CprE 310 / Sample
Midterm 1, Fall 2023

- Maximum score:
- Total duration: 75 minutes
- Please **write your name and netid** on the top of this page.
- You **cannot** consult your notes, textbook, your neighbor, or Google or Chegg (or equivalent).

---

1. **(5 points)** Are the statements $\neg(p \to q)$ and $\neg p \to \neg q$ logically equivalent? Justify your answer.

No. Set $p = q = T$

Then, $\neg(p \to q) = F$

But $\neg p \to \neg q = T$

2. **(10 points)** Consider the compound proposition $s = [\neg(p \to q)] \vee [\neg(p \vee q)]$.

- Construct a truth table for s.
- Find a simpler expression that is logically equivalent to s.

$\neg(p \Rightarrow q) = \neg(\neg p \vee q) = p \wedge \neg q$; $\neg(p \vee q) = \neg p \wedge \neg q$

$\therefore s = (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

if $p = T$, then $s = (T \wedge \neg q) \vee (F \wedge \neg q) = \neg q$

Likewise if $p = F$, $s = (F \wedge \neg q) \vee (T \wedge \neg q) = \neg q$

$\therefore s = \neg q$. Truth table can be generated

3. **(10 points)** Let the domain of discourse by the non-zero integers, i.e, $\mathbb{Z} \setminus \{0\}$. Let $P(x, y)$ be the predicate "x/y is an integer". Determine whether the following statements are true or false.

(a) $\forall y, \exists x, P(x, y)$.

True since one can choose x which is a multiple of y. e.g. $x = 2y$ will work since $\frac{2y}{y} = 2$. (integer).

(b) $\exists x, \forall y, P(x, y)$.

False, since this requires for all y, the existence of one x such that x/y is an integer. Note that you can cannot choose $x = 0$.

4. **(10 points)** Let a = "You can vote", b = "You are under 18 years old", and c = "You are from Mars".

   a. Translate the following sentence into propositional logic: "You can't vote if you are under 18 years of age or you are from Mars."

   $b \vee c \to \neg a$

   b. Give the contrapositive of this statement using the symbols a, b and c.

   $a \to \neg b \wedge \neg c$

   c. Give the contrapositive of this statement in English.

   If you can vote then you are over 18 and you are not from Mars.

5. **(20 points)** Use the rules of inference to show the conclusion from the premises. For each step clearly state which rule you are using and why the usage is valid.

a.

$$(\neg p \vee q) \rightarrow r$$
$$s \vee \neg q$$
$$\neg t$$
$$p \rightarrow t$$
$$\neg p \wedge r \rightarrow \neg s$$
$$------$$
$$\therefore \neg q$$

(1) $\begin{array}{l} \neg t \\ p \rightarrow t \\ \hline \therefore \neg p \end{array}$ Modus tollens

(2) $\therefore \neg p \vee q$ addition

(3) $\begin{array}{l} \neg p \vee q \rightarrow r \\ \hline \therefore r \end{array}$ modus ponens.

(4) $\begin{array}{l} \neg p \\ r \\ \hline \therefore \neg p \wedge r \end{array}$ conjunction

(5) $\begin{array}{l} \neg p \wedge r \rightarrow \neg s \\ \hline \therefore \neg s \end{array}$ modus ponens.

(6) $\begin{array}{l} \neg s \\ s \vee \neg q \\ \hline \therefore \neg q \end{array}$ disjunctive syllogism

b.

$$\neg p \rightarrow r \wedge \neg s$$
$$t \rightarrow s$$
$$u \rightarrow \neg p$$
$$\neg w$$
$$u \vee w$$
$$-------$$
$$\therefore \neg t$$

(1) $\begin{array}{l} \neg w \\ w \vee u \\ \hline \therefore u \end{array}$ disjunctive syllogism

(2) $\begin{array}{l} u \\ u \rightarrow \neg p \\ \hline \therefore \neg p \end{array}$ modus ponens

(3) $\begin{array}{l} \neg p \\ \neg p \rightarrow r \wedge \neg s \\ \hline \therefore r \wedge \neg s \end{array}$ modus ponens

(4) $\begin{array}{l} r \wedge \neg s \\ \hline \therefore \neg s \end{array}$ Simplification

(5) $\begin{array}{l} \neg s \\ t \rightarrow s \\ \hline \therefore \neg t \end{array}$ modus tollens

6. **(10 points)** Let x and y be integers such that $3x + 5y = 153$. Then, using proof by contradiction prove that either x or y or both are odd.

Suppose that both x & y are even.

then $3x+5y$ will be even since $3x$ is even & $5y$ is even & sum of evens is even

But then we have a contradiction since $3x+5y$ is even & 153 is odd.

7. **(10 points)** Let a be odd and b be even. Show by direct proof that $7ab + 6a^3$ is even.

a is odd $\Rightarrow a = 2k_1+1$ for integer k_1.

b is even $\Rightarrow b = 2k_2$ " " k_2.

$\therefore 7(2k_1+1)2k_2 + 6(2k_1+1)^3$

$= 2[7(2k_1+1) + 3(2k_1+1)^3]$

i.e. it is a multiple of 2 $\therefore$ even.

8. **(10 points)** Prove (using contraposition) that if x is irrational then $1/x$ is irrational.

Contrapositive : $\frac{1}{x}$ is rational $\Rightarrow$ x is rational

Since $\frac{1}{x}$ is ~~irr~~ rational we have

$\frac{1}{x} = \frac{m}{n}$ for integer m & n & $n \neq 0$ & $m \neq 0$.

$\therefore x = \frac{n}{m}$ i.e., x can be expressed as the ratio of integers with denominator non-zero

$\Rightarrow$ x is rational.