

HW#1 Solution

9/12/2019

HW 16.7 Productivity improvement.

```
# HW 16.07
HW07 <- read.table(url("https://raw.githubusercontent.com/npmlldabook/Stat3119/master/Week2/CH16PR07.txt"))
# rename the variables
names(HW07)<- c("productivity", "expenditures", "firm")

HW07$expenditures<- as.factor(HW07$expenditures)
head(HW07)
```

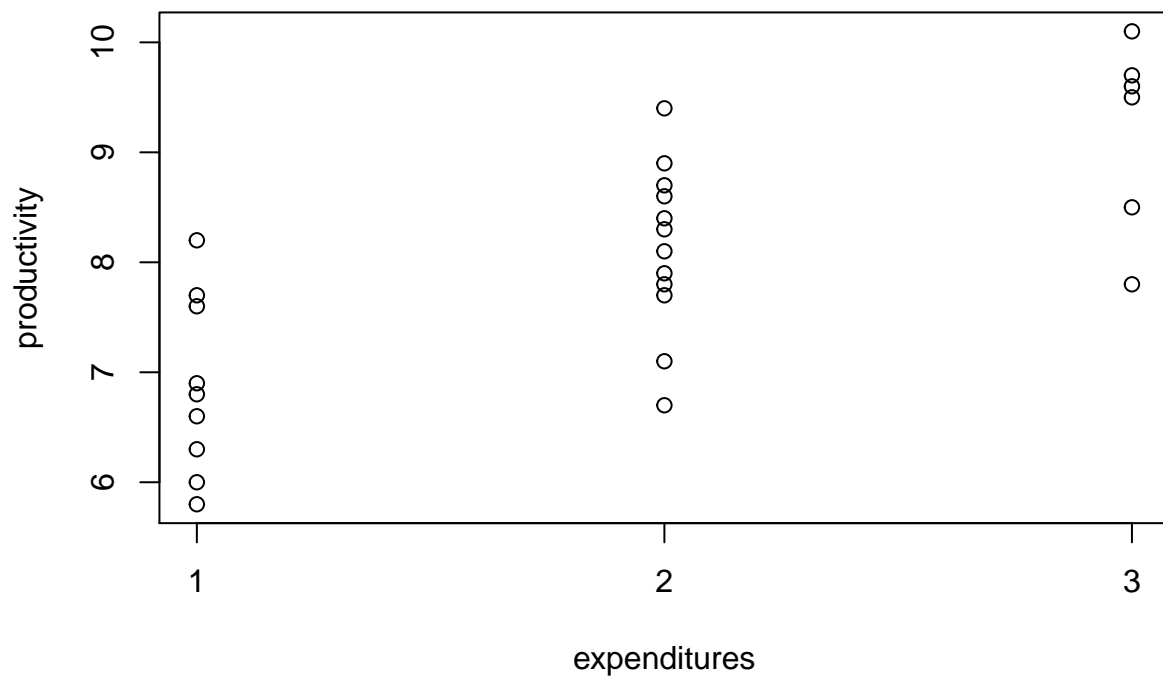
```
##   productivity expenditures firm
## 1          7.6            1    1
## 2          8.2            1    2
## 3          6.8            1    3
## 4          5.8            1    4
## 5          6.9            1    5
## 6          6.6            1    6
```

```
str(HW07)
```

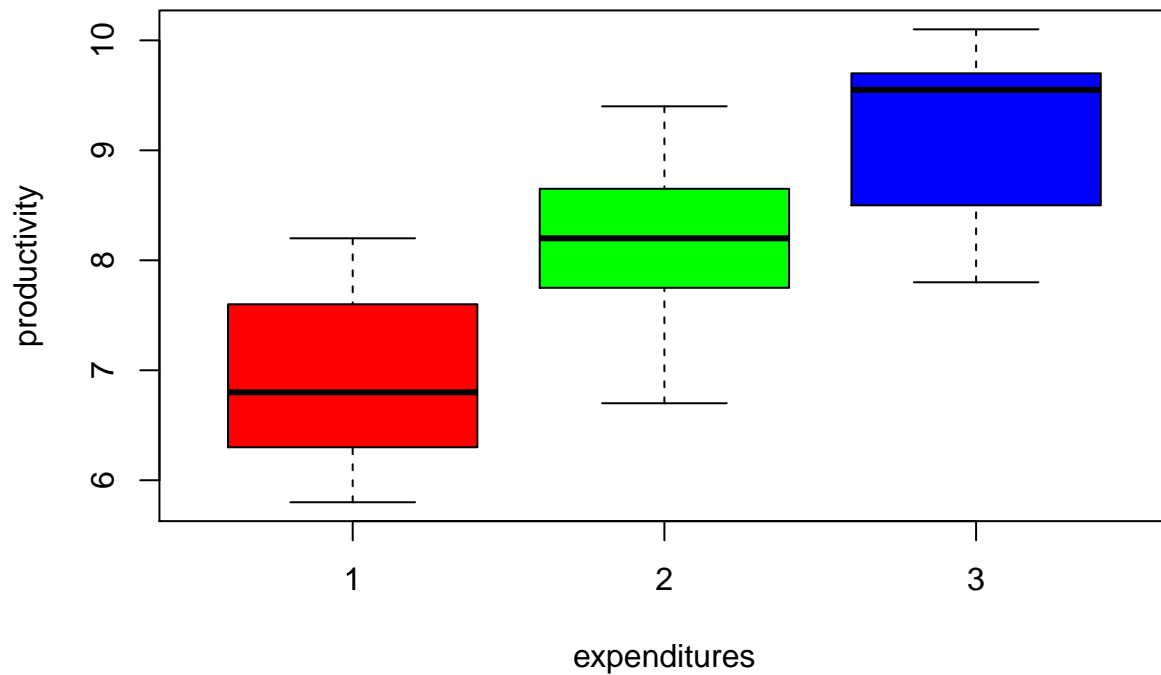
```
## 'data.frame':   27 obs. of  3 variables:
##  $ productivity: num  7.6 8.2 6.8 5.8 6.9 6.6 6.3 7.7 6 6.7 ...
##  $ expenditures: Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 2 ...
##  $ firm        : int  1 2 3 4 5 6 7 8 9 1 ...
```

a. aligned dot plots:

```
stripchart(productivity ~ expenditures, vertical = TRUE, pch=1, data = HW07, xlab="expenditures")
```



```
boxplot(productivity ~ expenditures, data = HW07, col=rainbow(3))
```



Results: the means appear to differ, and the varibilities appear to be similar.

b. Obtain the fitted values:

```
fit <- aov(productivity ~ expenditures, data = HW07)
# factor level means= fitted values
predict(fit, newdata = data.frame(expenditures = factor(1:3)))
```

```
##          1          2          3
## 6.877778 8.133333 9.200000
```

c. Obtain the residuals and sum:

```
# residuals
fit$residuals
```

```
##          1          2          3          4          5          6
## 0.7222222 1.3222222 -0.07777778 -1.07777778 0.02222222 -0.27777778
##          7          8          9         10         11         12
## -0.5777778 0.8222222 -0.8777778 -1.43333333 -0.03333333 1.26666667
##         13         14         15         16         17         18
```

```
## 0.46666667 -0.33333333 -0.43333333 0.76666667 -0.23333333 0.16666667
##      19      20      21      22      23      24
## 0.56666667 -1.03333333 0.26666667 -0.70000000 0.50000000 0.90000000
##      25      26      27
## -1.40000000 0.40000000 0.30000000
```

```
## sum of residuals
sum(fit$residuals)
```

```
## [1] 6.383782e-16
```

d. ANOVA table

```
summary(fit)
```

```
##           Df Sum Sq Mean Sq F value    Pr(>F)
## expenditures  2  20.12   10.06   15.72 4.33e-05 ***
## Residuals    24  15.36    0.64
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

e-f.

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

$$H_a : \text{not all } \mu_i \text{ are equal.}$$

Results: $F^* = 15.72$ with a p-value < 0.001 , we reject H_0 and conclude H_a .

HW 16.10 Cash offers.

```
# HW 16.10
HW10 <- read.table(url("https://raw.githubusercontent.com/npmlldabook/Stat3119/master/Week2/CH16PR10.txt"))
dim(HW10)
```

```
## [1] 36  3
```

```
# rename the variables
names(HW10) <- c("offer", "age", "dealer")
```

```
HW10$age <- as.factor(HW10$age)
```

```
str(HW10)
```

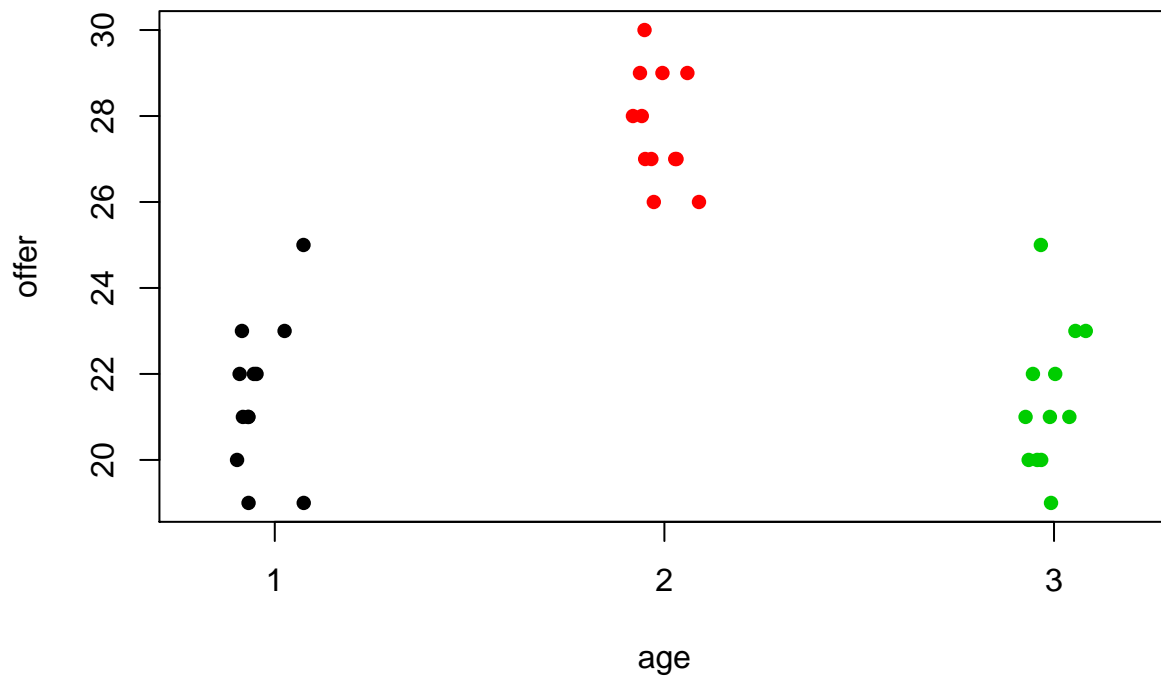
```
## 'data.frame':   36 obs. of  3 variables:
## $ offer : num  23 25 21 22 21 22 20 23 19 22 ...
## $ age   : Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 1 1 ...
## $ dealer: int   1 2 3 4 5 6 7 8 9 10 ...
```

```
head(HW10)
```

```
##   offer age dealer
## 1    23   1     1
## 2    25   1     2
## 3    21   1     3
## 4    22   1     4
## 5    21   1     5
## 6    22   1     6
```

a. aligned dot plots:

```
# add jitter since some values are the same
stripchart(offer ~ age, vertical = TRUE, data = HW10, xlab="age", method = "jitter", pch=16, col=1:3)
```



Results: the means appear to differ, and the variabilities appear to be similar.

b. Obtain the fitted values:

```
fit <- aov(offer ~ age, data = HW10)

# factor level means= fitted values
predict(fit, newdata = data.frame(age = factor(1:3)))
```

```
##          1          2          3
## 21.50000 27.75000 21.41667
```

c. Obtain the residuals :

```
# residuals
fit$residuals
```

```
##          1          2          3          4          5          6
## 1.5000000 3.5000000 -0.5000000 0.5000000 -0.5000000 0.5000000
##          7          8          9         10         11         12
## -1.5000000 1.5000000 -2.5000000 0.5000000 -2.5000000 -0.5000000
##          13         14         15         16         17         18
## 0.2500000 -0.7500000 -0.7500000 1.2500000 -1.7500000 1.2500000
##          19         20         21         22         23         24
## -0.7500000 2.2500000 0.2500000 -0.7500000 -1.7500000 1.2500000
##          25         26         27         28         29         30
## 1.5833333 -1.4166667 3.5833333 -0.4166667 0.5833333 1.5833333
##          31         32         33         34         35         36
## -0.4166667 -1.4166667 -2.4166667 -1.4166667 0.5833333 -0.4166667
```

d-f.

```
summary(fit)
```

```
##          Df Sum Sq Mean Sq F value    Pr(>F)
## age          2   316.7   158.36    63.6 4.77e-12 ***
## Residuals    33    82.2     2.49
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

$$H_a : \text{not all } \mu_i \text{ are equal.}$$

Results: $F^* = 63.6$ with a p-value < 0.001 , we reject H_0 and conclude H_a . The mean cash offers are significantly different among the different age groups.

HW 16.25

```
mu <- c(7,8,9)
sigma2<- 0.9^2
power.anova.test(groups = length(mu), n = 9, between.var = var(mu), within.var = sigma2)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##      groups = 3
##      n = 9
##      between.var = 1
##      within.var = 0.81
##      sig.level = 0.05
##      power = 0.9830563
##
## NOTE: n is number in each group
```

Results: The power of the test is 0.983.

HW 16.27

```
mu <- c(22,28,22)
sigma2<- 1.6^2
power.anova.test(groups = length(mu), n = 12, between.var = var(mu), within.var = sigma2)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##      groups = 3
##      n = 12
##      between.var = 12
##      within.var = 2.56
##      sig.level = 0.05
##      power = 1
##
## NOTE: n is number in each group
```

Results: The power of the test is about 1.

HW 16.29

We can define a power function that changes with Delta and Alpha

```
Power_Fun <- function( Delta=10 , Alpha=0.05){
  sigma = 10
  r = 5
  power.anova.test(groups = r, between.var = Delta^2/(2*(r-1)),
                    within.var = sigma^2, power = 0.95, sig.level = Alpha)
}
```

(a) Call the function for Delta= 10, 15,20,30 with with Alpha = 0.01

```
Power_Fun(Delta=10, Alpha=0.01)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##      groups = 5
##      n = 49.99829
##      between.var = 12.5
##      within.var = 100
##      sig.level = 0.01
##      power = 0.95
##
## NOTE: n is number in each group
```

```
Power_Fun(Delta=15, Alpha=0.01)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##      groups = 5
##      n = 22.98059
##      between.var = 28.125
##      within.var = 100
##      sig.level = 0.01
##      power = 0.95
##
## NOTE: n is number in each group
```

```
Power_Fun(Delta=20, Alpha=0.01)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##      groups = 5
##      n = 13.53809
##      between.var = 50
##      within.var = 100
##      sig.level = 0.01
##      power = 0.95
##
## NOTE: n is number in each group
```

```
Power_Fun(Delta=30, Alpha=0.01)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##      groups = 5
```



```
##           n = 6.824308
##   between.var = 112.5
##   within.var = 100
##   sig.level = 0.01
##   power = 0.95
##
## NOTE: n is number in each group
```

Results: For $\delta = 10, 15, 20, 30$ with $\alpha = 0.01$, the required sample sizes are 50, 23, 14, and 7, respectively, It suggested the sample size is decreasing with delta.

(b) Call the function for Delta= 10, 15,20,30 with Alpha = 0.05 (default in the function)

```
Power_Fun(Delta=10)
```

```
##
##   Balanced one-way analysis of variance power calculation
##
##   groups = 5
##   n = 38.10632
##   between.var = 12.5
##   within.var = 100
##   sig.level = 0.05
##   power = 0.95
##
## NOTE: n is number in each group
```

```
Power_Fun(Delta=15)
```

```
##
##   Balanced one-way analysis of variance power calculation
##
##   groups = 5
##   n = 17.4883
##   between.var = 28.125
##   within.var = 100
##   sig.level = 0.05
##   power = 0.95
##
## NOTE: n is number in each group
```

```
Power_Fun(Delta=20)
```

```
##
##   Balanced one-way analysis of variance power calculation
##
##   groups = 5
##   n = 10.28883
##   between.var = 50
##   within.var = 100
```

```
##      sig.level = 0.05
##      power = 0.95
##
## NOTE: n is number in each group
```

```
Power_Fun(Delta=30)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##      groups = 5
##      n = 5.187295
##      between.var = 112.5
##      within.var = 100
##      sig.level = 0.05
##      power = 0.95
##
## NOTE: n is number in each group
```

Results: For $\delta = 10, 15, 20, 30$ with $\alpha = 0.05$, the required sample sizes are 39, 18, 11, and 6, respectively, It suggested that the sample size is increasing with smaller α (type I error) if all the other assumptions are the same.