CprE 310 Homework 1

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1) (1.1.10) p: I bought a lottery ticket this week

q: I won the million dollar jackpot

a) $\neg p$ = I did not buy a lottery ticket this week

b) $p \vee q$ = I bought a lottery ticket this week or I won the jackpot

c) $p \to q$ = If I bought a lottery ticket this week, then I won the jackpot

d) $p \wedge q$ = I bought a lottery ticket this week and I won the jackpot

e) $p \leftrightarrow q$ = I bought a lottery ticket this week if and only if I won the jackpot.

f) $\neg p \to \neg q$ = If I did not buy a lottery ticket, then I did not win the jackpot

g) $\neg p \wedge \neg q$ = I did not buy a ticket this week and I did not win the jackpot

h) $\neg p \vee (p \wedge q)$ = I did not buy a ticket this week or I bought a ticket and won the jackpot

2) (1.1.14)

p: You have the flu q: You missed the final exam
r: You pass the course

a) $p \to q$ = if you have the flu, then you miss the final exam

b) $\neg q \leftrightarrow r$ = You do not miss the final exam if and only if you pass the course

c) $q \to \neg r$ = if you miss the final, then you do not pass the course

d) $p \vee q \vee r$ = You have the flu or you miss the final or you pass the course

e) $(p \to \neg r) \vee (q \to \neg r)$ either if you have the flu, you do not pass the course or if you miss the final you do not pass the course

f) $(p \wedge q) \vee (\neg q \wedge r)$ = Either you have the flu and you miss the final or you do not miss the fina and you pass the course.

3) (1.1.29)

a) If it snows today, I will ski tomorrow

Converse: If I will ski tomollow, then it showed today

Contrapositive: If I will not ski tomollow, then it did not snow today

Inverse: If it did not snow today, I will not ski tomorrow

b) I come to class whenever there is going to be a quiz

Converse: If there is going to be a quiz, I will come to class

Contrapositive: If I dont come to class, then there is not going to be a quiz

Inverse: If there is not going to be a quiz, I won't come to class.

c) A positive Integer is a prime only if it has no divisors other than 1 and itself

Converse: If a positive integer has no divisors other than 1 and itself, then it is prime.

Contrapositive: If a positive integer is not a prime, then it has divisors other than 1 and itself.

Inverse: If a positive integer has divisors other than 1 and itself then it is not a prime.

4) (1.1.34) Construct a Truth Table

a) $p \to \neg p =$

p	$\neg p$	$p \to \neg p$
F	T	T
T	F	F

b) $p \leftrightarrow \neg p =$

p	$\neg p$	$p \leftrightarrow \neg p$
F	T	F
T	F	F

c) $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
F	F	F	F
F	T	T	T
T	F	T	F
T	T	T	T

d) $(p \wedge q) \to (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
F	F	F	F	T
F	T	F	T	T
T	F	F	T	T
T	T	T	T	T

e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$q \to \neg p$	$p \leftrightarrow q$	$(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
F	F	T	T	T	T
F	T	T	T	F	F
T	F	F	F	F	T
T	T	F	T	T	F

f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
F	F	T	T	F	T
F	T	F	F	T	T
T	F	T	F	T	T
T	T	F	T	F	T

5) (1.3.12) Show Tautology using Truth Tables

a) $[\neg p \wedge (p \vee q)] \to q$

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
F	F	T	F	F	T
F	T	T	T	T	T
T	F	F	T	F	T
T	T	F	T	F	T

b) $[(p \to q) \wedge (q \wedge r)] \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$(p \to q) \wedge (q \to r) \to (p \to r)$
F	F	F	T	T	T	T
F	F	T	T	T	T	T
F	T	F	T	F	F	T
F	T	T	T	T	T	T
T	F	F	F	T	F	T
T	F	T	F	T	F	T
T	T	F	T	F	F	T
T	T	T	T	T	T	T

c) $[p \wedge (p \to q)] \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

D) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
F	F	F	F	T	T	F	T
F	F	T	F	T	T	F	T
F	T	F	T	T	F	F	T
F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	T
T	F	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

6) (1.4.8) $R(x)$ is "x is a rabbit" $H(x)$ is "x hops"

a) $\forall x (R(x) \to H(x))$: "Every animal that is a rabbit hops."

b) $\forall x (R(x) \wedge H(x))$: "Every animal that is a rabbit also hops."

c) $\exists x (R(x) \to H(x))$: "There exists an animal such that if it is a rabbit, it hops."

d) $\exists x (R(x) \wedge H(x))$: "There exists an animal that is both a rabbit and hops."

7) (1.4.9) $P(x)$ = "x can speak Russian" $Q(x)$ = "x knows the computer language C++"

a) There is a student at your school who can speak Russian and knows C++
$\boxed{\equiv \exists x (S(x) \wedge P(x) \wedge Q(x))}$

b) There is a student at your school who can speak Russian but who doesn't know C++
$\boxed{\equiv \exists x (S(x) \wedge P(x) \wedge \neg Q(x))}$

c) every student at your school either speaks Russian or knows C++
$\boxed{\equiv \forall x (S(x) \to (P(x) \vee Q(x)))}$

d) No student at your school speaks Russian or knows C++
$\boxed{\equiv \neg \exists x (S(x) \wedge (P(x)) \vee (Q(x)))}$

8) (1.4.36) Express negation without using the negative symbol (HW

a) $\forall x(-2<x<3)$: There exists an x such that -2 is not less than x or x is not less than 3

b) $\forall x(0 \leq x < 5)$: There exists an x such that x is not greater than or equal to 0 or x is not less than 5.

c) $\exists x(-4 \leq x \leq 1)$: For all x, x is either less than -4 or greater than 1.

D) $\exists x(-5<x<-1)$: For all x, x is either not less than -5 or not greater than

9) (1.4.38)

a) $\forall x(x^2 \neq x)$: Counterexample: $\boxed{\text{when } x=3}$ $x^2=9$ but $x=3$ $9 \neq 3$

b) $\forall x(x^2 \neq 2)$: $x=\sqrt{2}$; $\sqrt{2}^2=2$ making $x^2 \neq 2$ false however the $\sqrt{2}$ is not considered a rational number it is a real number

c) $\forall x(|x|>0)$: absolute value for any real number >0 except for when $\underline{x=0}$ $\underline{|0|=0}$

10) (1.5.28) for all real numbers

a) $\forall x \exists y(x^2=y)$ True for any real # x you can take $y=x^2$
b) $\forall x \exists y(x=y^2)$ False when $x=-1$ there is no real # y that $=$ to -1
c) $\exists x \forall y(xy=0)$ True $x=0$ any real # y makes $xy=0$

d) $\exists x \exists y(x+y \neq y+x)$ False
e) $\forall x(x \neq 0 \rightarrow \exists y(xy=1))$ True

f) $\exists x \forall y(y \neq 0 \rightarrow xy=1)$ False
g) $\forall x \exists y(x+y=1)$ True

h) $\exists x \exists y(x+2y=2 \wedge 2x+4y=5)$ False
i) $\forall x \exists y(x+y=2 \wedge 2x-y=1)$ True

j) $\forall x \forall y \exists z(z=(x+y)/2)$ True