

1. Random effects model for one-factor study vs. Fixed effects study:

$$Y_{ij} = \mu. + \tau_i + \epsilon_{ij} \quad (25.38)$$

where

- $\mu.$ is the constant (population mean)
- τ_i are independent $N(0, \sigma_\mu^2)$ (with a mean of zero), representing the effect of the i th factor levels
- ϵ_{ij} are independent $N(0, \sigma^2)$
- τ_i and ϵ_{ij} are independent random variables, $i = 1, \dots, r$, and $j = 1, \dots, n$.

Parameters	Random effects model	Fixed effects model
Variance of Y_{ij}	$\sigma_\mu^2 + \sigma^2$	σ^2
Observations	Correlated when <u>from same level</u> w. correlation = $\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma^2}$	Independent with each other
Descript H_0 in terms fixed parameter	$H_0 : \sigma_\mu^2 = 0$	$\tau_1 = \tau_2 = \dots = \tau_r$

2. Random effects models for 2-factor study vs. Fixed effects study:

$$Y_{ijk} = \mu.. + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \quad (25.39)$$

where

- $\mu..$ is a constant
- α_i , β_j and $(\alpha\beta)_{ij}$ are **independent** zero-mean normal random variables $N(0, \sigma_\alpha^2)$, $N(0, \sigma_\beta^2)$, and $N(0, \sigma_{\alpha\beta}^2)$
- ϵ_{ijk} are independent $N(0, \sigma^2)$.
- α_i , β_j , $(\alpha\beta)_{ij}$ and ϵ_{ijk} are pairwise independent
- $i = 1, \dots, a$, $j = 1, \dots, b$ and $k = 1, \dots, n$.

Parameters	Random effects model A,B are both random	Fixed effects model
Variance of Y_{ij}	$\sigma_\alpha^2 + \sigma_\beta^2 + \sigma_{\alpha\beta}^2 + \sigma^2$	σ^2
Observations	Independent when <u>they are from different levels</u>	Independent with each other

3. For balanced two factor study with 1-2 factors are random, the ANOVA table (SS, MS, df) are the same but the F-test statistics and their distributions depend on the type of the model. *Need to look Table 25.5 to compare EMS to set the F-test correctly.*

a) ANOVA model II:

Source of variation	df	MS	F-test statistics (in terms of MS)	Distribution F(df1, df2)
Factor A (random)	3	MSA	MSA/MSAB	F(3, 12)
Factor B (random)	4	MSB	MSB/MSAB	F(4, 12)
AB interaction	12	MSAB	MSAB/MSE	F(12, 60)
Error	60	MSE		

b) ANOVA model III:

Source of variation	df	MS	F-test statistics (in terms of MS)	Distribution F(df1, df2)
Factor A (random)	3	MSA	MSA/MSE	F(3, 60)
Factor B (fixed)	4	MSB	MSB/MSAB	F(4,12)
AB interaction	12	MSAB	MSAB/MSE	F(12, 60)
Error	60	MSE		

4. Balanced Three-factor ANOVA Model II (all three factors are random):

TABLE 25.11
ANOVA Table
for Random
Three-Factor
Study ($a = 3$,
 $b = 2$, $c = 5$,
 $n = 3$).

Source of Variation	SS	df	MS
Factor A (operators)	17.3	2	8.65
Factor B (machines)	4.2	1	4.20
Factor C (batches)	24.8	4	6.20
AB interactions	4.8	2	2.40
AC interactions	31.7	8	3.96
BC interactions	12.5	4	3.13
ABC interactions	11.9	8	1.49
Error	137.7	60	2.30
Total	244.9	89	

Test for the effects	F-test statistics (in terms of MS)	Distribution F(df1, df2)
ABC interaction	$F = MS_{ABC}/MSE$	F(8, 60)
AC interaction	$F = MS_{AC}/MS_{ABC}$	F(8,8)
Factor C	$F^* = MS_C/MS^*$ $MS^* = MS_{AC} + MS_{BC} - MS_{ABC}$	F(2, df), where $df = \frac{(MS^*)^2}{\frac{MS_{AC}^2}{8} + \frac{MS_{BC}^2}{4} + \frac{MS_{ABC}^2}{8}}$