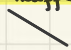
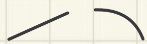
ECON 3030 - Section 2

January 31, 2025

→ Utility function $U(x,y)$

→ Budget constraint $B = p_x x + p_y y$

① Utility

Types	Indifference curve	Examples
Perfect substitutes		$U(x,y) = x+y$
Perfect complements		$U(x,y) = \min(x,y)$
Inbetweens		$U(x,y) = xy^2$
Others (specials)		$U(x,y) = x-y$

② Budget constraint

$B = p_x x + p_y y$

③ Monotone, decreasing, convex preference

(annotated notes + Eds)

④ Calculus

$$\frac{df(x)}{dx} = f'(x) = \text{derivative of } f(x)$$

$$\frac{df(x,y)}{dx} = f_x(x,y) = \text{partial derivative of } f(x,y) \text{ w.r.t. } x$$

$$\frac{d^2f(x,y)}{dx^2} = \frac{d}{dx}\left(\frac{df(x,y)}{dx}\right) = f_{xx}(x,y) = \text{partial derivative of } f_x(x,y)$$

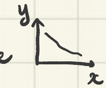
$$\frac{d^2f(x,y)}{dxdy} = \frac{d^2f(x,y)}{dydx} = \frac{d}{dx}\left(\frac{df(x,y)}{dy}\right) = \frac{d}{dy}\left(\frac{df(x,y)}{dx}\right) = f_{xy}(x,y) = f_{yx}(x,y)$$

1. MRS
2. MRT
3. Optimizations

1. MRS → using utility function to construct ($U(x,y) = \dots$)
   - → rate at which I'm willing to substitute one thing with another
   - → how many y can I substitute with 1 x to be equally happy ?

Mathematically :

$$MRS = \frac{dy}{dx} = -\frac{U_x}{U_y} = -\frac{dU}{dx} \Big/ \frac{dU}{dy} = -\frac{MU_x}{MU_y} = \text{Slope indiff curve}$$

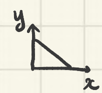


Example : $MRS = -2 \Rightarrow$ willing to give up 2 y for 1 x

if $= \frac{dy}{dx}$

2. MRT → using budget constraint to construct ($B = p_x x + p_y y$)
   - → rate at which I can buy one thing by giving up on another
   - → how many y can I buy if I give up 1 x ?

$$MRT = \frac{dy}{dx} = -\frac{B_x}{B_y} = -\frac{dB}{dx} \Big/ \frac{dB}{dy} = -\frac{p_x}{p_y} = \text{Slope budget}$$



③ Optimization :

- $\max_{\{x,y\}} U(x,y)$ s.t. $B = p_x x + p_y y$
- $\max_{\{x,y\}} f(x,y)$
- ...

→ We mostly see the first one. What to do ?
Two methods : Lagrangean vs Substitution.
Goal : end up with one function only
Steps : FOC (first order condition)
SOC (second order condition)

Method 1 : Lagrange

$\max \mathcal{L}(x, y, \lambda) = U(x,y) + \lambda(\text{budget constraint})$

< example 1 : $\mathcal{L}(x,y,\lambda) = xy + \lambda(5x + 6y - 100)$ >

Method 2 : substitution

From $B = p_x x + p_y y$, we have $x = \dfrac{B - p_y y}{p_x}$

Subs to U to get

$\max U(x,y) = U\left(\dfrac{B - p_y y}{p_x}, y\right)$

< example 2 : $U(x,y) = xy = \left(\dfrac{B - p_y y}{p_x}\right) y$ >

Now, for both methods :

FOC : $\dfrac{\partial \text{ function}}{\partial \text{ each variable}} = 0$

$$\left\langle \begin{array}{l} \text{example 1: } \dfrac{d\mathcal{L}}{dx} = 0, \quad \dfrac{d\mathcal{L}}{dy} = 0, \quad \dfrac{d\mathcal{L}}{d\lambda} = 0 \\[2ex] \text{example 2: } \dfrac{dU}{dy} = 0 \end{array} \right\rangle$$

SOC : check additional conditions

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ECON 3030 - Section 3

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A person's **Utility** $\langle u(x,y) \rangle$ → **Budget constraint** $\langle p_x x + p_y y = I \rangle$

↓

Utility maximization problem subjected to limited \$

↓ ⟨Solve by substitution or Lagrangean⟩

⟨Solution *normally* satisfy $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$⟩

↳ not always

Solution x & y

⟨"Marshallian demand"⟩

Why is it called "demand?"

Normally, when we plug in p_x, p_y, and I in the maximization problem, we get numbers such as $x = 5$ & $y = 10$ (which means this person "demand" 5 of x and 10 of y)

But, what if we don't plug in p_x, p_y, and I? We can still solve for the maximization problem, but get for example

$$x = I + 5p_y - 6p_x \quad \text{and} \quad y = \dots$$

Does this look familiar?

→ In intro to econ, we see the demand function

$$Q = 5 - 6P$$

$$\sim \quad x = \dots - 6p_x \qquad \text{(similar thing!)}$$

Overall, we write $x = x(p_x, p_y, I)$

→ Demand for x depends on p_x, p_y, I !

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So, what can we tell about x, knowing the person's demand for x ?

(1) → How sensitive that person is to price of x ...

(2) → If I own the x company, should I raise my price to ↑ revenue ?

(3) → Whether it's a normal or inferior good

(4) → If other goods get cheaper / more expensive, would I buy more x ?

(1) is answered using "**price elasticity of demand**"

$$e = \frac{p_x}{x} \cdot \frac{\partial x}{\partial p_x} = \frac{\%\text{ change in } x}{\%\text{ change in } p_x}$$

For example, if $e = \frac{-1}{2}$ ⇒ "if price of x increases by 2%, I'll buy 1% less of x"

$e = 6 = \frac{6}{1}$ ⇒ "if price of x increases by 1%, I'll buy 6% more of x"

(2) is answered using the calculated "**price elasticity of demand**"

$|e| > 1$ — 1 — $|e| < 1$ →

$|e| > 1$: elastic (people spend less $p_x x$ when p_x increase)

$|e| = 1$: unit elastic (people spend the same $p_x x$ when p_x changes)

$|e| < 1$: inelastic (people spend more $p_x x$ when p_x increase)

Since revenue $= p_x x$, we can infer information about revenue of e

But: be careful, since revenue ≠ profit !!

(3) is answered using "**income elasticity of demand**"

$$e_{I,x} = \frac{I}{x} \cdot \frac{\partial x}{\partial I} = \frac{\%\text{ change in } x}{\%\text{ change in } I}$$

For example, if $e_I = \frac{-1}{2}$ ⇒ "if my income increases by 2%, I'll by 1% less of x"

If $e_I > 0$ ⇒ normal good

$e_I < 0$ ⇒ inferior good

(4) is answered by using "cross - price elasticity of demand"

$$e_{x,p_y} = \frac{p_y}{x} \cdot \frac{\partial x}{\partial p_y} = \frac{\text{\% change in } x}{\text{\% change in } y}$$

If $e_{x,p_y} > 0$ (meaning $p_y\uparrow \rightarrow x\uparrow$, for example $x^{\uparrow} = I + 5p_y^{\uparrow} - 6p_x$)

$\rightarrow$ x and y are substitutes

If $e_{x,p_y} < 0$ (meaning $p_y\uparrow \rightarrow x\downarrow$, for example $x^{\downarrow} = I - \sqrt{p_y^{\uparrow}} - 6p_x$)

$\rightarrow$ x and y are complements

ECON 3030 - Section 4

Feb. 14, 2025

1. The dual problem

Utility
$U(x,y)$

Budget
$p_x x + p_y y = I$

I is fixed

Utility maximization

$$\max_{x,y} u(x,y) \quad s.t. \quad p_x x + p_y y = I$$

Lagrangean: $\mathcal{L}(x,y,\lambda) = u(x,y) + \lambda(p_x x + p_y y - I)$

solve

Solution: $x^* = x(p_x, p_y, I)$
$y^* = y(p_x, p_y, I)$

→ "Mashallian demand"

⟨Given p_x, p_y, I, these are the quantities of x & y that we demand⟩

If plug this x, y back to utility, we get:
$u(x^*, y^*) \equiv v(p_x, p_y, I)$

→ "Indirect utility"

⟨Given p_x & p_y, then v is the maximum utility we can get with budget I⟩

Utility
$U(x,y) = \bar{u}$

Expenditure
$p_x x + p_y y$

$\bar{u}$ is fixed

Expenditure minimization

$$\min_{x,y} p_x x + p_y y \quad s.t. \quad u(x,y) \geq \bar{u}$$

Lagrangean: $\mathcal{L}(x,y,\lambda) = p_x x + p_y y + \lambda(u(x,y) - \bar{u})$

solve

Solution: $x^* = x^c(p_x, p_y, \bar{u})$
$y^* = y^c(p_x, p_y, \bar{u})$

→ "Hicksian demand"

⟨Given $p_x, p_y, \bar{u}$, these are the quantities of x & y that I demand⟩

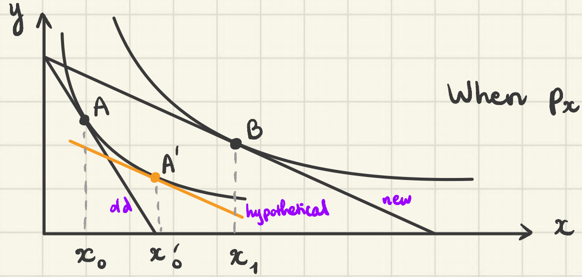
If I plug this x, y back to expenditure:
$p_x x^* + p_y y^* \equiv E(p_x, p_y, \bar{u})$

→ "Expenditure function"

⟨Given p_x & p_y, then E is the minimum cost we must pay to obtain $\bar{u}$ utility⟩

2. Slutsky equation

- The key concern: If p_x change, how would demand for x change?
- Back to intro to micro:



When p_x ↓

Total effect: $x_0 \longrightarrow x_1$, from bundle A to B

But we can decompose:

$$\text{total effect} \leftarrow \left\{ \begin{array}{l} x_0 \\ \downarrow \\ x_0' \\ \downarrow \\ x_1 \end{array} \right.$$

→ **Substitution effect**: from point A to A', price changes but utility doesn't

→ **Income effect**: from point A' to B, income changes leading to utility changes

- How do we measure *exactly* each of these effect? Slutsky equation!

$$\frac{\partial x}{\partial p_x} \leftarrow \left\{ \begin{array}{l} x_0 \\ \downarrow \\ x_0' \\ \downarrow \\ x_1 \end{array} \right.$$

→ $\dfrac{\partial x^c}{\partial p_x}$: response to price change but utility does not change

→ $-\dfrac{\partial x}{\partial I}x$: response to income changes leading to utility changes

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3. Compensating variation

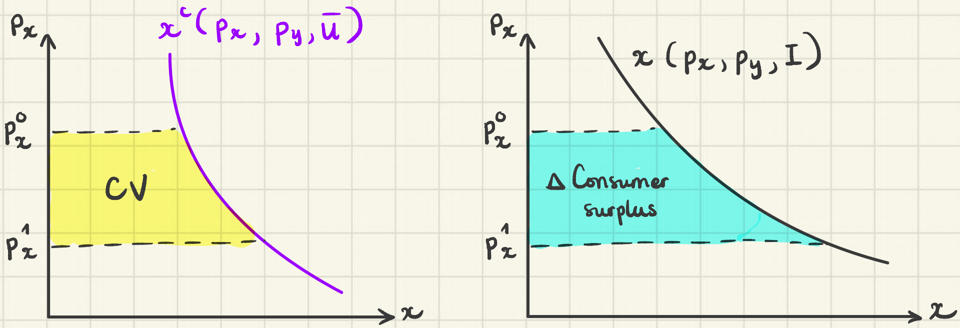
Welfare analysis for when p_x changes

- Before price change : it costs $E(p_x^0, p_y, \bar{u})$ to achieve utility $\bar{u}$
- After price change : it costs $E(p_x^1, p_y, \bar{u})$ to achieve utility $\bar{u}$

$$\Rightarrow \quad CV = E(p_x^0, p_y, \bar{u}) - E(p_x^1, p_y, \bar{u})$$

→ How much money we need to be compensated with to be as happy as before the price change .

- Graphically :



- Proof (self read)

Shephard's Lemma : $dE(p_x, p_y, \bar{u}) / dp_x = x^c(p_x, p_y, \bar{u})$

$$\Rightarrow \quad dE(p_x, p_y, \bar{u}) = x^c(p_x, p_y, \bar{u})\, dp_x$$

$$\Rightarrow \quad E(p_x^1, p_y, \bar{u}) - E(p_x^0, p_y, \bar{u}) = \int_{p_x^0}^{p_x^1} x^c(p_x, p_y, \bar{u})\, dp_x = \quad . \square$$

Δ = "change in"

- Why welfare = CV instead of Δ Consumer surplus ?

When p_x increases, you would expect they are exactly worse off by the decrease in purchase times the change in price ($\Delta x * \Delta p_x =$)

But : people aren't that much worse off , since the substitute x with other products .

ECON 3030: Section 5

February 21, 2025

Review

1. Utility & indifference curve & budget (Section 1 + 2 notes)

Utility can be : monotone (eg : more ~ better)

not always

quasi-concave (~ convex preference ~ convex upper-contour set for indiff curve)

Simple utility functions & their indifference curve

Indifference curve :  etc. (are these quasi-concave utilities ?)

Budget constraint : $xp_x + yp_y \leq I$

MRT , MRS, MU (section 2)

For interior solution & when indifference curve is smooth : $\dfrac{MU_x}{MU_y} = \dfrac{p_x}{p_y}$

2. Dual problem (Section 4)

Utility maximization , expenditure minimization

Marshallian vs Hicksian demand

Indirect utility vs Expenditure function

3. Elasticities (Section 3)

(Own) Price elasticity of demand : $p_x \uparrow \rightarrow x$?

Income elasticity of demand : $I \uparrow \rightarrow x$?

Cross - Price elasticity of demand : $p_y \uparrow \rightarrow x$?

4. Solving for max / min problem

- Special utility ($x+y$, or $\min\{x, y\}$) → special solutions : what are they graphically & can you solve ?
- Other utility : use either Lagrangian or substitution

  How to set up

  Steps to solve (FOC , SOC)

5. Welfare & price change

Slutsky eq : decompose change in x into income & substitution effects

CV & EV : meaning & calculation

6. Taxation

- Two types: lump. sum vs per unit
- Per unit tax: what does it do? Assume tax on x only

    Change price: p_x to $p_x + t$ for example

    Raise goverment revenue: $R =$ tax on x * quantity x demanded

    But, how to calculate?

    → When p_x changes, people demand less / more x → hard to calculate R

    Solution: Use the Mashallian demand:

$$R = \underbrace{t}_{\text{per unit tax}} * x^{new}(p_x + t, p_y, I)$$

consumption in response to price change