HW#7 Solution (week8 HW)

Due 10/31/2019

HW 21.5 Auditor training.

```
HW21<- read.table(url("https://raw.githubusercontent.com/npmldabook/Stat3119/master/Week8/CH21PR05.txt"))
head(HW21,6)
```

```
##   V1 V2 V3
## 1 73  1  1
## 2 81  1  2
## 3 92  1  3
## 4 76  2  1
## 5 78  2  2
## 6 89  2  3
```

```
names(HW21) = c("Response", "Block", "Training")
HW21$Block = as.factor(HW21$Block)
HW21$Training = as.factor(HW21$Training)
str(HW21)
```

```
## 'data.frame':    30 obs. of  3 variables:
##  $ Response: num  73 81 92 76 78 89 75 76 87 74 ...
##  $ Block   : Factor w/ 10 levels "1","2","3","4",..: 1 1 1 2 2 2 3 3 3 4 ...
##  $ Training: Factor w/ 3 levels "1","2","3": 1 2 3 1 2 3 1 2 3 1 ...
```

a.

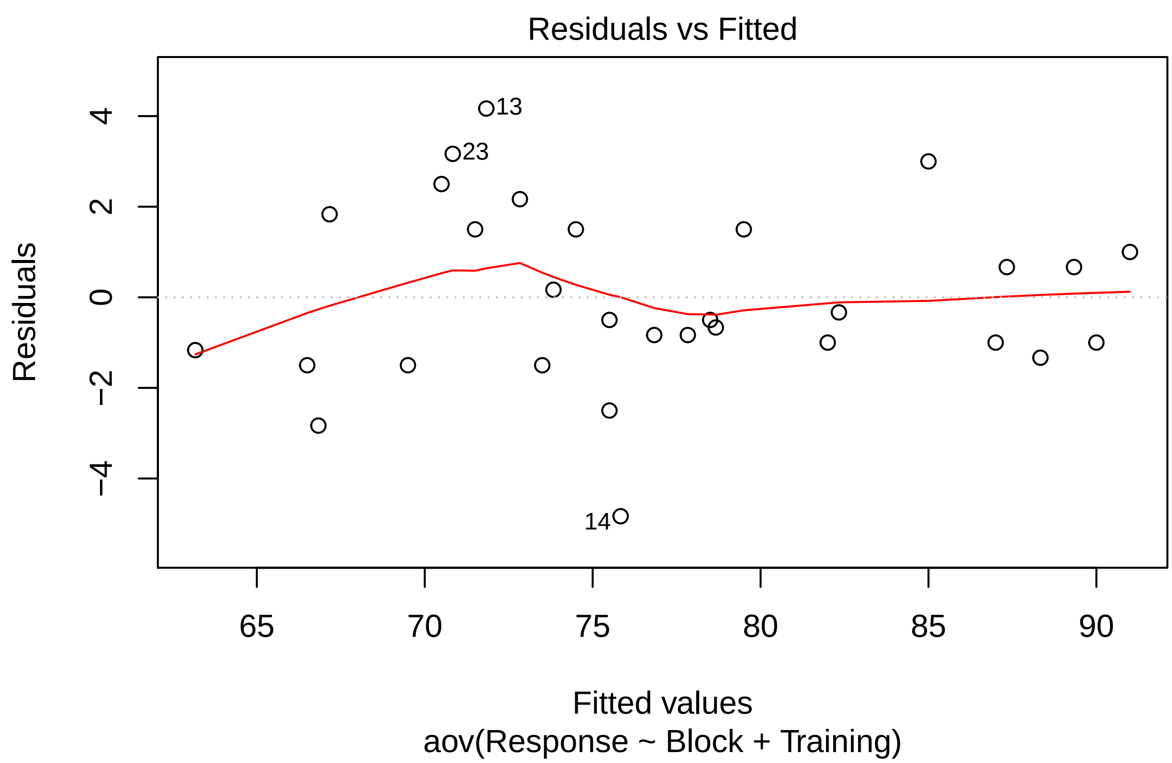
Answer: To group the subjects with similar responses into blocks and reduce the experimental errors.

b.

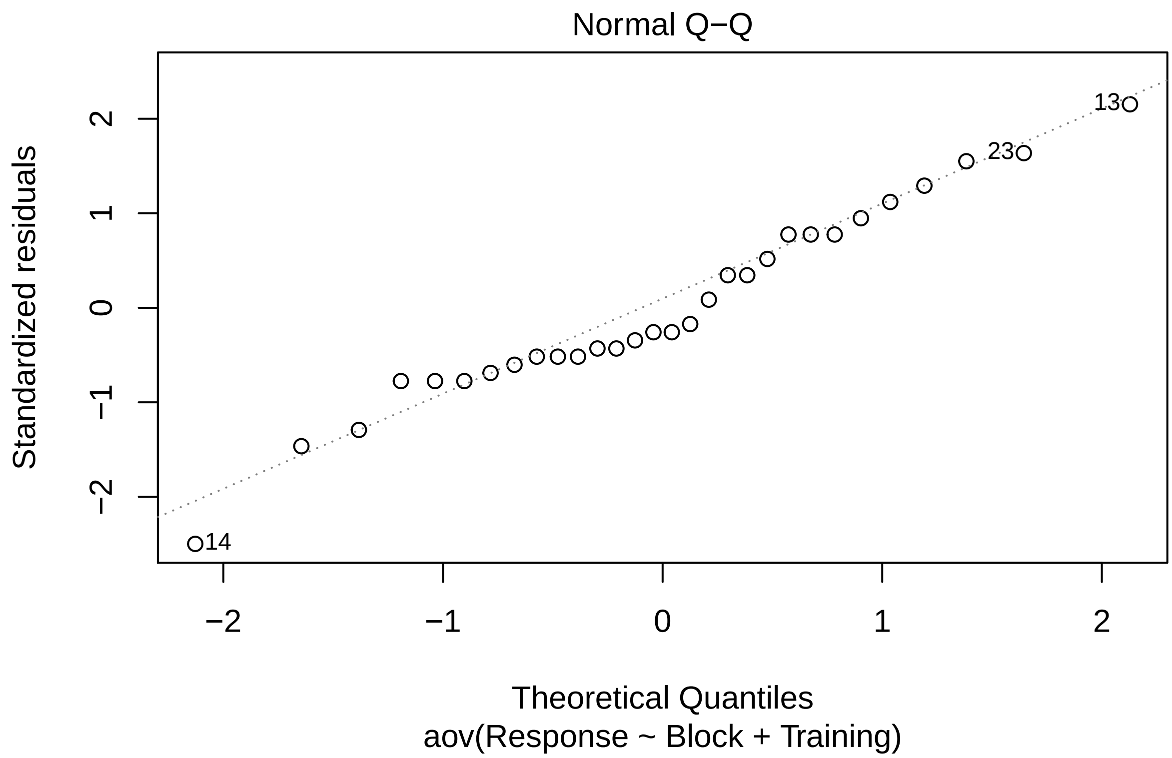
```
fit = aov(Response~ Block + Training, data= HW21 )
#summary(fit)
Resid = fit$residuals
matrix(Resid, nrow=10, ncol=3, byrow=T)
```

```
##             [,1]       [,2]       [,3]
##  [1,] -2.5000000  1.5000000  1.0000000
##  [2,]  1.5000000 -0.5000000 -1.0000000
##  [3,]  2.1666667 -0.8333333 -1.3333333
##  [4,]  0.1666667 -0.8333333  0.6666667
##  [5,]  4.1666667 -4.8333333  0.6666667
##  [6,]  1.5000000 -0.5000000 -1.0000000
##  [7,] -1.5000000 -1.5000000  3.0000000
##  [8,] -2.8333333  3.1666667 -0.3333333
##  [9,] -1.5000000  2.5000000 -1.0000000
## [10,] -1.1666667  1.8333333 -0.6666667
```

```r
plot(fit, 1)
```




```r
plot(fit,2)
```

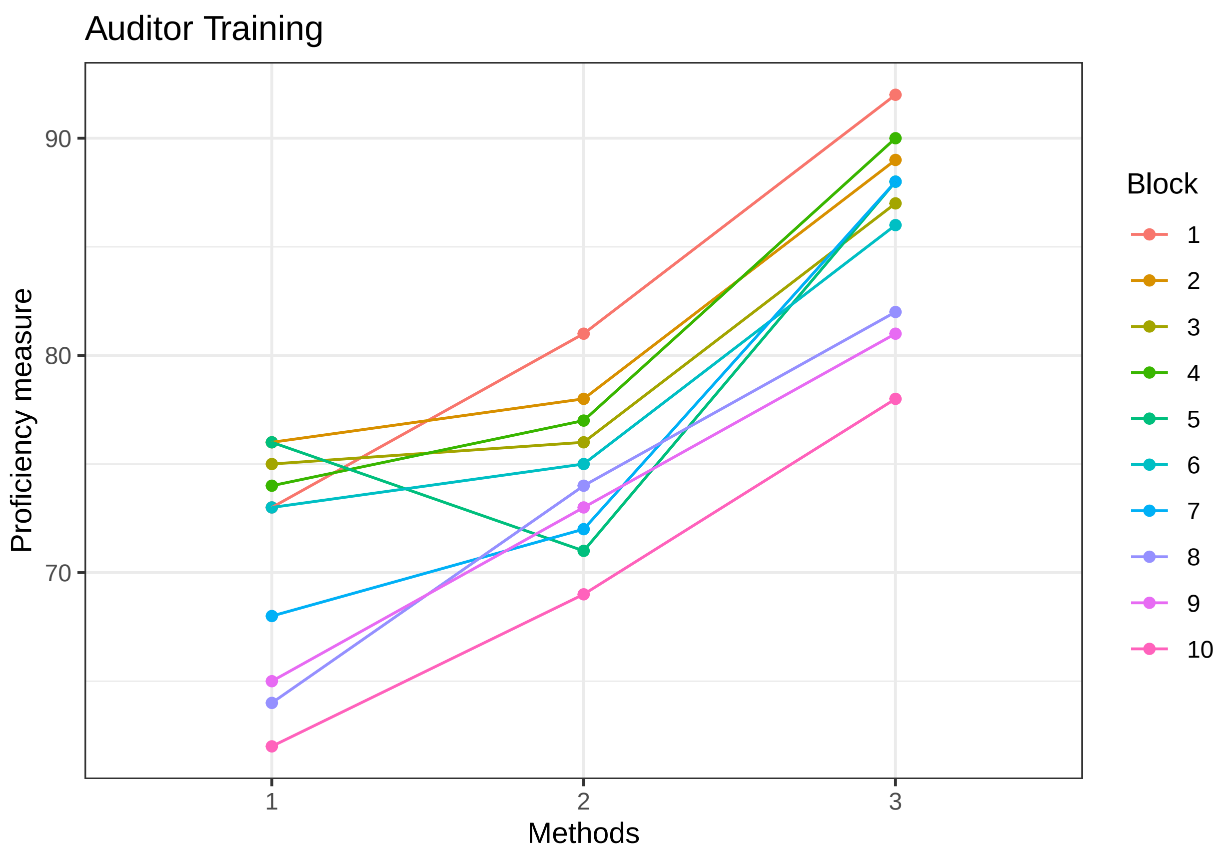


Answer: The Assumptions (1) constant variances and (2) normal errors seem to be reasonable. There are no big departure from these assumptions by residual analysis.

c. group by block plots

```
# A basic plot can be done from interaction.plot()
# with(HW21, interaction.plot(x.factor = Training, trace.factor = Block, response = Response))

# Or use ggplot
library(ggplot2)
ggplot(HW21, aes(x = Training, y = Response)) +
  geom_point( aes(group = Block, color = Block))+
  geom_line( aes(group = Block, color = Block)) +
  theme_bw() +
  labs( title = "Auditor Training",
    x = "Methods",
    y = "Proficiency measure"
  )
```



Answer: This plot suggest about the no-interaction assumption seems appropriate since the treatment means are mostly parallel across blocks , after accounting for some measurment errors.

d. Tukey test, alpha=0.1

```
library(additivityTests)
## make the data into matrix A*B Form (dim a * b)
(HW21m = matrix( HW21$Response, nrow=10, ncol=3, byrow=T ))
```

```
##       [,1] [,2] [,3]
##  [1,]   73   81   92
##  [2,]   76   78   89
##  [3,]   75   76   87
##  [4,]   74   77   90
##  [5,]   76   71   88
##  [6,]   73   75   86
##  [7,]   68   72   88
##  [8,]   64   74   82
##  [9,]   65   73   81
## [10,]   62   69   78
```

```
tukey.test( HW21m, alpha = 0.01)
```

```
##
```

```
## Tukey test on 1% alpha-level:
##
## Test statistic: 0.01918
## Critival value: 8.4
## The additivity hypothesis cannot be rejected.
```

```r
# compute P-value
(Pv= 1-pf(0.01918, df1=1, df2=  17 ))
```

```
## [1] 0.891479
```

Answer: We don't reject the additivity hypothes at tje leel of 0.01. The P-value of the test $P(F > F*) = P(F > 0.01918) = 0.89$, where F is the random variable with F(df1=1, df=17) distribution.

HW 21.6

a. ANOVA table

```r
summary(fit)
```

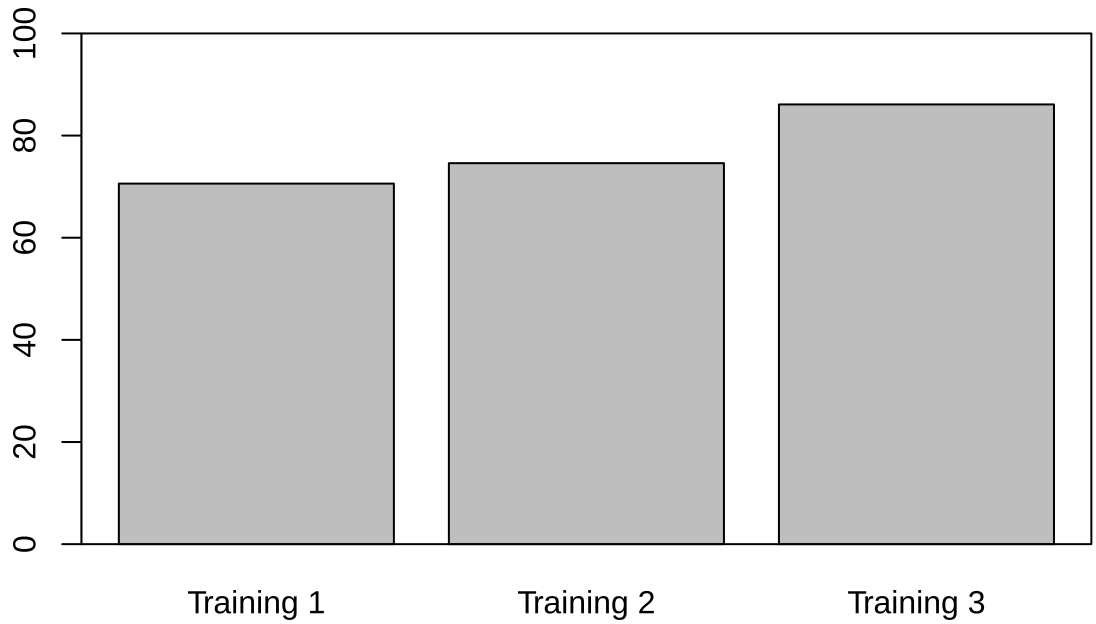
```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## Block        9  433.4    48.2   7.716 0.000132 ***
## Training     2 1295.0   647.5 103.754 1.32e-10 ***
## Residuals   18  112.3     6.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b. treatment means

```r
library(emmeans)
fit.emm <- emmeans(fit, ~ Training)

 # Extract means from emmeans output
Mean<- confint(fit.emm)[,2] # extract mean

# Or mean can be obtained from
# Mean= as.numeric(with(HW21, by (Response,Training,mean)))
names(Mean) <- paste("Training", 1:3)
barplot(Mean, ylim=c(0,100)); box()
```



c.

```r
# find the critical value
qf(.95, 2,18)
```

```
## [1] 3.554557
```

Answer:

1) Alternative hypothesis: The mean proficiency is not the same for the three training methods.
2) Decision rule: We reject null if the F-statistic >= 3.554557.
3) Results: For training effect, F-statistic= 103.75 with p-value <0.0001, highly significant.

d. Pairwise comparison,

```r
(pairtest= pairs(fit.emm, adjust = "Tukey"))
```

```
##  contrast estimate   SE df t.ratio p.value
##  1 - 2        -4.0 1.12 18  -3.580 0.0058
##  1 - 3       -15.5 1.12 18 -13.874 <.0001
##  2 - 3       -11.5 1.12 18 -10.294 <.0001
```

```
##
## Results are averaged over the levels of: Block
## P value adjustment: tukey method for comparing a family of 3 estimates
```

```
confint(pairtest, level = .90)
```

```
##  contrast estimate   SE df lower.CL upper.CL
##  1 - 2        -4.0 1.12 18    -6.45    -1.55
##  1 - 3       -15.5 1.12 18   -17.95   -13.05
##  2 - 3       -11.5 1.12 18   -13.95    -9.05
##
## Results are averaged over the levels of: Block
## Confidence level used: 0.9
## Conf-level adjustment: tukey method for comparing a family of 3 estimates
```

Answer: With a 90 percent family confidence coefficient, the responses were different beteen any pairs of two methods. The subject by method 3 had higher mean response values compared to those by the method 2; and the subject by method 2 had higher mean response values compared to those by the method 1.
The estimated and CI of the parwise difference can be found in the above output.

e. block effect

```
# find the critical value
qf(.95, 9,18)
```

```
## [1] 2.456281
```

Answer:

1) Alternative hypothesis: there are some block effects.

2) Decision rule: We reject null if the corresponding F-statistic >= 2.45628

3) Results: For block effect, F-statistic= 7.716 with p-value= 0.000132, highly significant. We reject the null hypothesis of no block effect.

HW 21.14: power of the test.

We consider the block as the number of replications with the same treatment level.

```
mu <- c(70, 73, 76)
sigma2 <- 2.5^2
power.anova.test(groups = length(mu), n = 10, between.var = var(mu), within.var = sigma2)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##          groups = 3
##               n = 10
##     between.var = 9
```

```
##      within.var = 6.25
##       sig.level = 0.05
##           power = 0.9970791
##
## NOTE: n is number in each group
```

Answer: Power is > .99.

HW 21.16: Sample size

Problems: make all pairwise treatment comparisons with precision ±1.5 with; a 99 percent family confidence coefficien; sigma (SD)

Write a function for pairwise CI with Tukey

```
# write a function for pairwise CI with Tukey
CIwidth <- function(nb){
r=3
newdf= (nb-1)*(r-1)
sigma= 2.5
alpha=0.01

# Tukey mutiple
T =1/sqrt(2)* qtukey(1-alpha, nm=r, newdf)

# SE(L)
SE.L= sqrt(2/nb)*sigma
# CI width
unlist(list(T=T, SE.L= SE.L, CIwidth= T*SE.L))
}
```

Try with several sample size to get half CI width <=1.5

```
CIwidth(nb=40)
```

```
##        T     SE.L  CIwidth
## 3.000894 0.559017 1.677550
```

```
CIwidth(nb=49)
```

```
##         T      SE.L   CIwidth
## 2.9841738 0.5050763 1.5072354
```

```
CIwidth(nb=50)
```

```
##        T     SE.L  CIwidth
## 2.982703 0.500000 1.491351
```

Answer: Nb=50 blocks are needed.

HW 21.18: relative efficiency

I think the (21.13) is actually referring to the formula (21.14)

```
nb=10
MSBL= 48.2
r=3
MSBL.TR=6.2

(ReEff= ((nb-1)*MSBL + nb*(r-1)* MSBL.TR)/((nb*r-1)*MSBL.TR) )
```

```
## [1] 3.102336
```

Answer: relative efficiency = 3.1, suggesting we would have required more than 3 times as many replications per training level with a completely randomized design to achieve the same variance of any estimated contrast as is obtained with blocking.