Wohlgemuth

Helpful Math Hints
Rules for Exponents

1. X^a = X times itself a times $\quad 5^3 = 5 * 5 * 5$
   X is called the base, a is called the exponent

2. $X^0 = 1$ by definition anything raised to the zero power equals 1.
   and $X^1 = X \quad 3^1 = 3$

3. $X^{-a} = 1/(X^a) \quad 3^{-2} = 1/(3^2) = 1/9$

4. $X^a X^b = X^{a+b} \quad 6^3\, 6^4 = (6*6*6) * (6*6*6*6) = 6^7 = 279{,}936$
   (like bases – add exponent)

5. $\dfrac{X^a}{X^b} = X^{a-b} \quad (4^3)/(4^2) = (4*4*4)/(4*4) = 4^1 = 4$

6. $X^{(1/a)}$ = the a^{th} root of X $\quad 4^{(1/2)} = 2$ (square root of 4 = 2, since 2*2=4)

7. $X^aY^a = (XY)^a \quad 2^2 * 3^2 = 2*2 \; * 3*3 = 2*3 * 2*3 = 6 * 6 = 6^2 = 36$
   (like exponents, multiply bases)

8. $(X^a)^b = X^{ab} \quad (4^3)^2 = (4*4*4)^2 = (4*4*4) * (4*4*4) =$
   $(4*4*4*4*4*4) = 4^6$

9. Summation Notation: $\sum_{t=0}^{3} \frac{1}{(1+i)^t} = \frac{1}{(1+i)^0} + \frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3}$
   if i = 10%, then

$$\sum_{t=0}^{3} \frac{1}{(1+.1)^t} = \frac{1}{(1.1)^0} + \frac{1}{(1.1)^1} + \frac{1}{(1.1)^2} + \frac{1}{(1.1)^3} = 1 + 0.909 + 0.826 + 0.751 = 3.487$$

Rules for Findings Slopes (a. k. a. Partial Derivatives)

Derivative of a constant: Y = a $\frac{dY}{dX} = 0$

Example: y = 30 The derivative of Y with respect to (wrt) X is $\frac{dY}{dX} = 0$

Power Rule: $Y = aX^b$ $\frac{dY}{dX} = baX^{b-1}$

Example: $\pi = 30Q^2$. The derivative of profit (π) wrt Q is

$$\frac{d\pi}{dQ} = 2 * 30Q^{2-1} = 60Q^1 = 60Q$$

Derivatives of a Sum: $Y = f(X) + g(X)$ $\frac{dY}{dX} = \frac{df}{dX} + \frac{dg}{dX}$

Example: $\pi = 40 + 2Q + 0.5Q^3$. The derivative of profit wrt Q is

$$\frac{d\pi}{dQ} = 0 + 2Q^{1-1} + 3 * 0.5Q^{3-1} = 2 + 1.5 * Q^2$$

For those who have had a calculus course – the following may be helpful refreshers.

Derivative of Products: Y=f(X)g(X) $\frac{dY}{dX} = f(x)\frac{dg}{dX} + g(x)\frac{df}{dX}$

Example: $C = (40 + Q)Q^2$. The derivative of C wrt Q is:

$$\frac{dC}{dQ} = (40 + Q) * 2 * Q + Q^2 * (1) = 80Q + 2Q^2 + Q^2 = 80Q + 3Q^2$$

Chain Rule: Y=f(Z) and Z=g(X) then Y=f(g(X)). The derivative of Y wrt X is

$$\frac{dY}{dX} = \frac{dY}{dZ}\frac{dZ}{dX}$$

Example: $Q = 20B^2 + 2 \quad and \quad B = -2P + 5P^2 \quad then \quad Q = 20(-2P + 5P^2)^2 + 2$

The derivative of Q wrt B is $\frac{dQ}{dB} = 2 * 20B$

The derivative of B wrt P is $\frac{dB}{dP} = -2 + 2 * 5P$

The derivative of Q wrt P is:

$$\frac{dQ}{dP} = \frac{dQ}{dB}\frac{dB}{dP} = [2 * 20\,(-2P + 5P^2)](-2 + 10P) = 160P - 1200P^2 + 2000P^3$$

(Suggestion: In many cases, you can avoid using the chain rule. First simply the function so that Q = f(P). Then find the slope of that simplified function. Both arrive at the same equation for the slope.)