

CHAPTER :SEMI-STABLE LATTICE IN HIGHER RANK

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In this chapter, we will establish the notion of semi-stable lattice. Heuristically, this is the lattice that achieve all the successive minima at the same time, see [?].

We will provide two different definitions of semi -stable lattice: one is geometric - which follows Grayson's idea of utilizing the canonical plot, and one is purely algebraic, which make use of the maximal standard parabolic subgroups. The toy model will be the moduli space of 2-dimensional lattice, which is essential the upper half plane in the complex field. At the end, we will show that the two definitions coincide.

1 Lattices and semi-stable lattice in higher rank

For each z with $\Im(z) > 0$, we can attach to z a lattice structure $L_z = \mathbb{Z}z \oplus \mathbb{Z}$. Roughly speaking a lattice is a discrete subgroup that is generated by a k -basis of the k -space V . In particular, we will only work with the real vector space V . Grayson works with lattice over a ring of algebraic integers, but we will restrict to just the lattice that has the underlying structure as a \mathbb{Z} -module. The precise definition of a lattice is as follows:

Definition 1.1.