

# HW#2 Solution

9/19/2019

## HW 17.8 Productivity improvement.

```
# HW 16.07
HW07 <- read.table(
  url("https://raw.githubusercontent.com/npmlldabook/Stat3119/master/Week2/CH16PR07.txt"))
# rename the variables
names(HW07)<- c("productivity", "expenditures", "firm")

HW07$expenditures<- as.factor(HW07$expenditures)
head(HW07)
```

```
##   productivity expenditures firm
## 1          7.6             1    1
## 2          8.2             1    2
## 3          6.8             1    3
## 4          5.8             1    4
## 5          6.9             1    5
## 6          6.6             1    6
```

```
str(HW07)
```

```
## 'data.frame':   27 obs. of  3 variables:
##  $ productivity: num  7.6 8.2 6.8 5.8 6.9 6.6 6.3 7.7 6 6.7 ...
##  $ expenditures: Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 2 ...
##  $ firm        : int  1 2 3 4 5 6 7 8 9 1 ...
```

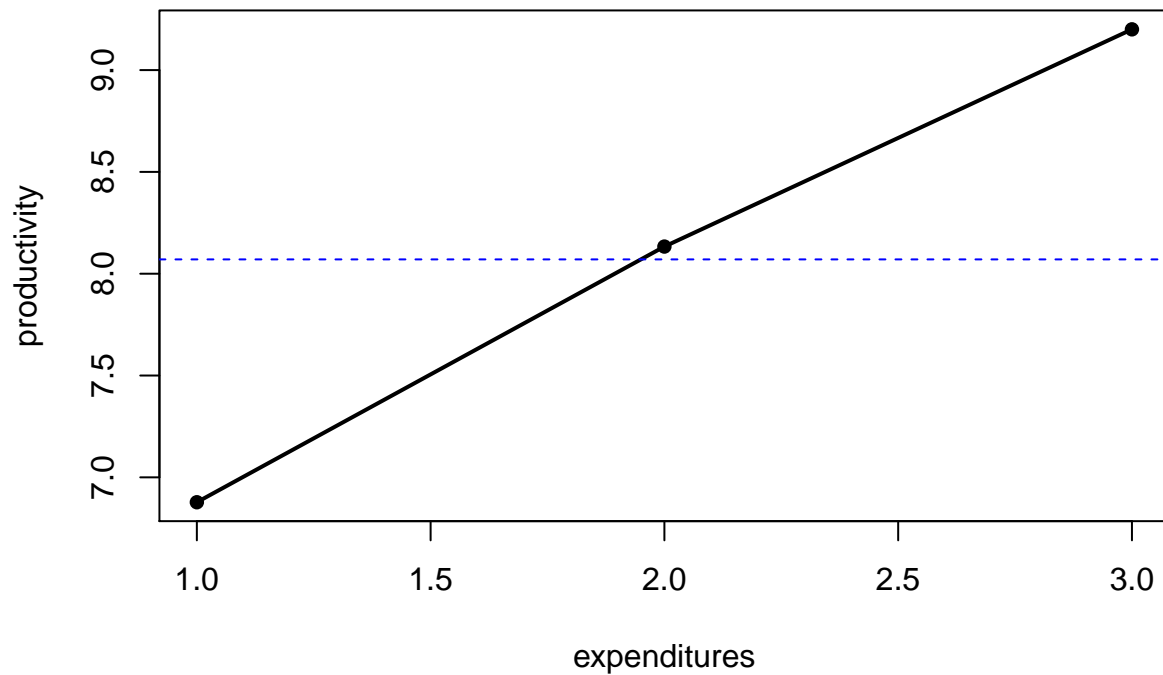
### a. Line plot

```
fit <- aov(productivity ~ expenditures, data = HW07)
(Factor_means = predict(fit, newdata = data.frame(expenditures = factor(1:3))))
```

```
##           1           2           3
## 6.877778 8.133333 9.200000
```

```
Overall_means = mean(Factor_means)
plot(1:3, Factor_means, type='o', pch=16, lwd=2,
     xlab="expenditures", ylab="productivity", main="Main Effects Plot")
abline(h= Overall_means, lty=2, col='blue')
```

## Main Effects Plot



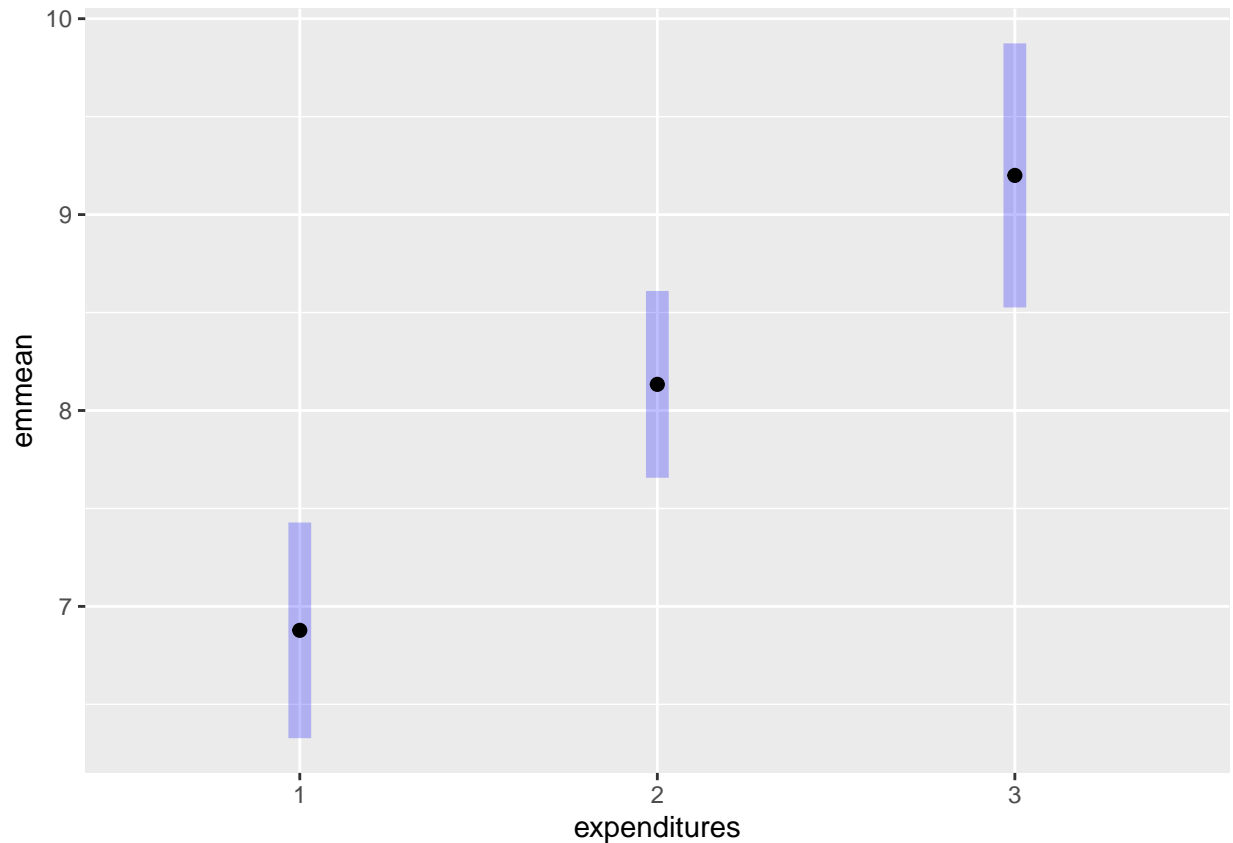
Results: It suggests productivity is increasing with expenditures.

b. Estimate mean and 95% CI for  $\mu_3$

```
library(emmeans)
(Est.mean<- emmeans(fit, ~ expenditures))
```

```
## expenditures emmean    SE df lower.CL upper.CL
## 1             6.88 0.267 24    6.33    7.43
## 2             8.13 0.231 24    7.66    8.61
## 3             9.20 0.327 24    8.53    9.87
##
## Confidence level used: 0.95
```

```
plot(Est.mean, horizontal=F)
```



#### Results: The mean is 9.2 with 95% CI (8.53-9.87).

c. Obtain the 95% CI for  $\mu_2 - \mu_1$

```
pairs(Est.mean, adjust = "none" )
```

```
## contrast estimate SE df t.ratio p.value
## 1 - 2          -1.26 0.353 24 -3.559 0.0016
## 1 - 3          -2.32 0.422 24 -5.507 <.0001
## 2 - 3          -1.07 0.400 24 -2.666 0.0135
```

Results: We can first obtain estimate is  $-(-1.26)=1.26$ , with  $SE = 0.53$ ,  $df=24$ . Then we get CI as follows:

```
LCI= 1.26 - qt(.975, 24)*0.353
UCI= 1.26 + qt(.975, 24)*0.353
paste("95% CI is (", round(LCI,2), round(UCI,2), ").")
```

```
## [1] "95% CI is ( 0.53 1.99 )."
```

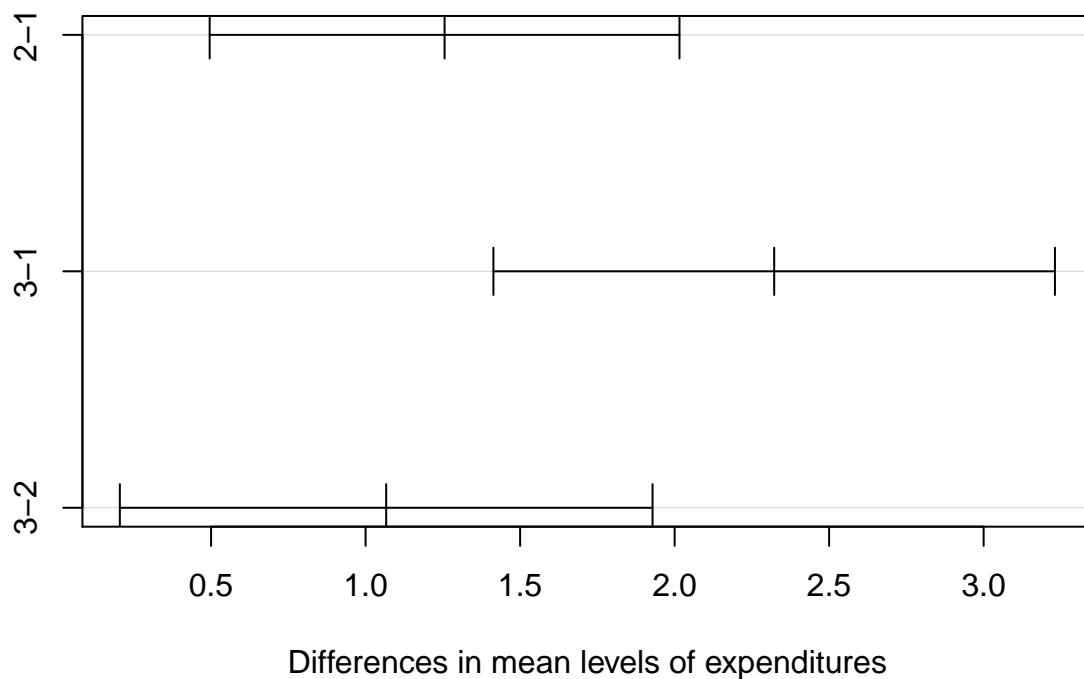
d. Tukey

```
TukeyHSD(fit, conf.level = 0.90)
```

```
## Tukey multiple comparisons of means
## 90% family-wise confidence level
##
## Fit: aov(formula = productivity ~ expenditures, data = HW07)
##
## $expenditures
##      diff      lwr      upr    p adj
## 2-1 1.255556 0.4954165 2.015695 0.0043755
## 3-1 2.322222 1.4136823 3.230762 0.0000335
## 3-2 1.066667 0.2047500 1.928583 0.0347870
```

```
plot(TukeyHSD(fit, conf.level = 0.90))
```

### 90% family-wise confidence level



e. compute the three multiples

```
#Tukey multiple
1/sqrt(2)* qtkey(.90, nm=3, df=24)
```

```
## [1] 2.154636
```

```
#Bonferroni multiple,
g=3 #
qt(1- 0.1/(2*g), 24)
```

```
## [1] 2.25775
```

```
# Scheffe multiple r-1=2, n_T-r=24
sqrt(2*qf(.90, 2, 24))
```

```
## [1] 2.253145
```

Results: Yes. Tukey multiple =2.15, which is less than Bonferroni multiple=2.257 and Scheffe multiple =2.253.

## HW 17.11 Cash offers data

```
# HW 16.10
HW10 <- read.table(
  url("https://raw.githubusercontent.com/npmldabook/Stat3119/master/Week2/CH16PR10.txt"))
dim(HW10)
```

```
## [1] 36 3
```

```
# rename the variables
names(HW10)<- c("offer", "age", "dealer")

HW10$age<- as.factor(HW10$age)

str(HW10)
```

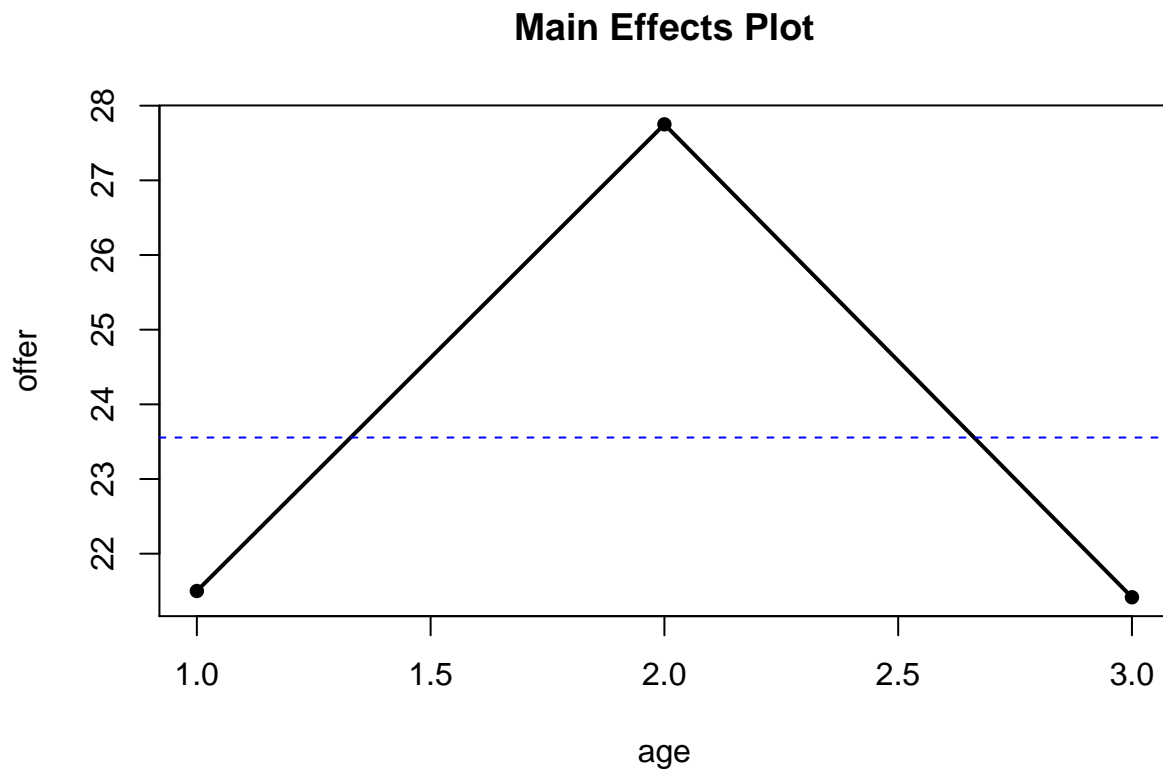
```
## 'data.frame': 36 obs. of 3 variables:
## $ offer : num 23 25 21 22 21 22 20 23 19 22 ...
## $ age : Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 1 1 ...
## $ dealer: int 1 2 3 4 5 6 7 8 9 10 ...
```

### a. main effect plots

```
fit <- aov(offer ~ age, data = HW10)
(Factor_means = predict(fit, newdata = data.frame(age = factor(1:3))))
```

```
##          1          2          3
## 21.50000 27.75000 21.41667
```

```
Overall_means = mean(Factor_means)
plot(1:3, Factor_means, type='o', pch=16, lwd=2,
     xlab="age", ylab="offer", main="Main Effects Plot")
abline(h= Overall_means, lty=2, col='blue')
```



Results: It suggests a nonlinear relationship. The middle age group seems to get a larger case offer..

b. Obtain 99% CI for younger age (mu1)

```
Est.mean<- emmeans(fit, ~ age)
confint(Est.mean, adjust="none", level=.99)
```

```
## age emmean SE df lower.CL upper.CL
## 1 21.5 0.456 33 20.3 22.7
## 2 27.8 0.456 33 26.5 29.0
## 3 21.4 0.456 33 20.2 22.7
##
## Confidence level used: 0.99
```

Results: For younger age , estimate=21.5, 99% CI is (20.3. 22.7)

c. Get 99% CI for mu3-mu1

```
pairs(Est.mean, adjust = "none" )
```

```
## contrast estimate SE df t.ratio p.value
## 1 - 2 -6.2500 0.644 33 -9.702 <.0001
## 1 - 3 0.0833 0.644 33 0.129 0.8979
## 2 - 3 6.3333 0.644 33 9.831 <.0001
```

Results: We can first obtain estimate is  $D1 = \mu_3 - \mu_1 = -0.0833$ ,  $SE = 0.644$ ;  $df = 33$

```
# For D1
D1 = -0.0833
SE = 0.644
LCI= D1 - qt(1-0.01/2, 33)* 0.644
UCI= D1 + qt(1-0.01/2, 33)* 0.644
paste("99% CI is D1 , (", round(LCI,2), round(UCI,2), ").")
```

```
## [1] "99% CI is D1 , ( -1.84 1.68 )."
```

d. test if a contrast  $= 0$ ,

$$H_0 : L = \mu_1 - 2\mu_2 + \mu_3 = 0$$

$$H_0 : L = \mu_1 - 2\mu_2 + \mu_3 \neq 0$$

Rejection rule,  $t^* = \hat{L}/s(\hat{L})$ , and we reject null if  $|t^*| > t(1 - \alpha/2, n_T - r)$ .

```
L = list(L1= c(1,-2,1))
contrast(Est.mean, L, adjust="none")
```

```
## contrast estimate SE df t.ratio p.value
## L1 -12.6 1.12 33 -11.278 <.0001
```

Results: we reject the null hypothesis that this contrast is zero, or equivalently,

$$\mu_2 - \mu_1 = \mu_3 - \mu_2.$$

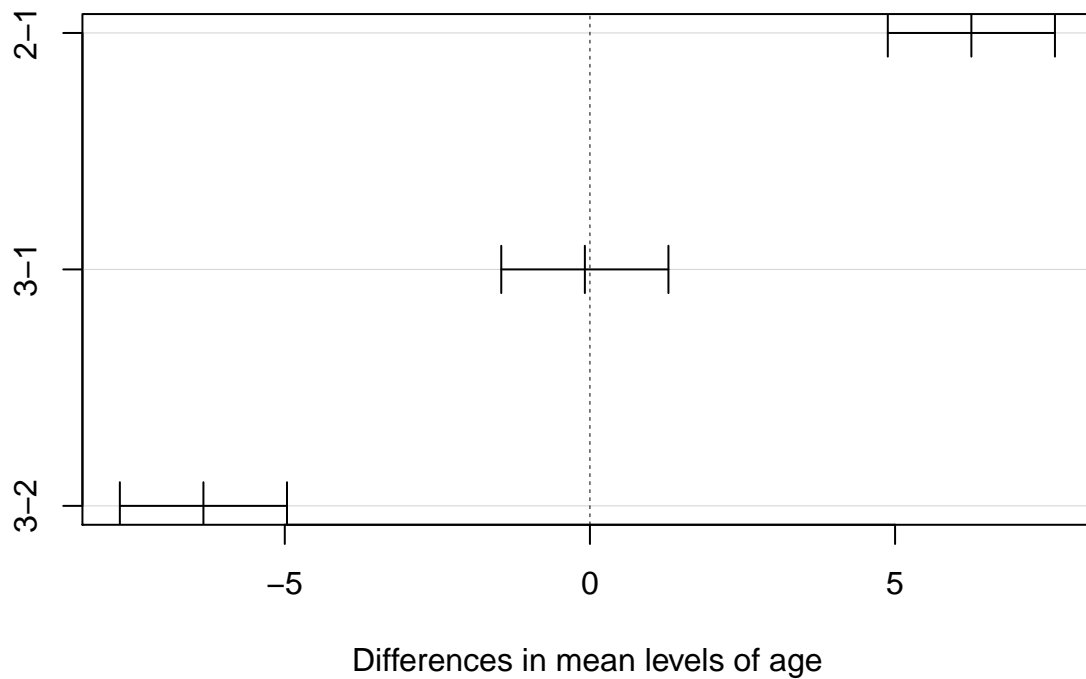
e. Tukey

```
TukeyHSD(fit, conf.level = 0.90)
```

```
## Tukey multiple comparisons of means
## 90% family-wise confidence level
##
## Fit: aov(formula = offer ~ age, data = HW10)
##
## $age
## diff lwr upr p adj
## 2-1 6.25000000 4.880508 7.619492 0.0000000
## 3-1 -0.08333333 -1.452825 1.286158 0.9908192
## 3-2 -6.33333333 -7.702825 -4.963842 0.0000000
```

```
plot(TukeyHSD(fit, conf.level = 0.90))
```

### 90% family-wise confidence level



f. compute the 2 multiples

```
#Tukey multiple
1/sqrt(2)* qtukey(.90, nm=3, df=33)
```

```
## [1] 2.125907
```

```
#Bonferroni multiple,
g=3 #
qt(1- 0.1/(2*g), 33)
```

```
## [1] 2.220913
```

Results: No. Tukey multiple =2.13, which is less than Bonferroni multiple=2.22.