STAT 3119

Week5: 9/26/2019 @GWU

Outline :

- Review ANOVA diagnostic and Remedial measure
- Nonparametric test & Case study
- Basic concepts for muti- factor studies

Review ANOVA diagnostic and Remedial measure (Ch 18)

- ANOVA Model assumptions
  - homogeneity or constant variances for different factor levels
  - data or errors are from normal distribution
- Residual plots from an ANOVA fit
  - Residual vs. fitted value plot
  - Normal QQ plot
- Test for normality
  - Shapiro–Wilk test test if a sample from a normal distribution
- Tests for homogeneity of variances
  - Hartley test (equal ni, normal data)
  - Bartlett Test (normal data)
  - Brown-Forsythe test (robust alternative)
- Remedial measures
  - Regression model with WLS fit (normal data, unequal variance)
  - Transformation (simple guide, or more general form: box-cox transformation to search a best power parameter including log transformation)

The Alternative or Nonpametric approaches (Ch 18.7)

- (As the 3rd remedial measure:) When there are major departures from ANOVA model and transformations are not successful in stabilizing the error variance and bringing the error distribution close to normal, **a nonparametric rank-based test for the equality of the factor level means** may be used instead of the standard F test to fit the observed data.
- The nonparametric procedures here do not depend on the distribution of the error terms; it only assumes that the r populations under study have the same kind of **continuous** distributions that differ only with respect to **location**. We are using these nonparametric procedures to test whethether the r populations (i.e. r factor levels in a one-factor study) have the same population means or medians or not.
- We consider two closely related procedures.

Rank-based F-test

- The test procedure is very simple. All n_T observations are ranked from 1 to n_T in ascending order. Then, the usual F^* test statistic for the ANOVA table is calculated, but now based on the ranks R_{ij}, and the F test is carried out in the ordinary manner. When the treatment means are the same, the rank-based test statistic F_R follows approximately the $F(r1, n_T r)$ distribution.
- We are still taking the same hypotheses for the equality of means, the formulas and the decisions rules are the same excepting we are replacing the observation Y_{ij} with its rank R_{ij}.

Kruskal-Wallis test

- Kruskal-Wallis test is a widely used nonparametric test for testing the equality of treatment means, is based on a test statistic that is closely related to the rank F test statistic.

$$\chi^2_{KW} = \frac{SSTR^*}{SSTO^*/(n_T - 1)}$$

where $SSTR^*$ and $SSTO^*$ are the SS using the same definition as before based on the rank R_{ij}. The Kruskal-Wallis test statistic follows $\chi^2(r-1)$ distribution, and reject the null hypothesis for large value, i.e., if

$$\chi^2_{KW} > \chi^2(1-\alpha, r-1)$$

.

Two tests are closely-related

The F_R^* and X^2_{KW} test statistics are equivalent, being related as follows:

$$F_R^* = \frac{(n_T - r)X^2_{KW}}{(r-1)(n_T - 1 - X^2_{KW})} \tag{18.29}$$

■

- Large K-W stat statistic <==> Large rank F- statistic.

Example in Table 18.5 :

- Table 18.5 contains the **lengths of time between computer failures** (response) for the **three locations** (factor under study).

TABLE 18.5 Time between Computer Failures at Three Locations (in hours)—Servo-Data Example.

Failure Interval	Location (i)					
	1		2		3	
j	Y_{1j}	R_{1j}	Y_{2j}	R_{2j}	Y_{3j}	R_{3j}
1	4.41	2	8.24	4	106.19	14
2	100.65	13	81.16	11	33.83	7
3	14.45	6	7.35	3	78.88	10
4	47.13	9	12.29	5	342.81	15
5	85.21	12	1.61	1	44.33	8

i	$\bar{Y}_{i\cdot}$	s_i^2	i	$\bar{R}_{i\cdot}$	s_i^2
1	50.4	1,789	1	8.4	20.3
2	22.1	1,103	2	4.8	14.2
3	121.2	16,167	3	10.8	12.7
	$\bar{Y}_{\cdot\cdot} = 64.6$			$\bar{R}_{\cdot\cdot} = 8.00$	

Implementation with R

read the data

```
# read data from week5 folder online
Ex18B =read.table(
        url("https://raw.githubusercontent.com/npmldabook/Stat3119/master/Week5/CH18TA05.txt"))
names(Ex18B) =  c("response", "Location", "units")
# make another categorical variable
Ex18B$Location =  as.factor(Ex18B$Location)

str(Ex18B)
```

```
## 'data.frame':    15 obs. of  3 variables:
##  $ response: num  4.41 100.65 14.45 47.13 85.21 ...
##  $ Location: Factor w/ 3 levels "1","2","3": 1 1 1 1 1 2 2 2 2 2 ...
##  $ units   : int  1 2 3 4 5 1 2 3 4 5 ...
```

Last class, we find the log-transformation is a good choice.

```
fit4 =  aov( log(response)~ Location, data= Ex18B )
summary(fit4)
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## Location     2  11.45   5.726   3.789  0.053 .
## Residuals   12  18.14   1.511
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Rank-based F-test

```r
# make the rank variable
Ex18B$rank <- rank(Ex18B$response)

fit5 =  aov( rank~ Location, data= Ex18B )
summary(fit5)
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## Location     2   91.2   45.60   2.898  0.094 .
## Residuals   12  188.8   15.73
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Results: Apply ANOVA model to rank R_{ij}, we obtain $F_R = 2.898$ with a p-value=0.094. This is similar to the results that we applied to the transformed (log) data. If we use $\alpha = 0.05$, the results only suggest a trend for difference between the locations. If the significant level is set as 0.1 (this should be pre-specified before looking at the data), then the results show the responses differed by the location.

Kruskal-Wallis test

We can calculate the test statistic by the defintion (the SS from ANOVA on ranks), then to compare with the critical value from Chi-square table

```r
SSTR = 91.2
SSTO = 91.2+ 188.8
(KW.stat = SSTR/(SSTO/14))
```

```
## [1] 4.56
```

Or an easier way is to use the build-in function **kruskal.test()** directly. The results from Kruskal-Wallis is very similar to the rank-based F-test.

```r
kruskal.test(response~ Location, data= Ex18B)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data:  response by Location
## Kruskal-Wallis chi-squared = 4.56, df = 2, p-value = 0.1023
```

Multiple Pairwise Testing Procedure uisng ranks

- If it is not easy to apply a transformation to stablize the variance and make the data to be approximately normal, then we can make the inference using the nonparametric procedure based on the ranks of the original observations.
- Since Bonferroni procedure does not depends on the distribution assumption, it is directly divide the α based on the number of tests or comparisons. We can still use Bonferroni procedure for the simultaneous inference among the factor levels means based on ranks.

- To make inferences about the pairwise comparisons of the factor level means using their ranks, we can use the simultaneous confidence intervals of the pairwise differences for the mean ranks $\overline{R}_{i\cdot}$. If the confidence inntervals include zero, we conclude that the corresponding treatment means do not differ.

$$(\bar{R}_{i\cdot} - \bar{R}_{i'\cdot}) \pm B\left[\frac{n_T(n_T+1)}{12}\left(\frac{1}{n_i}+\frac{1}{n_{i'}}\right)\right]^{1/2} \quad \textbf{(18.30)}$$

where:

$$B = z(1-\alpha/2g) \quad \textbf{(18.30a)}$$

$$g = \frac{r(r-1)}{2} \quad \textbf{(18.30b)}$$

Example for CIs of the pairwise mean rank differences (page 797):

```
library(emmeans)
Est.mean = emmeans(fit5, ~ Location)

pairmean =  pairs(Est.mean, adjust = "Bonferroni" )
(CI90= confint(pairmean, level = .90))
```

```
##  contrast estimate   SE df lower.CL upper.CL
##  1 - 2         3.6 2.51 12    -2.43   9.6284
##  1 - 3        -2.4 2.51 12    -8.43   3.6284
##  2 - 3        -6.0 2.51 12   -12.03   0.0284
##
## Confidence level used: 0.9
## Conf-level adjustment: bonferroni method for 3 estimates
```

Results: As all three CIs include 0, we can't reject the null hypotheses and find any two means were significantly different based on this nonparametric procedure.

Note: In the last class, that when the Bonferroni pairwise comparison procedure was conducted on the log of the responses, we concluded that a significant difference existed between the means of locations 2 and 3. This is because, when the sample size is small, the nonparametric procedure is often more conservative while the parametric ANOVA model on the transformed variable is more powerful. However, when sample size is relatiely large, the results for the nonparametric model and parametric model on the transformed response should be very similar.

Case Example (Ch 18.8)

Example:

In heart transplant surgery, the similarity of the donor's tissue type and that of the recipient is of importance because large differences may increase the probability that the transplanted heart is rejected. Table 18.7 shows a portion of the **survival times (in days)** obtained from an observational study of 39 patients following heart transplant surgery. The data are grouped into three categories, according to the **degree of mismatch** between the donor tissue and the recipient tissue. Investigators would like to determine if the mean survival time changes with the degree of mismatch.

TABLE 18.7 Survival Times of Patients Following Heart Transplant Surgery—Heart Transplant Example.

	Degree of Tissue Mismatch (i)		
Case j	Low $i=1$	Medium $i=2$	High $i=3$
1	44	15	3
2	551	280	136
3	127	1,024	65
...	...	...	...
12	47	836	48
13	994	51	
14	26		

1. read the data

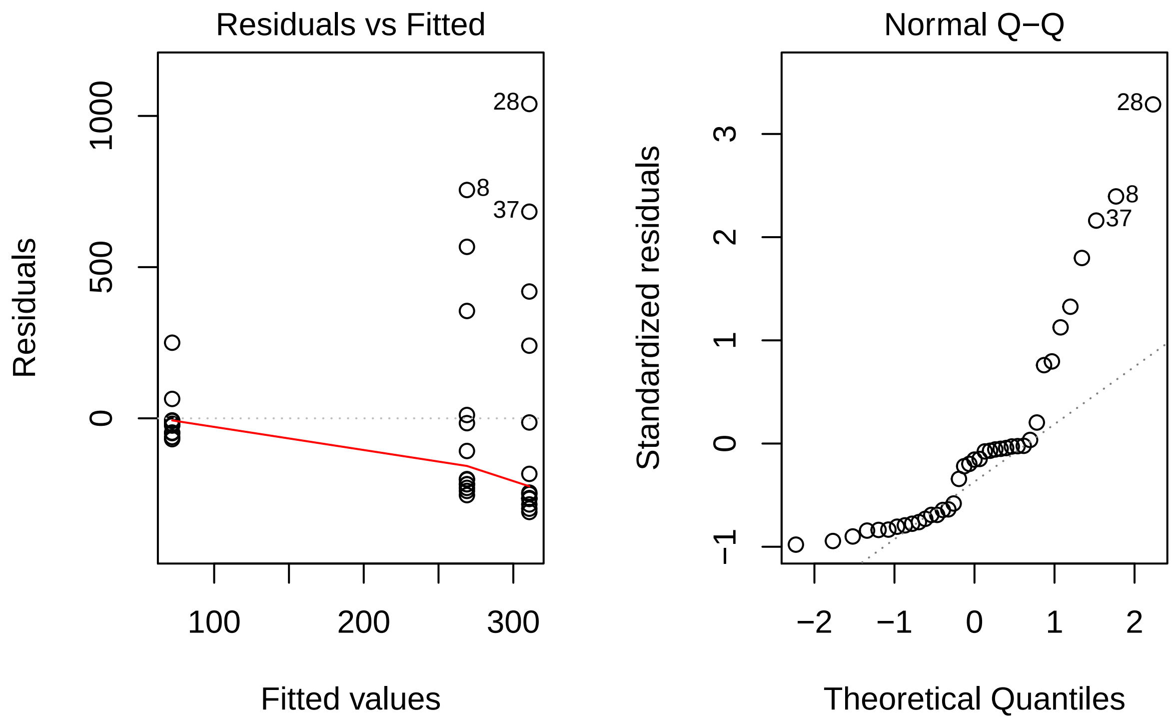
```r
# read data from week5 folder online
Ex18C =read.table(
        url("https://raw.githubusercontent.com/npmldabook/Stat3119/master/Week5/CH18TA07.txt"))
names(Ex18C) =  c("response", "Tissue")
# make another categorical variable
Ex18C$Tissue =  as.factor(Ex18C$Tissue)

str(Ex18C)
```

```
## 'data.frame':    39 obs. of  2 variables:
##  $ response: int  44 15 3 551 280 136 127 1024 65 1 ...
##  $ Tissue  : Factor w/ 3 levels "1","2","3": 1 2 3 1 2 3 1 2 3 1 ...
```

2. The plots of observations and residuals are similar as before.

YOU can follow the previous steps to run the ANOVA diagnostics and compare with those from the textbook.

```r
fit  =  aov(response ~ Tissue  , data = Ex18C)
par(mfrow=c(1,2))
plot(fit,1)
plot(fit,2)
```

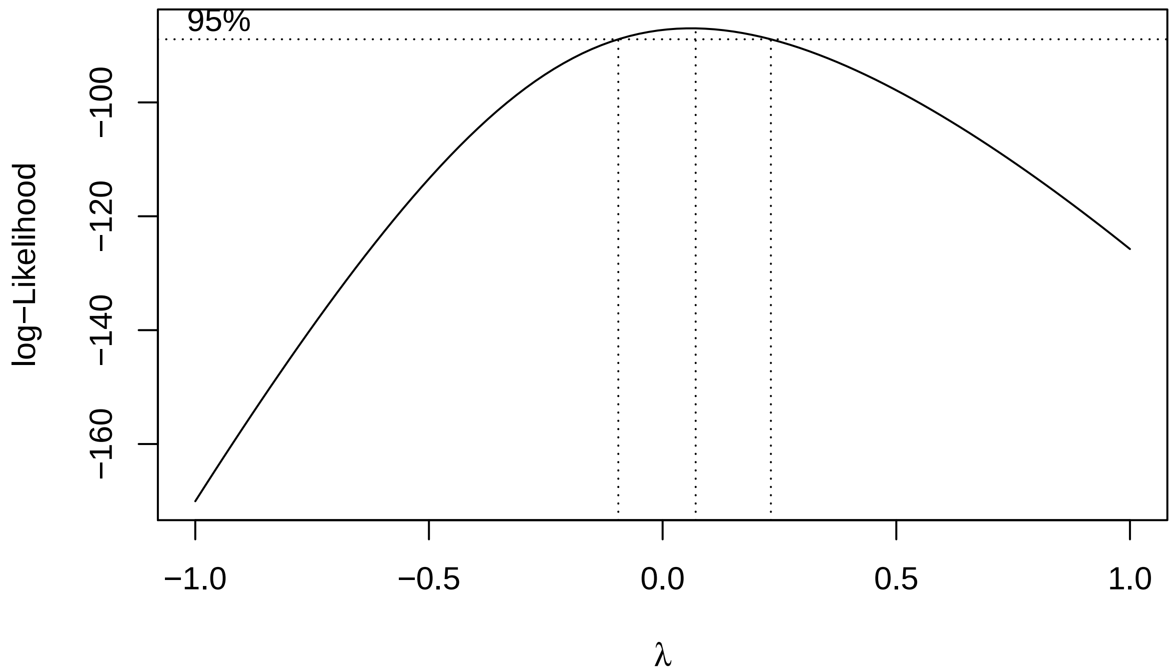



```
shapiro.test(fit$residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  fit$residuals
## W = 0.8021, p-value = 9.29e-06
```

3. Apply the boxcox procedure and ANOVA for the transformed data

```
library(MASS)

#  we can call boxcox function and use anova fitted model object
boxcox(fit,  lambda=seq(-1,1, by=0.1) )
```




```r
# apply transformation for the response
fit2 =  aov( log(response)~ Tissue, data= Ex18C )
summary(fit2)
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## Tissue       2   7.61   3.806    1.56  0.224
## Residuals   36  87.83   2.440
```

4. Apply a nonparametric procedure

```r
kruskal.test(response~ Tissue, data= Ex18C)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data:  response by Tissue
## Kruskal-Wallis chi-squared = 2.8347, df = 2, p-value = 0.2424
```

Results: Therefore, from the above procedure (3) or (4), we can't reject null and we conclude the mean survival time for heart transplant patients with the characteristics of those included in the study did not depend on the degree of tissue mismatch in this study.

Two- Factor Studies with Equal Sample sizes (Chapter 19.1)

- In the last 3 chapters, we considered the design and analysis of experimental and observational studies in which the effects ofone factor are investigated. Next, we are concerned with investigations of the simultaneous effects of two or more factors.
- Two-factor studies, like single-factor studies, can be based on experimental or observational data.

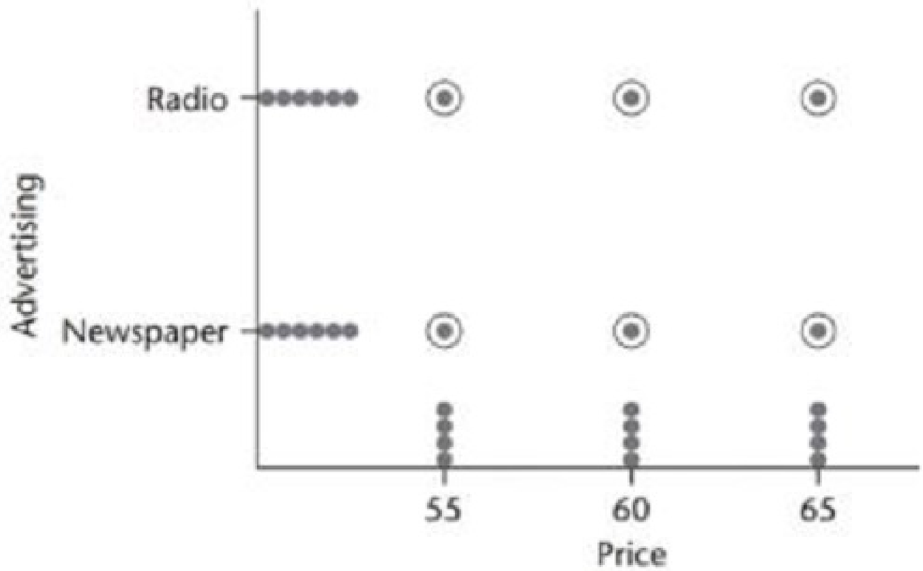
Example 1:

A company investigated the effects of selling price and type of promotional campaign on sales of one of its products.

- Three selling prices (55 cents, 60 cents, 65 cents) were studied, as were two types of promotional campaigns (radio advertising, newspaper advertising).
- Let us consider selling price to be factor A and promotional campaign to be factor B. Factor A here was studied at three price levels;
- In general, we use the symbol a to denote the number of levels of factor A investigated, $a = 3$. Factor B was here studied at two levels, we use the symbol b to denote the number of levels of factor B, $b = 2$.
- Each combination of a factor level of A and a factor level of B is a **treatment**. Thus, there are $3\times2 = 6$ treatments here altogether. In general, the total number of possible treatments in a two-factor study is ab.

Treatment	Description
1	55 price, radio advertising
2	60 price, radio advertising
3	65 price, radio advertising
4	55 price, newspaper advertising
5	60 price, newspaper advertising
6	65 price, newspaper advertising

FIGURE 19.1 Experimental Layout—Example 1.



- This can be an experimental study if experiment units were randomly assigned into the different treatments (combination of the factor A and factor B levels).

More examples

Example 2: Family income or certain type of families

Example 2 An analyst studied the effects of family income (under \$15,000, \$15,000–\$29,999, \$30,000–\$49,999, \$50,000 and more) and stage in the life cycle of the family (stages 1, 2, 3, 4) on appliance purchases. Here, $4 \times 4 = 16$ treatments are defined. These are in part:

Treatment	Description
1	Under \$15,000 income, stage 1
2	Under \$15,000 income, stage 2
⋮	⋮
16	\$50,000 and more income, stage 4

The analyst selected 20 families with the required income and life-cycle characteristics for each of the "treatment" classes for this study, yielding 320 families for the entire study.

This study is an observational one because the data were obtained without assigning income and life-cycle stage to the families. Rather, the families were selected because they had the specified characteristics.

Example 3. Drug (can be randomly assigned) and gender (observatinal, block factor), randomize drug within the blocks

Example 3 A medical investigator studied the relationship between the response to three blood pressure lowering drug types for hypertensive males and females. Here, $3 \times 2 = 6$ treatments are defined. These are:

Treatment	Description
1	Drug type 1, males
2	Drug type 1, females
3	Drug type 2, males
4	Drug type 2, females
5	Drug type 3, males
6	Drug type 3, females

The investigator selected 30 adult males and 30 adult females and randomly assigned 10 males and 10 females to each of the three drug types, yielding 60 total subjects.

This study has one observational factor, gender, and one experimental factor, drug type. This design is referred to as a randomized complete block design where the gender factor is called a block. This design will be discussed in Chapter 21.

Problems with One-Factor-at-a-Time (OFAAT) Approach

- The OFAAT approach does not explore the entire space of treatment combinations, and important treatment combinations may therefore be missed
- Interactions cannot be estimated
- A full randomization is not possible for the OFAAT approach

Advantages of Crossed, Multi-Factor Designs

- More efficient and easy to study the interactions between the factors.
- Strengthen the validity ofthe findings: Multifactor studies can include some factors of secondary importance to permit inferences and better understanding about the primary factors with a greater range of validity.

Meaning of ANOVA Model Elements (Ch 19.2)

Example: We consider a simple two-factor study in which the effects of **gender and age on learning of a task are of interest**. For sim-plicity, the age factor has been defined in terms of only three factor levels (young, middle, old), as shown in Table 19.1a.

TABLE 19.1
Age Effect but No Gender Effect, with No Interactions—Learning Example.

(a) Mean Learning Times (in minutes)

	Factor B—Age			
Factor A—Gender	$j=1$ Young	$j=2$ Middle	$j=3$ Old	Row Average
$i=1$ Male	9 (μ_{11})	11 (μ_{12})	16 (μ_{13})	12 ($\mu_{1\cdot}$)
$i=2$ Female	9 (μ_{21})	11 (μ_{22})	16 (μ_{23})	12 ($\mu_{2\cdot}$)
Column average	9 ($\mu_{\cdot 1}$)	11 ($\mu_{\cdot 2}$)	16 ($\mu_{\cdot 3}$)	12 ($\mu_{\cdot\cdot}$)

(b) Main Gender Effects (in minutes)

$\alpha_1 = \mu_{1\cdot} - \mu_{\cdot\cdot} = 12 - 12 = 0$
$\alpha_2 = \mu_{2\cdot} - \mu_{\cdot\cdot} = 12 - 12 = 0$

(c) Main Age Effects (in minutes)

$\beta_1 = \mu_{\cdot 1} - \mu_{\cdot\cdot} = 9 - 12 = -3$
$\beta_2 = \mu_{\cdot 2} - \mu_{\cdot\cdot} = 11 - 12 = -1$
$\beta_3 = \mu_{\cdot 3} - \mu_{\cdot\cdot} = 16 - 12 = 4$

Treatment Means

The mean response for a given treatment in a two-factor study is denoted by $_{ij}$, where i refers to the level of factor A ($i = 1, ..., a$) and j refers to the level of factor B ($j = 1, ..., b$). e.g. $_{11} = 9$, which indicates that the mean learning time for young males is 9 minutes. Similarly, we see that $_{22} = 11$, so that the mean learning time for middle-aged females is 11 minutes.

Factor level means:

In general, the column average for the jth column is denoted by $\mu_{\cdot j}$:

$$\mu_{\cdot j} = \frac{\sum_{i=1}^{a} \mu_{ij}}{a} \tag{19.1}$$

and the row average for the ith row is denoted by $\mu_{i\cdot}$:

$$\mu_{i\cdot} = \frac{\sum_{j=1}^{b} \mu_{ij}}{b} \tag{19.2}$$

The overall mean learning time for all ages and both genders is denoted by $\mu_{\cdot\cdot}$, and is defined in the following equivalent fashions:

$$\mu_{\cdot\cdot} = \frac{\sum_i \sum_j \mu_{ij}}{ab} \tag{19.3a}$$

$$\mu_{\cdot\cdot} = \frac{\sum_i \mu_{i\cdot}}{a} \tag{19.3b}$$

$$\mu_{\cdot\cdot} = \frac{\sum_j \mu_{\cdot j}}{b} \tag{19.3c}$$

Main effects for factor A and B

General Definitions. In general, we define the main effect of factor A at the ith level as follows:

$$\alpha_i = \mu_{i\cdot} - \mu_{\cdot\cdot} \tag{19.4}$$

Similarly, the main effect of the jth level of factor B is defined:

$$\beta_j = \mu_{\cdot j} - \mu_{\cdot\cdot} \tag{19.5}$$

It follows from (19.3b) and (19.3c) that:

$$\sum_i \alpha_i = 0 \qquad \sum_j \beta_j = 0 \tag{19.6}$$

Thus, the sum of the main effects for each factor is zero.

Note again that a main effect indicates how much the factor level mean deviates from the overall mean. The greater the main effect, the more the factor level mean differs from the overall mean response averaged over the factor levels for both factors.

A. Additive model (no interaction between A and B)

The factor effects in Table 19.1 have an interesting property. Each mean response $_{ij}$ can be obtained by adding the respective gender and age main effects to the overall mean.

This suggest an addtive model for the two main effects, then the treatment means satistfy these equations:

In general, we have for Table 19.1a:

$$\mu_{ij} = \mu_{..} + \alpha_i + \beta_j \qquad \text{Additive factor effects} \tag{19.7}$$

which can be expressed equivalently, using the definitions of α_i in (19.4) and of β_j in (19.5), as:

$$\mu_{ij} = \mu_{i.} + \mu_{.j} - \mu_{..} \qquad \text{Additive factor effects} \tag{19.7a}$$

It can also be shown that each treatment mean μ_{ij} in Table 19.1a can be expressed in terms of three other treatment means:

$$\mu_{ij} = \mu_{ij'} + \mu_{i'j} - \mu_{i'j'} \qquad \text{Additive factor effects} \qquad i \neq i', j \neq j' \tag{19.7b}$$

- The significance of no factor interactions is that the effect of either factor does not depend on the level of the other factor.
- The difference between the mean responses for any two levels of factor B (or A) is the same for all levels of factor A (or B).
- The curves of the mean responses for the different levels of a factor are all parallel.

B. Interacting Factor Effect

Table 19.3a contains an illustration for the learning example where the factor effects do interact. The mean learning times for the different gender-age combinations in Table 19.3a indicate that gender has no effect on learning time for young persons but has a substantial effect for old persons. This differential influence of gender, which depends on the age of the person, implies that the age and gender factors interact in their effect on learning time.

TABLE 19.3 Age and Gender Effects, with Interactions—Learning Example.

(a) Mean Learning Times (in minutes)

	Factor B—Age				
Factor A—Gender	$j=1$ Young	$j=2$ Middle	$j=3$ Old	Row Average	Main Gender Effect
$i=1$ Male	9 (μ_{11})	12 (μ_{12})	18 (μ_{13})	13 ($\mu_{1.}$)	1 (α_1)
$i=2$ Female	9 (μ_{21})	10 (μ_{22})	14 (μ_{23})	11 ($\mu_{2.}$)	−1 (α_2)
Column average	9 ($\mu_{.1}$)	11 ($\mu_{.2}$)	16 ($\mu_{.3}$)	12 ($\mu_{..}$)	
Main age effect	−3 (β_1)	−1 (β_2)	4 (β_3)		

(b) Interactions (in minutes)

	$j=1$	$j=2$	$j=3$	Row Average
$i=1$	−1	0	1	0
$i=2$	1	0	−1	0
Column average	0	0	0	0

The difference between the treatment mean μ_{ij} and the value $\mu_{..} + \alpha_i + \beta_j$ that would be expected if the two factors were additive is called the *interaction effect,* or more simply the *interaction,* of the ith level of factor A with the jth level of factor B, and is denoted by $(\alpha\beta)_{ij}$. Thus, we define $(\alpha\beta)_{ij}$ as follows:

$$(\alpha\beta)_{ij} = \mu_{ij} - (\mu_{..} + \alpha_i + \beta_j) \qquad \textbf{(19.8)}$$

Replacing α_i and β_j by their definitions in (19.4) and (19.5), respectively, we obtain an alternative definition:

$$(\alpha\beta)_{ij} = \mu_{ij} - \mu_{i.} - \mu_{.j} + \mu_{..} \qquad \textbf{(19.8a)}$$

To repeat, the interaction of the ith level of A with the jth level of B, denoted by $(\alpha\beta)_{ij}$, is simply the difference between the treatment mean μ_{ij} and the value that would be expected if the factors were additive. If in fact the two factors are additive, all interactions equal zero; i.e., $(\alpha\beta)_{ij} \equiv 0$.

The interactions for the learning example in Table 19.3a are shown in Table 19.3b. We have, for instance:

$$\begin{aligned}(\alpha\beta)_{13} &= \mu_{13} - (\mu_{..} + \alpha_1 + \beta_3) \\ &= 18 - (12 + 1 + 4) \\ &= 1\end{aligned}$$

By definition, we also have

$$\sum_i (\alpha\beta)_{ij} = 0 \qquad j = 1, \ldots, b \qquad \textbf{(19.9a)}$$

$$\sum_j (\alpha\beta)_{ij} = 0 \qquad i = 1, \ldots, a \qquad \textbf{(19.9b)}$$

Consequently, the sum of all interactions is also zero:

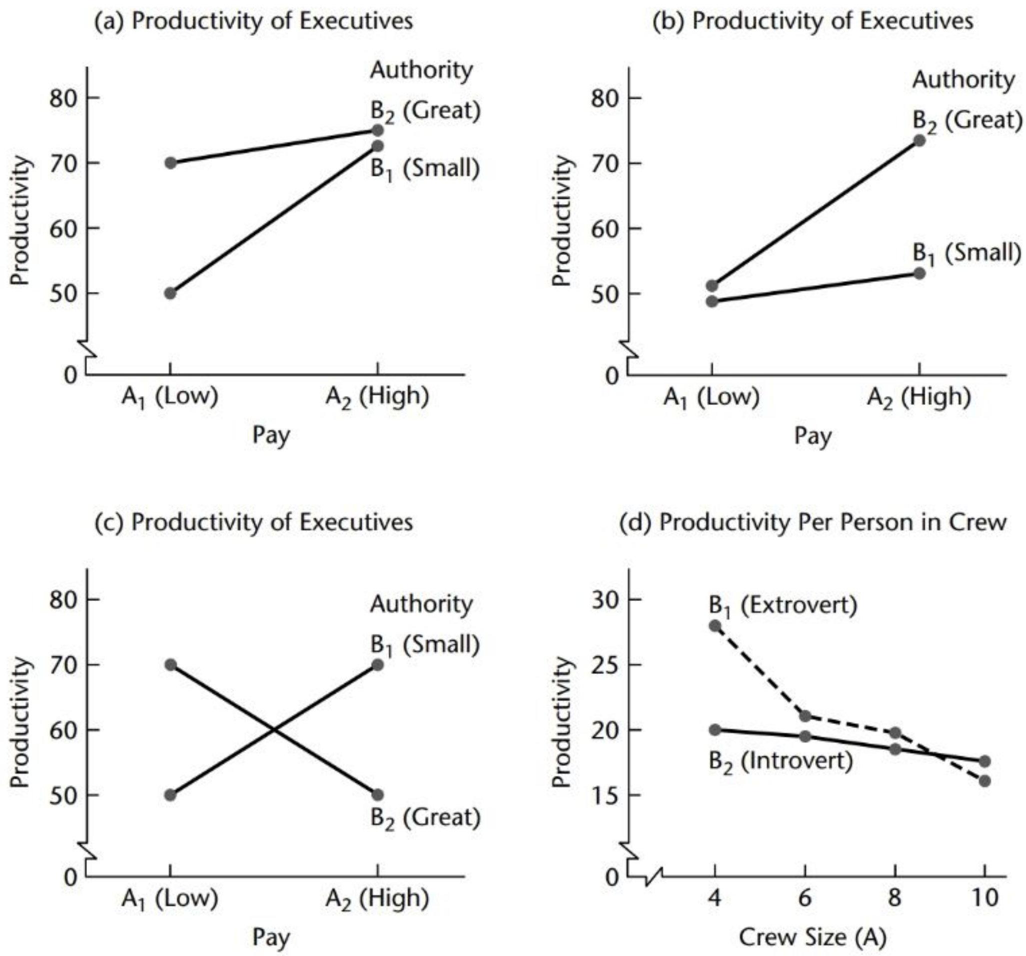
$$\sum_i \sum_j (\alpha\beta)_{ij} = 0 \qquad \textbf{(19.9c)}$$

Recognition of Interactions:

1. By examining whether all ij can be expressed as the sums .. + i + j .
2. By examining whether the difference between the mean responses for any two levels of factor B (or A) is the same for all levels of factor A (or B)
3. By examining whether the treatment means curves for the different factor levels in a treatment means plot are parallel.

Example of treatment effect plots to show interactions:

FIGURE 19.7 **Treatment Means Plots—Examples of Interactions from Table 19.6.**



Summary this week

- Reading: Chapter 18 (skip ch 18.6); Chapter 19.1-19.2
- HW assignment: Problem 18.17, Problem 18.18 (**with addition of questions d and e**).
- CH18PR17.txt is posted in week5 folder

*18.18. Refer to **Winding speeds** Problem 18.17. The researcher decided to apply the logarithmic transformation $Y' = \log_{10} Y$ and investigate its effectiveness.

a. Obtain the transformed response data, fit ANOVA model (16.2), and obtain the residuals.

b. Prepare suitable plots of the residuals to study the equality of the error variances of the transformed response variable for the four winding speeds. Also obtain a normal probability plot and the coefficient of correlation between the ordered residuals and their expected values under normality. What are your findings about the effectiveness of the transformation?

c. Test by means of the Brown-Forsythe test whether or not the treatment error variances for the transformed response variable are equal; use $\alpha = .05$. State the alternatives, decision rule, and conclusion. Are your findings in part (b) consistent with your conclusion here?

d. Test the homogeneity of the variances for the transformed variable using both the Hartley test and the Bartlett Test. Do you find the results are similar to those obtained in question c.?

e. Without doing the data transformation, apply a nonparametric test using the rank-based F-test or the Kruskal-Wallis test. What are your findings? Similar to those obtained in question a. ?