetry since x does not = y can still hold.

Transitive: Yes, because if (x,y) is in R and (y,z) is in R, it implies either x = 1 or y = 1 for the first pair and y = 1 or z = 1 for the second pair. therefore, it follows that either x=1 or z=1 for the composite pair (x, z).

3. Let R be the relation on the set of ordered pairs of positive integers such that ((a, b), (c, d)) ∈ R if and only if a + d = b + c. Show that R is an equivalence relation.

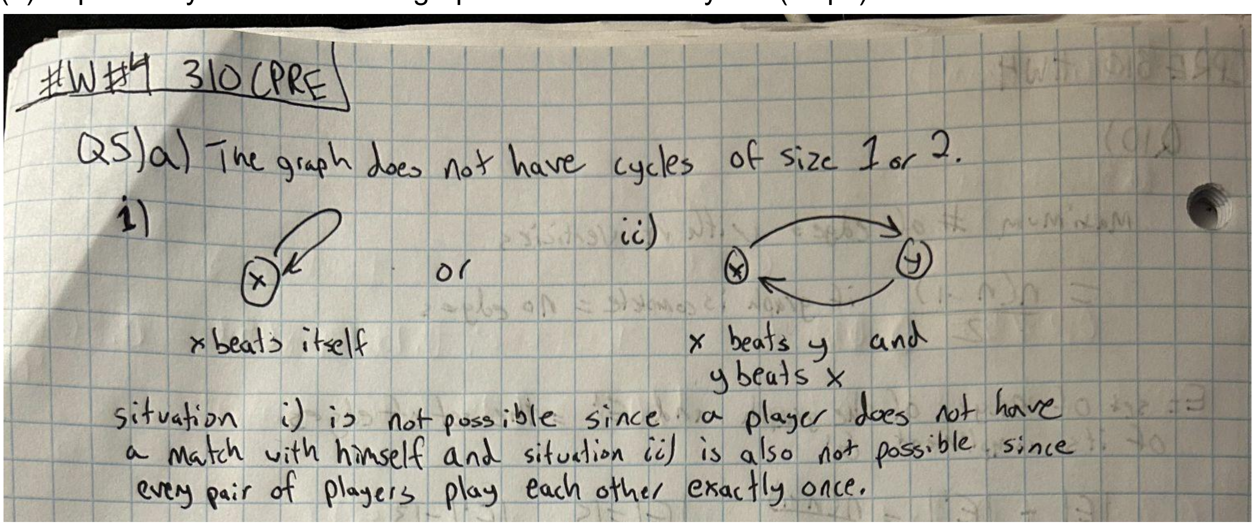
1) Reflexive:- for every ordered pair (a,b) of positive integers a,b {(a,b),(a,b)}∈R Since, a+b= b+a therefore, R is Reflexive.

2) Symmetric:- We have, {(a,b),(c,d)}∈R Since, a+d=b+c ⇒c+b=d+a ⇒{(c,d),(a,b)}∈R therefore, R is symmetric.

3) Transitive:- Let {(a,b),(c,d)}∈R and {(c,d),(e,f)}∈R Then,
A+d = b+c
C+f = d+e
⇒{(a,b), (e,f)} exists in R. So R is transitive

4. Which of these collections of subsets are partitions of {1, 2, 3, 4, 5, 6}?
   a){1, 2}, {2, 3, 4}, {4, 5, 6}
   NOT a partition. For subsets to be in partition, every element is inserted in exactly one subset, but here, element 2 is present in the 1st and 2nd subsets.
   b){1}, {2, 3, 6}, {4}, {5}
   YES is a partition, The above collection is a partition because the union of the elements of the subsets forms the given main set, and the intersection of each subsets is Ø.
   c){2, 4, 6}, {1, 3, 5}
   YES is a partition, The above collection is a partition because the union of the elements of the subsets forms the given main set, and the intersection of each subsets is Ø.
   d){1, 4, 5}, {2, 6}
   NOT a partition. The above collection is not a partition because the union of all the elements of the subsets does not give the main set.
5. Consider an n-player round-robin tournament where every pair of players play each other exactly once. Assume that there are no ties and every game has a winner. Then, the tournament can be represented via a directed graph with n nodes where the edge x → y means that x has beaten y in their game.

(a) Explain why the tournament graph does not have cycles (loops) of size 1 or 2.


HW#4 310 CPRE

Q5)a) The graph does not have cycles of size 1 or 2.

i) x beats itself

or

ii) x beats y and y beats x

situation i) is not possible since a player does not have a match with himself and situation ii) is also not possible since every pair of players play each other exactly once.

(b) We can interpret this graph in terms of a relation where the domain of discourse is the set of n players. Explain whether the “beats” relation for any given tournament is always/sometimes/never:

(i) asymmetric
(ii) reflexive
(iii) irreflexive
(iv) transitive.

b) i) asymmetric)

R is Always asymmetric : Let xRy (x has beaten y) but we can't have yRx since every pair of players play each other exactly once.

ii) reflexive)

R is Never Reflexive $\rightarrow$ xRx if x has beaten x. which is impossible

iii) Irreflexive)

R is Always Irreflexive $\rightarrow$ Opposite of part ii)

iv) Transitive)

xRz is Sometimes true

let xRy and yRz if x has beaten y and y has beaten z then it is possible that x has beaten z.

6. (20 points) Let W be the set of all words in the sentence, “The sky above the port was the color of television, tuned to a dead channel.” Define a relation R on W as follows: for any words w1, w2 ∈ W, w1Rw2 if the first letter of w1 is the same as the first letter of w2 without regard to upper or lower cases.

(a) Prove that R is an equivalence relation.

Reflexive :

If we let w1 belong to W, then it is obvious that the first letter of w1 is the same as the first letter of w2.

therefore (W1, W1) ER, therefore R is reflexive.

Symmetric :

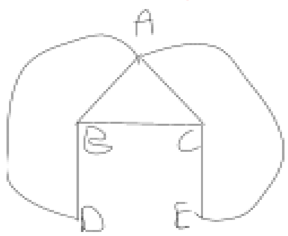
Let (w1, w2) belong to R, where w1 and w2 belong to W. Since (W1, W2) ER implies, the first letter of w1 is the same as the first letter of w2. then it is true that the first letter of w2 is the same as the first letter of w1. therefore (W2, W) ER therefore, R is symmetric.

Transitive :

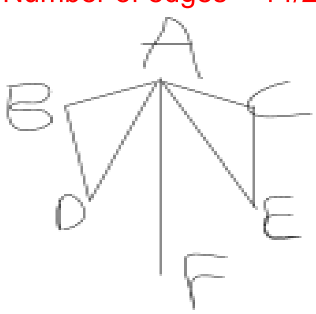
here we let w1, w2 and w3 belongs to W and let (w1,w2),(w2,w3) in R. Since (W1, W2) ER then first letter of w1 = first letter of w2. and when (w_{2},w_{3 })\in R then first letter of w2 = first letter of w3. therefore first letter of w1 = first letter of w2 = first letter of w3. therefore first letter of w1 = first letter of w3. then (W1, W3) ER therefore if (W1, W2), (W2, W3) ER then (w1, W3) E R Therefore, R is transitive.

From the above three properties, R is reflexive, symmetric and transitive.
(b) Enumerate all possible equivalence classes in R. (As per lecture, any equivalence class is the set of all elements in W that are related to each other via R.)
Let w1 is element of W then by definition of equivalence classes : {x exists in W}
This equivalence class of all elements in W is C(w) = {x exists in W} where w is any random element of W.

7. (20 points) How many edges does a simple undirected graph have if its degree sequence is
(i) 4, 3, 3, 2, 2 (5 vertices)
Total degree of graph: 4+3+3+2+2= = 14
Number of edges = 14/2 = 7 (no vertices with degree 0)



(ii)5, 2, 2, 2, 2, 1? (6 vertices)
The total degree of graph: 5+2+2+2+2+1 = 14
Number of edges = 14/2 = 7



In both cases, draw the corresponding graphs.

8. (20 points) Let G be a simple undirected graph with n vertices and m edges. Let c1 and c2 be the minimum and maximum vertex degrees in G. Show that c2 ≥ 2m/n ≥ c1
Number of vertices = n: number of edges = m:
Adding all together=
C1 ≤ deg(vi) ≤ C2 for i=1, 2, n
But Σdeg (Vi) ≤ n *C2 when i = 1
So 1 gives nC1 ≤ 2m ≤ nC2 //divide by n
= C2 ≥ 2m/n ≥ C1

9. (10 points) A simple undirected graph is called regular if every vertex has the same degree. How many vertices does a regular graph of degree four with 10 edges have?

Since the degree of every vertex is 4, therefore the sum of the degree of all vertices can be written as $N \times 4$

How m$\Rightarrow N \times 4 = 2|E|$

$\Rightarrow N = (2 \times 10)/4$

$\Rightarrow N = 5$

10. (20 points) Let G be a simple undirected graph. $\overline{G}$ represents the complementary graph of G. This graph has the same vertex set as G and is obtained as follows. If (u, v) is an edge in G, then (u, v) is not an edge in G. Conversely, if (u, v) is not an edge in G, then (u, v) is an edge in G.

• Suppose that G has 15 edges and $\overline{G}$ has 13 edges. Then, how many vertices does G have?

CPRE 310 HW4

Q10)

Maximum # of edges with n verticies

$= \frac{n(n-1)}{2}$ if graph is complete = no edges

E = set of edges of the graph and E' = the set of edges of its compliment G'

$|E| + |E'| = \frac{n(n-1)}{2}$ $|E| = 15$ $|E'| = 13$

$15 + 13 = \frac{n(n-1)}{2} \rightarrow 28 = \frac{n(n-1)}{2} \rightarrow n(n-1) = 56$

$n^2 - n - 56 = 0$ $(n-8)(n+7)$

$n = 8, -7$

$\therefore$ # of verticies = 8 (cant have negative verticies)