Gerichtete Graphen

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Diskrete Mathematik und Stochastik, 2019

Fachbereich Wirtschaft und Informationstechnik
Westfälische Hochschule Bocholt

Outline

First Main Section

First Slide Title

- My first point.
- My second point.

Second Slide Title

- First item.

Second Slide Title

- First item.
- Second item.

Second Slide Title

- First item.
- Second item.
- Third item.

Second Slide Title

- First item.
- Second item.
- Third item.
- Fourth item.

Second Slide Title

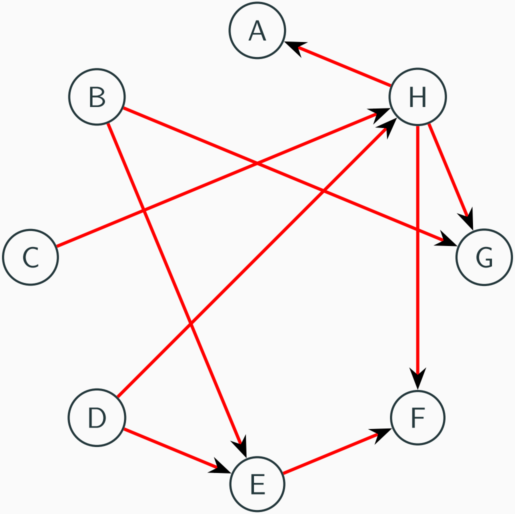
- First item.
- Second item.
- Third item.
- Fourth item.
- Fifth item.

Second Slide Title

- First item.
- Second item.
- Third item.
- Fourth item.
- Fifth item. Extra text in the fifth item.

Topologisches Sortieren

Beispiel: Topologisches Sortieren



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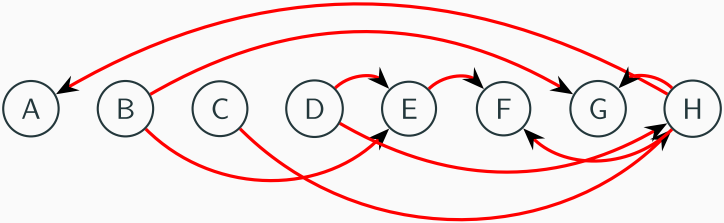
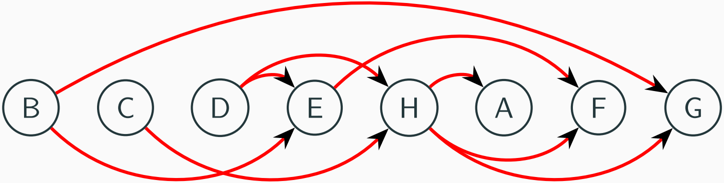
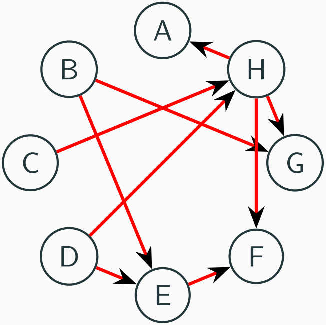




Kahn's Algorithmus

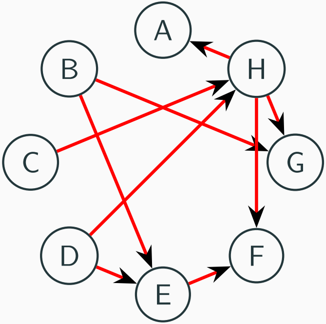
jo

Beispiel: Kahn's Algorithmus



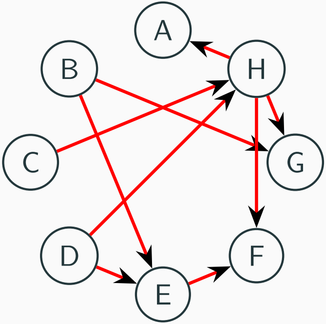
i	v	$A(v)$
	A	
	B	
	C	
	D	
	E	
	F	
	G	
	H	

Beispiel: Kahn's Algorithmus



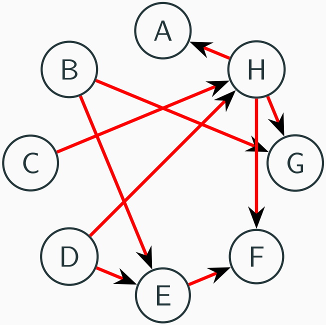
i	v	$A(v)$
	A	{H}
	B	
	C	
	D	
	E	
	F	
	G	
	H	

Beispiel: Kahn's Algorithmus



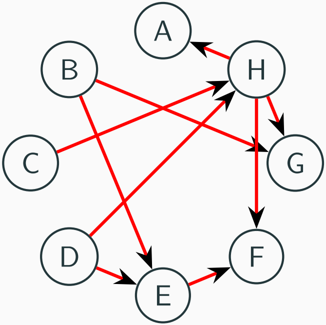
i	v	$A(v)$
	A	{H}
	B	$\varnothing$
	C	
	D	
	E	
	F	
	G	
	H	

Beispiel: Kahn's Algorithmus



i	v	$A(v)$
	A	{H}
	B	∅
	C	∅
	D	
	E	
	F	
	G	
	H	

Beispiel: Kahn's Algorithmus



i	v	$A(v)$
	A	{H}
	B	$\varnothing$
	C	$\varnothing$
	D	$\varnothing$
	E	
	F	
	G	
	H	

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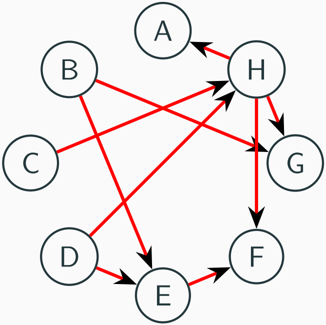
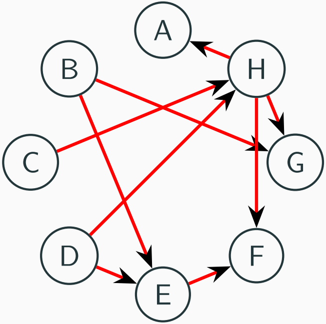


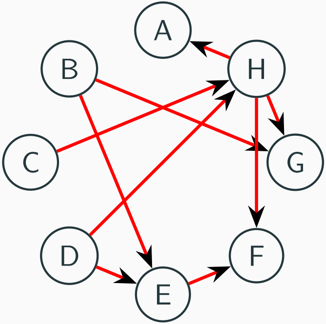
i	v	$A(v)$
	A	{H}
	B	$\varnothing$
	C	$\varnothing$
	D	$\varnothing$
	E	{B, D}
	F	
	G	
	H	

Beispiel: Kahn's Algorithmus



i	v	$A(v)$
	A	{H}
	B	$\varnothing$
	C	$\varnothing$
	D	$\varnothing$
	E	{B, D}
	F	{E, H}
	G	
	H	

Beispiel: Kahn's Algorithmus



i	v	$A(v)$
	A	$\{H\}$
	B	$\varnothing$
	C	$\varnothing$
	D	$\varnothing$
	E	$\{B, D\}$
	F	$\{E, H\}$
	G	$\{B, H\}$
	H	

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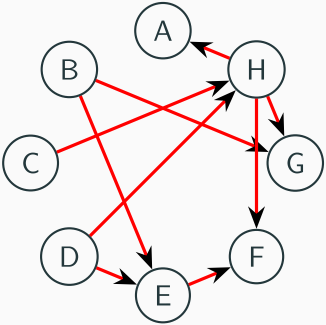

i	v	$A(v)$
	A	{H}
	B	$\varnothing$
	C	$\varnothing$
	D	$\varnothing$
	E	{B, D}
	F	{E, H}
	G	{B, H}
	H	{C, D}

Beispiel: Kahn's Algorithmus

i	v	$A(v)$
	A	{H}
	B	$\varnothing$
	C	$\varnothing$
	D	$\varnothing$
	E	{B, D}
	F	{E, H}
	G	{B, H}
	H	{C, D}

Beispiel: Kahn's Algorithmus

i	v	$A(v)$
	A	$\{H\}$
1	B	$\emptyset$
	C	$\emptyset$
	D	$\emptyset$
	E	$\{B, D\}$
	F	$\{E, H\}$
	G	$\{B, H\}$
	H	$\{C, D\}$

Beispiel: Kahn's Algorithmus

i	v	$A(v)$
	A	{H}
1	B	$\varnothing$
	C	$\varnothing$
	D	$\varnothing$
	E	{$\cancel{\text{B}}$, D}
	F	{E, H}
	G	{$\cancel{\text{B}}$, H}
	H	{C, D}

Beispiel: Kahn's Algorithmus

i	v	$A(v)$
	A	{H}
1	B	$\varnothing$
	C	$\varnothing$
	D	$\varnothing$
	E	{~~B~~, D}
	F	{E, H}
	G	{~~B~~, H}
	H	{C, D}

Beispiel: Kahn's Algorithmus

i	v	$A(v)$
	A	$\{\text{H}\}$
1	B	$\varnothing$
2	C	$\varnothing$
	D	$\varnothing$
	E	$\{\cancel{\text{B}}, \text{D}\}$
	F	$\{\text{E}, \text{H}\}$
	G	$\{\cancel{\text{B}}, \text{H}\}$
	H	$\{\text{C}, \text{D}\}$

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i	v	$A(v)$
	A	$\{\text{H}\}$
1	B	$\varnothing$
2	C	$\varnothing$
	D	$\varnothing$
	E	$\{\cancel{\text{B}}, \text{D}\}$
	F	$\{\text{E}, \text{H}\}$
	G	$\{\cancel{\text{B}}, \text{H}\}$
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i	v	$A(v)$
	A	{H}
1	B	$\varnothing$
2	C	$\varnothing$
	D	$\varnothing$
	E	$\{\cancel{B}, D\}$
	F	{E, H}
	G	$\{\cancel{B}, H\}$
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Beispiel: Kahn's Algorithmus

i	v	$A(v)$
	A	$\{\mathsf{H}\}$
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
	E	$\{\cancel{\mathsf{B}}, \mathsf{D}\}$
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Beispiel: Kahn's Algorithmus

i	v	$A(v)$
	A	{H}
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
	E	$\{\cancel{B}, \cancel{D}\}$
	F	{E, H}
	G	$\{\cancel{B}, H\}$
	H	$\{\cancel{C}, \cancel{D}\}$

Beispiel: Kahn's Algorithmus

i	v	$A(v)$
	A	$\{\mathsf{H}\}$
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
	E	$\{\cancel{\mathsf{B}}, \cancel{\mathsf{D}}\}$
	F	$\{\mathsf{E}, \mathsf{H}\}$
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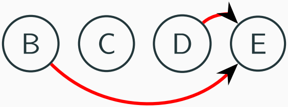

i	v	$A(v)$
	A	$\{H\}$
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
4	E	$\{\cancel{B}, \cancel{D}\}$
	F	$\{E, H\}$
	G	$\{\cancel{B}, H\}$
	H	$\{\cancel{C}, \cancel{D}\}$

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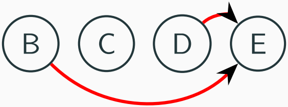

i	v	$A(v)$
	A	{H}
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
4	E	$\{\cancel{B}, \cancel{D}\}$
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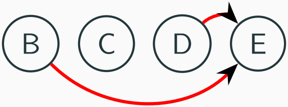

i	v	$A(v)$
	A	{H}
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2	C	$\varnothing$
3	D	$\varnothing$
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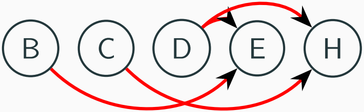

i	v	$A(v)$
	A	$\{H\}$
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
4	E	$\{\cancel{B}, \cancel{D}\}$
	F	$\{\cancel{E}, H\}$
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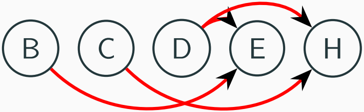

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	A	$\{\cancel{H}\}$
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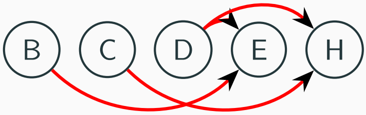

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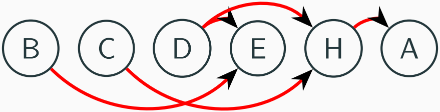

i	v	$A(v)$
6	A	$\{\cancel{H}\}$
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
4	E	$\{\cancel{B}, \cancel{D}\}$
	F	$\{\cancel{E}, \cancel{H}\}$
	G	$\{\cancel{B}, \cancel{H}\}$
5	H	$\{\cancel{C}, \cancel{D}\}$

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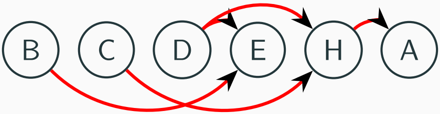

i	v	$A(v)$
6	A	$\{\cancel{H}\}$
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
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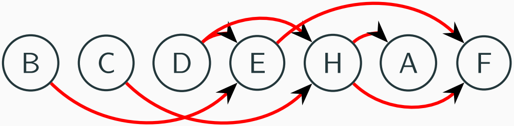

i	v	$A(v)$
6	A	$\{\cancel{H}\}$
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
4	E	$\{\cancel{B}, \cancel{D}\}$
7	F	$\{\cancel{E}, \cancel{H}\}$
	G	$\{\cancel{B}, \cancel{H}\}$
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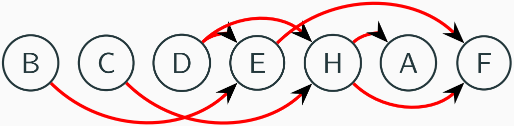

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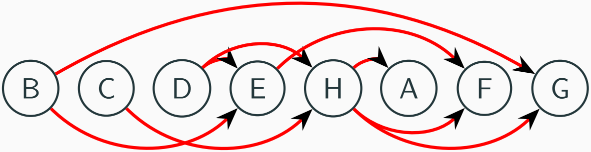


i	v	$A(v)$
6	A	$\{\cancel{H}\}$
1	B	$\varnothing$
2	C	$\varnothing$
3	D	$\varnothing$
4	E	$\{\cancel{B}, \cancel{D}\}$
7	F	$\{\cancel{E}, \cancel{H}\}$
8	G	$\{\cancel{B}, \cancel{H}\}$
5	H	$\{\cancel{C}, \cancel{D}\}$

Wege in Digraphen

Wege in Digraphen

- Existenz von Wegen zwischen beliebigen Knoten
- Adjazenzmatrix $\mathbf{M} \longrightarrow$ Erreichbarkeitsmatrix $\mathbf{M}^*$
- Kantenrelation $E \longrightarrow$ transitiver Abschluss E^*

Wege als transitiver Abschluss der Kantenrelation

- Kantenrelation E: Menge von Wegen der Länge 1 zwischen Knoten
- Menge von Wegen der Länge 2 als Komposition von E mit sich selber:
  $E \circ E = \{(v_1, v_2) \in V^2 | \exists u \in V : (v_1, u) \in E \wedge (u, v_2) \in E\}$
- Menge von Wegen einer Länge k: $\underbrace{E \circ \cdots \circ E}_{k \text{ mal}}$
- Transitiver Abschluss als Vereinigung dieser Mengen:
  $E^* = E \cup E \circ E \cup \cdots \cup \underbrace{E \circ \cdots \circ E}_{n \text{ mal}}$

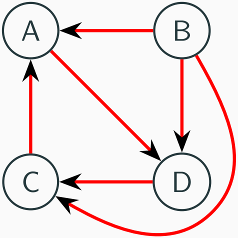
Potenzieren der Adjazenzmatrix

- Komposition von E als logisches Matrixprodukt der Adjazenzmatrix $\mathbf{M}$
- Weitere Komposition durch das Potenzieren von $\mathbf{M}$
- Vereinigen der Mengen durch das logische **oder** der Matrizen
- Logisches **oder** zweier Matrizen wird komponentenweise gebildet
  - $(\mathbf{A} \vee \mathbf{B})_{ij} = \mathbf{A}_{ij} \vee \mathbf{B}_{ij}$

Berechnung der Erreichbarkeitsmatrix $\mathbf{M}^*$

$$\mathbf{M}^* = \mathbf{M} \vee \mathbf{M}^2 \vee \cdots \vee \mathbf{M}^n$$

Beispiel: Potenzieren der Adjazenzmatrix



$$\mathbf{M} = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{M}^2 = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{M}^3 = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \qquad \mathbf{M}^4 = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{M}^* = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

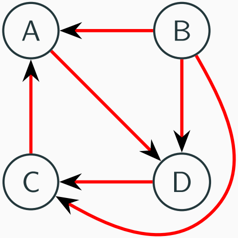
Warshall's Algorithmus

- Matrixmultiplikation ist aufwändig
  - Berechnen der Potenzen bei Graphen mit vielen Knoten ist problematisch
- Warshall ermittelt $\mathbf{M}^*$ in Worst-Case $\Theta(n^3)$ Zeit
- Warshall's Algorithmus arbeitet *In-Place*
  - $\mathbf{M}$ wird iterativ in $\mathbf{M}^*$ überführt
- Warshall erzeugt Matrizen $\mathbf{W}_0, \mathbf{W}_1, \ldots, \mathbf{W}_n$
  - Wobei $\mathbf{W}_0 = \mathbf{M}$ und $\mathbf{W}_n = \mathbf{M}^*$
- $\mathbf{W}_k[i,j] = 1 \iff$ Es existiert ein Weg von v_i nach v_j, wobei alle Zwischenknoten $\in \{v_1, v_2, \ldots, v_k\}$ sind

Pseudocode: Warshall's Algorithmus

Algorithm 1 Warshall's Algorithmus

```
function WARSHALL(M)
    W ← M
    for k = 1 to n do
        for i = 1 to n do
            if W[i, k] then
                for j = 1 to n do
                    W[i, j] ← W[i, j] ∨ W[k, j]
                end for
            end if
        end for
    end for
    return W
end function
```

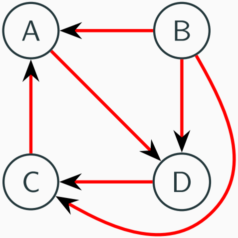


$$\mathbf{W}_0 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus



$$\mathbf{W}_0 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 0 & 0 & 1 \\ B & 1 & 0 & 1 & 1 \\ C & 1 & 0 & 0 & 0 \\ D & 0 & 0 & 1 & 0 \end{array}$$

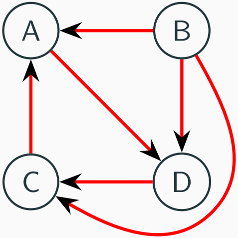
$$\mathbf{W}_1 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

$$\mathbf{W}_3 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

Beispiel: Warshall's Algorithmus

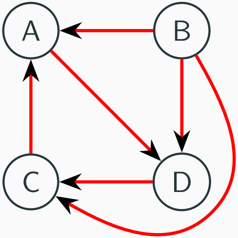


$$\mathbf{W}_0 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ & & & \\ & & & \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

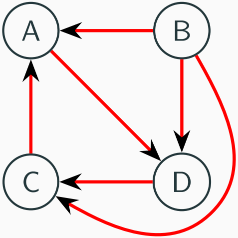


$$\mathbf{W}_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ & & & \\ & & & \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus



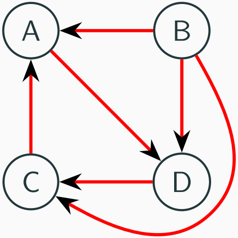
$$\mathbf{W}_0 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 0 & 0 & 1 \\ B & 1 & 0 & 1 & 1 \\ C & 1 & 0 & 0 & 0 \\ D & 0 & 0 & 1 & 0 \end{array}$$

$$\mathbf{W}_1 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 0 & 0 & 1 \\ B & 1 & 0 & 1 & 1 \\ C & & & & \\ D & 0 & 0 & 1 & 0 \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

$$\mathbf{W}_3 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

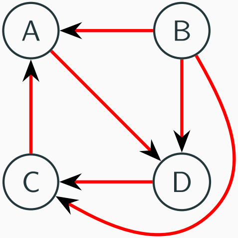


$$\mathbf{W}_0 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ & & & \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

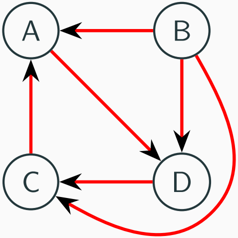


$$\mathbf{W}_0 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 0 & 0 & 1 \\ B & 1 & 0 & 1 & 1 \\ C & 1 & 0 & 0 & 0 \\ D & 0 & 0 & 1 & 0 \end{array} \qquad \mathbf{W}_1 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 0 & 0 & 1 \\ B & 1 & 0 & 1 & 1 \\ C & 1 & 0 & 0 & 1 \\ D & 0 & 0 & 1 & 0 \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array} \qquad \mathbf{W}_3 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

Beispiel: Warshall's Algorithmus

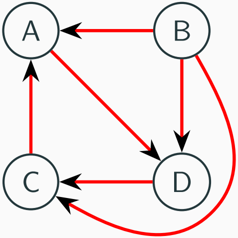


$$\mathbf{W}_0 = \begin{array}{cc} & \begin{matrix} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{matrix} \\ \begin{matrix} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{cc} & \begin{matrix} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{matrix} \\ \begin{matrix} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{cc} & \begin{matrix} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{matrix} \\ \begin{matrix} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{matrix} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{cc} & \begin{matrix} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{matrix} \\ \begin{matrix} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{matrix} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{cc} & \begin{matrix} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{matrix} \\ \begin{matrix} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{matrix} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

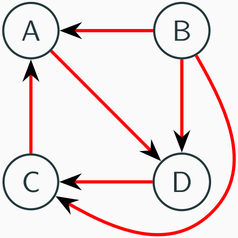


$$\mathbf{W}_0 = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} & & & \\ \\ \\ \\ \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} & & & \\ \\ \\ \\ \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} & & & \\ \\ \\ \\ \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus



$$\mathbf{W}_0 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

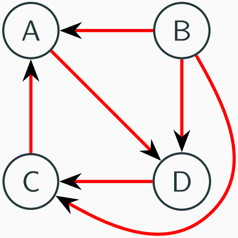
$$\mathbf{W}_1 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_3 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} \; & \; & \; & \; \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{cc} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} \; & \; & \; & \; \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

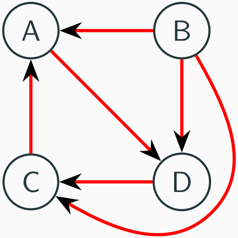


$$\mathbf{W}_0 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

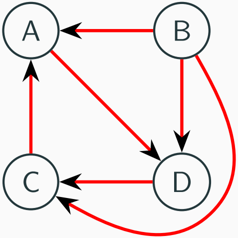


$$\mathbf{W}_0 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ \\ \\ \\ \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ \\ \\ \\ \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus



$$\mathbf{W}_0 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

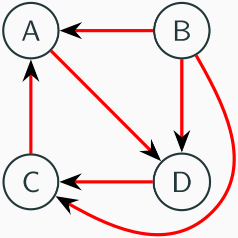
$$\mathbf{W}_1 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_3 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ & & & \\ 1 & 0 & 0 & 1 \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus



$$\mathbf{W}_0 = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

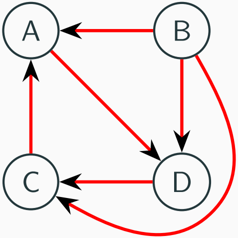
$$\mathbf{W}_1 = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_3 = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ & & & \\ 1 & 0 & 0 & 1 \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

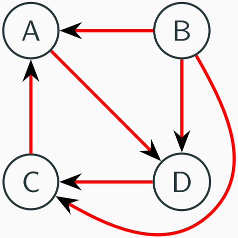


$$\mathbf{W}_0 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ & & & \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

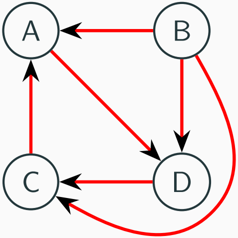


$$W_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad W_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$W_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad W_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ & & & \end{bmatrix} \end{array}$$

$$W_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

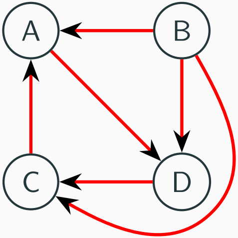


$$\mathbf{W}_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

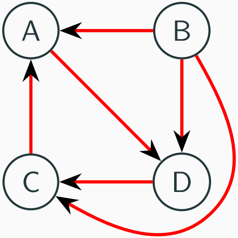


$$\mathbf{W}_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

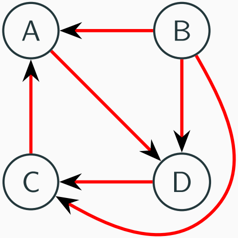


$$\mathbf{W}_0 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c c} & \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

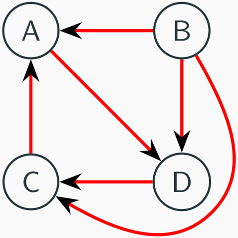


$$\mathbf{W}_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

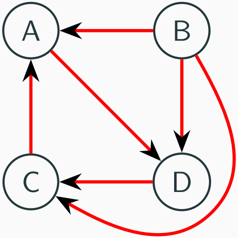


$$\mathbf{W}_0 = \begin{array}{c} \\ \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} \begin{array}{c} \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c} \\ \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} \begin{array}{c} \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} \begin{array}{c} \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c} \\ \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} \begin{array}{c} \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} \begin{array}{c} \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

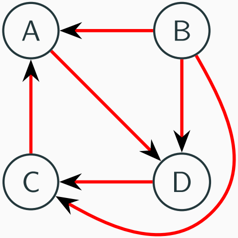


$$\mathbf{W}_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

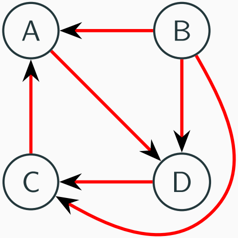


$$\mathbf{W}_0 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad \mathbf{W}_1 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad \mathbf{W}_3 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

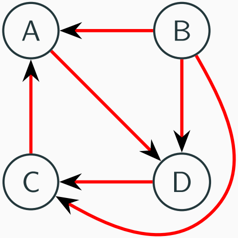


$$\mathbf{W}_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus



$$\mathbf{W}_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

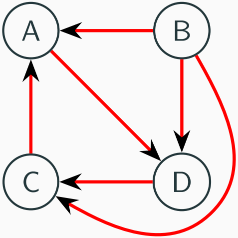
$$\mathbf{W}_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

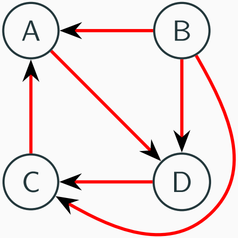


$$\mathbf{W}_0 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ & & & \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus

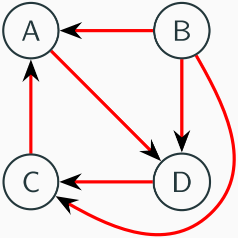


$$\mathbf{W}_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{W}_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Beispiel: Warshall's Algorithmus



$$\mathbf{W}_0 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{W}_3 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{W}_4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Kürzeste Wege

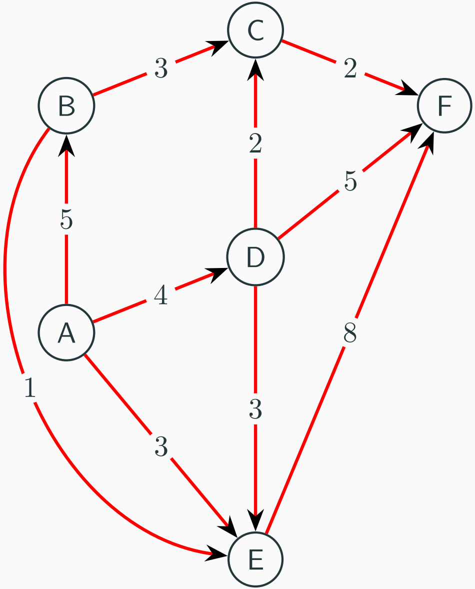
Kürzeste Wege

yo

Dijkstra's Algorithmus

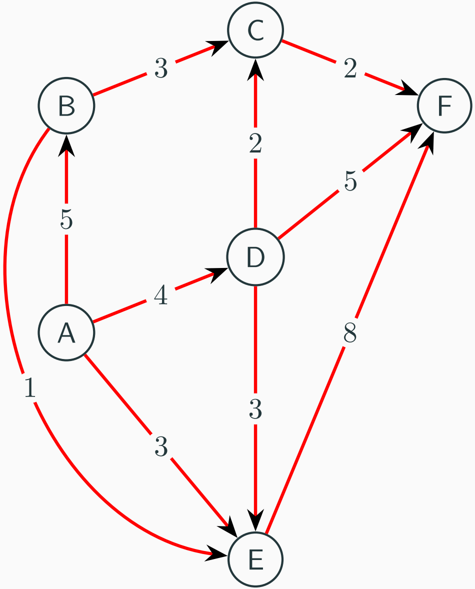
jawohl

Beispiel: Dijkstra's Algorithmus



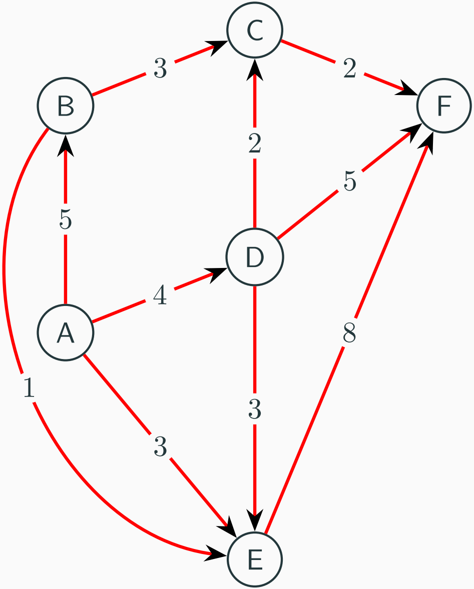
v	$d(v)$	$p(v)$
A		
B		
C		
D		
E		
F		

Beispiel: Dijkstra's Algorithmus



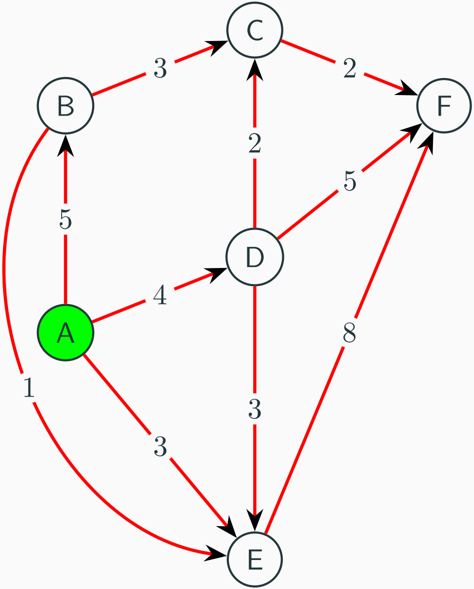
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B		
C		
D		
E		
F		

Beispiel: Dijkstra's Algorithmus



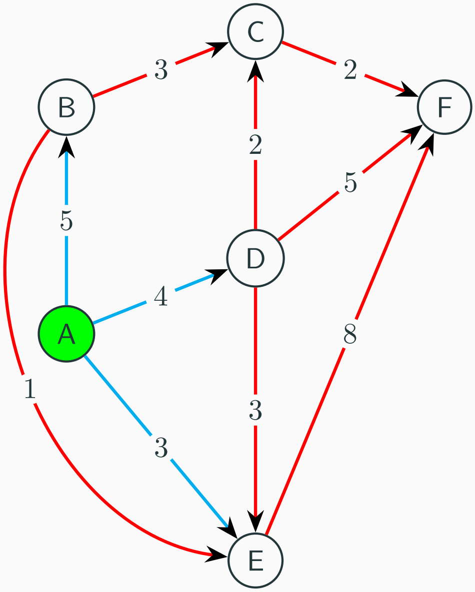
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	∞	$\varnothing$
C	∞	$\varnothing$
D	∞	$\varnothing$
E	∞	$\varnothing$
F	∞	$\varnothing$

Beispiel: Dijkstra's Algorithmus



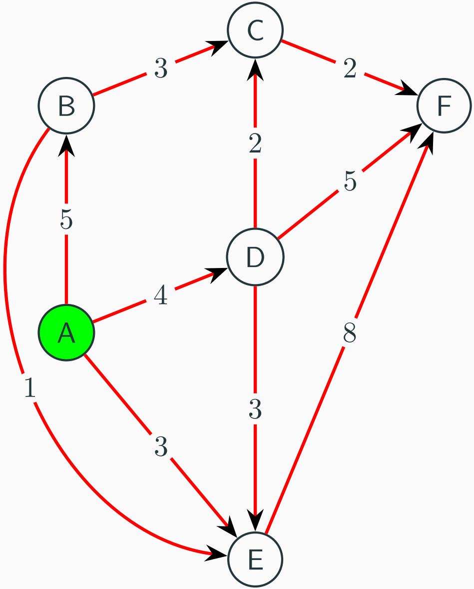
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	∞	$\varnothing$
C	∞	$\varnothing$
D	∞	$\varnothing$
E	∞	$\varnothing$
F	∞	$\varnothing$

Beispiel: Dijkstra's Algorithmus



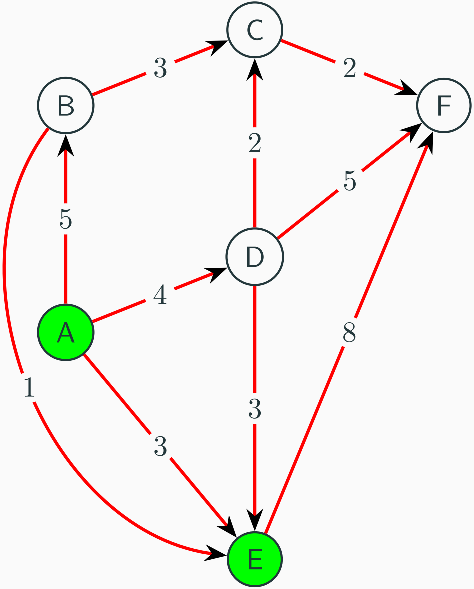
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	∞	$\varnothing$
C	∞	$\varnothing$
D	∞	$\varnothing$
E	∞	$\varnothing$
F	∞	$\varnothing$

Beispiel: Dijkstra's Algorithmus



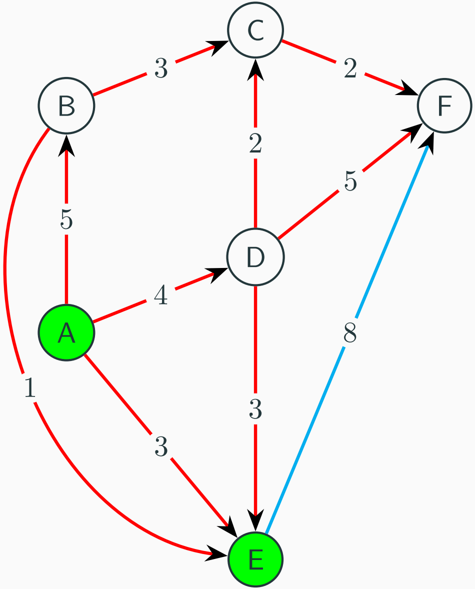
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	∞	$\varnothing$
D	4	A
E	3	A
F	∞	$\varnothing$

Beispiel: Dijkstra's Algorithmus



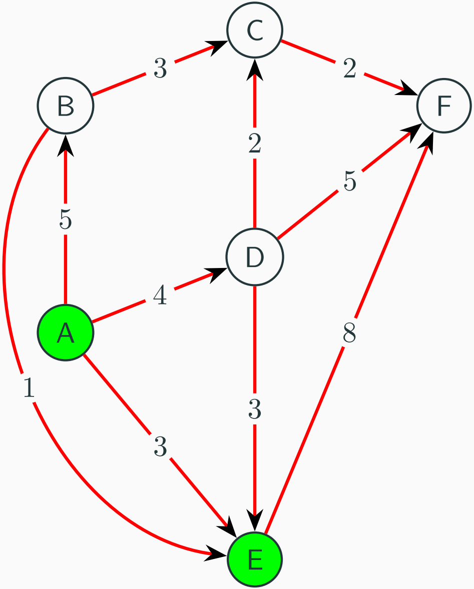
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	∞	$\varnothing$
D	4	A
E	3	A
F	∞	$\varnothing$

Beispiel: Dijkstra's Algorithmus



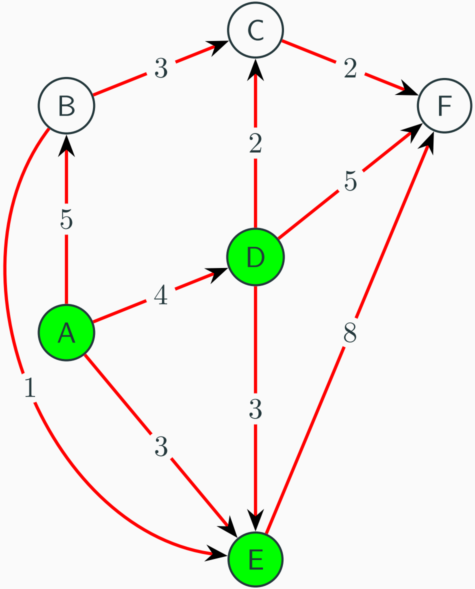
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	∞	$\varnothing$
D	4	A
E	3	A
F	∞	$\varnothing$

Beispiel: Dijkstra's Algorithmus



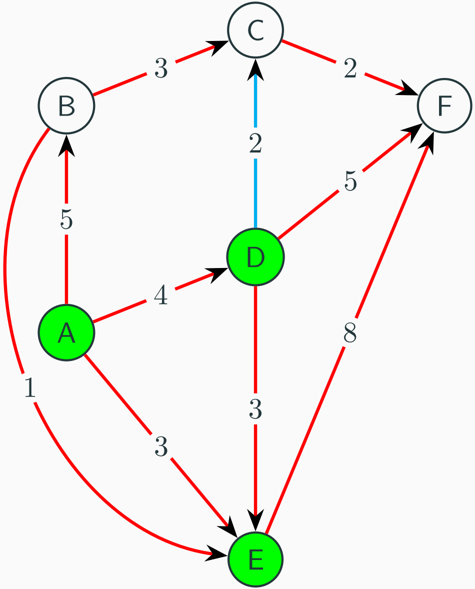
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	∞	$\varnothing$
D	4	A
E	3	A
F	11	E

Beispiel: Dijkstra's Algorithmus



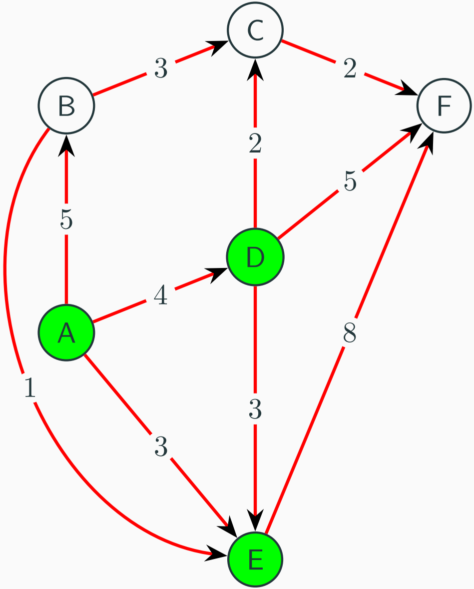
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	∞	$\varnothing$
D	4	A
E	3	A
F	11	E

Beispiel: Dijkstra's Algorithmus



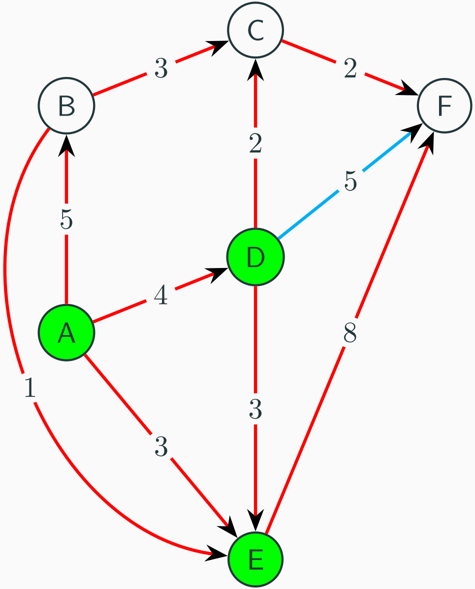
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	∞	$\varnothing$
D	4	A
E	3	A
F	11	E

Beispiel: Dijkstra's Algorithmus



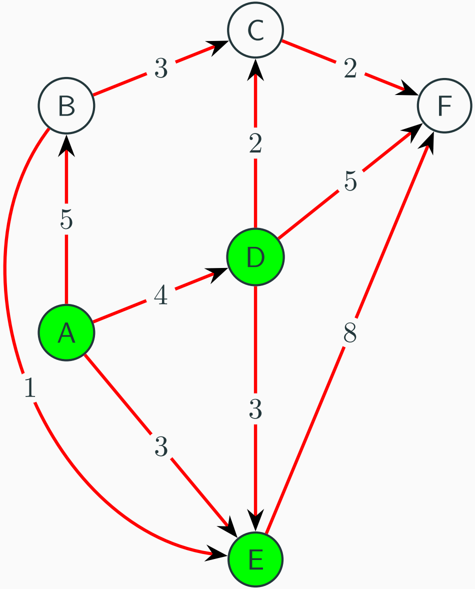
v	$d(v)$	$p(v)$
A	0	$\emptyset$
B	5	A
C	6	D
D	4	A
E	3	A
F	11	E

Beispiel: Dijkstra's Algorithmus



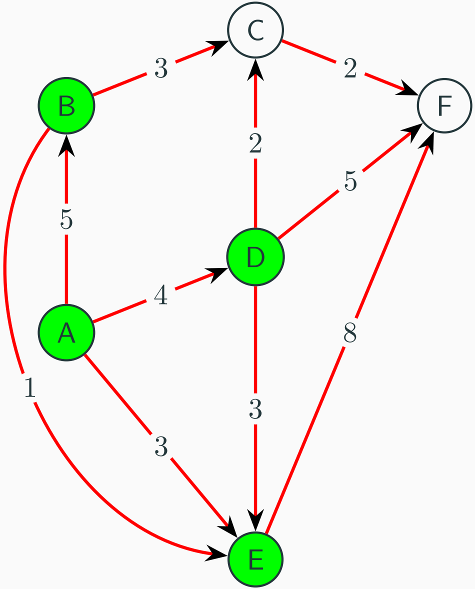
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	11	E

Beispiel: Dijkstra's Algorithmus



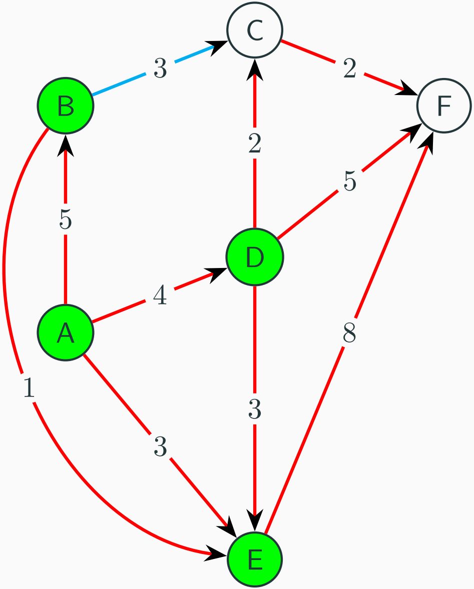
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	9	D

Beispiel: Dijkstra's Algorithmus



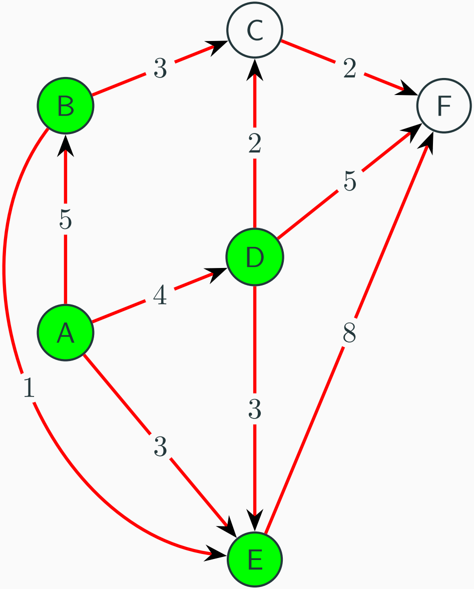
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	9	D

Beispiel: Dijkstra's Algorithmus



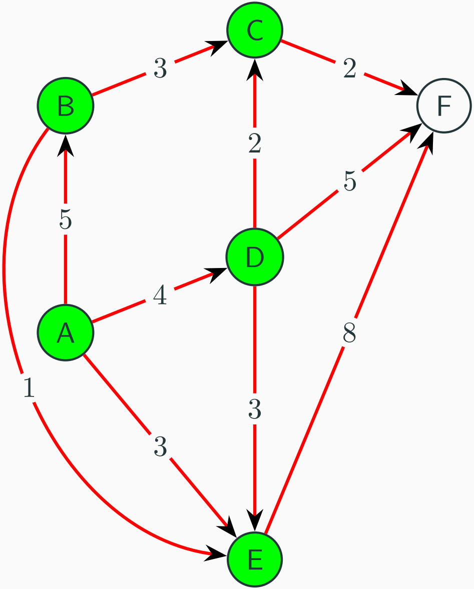
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	9	D

Beispiel: Dijkstra's Algorithmus



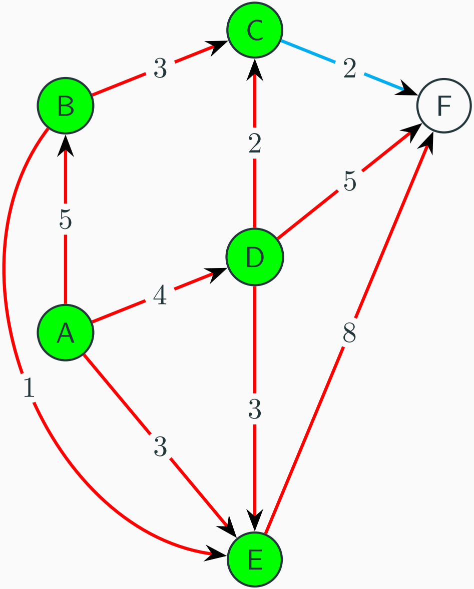
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	9	D

Beispiel: Dijkstra's Algorithmus



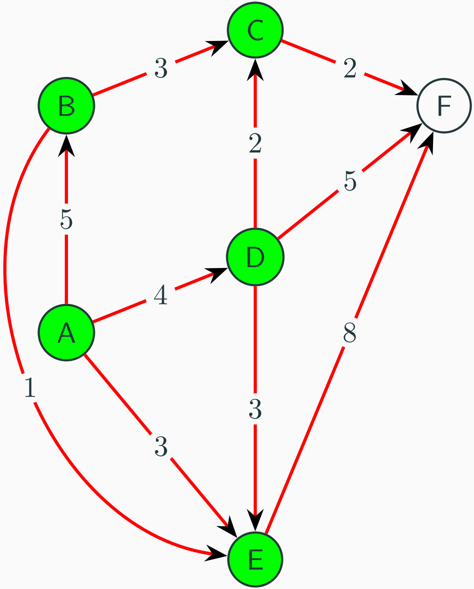
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	9	D

Beispiel: Dijkstra's Algorithmus



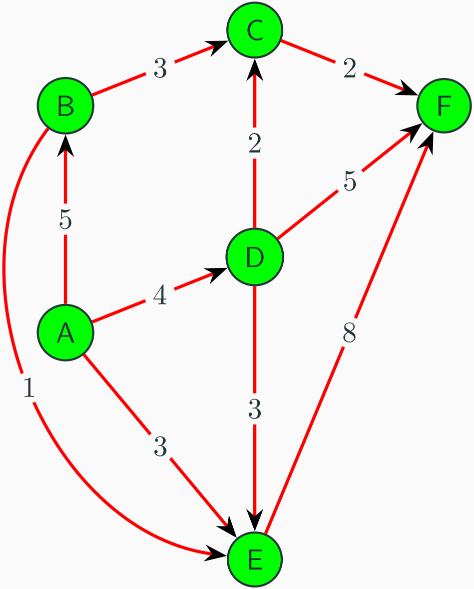
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	9	D

Beispiel: Dijkstra's Algorithmus



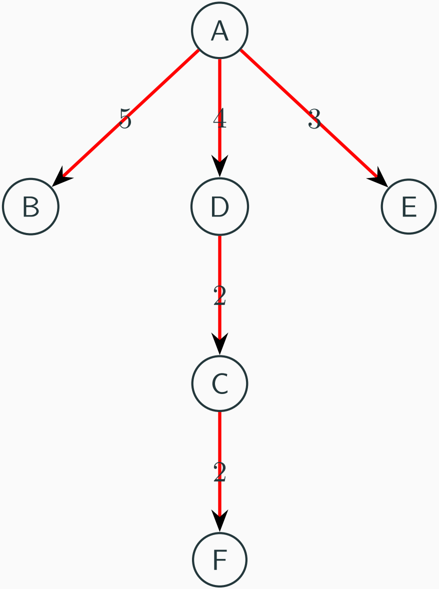
v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	8	C

Beispiel: Dijkstra's Algorithmus



v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	8	C

Shortest-Path Baum



v	$d(v)$	$p(v)$
A	0	$\varnothing$
B	5	A
C	6	D
D	4	A
E	3	A
F	8	C

Block Title

You can also highlight sections of your presentation in a block, with it's own title

Theorem

There are separate environments for theorems, examples, definitions, lemmas and proofs.

Lemma

$$\sum_{v \in V} \deg^{-}(v) = \sum_{v \in V} \deg^{+}(v) = |E|$$

Proof.

Left as an exercise to the reader. $\square$

Example

Here is an example of an example block.

Summary

Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.

Appendix

For Further Reading i

A. Author.
Handbook of Everything.
Some Press, 1990.

S. Someone.
On this and that.
Journal of This and That, 2(1):50–100, 2000.