CprE 310 / Sample Midterm 1, Fall 2023

- Maximum score:
- Total duration: 75 minutes
- Please write your name and netid on the top of this page.
- You **cannot** consult your notes, textbook, your neighbor, or Google or Chegg (or equivalent).
- 1. **(5 points)** Are the statements $\neg(p \to q)$ and $\neg p \to \neg q$ logically equivalent? Justify your answer.

No. Set $p=Q_1=T$ Then, $\gamma(p \Rightarrow q) = F$ But $\gamma_p \Rightarrow \gamma_{Q_1} = T$

- 2. (10 points) Consider the compound proposition $s = [\neg(p \to q)] \lor [\neg(p \lor q)]$.
 - Construct a truth table for s.
 - Find a simpler expression that is logically equivalent to s.

7(p>q)=7(7pVq)=pMq;7(pVa)=7pMq .. S = (PM2) V (7PM2) if p=T, then S= (TMQ) V (F MQ) = 7Q Chewise of p=F, s= (FMR) U (TMA) = Te .. S=791. Town table can be generated

- 3. (10 points) Let the domain of discourse by the non-zero integers, i.e. $\mathbb{Z}\setminus\{0\}$. Let P(x,y) be the predicate "x/y is an integer". Determine whether the following statements are true or false.
- (a) $\forall y, \exists x, P(x, y)$.

True since one can choose a which is a multiple of y. e.g. x=2y will wonksince 2y =2. (integer).

(b) $\exists x, \forall y, P(x, y).$

Folse, since this magnines for all of the existence of one or such that

My is an integen. Note that

you can cannot choose X=0.

- 4. (10 points) Let a= "You can vote", b= "You are under 18 years old", and c= "You are from Mars".
 - a. Translate the following sentence into propositional logic: "You can't vote if you are under 18 years of age or you are from Mars."

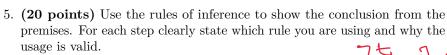
bVc > Ta

b. Give the contrapositive of this statement using the symbols $\it a, \it b$ and $\it c.$

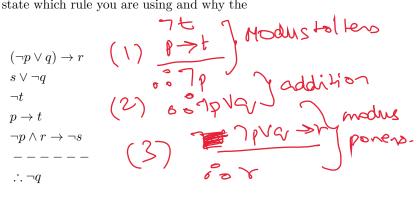
a >> 76 M7c

c. Give the contrapositive of this statement in English.

If you can vote then you are over 18 and you are not from Mars.



a.



(6) 7s disjunctive SV78 Syllogism

b.

$$\begin{array}{c}
 \neg p \rightarrow r \wedge \neg s \\
 t \rightarrow s \\
 u \rightarrow \neg p \\
 \neg w \\
 u \vee w \\
 ------\\
 \vdots \neg t
 \end{array}$$

- (1) 7 w disjunctive

 w/u disjunctive

 syllosism
- (2) Unadus U77P Jones
- (2) TP modus

 77-> V/1s y ponens

- (h) mas J Simplification
- (5) 7s 1 modules tollers

 5.7t

4

6. (10 points) Let x and y be integers such that 3x + 5y = 153. Then, using proof by contradiction prove that either x or y or both are odd.

Suppose that both x by are even.

Then Brothy will be even sine 3n is even

by is even & sum of evens is even

But then we have a continction

Since Sur Sy is even & 153 is odd.

7. (10 points) Let a be odd and b be even. Show by direct proof that $7ab + 6a^3$ is even.

a bodd = 0 a= $2k_1+1$ for indegent,.

biseren = 0 b= $2k_2$ 11 h k_2 .

1 ($2k_1+1$) $2k_2+1$ ($2k_1+1$)3-2 [7 ($2k_1+1$) + 3 ($2k_1+1$)3i.e. it is a multiple of 2 ... even.

8. (10 points) Prove (using contraposition) that if x is irrational then 1/x is irrational.

Contrapositive: I is radional = Trisrational

Since I is protional we have

I z m for integer in fin

in z n ie-, in a transpressed

as the ratio of integers

with denominator non-zero

— Trisrational.