PROBLEM SET 2

(1) Let's denote $\mathbb{D} = \{|z| < 1\}$ and $\mathbb{H} = \{\Im z > 0\}$. Show that
- $\mathbb{H} \cong \mathbb{D}$ as Riemann surfaces via the map $z \mapsto \dfrac{z-i}{z+i}$.
- Show that the complex plane $\mathbb{C}$ is not isomorphic to $\mathbb{H}$.

(2) Show that the projective curves $X^2 + Y^2 = Z^2$ is non - singular and is isomorphic to the complex projective line $\mathbb{P}^1$ as Riemann surfaces.

(3) Let $f\colon X \to \mathbb{C}$ is a non constant analytic map of Riemann surfaces. Suppose that, for fixed $p \in X$, f has two local expressions of the form $F(z) = z^k$ and $\tilde{F}(\tilde{z}) = z = \tilde{z}^{\tilde{k}}$. Show that $k = \tilde{k}$.

(4) Let $f\colon X \to \mathbb{C}$ be a non-constant analytic map of Riemann surfaces and $p \in X$. From the theorem about local form, we could see that the power of the local form is a function depending on p, namely, $F(z) = z^{k(p)}$. We call $k(p)$ the ramification index of f at p. Show that there exists a neighborhood U of p such that the ramification index of any $x \in U \setminus \{p\}$ is 1.