ECON 3030 - Section 13

April, 2025

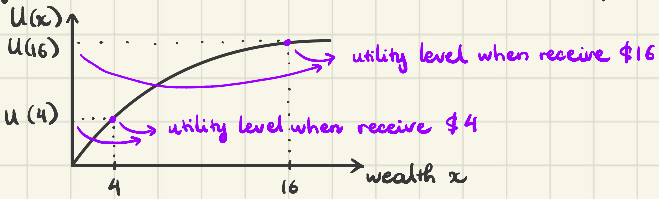
Today:

1. Risk graph
2. Insurance setup
3. Why insurance work - graphical presentation

① Understanding the risk graph

Assume your attitude towards outcome x is $U(x)$. Let's plot **wealth x vs util U**

eg:

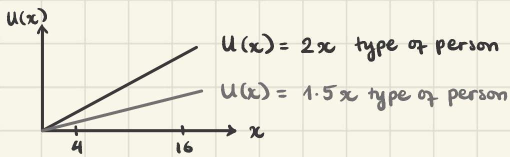


Now let's demonstrate different concepts on this (wealth, utility) space

a. **Risk neutral**: a preference type, demonstrated by a linear utility function

In general: Mathematically, U is risk neutral if $\frac{\partial^2 u}{\partial x^2} = u''(x) = 0$

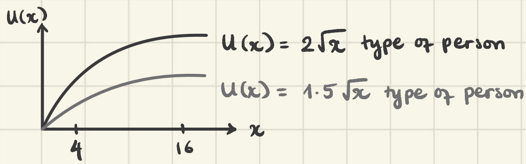
eg:



b. **Risk averse**: a preference type, demonstrated by a concave utility function

In general: Mathematically, U is risk averse if $\frac{\partial^2 u}{\partial x^2} = u''(x) < 0$

eg:



c. **Risk seeking**: a preference type, demonstrated by a convex utility function

In general: Mathematically, U is risk seeking if $\frac{\partial u^2}{\partial x^2} = u''(x) > 0$

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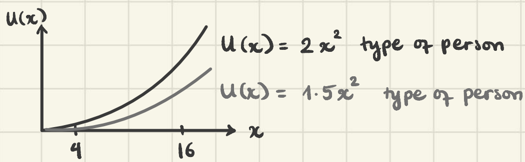
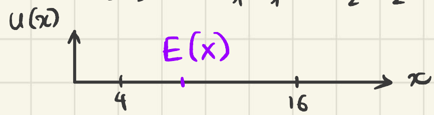


d. **Expected wealth**: $E(X)$ is expected final raw outcome such as money (not utility)

eg: If a person expect to get x_1 with π_1 probability
x_2 with π_2 probability

then $E(X) = \pi_1 x_1 + \pi_2 x_2$

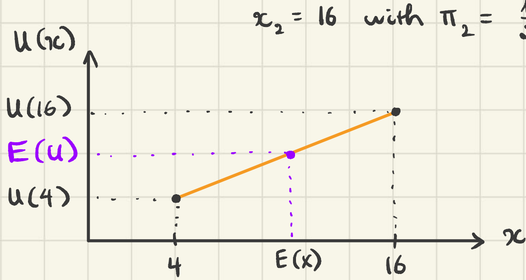


d'. **Expected utility**: $E(U)$, the utility a person expect to gain when they get x_1 at probability π_1 and x_2 at probability π_2.

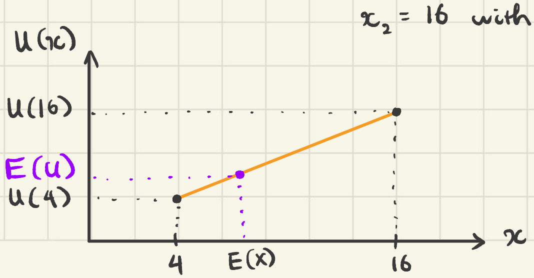
In general: Mathematically, $E(U) = \pi_1 U(x_1) + \pi_2 U(x_2)$

$E(U)$ will be a point on the <u>straight</u> line connecting 2 scenarios and corresponds to $E(X)$ on that line!

eg: A person faces $x_1 = 4$ with $\pi_1 = \frac{1}{2} = 50\%$
$x_2 = 16$ with $\pi_2 = \frac{1}{2} = 50\%$



eg: A same person faces $x_1 = 4$ with $\pi_1 = 3/4 = 75\%$
$x_2 = 16$ with $\pi_2 = 1/4 = 25\%$



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e. **Certainty equivalence** (x_{CE}): the level of x such that having x for sure is equivalent to having x_1 at probability π_1 and x_2 at probability π_2.

new notation

In general, mathematically, $U(x_{CE}) = \pi_1 U(x_1) + \pi_2 U(x_2)$ (*)

You'll need the shape of U to solve for x_{CE}. 2 ways to solve:

- **Method 1:** apply (*) to get exact numerical solution
  Calculate $\pi_1 U(x_1) + \pi_2 U(x_2)$ numerically
  Then plug in $U(x_{CE})$ to the utility function and solve

eg: A person faces $x_1 = 4$ with $\pi_1 = 3/4 = 75\%$
$x_2 = 16$ with $\pi_2 = 1/4 = 25\%$

They have $U(x) = \sqrt{x}$ (risk averse type of person)

What's x_{CE}?

Solution: Applying (*):

$$U(x_{CE}) = \pi_1 U(x_1) + \pi_2 U(x_2)$$

equivalently:

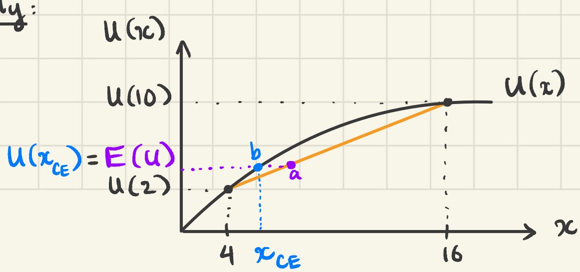
$$\sqrt{x_{CE}} = \frac{3}{4}\sqrt{4} + \frac{1}{4}\sqrt{16}$$

$$\Rightarrow x_{CE} = 2.5^2$$

- **Method 2:** (visual solution only)
  On the graph, find where $E(U)$ is.
  Then find x_{CE} such that $U(x_{CE}) = E(U)$.

eg: Same setup as above, find x_{CE}.

Visually:



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f. **Risk premium** : horizontal gap between point a and b above

Point b corresponds to x_{CE}, which is found using the method in part e

Point a corresponds to $E(X)$, ________________________ d

$\Rightarrow$ Risk premium is the gap between $E(X)$ and x_{CE} !

In general : Risk premium $= E(X) - x_{CE}$

<u>Bonus question</u>:

<u>Q</u>: If I'm risk neutral, what's my risk premium ? Does it depend on π_1, π_2?

<u>A</u>: ________________, risk premium $= 0$ (try to draw the graph urself and see that point a & b are same)

<u>Q</u>: Why is $x_{CE} < E(X)$ for the above person ?

<u>A</u>: Based on the shape of the utility function ( ), this person is risk-averse (afraid of risk). To this person, a small money won <u>for sure</u> (which is x_{CE}) is better than larger <u>expected</u> money ($E(X)$) that involves risk.

<u>Q</u>: When is $x_{CE} > E(X)$?

<u>A</u>: Reverse of above case: the person is risk - seeking.

② Insurance setup

Many ways to setup an insurance problem, but in general

<u>W/o insurance</u> : $\begin{cases} x_1 & \text{with } \pi_1 \quad (\text{"good"}) \\ x_2 & \text{with } \pi_2 \quad (\text{"bad"}) \end{cases}$

<u>With insurance</u> : Pay m to receive n in "bad" scenario

$$\text{new} \begin{cases} x_1' = x_1 - m & \text{with } \pi_1 \\ x_2' = x_2 - m + n & \text{with } \pi_2 \end{cases}$$

<u>eg</u> : Buy z insurance, each cost P but each payout 2 when "bad"

$$\Rightarrow \text{new} \begin{cases} x_1' = x_1 - zP & \text{with } \pi_1 \\ x_2' = x_2 - zP + z \cdot 2 & \text{with } \pi_2 \end{cases}$$

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- **Actuarially fair**: a case when the insurance polity satisfies: $E(X \text{ w/o insurance}) = E(X \text{ with insurance})$

③ Extra note for the section question

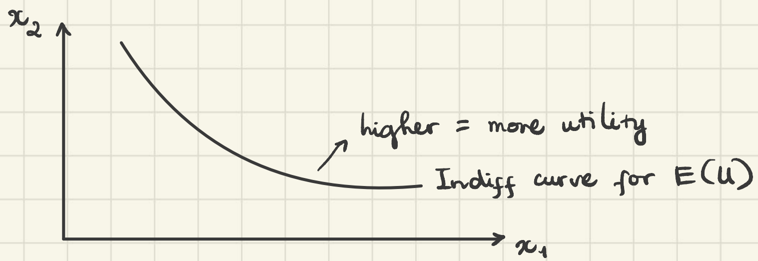
Why does insurance work? A graphical presentation

Let x_1 be payout when no flood, which is $x_1 = 64 - zP$ (*)

x_2 be payout when flood, which is $x_2 = 16 - zP + z$

a. Expected utility for Kevin is (graph 2 scenarios x_1, x_2 against each other)

$$E(U) = \pi_1 U(x_1) + \pi_2 U(x_2)$$

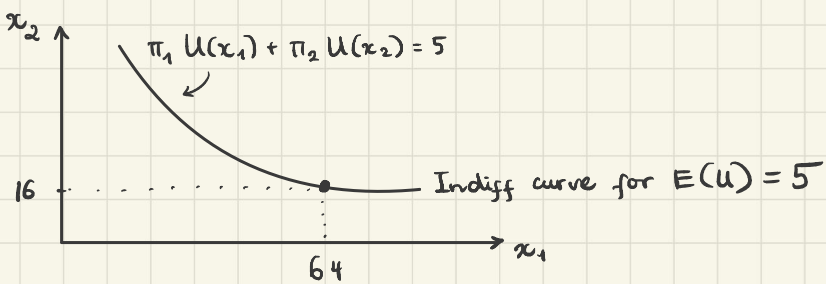


What indiff curve show the equivalent bundles to "no insurance" scenario?

$$E(U) = \pi_1 U(x_1) + \pi_2 U(x_2) = 5$$

(because

$$E(U, \text{no insurance}) = \pi_1 U(64) + \pi_2 U(16) = \frac{1}{4}\sqrt{64} + \frac{3}{4}\sqrt{16} = 5$$
)



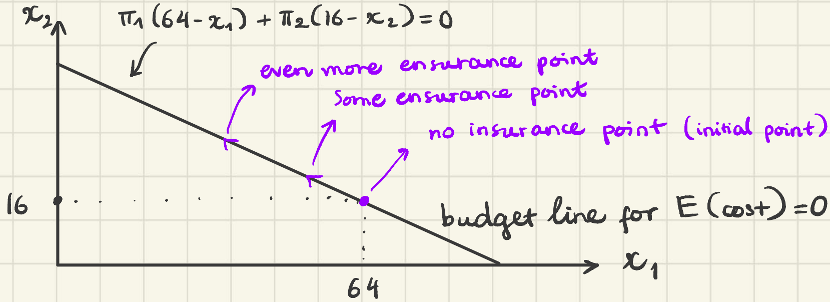
b. With insurance, the expected cost of buying insurance is

$$E(\text{cost}) = \pi_1 (zP) + \pi_2 (zP - z)$$
$$= \pi_1 (64 - x_1) + \pi_2 (16 - x_2) \quad \text{because of } (*)$$

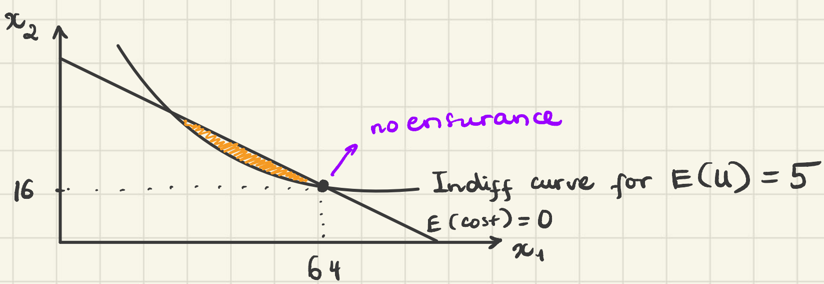
What budget line show the equivalent bundles to "no insurance" scenario ?

$$E(\text{cost}) = \pi_1 (64 - x_1) + \pi_2 (16 - x_2) = 0$$

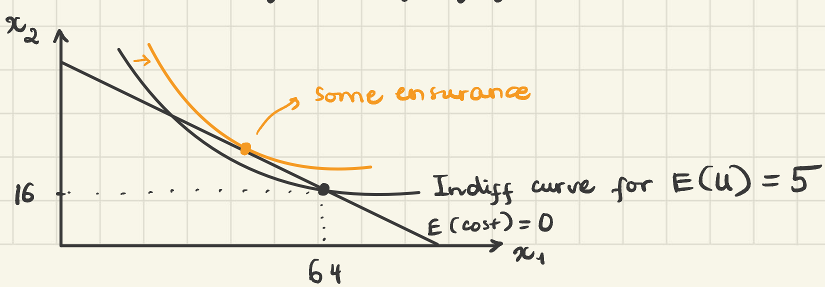
(because
$$E(\text{cost, no insurance}) = \frac{1}{4}(0) + \frac{3}{4}(0 - 0) = 0$$)



- Combining the two :



⇒ There's potential gain by buying some ensurance :



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