STAT3119-10 Quiz#4 (25 mins, total 100 points) Class: 11/7/2019

Name:___________________________________ GWID: _______________________________

1. (10points) What is the main objective to include a continuous covariate to study different treatment levels on the response in an ANCOVA analysis.

1) reduce experiment errors and make the estimate more precise/efficient
2) control/adjust for confounding variable

Students can get full credit if they answer one of the two reasons above.

2. (20points) Please state the two additional assumptions for ANCOVA models in addition to those assumptions for ANOVA.

1) _Linear relationship of covariate and response

2) _Constant slope of different treatment regression lines____________

3. **Data example**: A company studied the effects of **three** different types of promotions on sales of its crackers. **15** stores were selected for the study, and a completely randomized experimental design was utilized. Each store was randomly assigned one of the promotion types, with five stores assigned to each type of promotion. In the Table, ***Y***=data on the number of cases of the product sold during promotional, and *X*=data on the sales of the product in the preceding period. Sales in the preceding period are to be used as the concomitant variable.

Treatment	Store (j) 1		2		3		4		5	
i	Y_{i1}	X_{i1}	Y_{i2}	X_{i2}	Y_{i3}	X_{i3}	Y_{i4}	X_{i4}	Y_{i5}	X_{i5}
1	38	21	39	26	36	22	45	28	33	19
2	43	34	38	26	38	29	27	18	34	25
3	24	23	32	29	31	30	21	16	28	29

A. (10 points) Please write down the standard **single-factor ANCOVA model** (center the covariate around its overall mean).

__________ $Y_{ij} = \mu_{.} + \tau_i + \gamma(X_{ij} - \overline{X}_{..}) + \epsilon_{ij}$ ___________________

B. We can use an equivalent linear regression model with the same regression coefficients as the model parameters in the ANCOVA model.

a) (10 points) Please specify the regression model

_____ $Y_{ij} = \mu. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \varepsilon_{ij}$, where $x_{ij} = X_{ij} - \bar{X}_{..}$ ____

b) (10 points) Please specify the indicator functions use to define the three promotions (treatment levels).

$$I_1 = \begin{cases} 1 & \text{if store received treatment 1} \\ -1 & \text{if store received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$I_2 = \begin{cases} 1 & \text{if store received treatment 2} \\ -1 & \text{if store received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

Note: *For questions **C** below: please find out the values using regression outputs instead of only giving the estimator formula. If you show the correct formula and work, you can only get partial credit.*

C. If from the regression models, we obtained the following regression coefficients:

$$\hat{\mu}_{.} = 34, \quad \hat{\tau}_1 = 6, \quad \hat{\tau}_2 = 1, \quad \hat{\gamma} = .9$$

to estimate the overall mean (μ), treatment effects (τ_1, τ_2) and X~Y relationship (γ). Please calculate the adjusted mean response for three type of promotions when covariate X is at its overall mean (put your answers in the following table). (20 points)

Promotion type (treatment)	Adjusted mean response when $X = \overline{X}_{..}$
1	34+6 =40
2	34+1 = 35
3	34- (6+1)= 27

Formula: The mean response at $X = \overline{X}_{..}$ for ith treatment is $\mu_{.} + \tau_i$.;

$\hat{\tau}_3 = -(\hat{\tau}_1 + \hat{\tau}_2)$

If we also obtained the following variance and covariance estimates from the regression model:

$$\hat{\sigma}^2(\tau_1) = 0.5, \quad \hat{\sigma}^2(\tau_1) = 0.5, \quad \hat{\sigma}(\tau_1, \tau_2) = -0.3$$

Then given the same covariate value, please calculate the estimates of the following treatment differences and their correponding estimated variances (put your answers in the table). (20 points)

Comparison	Estimate	Estimated variance
τ_1 - τ_2 (the difference in the 1st and 2nd types of promotion)	6-1=5	0.5+0.5-2*(-0.3)=1+0.6=1.6
τ_1 - τ_3 (the difference in the 1st and 3rd types of promotion)	6-(-(6+1))=6+7=13	4*0.5+0.5+4(-0.3)= 1.3

Formula:

1) *When L=* τ_1 - τ*,* $\hat{L} = \hat{\tau}_1 - \hat{\tau}_2$

$$\widehat{var}(\hat{L}) = \widehat{var}(\hat{\tau}_1) + \widehat{var}(\hat{\tau}_2) - 2\widehat{cov}(\hat{\tau}_1, \hat{\tau}_2)$$

2) When *L*=τ_1 - $\tau_{3,}$

$$\hat{L} = \hat{\tau}_1 - \hat{\tau}_3 = \hat{\tau}_1 - (-\hat{\tau}_1 - \hat{\tau}_2) = 2\hat{\tau}_1 + \hat{\tau}_2$$

$$\widehat{var}(\hat{L}) = 4\widehat{var}(\hat{\tau}_1) + \widehat{var}(\hat{\tau}_2) + 4\widehat{cov}(\hat{\tau}_1, \hat{\tau}_2)$$