ECON 3030: Section 6

February 28, 2025

1. Two main players in the economy : Consumers & Firms

Similar concerns : Max utility / profit

Min costs

$f(x, y)$

$f(2x, 2y) = 2^0 f(x, y)$

2. Characteristics of a function

- Quasi-concave
- Quasi-linear
- Homogeneous of degree k : if $f(ax, ay) = a^k f(x, y)$
- Return to scale : compare $f(ax, ay)$ versus $af(x, y)$ with $a > 1$

If f is homogeneous of degree k

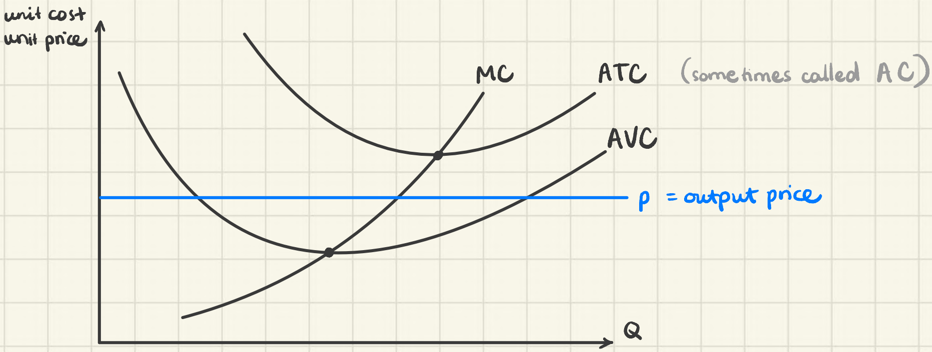
$k < 1$ → decreasing return to scale

$k = 1$ → constant return to scale

$k > 1$ → increasing return to scale

3. Marginal productivity (read supplement at the end)
4. Types of costs (will talk about next section)

MC, ATC, AVC : all are per-unit costs



This graph alone determines:

- Produce now or not ("short-run" choice)
- Produce at all or not ("long-run" choice)
- If produces, what quantity ?
- If produce, what revenue & profit ?

Firm's problem : Profit maximization

Two approaches to solve for <u>how much to produce</u> (q^*)

Input - oriented

Revenue & production

$R(f(x,y))$

$f(\cdot)$ is production function

$R(\cdot)$ is revenue function

example : $f(x,y) = xy$

$R(f(x,y)) = pf(x,y) = pxy$

Cost

$p_x x + p_y y$

Profit maximization

$$\max_{x,y} \; R(f(x,y)) - p_x x - p_y y$$

Solve using FOC, SOC

Solution : input $x^* = x(p_x, p_y, \underbrace{p}_{\text{price of outputs}}, \dots)$

$y^* = y(p_x, p_y, p, \dots)$

$\Rightarrow$ Quantity $q^* = f(x^*, y^*)$ by plug-in

Pros : Straight-forward

Cons : Many variables to solve for $(x, y, z, \dots)$

Not intuitive for firm owners

Output - oriented

Revenue

$R(q)$

q is quantity

$R(\cdot)$ is revenue function

example : $R(q) = pq$

Cost

$C(q)$

$C(\cdot)$ is cost function

example : $C(q) = 5q$

Profit maximization

$$\max_{q} \; R(q) - C(q)$$

Solve using FOC, SOC

Solution : Quantity q^*

Pros : Only one variable q to solve for.

Intuitive for firm owners

Cons : $C(q)$ is not always know

→ Sometimes, need to solve for $C(q)$ first

How ? Cost-minimization problem

$C(q)$ = Lowest cost to produce q

→ $\min_{x,y} \; p_x x + p_y y$ s.t. $f(x,y) \geq \bar{q}$

$C(p_x, p_y, \bar{q})$

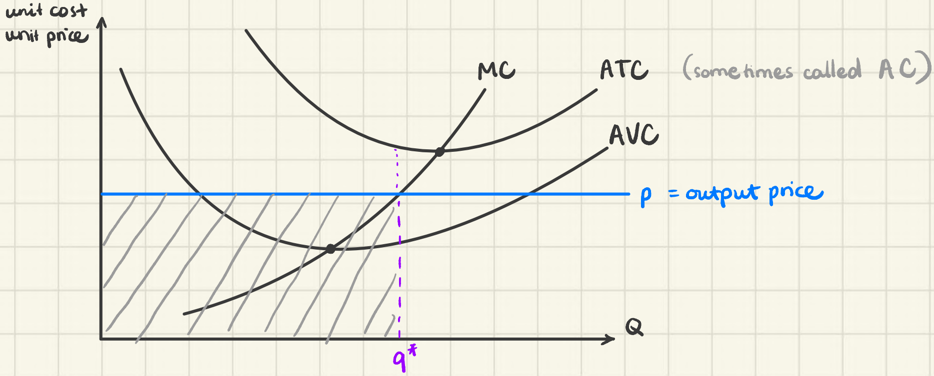
→ Identical to consumer's cost-min problem

→ Help calculate MC, ATC, AVC, etc.

ECON 3030 - Section 7

March 7, 2025

1. Review cost : Definition in lec 7, page 6.



a. Why ATC & AVC cross MC at their lowest point ?

<u>Proof</u> : Assume you want to know, at which q is AVC minimized

→ You solve for :

$$\min_q AVC = \frac{C(p_1, p_2, q)}{q}$$

FOC : $\frac{\partial C}{\partial q} \cdot \frac{1}{q} - C \cdot \frac{1}{q^2} = 0 \Rightarrow \frac{\partial C}{\partial q} = \frac{C}{q}$

By definition : MC = AVC

⇒ At their lowest point, AVC cuts MC

<u>Question</u> : Can you write the same proof for ATC ?

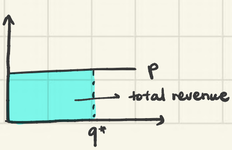
b. What info can you get from the above graph ?

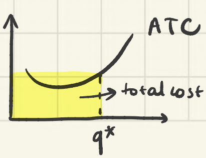
(i) If produce, best to choose q^* where MC cuts p

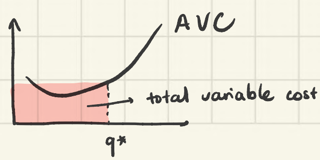
(ii) If chooses q^*, then total revenue is the rectangle where q^* cuts p

(iii) __________, then total cost for firm is the rectangle where q^* cuts ATC

(iv) __________, then total variable cost is the rectangle where q^* cuts AVC







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(v) Profit = Total revenue − Total cost

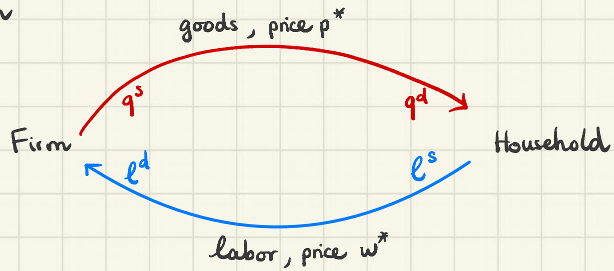
If Profit > 0 ⇒ produce in long + short run

If Profit < 0 ⇒ exit in the long run, but might still produce in short run (why ?)

(vi) If (Total revenue − Total variable cost) > 0

⇒ produce in the short run (why ?)

2. Equilibrium



3. GE vs PE

PE → one firm, one market, or one side of economy, etc.

GE → full effect when taking the entire economy into consideration.

<u>Example</u>

 ← me, a producer of rice

$$\underset{\{\text{input } x\}}{\text{Max}} \quad 5x^2 - 2x$$

price of input $x = 2$

price of output $= 5$

, people buy whatever I produce

Assume I'm twice productive

GE

$$\text{Max } ?(2x^2) - ?x$$

PE

$$\underset{\{x\}}{\text{Max}} \quad 5\,\underbrace{(2x^2)}_{} - 2x$$

ppl clear out my inventory

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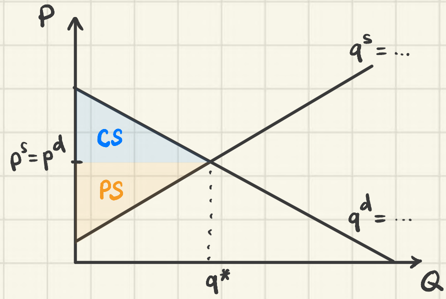
4. Partial equilibrium : one market case

Firm supply $q^s = q^s(p^s)$

Consumer demand $q^d = q^d(p^d)$

→ Note : same p



Consumer surplus CS = above p^d, below q^d

Producer surplus PS = below p^s, above q^s

a. Tax on producer : now $p^s \neq p^d$ such that :

$$p^s = \underbrace{p^d - t}$$

↳ producers get less than what consumers pay

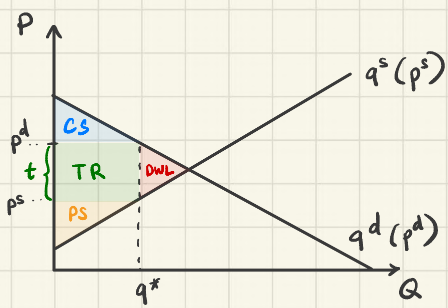
⇒ Need to modify supply & demand function :

$q^s = q^s(p^s = p^d - t)$ (replace p with $p^d - t$)

$q^d = q^d(p^d = p^d)$ (replace p with p^d)

⇒ Can solve for the new q^*, p^d

(Why solve for p^d and not p^s ? We care more about actual price, p^d)



There's tax revenue TR , but also Dead weigh loss DWL

ECON 3030 - Section 9

March 21, 2025

Review

1. Firms
2. Partial equilibrium + Welfare
3. General equilibrium + Trade + Welfare

1. Firms:

- 2 methods to solve for optimal production quantity q
- Costs, revenue, profit → Whether to operate after paying fixed cost (short-run) or to open business at all (long-run)
- Isoquant, homogeneous of degree k, increasing / constant / decreasing return to scale, etc.

2. Partial equilibrium:

- Given demand + supply → solve for q^*, p^*
  → calculate surplus (CS, PS)
- With tax / subsidies → what changes? New q^*, p^*
  → calculate tax revenue, DWL

3. General equilibrium - endowment economy

Setup:

countries / people / ... A, B, ...

- Many players with their own utility function and endowment
  ↳ $u^A, u^B, \dots$ ↳ $w_1^A, w_1^B, \dots$

good 1, 2, ...

- Many goods, with final consumptions and final prices
  ↳ $x_1^A, x_1^B, \dots$ ↳ $p_1^A, p_1^B, \dots$ or $p_1, p_2, \dots$ depending on setup

3.1 Closed economy solution

+ Solve for each player separately. Example for player A:

S1: Derive demand for player A by $\max u^A$ s.t. $x_1^A p_1^A + x_2^A p_2^A \leq I^A \,(= w_1^A p_1^A + w_2^A p_2^A)$

↳ $x_1^{A,demand} = x_1^A(p_1^A, p_2^A, I^A)$

$x_2^{A,demand} = x_2^A(p_1^A, p_2^A, I^A)$

Note: generally, solution satisfies $\frac{MU_1^A}{MU_2^A} = \frac{p_1^A}{p_2^A}$

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S_2: Derive supply for A

(Easy in endowment economy: $x_1^{A,supply} = w_1^A$, $x_2^{A,supply} = w_2^A$)

S_3: Market clearing for A

$$\left(\begin{array}{l} \text{Set } x_1^{A,demand} = x_1^{A,supply}; \quad x_2^{A,demand} = x_2^{A,supply} \Rightarrow \text{Solution } \frac{p_1^A}{p_2^A} = \dots \\ \text{Welfare}^A = u^A(x_1^A, x_2^A) \end{array} \right)$$

3.2. Competitive open economy solution

+ Solve for each player's demand separately but pull them together for market-clearing condition

S_1: Derive demand for player A by Max u^A s.t. $x_1^A p_1 + x_2^A p_2 \le I^A (= w_1^A p_1 + w_2^A p_2)$

↳ solution $x_1^{Ademand}$, $x_2^{A\,demand}$

Derive demand for player B by Max u^B s.t. $x_1^B p_1 + x_2^B p_2 \le I^A (= w_1^B p_1 + w_2^B p_2)$

↳ solution $x_1^{Bdemand}$, $x_2^{Bdemand}$

⇒ Derive total demand for each good

$$\left(\begin{array}{l} x_1^{demand} = x_1^{Ademand} + x_1^{Bdemand} = \text{world demand for good 1} \\ x_2^{demand} = x_2^{Ademand} + x_2^{Bdemand} = \text{world demand for good 2} \end{array} \right)$$

S_2: Derive total supply for each good

$$\left(\begin{array}{l} x_1^{supply} = w_1^A + w_1^B = \text{world endowment for good 1} \\ x_2^{supply} = w_2^A + w_2^B = \text{world endowment for good 2} \end{array} \right)$$

S_3: Market clearing for each good

$$\left(\begin{array}{l} x_1^{supply} = x_1^{demand} \\ x_2^{supply} = x_2^{demand} \quad \Rightarrow \text{Solution } \frac{p_1}{p_2} = \dots \\ \text{Welfare: } U^A, u^B \end{array} \right)$$

3.3 Social planner solution: prices no longer important

S_1: Determine welfare function (should be given to you)

$$\left(\begin{array}{l} \text{eg: } W = U^A + U^B \\ \text{eg: } W = 0.9\,U^A + 0.1\,U^B \\ \text{eg: } W = U^A U^B \end{array} \right)$$

N.Linh

$\underline{S2}$. Determine resource constraint

$$\begin{pmatrix} x_1^A + x_1^B = w_1^A + w_1^B = \text{world endowment for good 1} \\ x_2^A + x_2^B = w_2^A + w_2^B = \text{world endowment for good 2} \end{pmatrix}$$

$\underline{S3}$. Solve for optimal allocation

$$\max_{\{x_1^A, x_1^B, x_2^A, x_2^B\}} W \quad \text{s.t.} \quad \begin{matrix} x_1^A + x_1^B = \dots \\ x_2^A + x_2^B = \dots \end{matrix}$$

Set up Lagrange (with 2 constraints)

$$\mathcal{L}(x_1^A, x_1^B, x_2^A, x_2^B, \lambda_1, \lambda_2) = W + \lambda_1(x_1^A + x_1^B - \dots) + \lambda_2(x_2^A + x_2^B - \dots)$$

FOCs (6 of them)

$\rightarrow$ Solution: $x_1^A, x_1^B, x_2^A, x_2^B$

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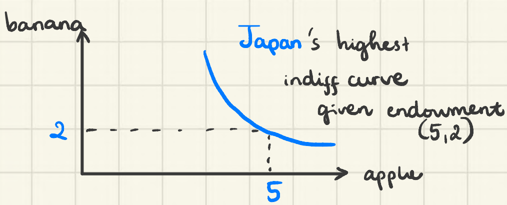
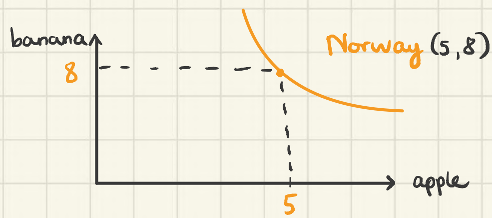
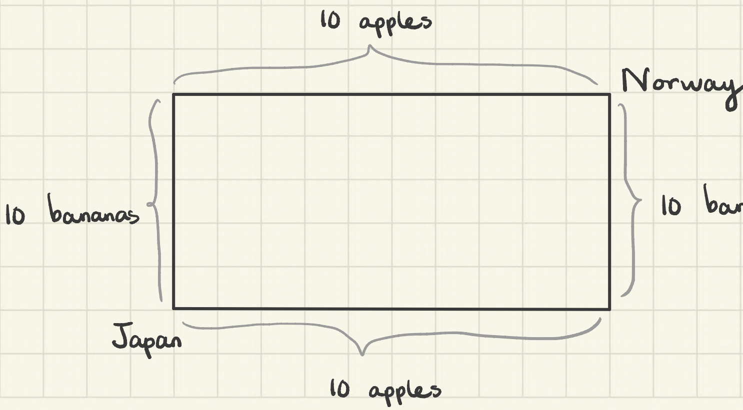
5. Gain from trade: visualization using Edgeworth box (Section 8 - cont.)



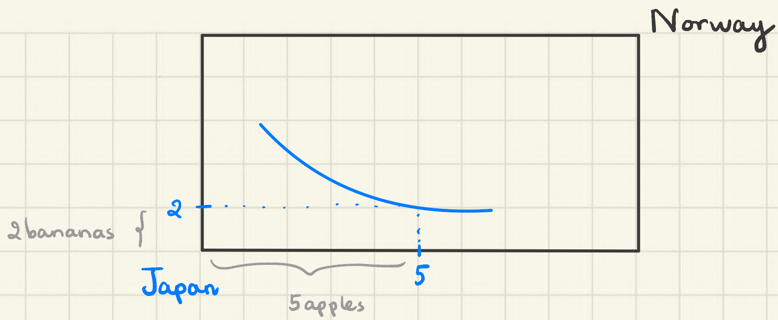


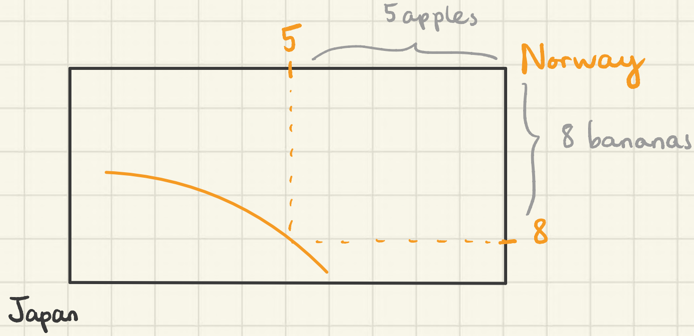
Edgeworth box: How to visualize world demand & world supply ?

$\rightarrow$ A box of all possible allocations + flip Norway

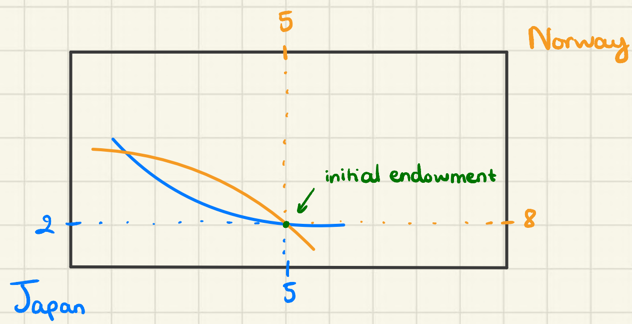


Before trade





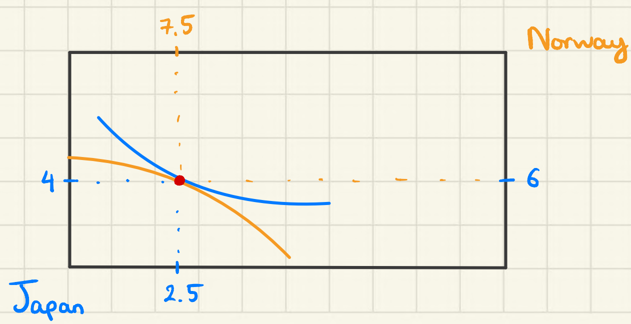
Combined →



After trade:

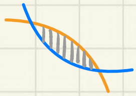
Japan = (2.5 , 4)

Norway = (7.5 , 6)

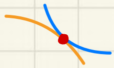


Definitions :

+ Lens :



+ Contract curve :
    + Whenever



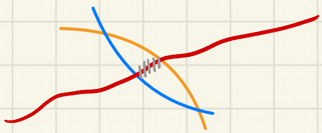
(lens = 0) ⇒

⇒

    + Mathematicaly :

$$\frac{MU_1^A}{MU_2^A} = \frac{MU_1^B}{MU_2^B}$$

+ Core : the part of contract curve inside lens



+ At the contract curve, the allocation is Pareto efficient

ECON 3030 - Section 11

April 2025

The problem of firm(s)

We have seen lots of firms' problems before
& formulas of how to solve them → General framework ???

But: firms' problems are very flexible
& vary based on the way one sets it up

In general, a firm care about
+ Output quantity (q)
+ Input use ($x_1, x_2, \dots$) ($l, k, \dots$)
+ Price to sell (p)

solve one leads to another (input / output oriented)

What they always do: Maximize profit
Note: there are many ways to write profit, for example
+ $pq - C(q)$ (output - oriented ; cost $C(q)$ is a function of q)
+ $p f(x_1, x_2) - (p_1 x_1 + p_2 x_2)$ (input - oriented)
+ $pq - MC \cdot q$ (for fixed marginal cost MC)
+ $p(q) \cdot q - C(q)$ (when price $p(q)$ is a function of q)
+ $p \cdot q(p) - C(q(p))$ (when output $q(p)$ is a function of p)

... & many other ways

1. Perfect competition : Price is given ; firm chooses quantity

eg: $\pi \text{ (profit)} = \max_q \; pq - C(q)$

→ Choose quantity q only

2. Monopoly : Firm choose price & quantity

eg: $\pi = \max_{p,q} \; pq - C(q)$

But p and q are connected by market demand

eg: $p = 5 - q$

N.Linh

$\Rightarrow$ Can substitute in p as a function of q, or vice versa

eg: $p = 5 - q$

$$\Rightarrow \max_{p,q} pq - C(q) = \max_{q} p(q)q - C(q)$$

$$= \max_{q} (5-q)q - C(q)$$

3. Oligopoly & competition : Firm choose price & quantity with limited power

eg: $$\pi_1 = \max_{p_1, q_1} p_1 q_1 - C(q_1)$$

But p and q are connected by market demand

eg: $p = 5 - q = 5 - (q_1 + q_2)$ (only 2 firms in oligopoly)

$\Rightarrow$ Can substitute in p as a function of q

eg: $$\pi_1 = \max_{p_1, q_1} p_1 q_1 - C(q_1)$$

$$= \max_{q_1} (5 - q_1 - q_2) q_1 - C(q_1)$$

Solution: $q_1 = q_1(q_2)$ depends on q_2

→ we call this "firm 1's best response function".

4. Oligopoly & collusion : Game theory style

		Firm 2: Collude	Firm 2: Not collude
Firm 1	Collude	Mono + share profit	
	No collude		Oligopoly + competition