STAT 3119

Week 12: 11/14/2019 @GWU

Outline

- Three factor ANOVA models

- Analysis of Factor Effects (Ch 24.5)

- Setting: $n = 1$ (one case per treatment) or unbalanced cases n_{ijk}

- Sample size planning (Ch 24.6)

- Class final project (questions and discussion)

- Recitation: Homework and Quiz #4

Review: ANOVA Model for Three-Factor Studies

Let Y_{ijkm} denote the observation for the mth case or trial ($m = 1, ..., n$) for the treatment consisting of the ith level of A ($i = 1, ..., a$), the jth level of B ($j = 1, ..., b$), and the kth level of C ($k = 1, ..., c$). Thus, the total number of cases in the study is: $n_T = nabc$. (If the study is unbalanced, we have n_{ijk} with the factor level combinations- treatments level.)

1. **Cell Means Model**:

   The ANOVA model for a three-factor study in terms of the cell (treatment) means μ_{ijk} with fixed factor levels is:

   $$Y_{ijkm} = \mu_{ijk} + \varepsilon_{ijkm} \qquad (24.12)$$

   where:

   μ_{ijk} are parameters

   ε_{ijkm} are independent $N(0, \sigma^2)$

   $i = 1, \ldots, a; \; j = 1, \ldots, b; \; k = 1, \ldots, c; \; m = 1, \ldots, n$

2. **Factor Effects Model**: An equivalent factor effects model can be developed that incorporates the factorial structure by expressing each treatment mean μ_{ijk} in terms of the various factor effects.

$$Y_{ijkm} = \mu_{...} + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkm} \quad (24.14)$$

Analysis of Factor Effects (ch 24.5)

Simiar to two-factor studies, the focus of the analysis is usually

- on factor level means when no important interactions are present,

- on various two-factor means $\mu_{ij.}, \mu_{i.k}, \mu_{.jk}$ or individual cell means μ_{ijk} when there are important interactions.

Since a simple, parsimonious model is preferred, strategy for Analysis for analyzing factor effects in three-factor studies can be summarized in a flowchart (7 steps on p. 1013-1014):
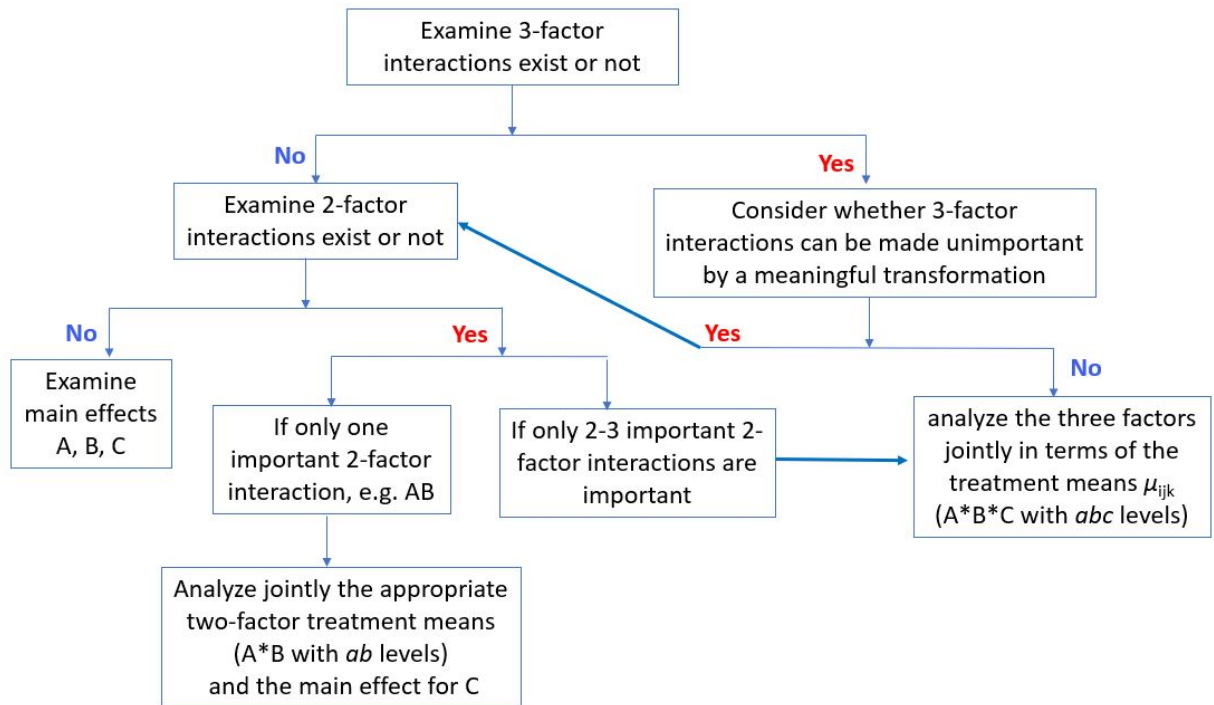
Example data: Stress Test

- Easy to idenfiy the one-factor level, two-factor level means and treatment (3-factor level) means from the data table

TABLE 24.4
Sample Data and Estimated Treatment and Factor Level Means for Three-Factor Study—Stress Test Example.

(a) Data

	Smoking History	
	$k = 1$ Light	$k = 2$ Heavy
$j = 1$ Low fat:		
$i = 1$ Male	24.1 (Y_{1111})	17.6 (Y_{1121})
	29.2 (Y_{1112})	18.8 (Y_{1122})
	24.6 (Y_{1113})	23.2 (Y_{1123})
$i = 2$ Female	20.0 (Y_{2111})	14.8 (Y_{2121})
	21.9 (Y_{2112})	10.3 (Y_{2122})
	17.6 (Y_{2113})	11.3 (Y_{2123})
$j = 2$ High fat:		
$i = 1$ Male	14.6 (Y_{1211})	14.9 (Y_{1221})
	15.3 (Y_{1212})	20.4 (Y_{1222})
	12.3 (Y_{1213})	12.8 (Y_{1223})
$i = 2$ Female	16.1 (Y_{2211})	10.1 (Y_{2221})
	9.3 (Y_{2212})	14.4 (Y_{2222})
	10.8 (Y_{2213})	6.1 (Y_{2223})

Analysis of Factor Effects when Factors Do Not Interact

Estimation of factor level means:

If two-way and three-way interactions are not important, we can estimate factor level means, separately. For example, for estimate the factor A level mean $\mu_{i..}$,

$$\hat{\mu}_{i..} = \bar{Y}_{i...} \tag{24.27}$$

The estimated variance of this estimator is:

$$s^2\{\bar{Y}_{i...}\} = \frac{MSE}{nbc} \tag{24.28}$$

Confidence limits for $\mu_{i..}$ are obtained by means of the t distribution with $(n - 1)abc$ degrees of freedom:

$$\bar{Y}_{i...} \pm t[1 - \alpha/2; (n - 1)abc]s\{\bar{Y}_{i...}\} \tag{24.29}$$

Note:

These inference procedures are essentially the same as before (one-factor study in Ch 17) except we need to replace the total sample size n_T, treatment levels (r), and the sample size for given factor level (n_i) on the 3-factor study setting.

Inferences for Contrast of Factor Level Means: e.g. for factor A.

3

Inference procedures for a contrast involving the factor A level means $\mu_{i..}$

$$L = \sum c_i \mu_{i..} \qquad \text{where} \qquad \sum c_i = 0 \qquad \text{(24.30)}$$

are easily developed. The $1 - \alpha$ confidence limits for L are:

$$\hat{L} \pm t[1 - \alpha/2; (n-1)abc]s\{\hat{L}\} \qquad \text{(24.31)}$$

where L is estimated unbiasedly by:

$$\hat{L} = \sum c_i \bar{Y}_{i...} \qquad \text{(24.31a)}$$

$$s^2\{\hat{L}\} = \frac{MSE}{nbc} \sum c_i^2$$

The test statistic and decision rule for the following alternatives concerning a contrast L in (24.30):

$$H_0: L = 0$$
$$H_a: L \neq 0 \qquad \text{(24.32)}$$

are:

$$t^* = \frac{\hat{L}}{s\{\hat{L}\}}; \text{If } |t^*| > t[1 - \alpha/2; (n-1)abc], \text{conclude } H_a \qquad \text{(24.33)}$$

When study multiple contrasts of factor level means, we can apply the Tukey, Scheffe, and Bonferroni procedure (the Tukey procedure concerns only pairwise comparisons).

To obtain simultaneous confidence interval estimates, the t multiple in the CI is replaced by the T, S, B multiple defined as follows:

Procedure	Multiple	
Tukey	$T = \frac{1}{\sqrt{2}}q[1 - \alpha; a, (n-1)abc]$	(24.34a)
Scheffé	$S^2 = (a-1)F[1 - \alpha; a-1, (n-1)abc]$	(24.34b)
Bonferroni	$B = t[1 - \alpha/2g; (n-1)abc]$	(24.34c)

Inference for factor level means and contrast for factors B or C is done similarly.

Analysis of Factor Effects with >one 2-way interactions or 3-way interaction

In such cases, the results of the study are typically analyzed in terms of the treatment means μ_{ijk}.

Estimation of Treatment Mean

$$\hat{\mu}_{ijk} = \bar{Y}_{ijk\cdot} \tag{24.36}$$

The estimated variance of $\bar{Y}_{ijk\cdot}$ is:

$$s^2\{\bar{Y}_{ijk\cdot}\} = \frac{MSE}{n} \tag{24.37}$$

Confidence limits for μ_{ijk} are:

$$\bar{Y}_{ijk\cdot} \pm t[1 - \alpha/2; (n-1)abc]s\{\bar{Y}_{ijk\cdot}\} \tag{24.38}$$

Inferences for Contrast of Treatment Means

The linear combination will have **triple** summations over the i, j, k levels for factor A, B and C.

$$L = \sum\sum\sum c_{ijk}\mu_{ijk} \quad \text{where} \quad \sum\sum\sum c_{ijk} = 0 \tag{24.39}$$

Confidence limits for L are:

$$\hat{L} \pm t[1 - \alpha/2; (n-1)abc]s\{\hat{L}\} \tag{24.40}$$

where:

$$\hat{L} = \sum\sum\sum c_{ijk}\bar{Y}_{ijk\cdot} \tag{24.40a}$$

$$s^2\{\hat{L}\} = \frac{MSE}{n}\sum\sum\sum c_{ijk}^2 \tag{24.40b}$$

The test statistic and decision rule for alternatives H_0: $L = 0$, H_a: $L \neq 0$ are:

$$t^* = \frac{\hat{L}}{s\{\hat{L}\}}; \text{If } |t^*| > t[1 - \alpha/2; (n-1)abc], \text{ conclude } H_a \tag{24.41}$$

Multiple Contrasts of Treatment Means

For simultaneous interval estimates of contrasts of treatment means μ_{ijk}, the t multiple in (24.40) is replaced by the T, S, B multiple defined as follows:

Procedure	Multiple	
Tukey	$T = \dfrac{1}{\sqrt{2}}q[1 - \alpha; ABC, (n-1)abc]$	(24.45a)
Scheffé	$S^2 = (abc - 1)F[1 - \alpha; abc - 1, (n-1)abc]$	(24.45b)
Bonferroni	$B = t[1 - \alpha/2g; (n-1)abc]$	(24.45c)

These formulas are also the same as before if we replace the $n_T = nabc$ and $r = abc$ to fit the 3 study setting.

Analysis of Factor Effects with Single Important 2-Factor Interaction

For example, when the only interactions present are the BC interactions, there may be interest in contrasts of the means $\mu_{\cdot jk}$ since we need to look at the joint effect of BC treatment levels.

The linear combination will have **double** summation over the j, k levels for factor B and C.

$$L = \sum\sum c_{jk}\mu_{\cdot jk} \qquad \text{where} \qquad \sum\sum c_{jk} = 0 \qquad (24.42)$$

Such contrasts are, of course, special cases of contrasts of the treatment means μ_{ijk} in (24.39). The estimator of the contrast in (24.42) can be obtained from (24.40a) and the estimated variance from (24.40b); they are:

$$\hat{L} = \sum\sum c_{jk}\overline{Y}_{\cdot jk\cdot} \qquad (24.43)$$

$$s^2\{\hat{L}\} = \frac{MSE}{na}\sum\sum c_{jk}^2 \qquad (24.44)$$

With the point estimate and SE, then we can make inferences to get CI and testing with necessary multiple comparison adjustment.

Example (continued): Stress Test (p.1018)

In the previous analysis, we find 1) no significant 3-way interaction; 2) the gender (factor A) had no significant interaction with factor B (body fat) and factor C (smoking history). 3) only one significant two-way interaction between factor B and C.

Now we can study nature of the BC interaction effects in this example: the researcher wished to estimate separately, for persons with high and low body fat (given j), the difference in mean fatigue time for light smokers and heavy smokers ($k = 1$ vs. $k = 2$). The desired contrasts are:

$$L_1 = \mu_{\cdot 11} - \mu_{\cdot 12}$$
$$L_2 = \mu_{\cdot 21} - \mu_{\cdot 22}$$

In addition, a single comparison between the factor level means for factor A is sufficient to analyze the factor A main effects since factor A has only two levels. This contrast of interest is:

$$L_3 = \mu_{1\cdot\cdot} - \mu_{2\cdot\cdot}$$

We can then estimate these contrast using one-factor or two factor level means, and using the Bonferroni method to adjust for mutiple comparisons $g = 3$.

$$\hat{L}_1 = \bar{Y}_{.11.} - \bar{Y}_{.12.} = 22.90 - 16.00 = 6.90$$

$$\hat{L}_2 = \bar{Y}_{.21.} - \bar{Y}_{.22.} = 13.07 - 13.12 = -.05$$

$$\hat{L}_3 = \bar{Y}_{1...} - \bar{Y}_{2...} = 18.98 - 13.56 = 5.42$$

The researcher obtained the estimated variances by using (24.44) and (24.31b) and the Bonferroni multiple for a 95 percent family confidence coefficient:

$$s^2\{\hat{L}_1\} = s^2\{\hat{L}_2\} = \frac{MSE}{na}[(1)^2 + (-1)^2] = \frac{9.335}{6}(2) = 3.112$$

$$s^2\{\hat{L}_3\} = \frac{MSE}{nbc}[(1)^2 + (-1)^2] = \frac{9.335}{12}(2) = 1.556$$

$$s\{\hat{L}_1\} = s\{\hat{L}_2\} = 1.764 \qquad s\{\hat{L}_3\} = 1.247$$

$$B = t(1 - .05/6; 16) = 2.673$$

R Implementation

1. Make BC as a combined factor

```r
str(Ex24)
```

```
## 'data.frame':    24 obs. of  5 variables:
##  $ Response: num  24.1 29.2 24.6 20 21.9 17.6 14.6 15.3 12.3 16.1 ...
##  $ Gender  : Factor w/ 2 levels "M","F": 1 1 1 2 2 2 1 1 1 2 ...
##  $ Body_Fat: Factor w/ 2 levels "Low","High": 1 1 1 1 1 1 2 2 2 2 ...
##  $ Smoking : Factor w/ 2 levels "Light","Heavy": 1 1 1 1 1 1 1 1 1 1 ...
##  $ units   : int  1 2 3 1 2 3 1 2 3 1 ...
```

```r
# make BC as a combination factor
Ex24$Body_Fat_Smoke[Ex24$Body_Fat=="Low"  &  Ex24$Smoking=="Light"] <- "1.Low-Light"
Ex24$Body_Fat_Smoke[Ex24$Body_Fat=="Low"  &  Ex24$Smoking=="Heavy"] <- "2.Low-Heavy"
Ex24$Body_Fat_Smoke[Ex24$Body_Fat=="High" & Ex24$Smoking=="Light" ] <- "3.High-Light"
Ex24$Body_Fat_Smoke[Ex24$Body_Fat=="High" & Ex24$Smoking=="Heavy" ] <- "4.High-Heavy"
table(Ex24$Body_Fat_Smoke)
```

```
## 
##  1.Low-Light  2.Low-Heavy 3.High-Light 4.High-Heavy
##            6            6            6            6
```

2. Get the inference of factor BC and factor A

```r
library(emmeans)
fit = lm( Response ~ Gender +Body_Fat_Smoke, data= Ex24)
Est.meanBC <- emmeans(fit, ~ Body_Fat_Smoke)
 # For L1 and L2
```

```
#Set the c1-c6 corresponding to treatment levels
L = list( L1= c( 1, -1, 0, 0),     #L1=mu11-mu12
          L2 = c(0, 0 ,1, -1))    #L1=mu21-mu22
contrast(Est.meanBC, L, adjust='none')
```

```
##  contrast estimate   SE df t.ratio p.value
##  L1             6.90 1.76 19  3.927  0.0009
##  L2            -0.05 1.76 19 -0.028  0.9776
##
## Results are averaged over the levels of: Gender
```

```
Est.meanA <- emmeans(fit, ~ Gender)
#L3
pairs(Est.meanA )
```

```
##  contrast estimate   SE df t.ratio p.value
##  M - F          5.42 1.24 19 4.367    0.0003
##
## Results are averaged over the levels of: Body_Fat_Smoke
```

Therefore we can get the point estimate and SE from functions in emmeans. We still the get the Bonferroni multiple, then get CI.

```
#Bonferroni multiple
g=3 #
df= 16
qt(1- .05/(2*g), df)
```

```
## [1] 2.673032
```

The desired confidence intervals using (24.40) therefore are:

$$2.2 = 6.90 - 2.673(1.764) \leq \mu_{\cdot 11} - \mu_{\cdot 12} \leq 6.90 + 2.673(1.764) = 11.6$$
$$-4.8 = -.05 - 2.673(1.764) \leq \mu_{\cdot 21} - \mu_{\cdot 22} \leq -.05 + 2.673(1.764) = 4.7$$
$$2.1 = 5.42 - 2.673(1.247) \leq \mu_{1\cdot\cdot} - \mu_{2\cdot\cdot} \leq 5.42 + 2.673(1.247) = 8.8$$

Finally, based on these inference resuots, we can conclude family confidence coefficient 0.95:

1) Among people with low body fat, those who have a light smoking history have a mean stress test endurance that is 2.2 to 11.6 minutes longer than the mean endurance for people with a heavy smoking history.

2) People with high body fat do not differ in mean stress test endurance whether they have a light or a heavy smoking history.

3) The mean stress test endurance for men is 2.1 to 8.8 minutes longer than the mean endurance for women.

These results can also be displayed in a graph for gender and combined body fat*smoking.

FIGURE 24.8 Key Findings from Stress Test Endurance Study.

(a) Effect of Gender

(b) Effects of Body Fat and Smoking History
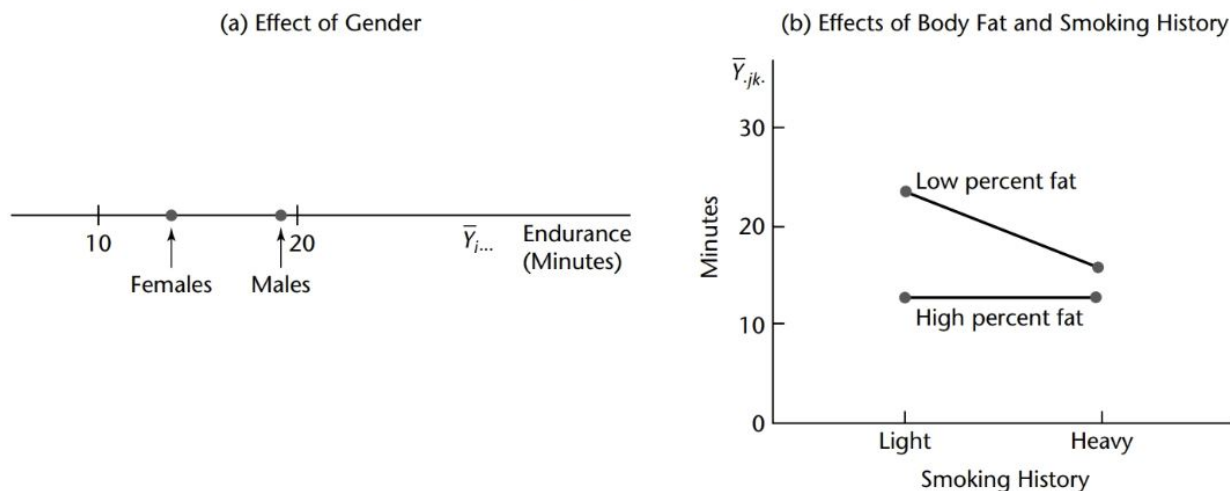
Special cases: $n = 1$ (p. 1011)

Similar to what we discussed in Chapter 20, due to constraints on cost, time, materials, etc, some studies only allow one replication/case per treatment (ABC combination).

In such situation, we can't estimate the MSE using the standard 3-way ANOVA table (no within treatment data to estimate the errors, used up the df). Then the analysis of variance tests can only be conducted if it is possible to assume that **some interactions** equal zero. Usually, the interactions most likely to equal zero are the three-factor interactions (keep the first order interactions). If it is possible to assume that all three-factor interactions equal zero, MSABC has expectation σ^2 and plays the role of the error mean square MSE. All mean squares are calculated in the usual manner, except that $n = 1$.

More general cases: Unbalanced 3-factor study with n_{ijk} (Ch 24.6)

We can follow the procedures explained in Sections 23.1–23.3 for two-factor studies with unequal treatment sample sizes should be followed with routine modifications.

- Indicator variables taking on the values 1, -1, 0, are designated for each factor, the number of such variables for each factor being one less than the number of factor levels. (If only 2 levels for the factor, need one indicator function with 1 and -1 to denote the two levels.)

- Interaction effects are represented by cross-product terms.

- Since the sums of squares are no longer orthogonal when the treatment sample sizes are unequal. We can use the F-test from MS ratio, and we will use different reduced models need to be fitted for the tests of interest (or use SS type III table generated from software).

Example: Stress test example of Table 24.4: observations Y_{1113} and Y_{2212} were missing. Since each of the three factors is at two levels, one indicator variable is required for each factor to set up a regression model as follows:

$$Y_{ijkm} = \mu_{...} + \alpha_1 X_{ijkm1} + \beta_1 X_{ijkm2} + \gamma_1 X_{ijkm3} + (\alpha\beta)_{11} X_{ijkm1} X_{ijkm2}$$
$$+ (\alpha\gamma)_{11} X_{ijkm1} X_{ijkm3} + (\beta\gamma)_{11} X_{ijkm2} X_{ijkm3}$$
$$+ (\alpha\beta\gamma)_{111} X_{ijkm1} X_{ijkm2} X_{ijkm3} + \varepsilon_{ijkm} \qquad \text{Full model} \qquad (24.47)$$

R implementation

```
#  Remove 2 observations (assume missing) to get unbalanced data
Ex24B = Ex24[-c(3,11),]

# set indicator function
Ind.A =  (Ex24B$Gender=="M")*1+ (Ex24B$Gender=="F")*(-1)
Ind.B =  (Ex24B$Body_Fat=="Low")*1+ (Ex24B$Body_Fat=="High")*(-1)
Ind.C =  (Ex24B$Smoking=="Light")*1+ (Ex24B$Smoking=="Heavy")*(-1)


Model.full = lm(Response~ Ind.A*Ind.B*Ind.C, data= Ex24B )
summary(Model.full)
```

```
##
## Call:
## lm(formula = Response ~ Ind.A * Ind.B * Ind.C, data = Ex24B)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.1000 -2.1333 -0.4667  2.4292  4.3667
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       16.5292     0.6725  24.578 6.47e-13 ***
## Ind.A              2.6250     0.6725   3.903 0.001592 **
## Ind.B              3.0917     0.6725   4.597 0.000415 ***
## Ind.C              1.9708     0.6725   2.931 0.010961 *
## Ind.A:Ind.B        1.0125     0.6725   1.506 0.154415
## Ind.A:Ind.C       -0.7667     0.6725  -1.140 0.273421
## Ind.B:Ind.C        1.6500     0.6725   2.453 0.027859 *
## Ind.A:Ind.B:Ind.C  0.5375     0.6725   0.799 0.437508
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.106 on 14 degrees of freedom
## Multiple R-squared:  0.7821, Adjusted R-squared:  0.6731
## F-statistic: 7.178 on 7 and 14 DF,  p-value: 0.0009318
```

We can set up a reduced model to test each effect with full model as discussed in Chapter 24.
Or equivalently, we can generate type III SS table to test all the factors.

```
library(car)
```

```
## Loading required package: carData
```

```
Anova(Model.full, type=3)
```

```
## Anova Table (Type III tests)
##
## Response: Response
##                  Sum Sq Df  F value     Pr(>F)
## (Intercept)      5828.6  1 604.0695 6.473e-13 ***
## Ind.A             147.0  1  15.2350 0.0015917 **
## Ind.B             203.9  1  21.1334 0.0004147 ***
## Ind.C              82.9  1   8.5879 0.0109607 *
## Ind.A:Ind.B        21.9  1   2.2666 0.1544151
## Ind.A:Ind.C        12.5  1   1.2996 0.2734211
## Ind.B:Ind.C        58.1  1   6.0194 0.0278585 *
## Ind.A:Ind.B:Ind.C   6.2  1   0.6388 0.4375082
## Residuals         135.1 14
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Unbalanced studies: Inferences for Contrasts of Factor Level Means

Estimation and testing of contrasts of factor level means in multi-factor studies with unequal sample sizes are conducted in similar fashion as for two-factor studies. The formulas in Table 23.5 (Lecture 11A) need simply be extended to three or more factors. The linear combination of the factor or treatment level means are still estimated based on sample means, instead of n within treatment and we just need to replace it with n_{ijk} and use the correct df for MSE.

For example, to get the paired comparison for factor level A:

$$D = \mu_{i..} - \mu_{i'..} \qquad\qquad\qquad (24.48a)$$

$$\hat{D} = \hat{\mu}_{i..} - \hat{\mu}_{i'..} \qquad \text{where} \qquad \hat{\mu}_{i..} = \frac{\sum_j \sum_k \bar{Y}_{ijk\cdot}}{bc} \qquad (24.48b)$$

$$s^2\{\hat{D}\} = \frac{MSE}{b^2 c^2} \sum_j \sum_k \left(\frac{1}{n_{ijk}} + \frac{1}{n_{i'jk}} \right) \qquad (24.48c)$$

The appropriate degrees of freedom associated with MSE are $n_T - abc$.

Sample size planning (Ch 24.7)

We considered the planning of sample sizes for single-factor studies with power approach and estimation approach in Chapters 16 and 17. Then we considered the planning of sample sizes for two-factor studies in Chapter 19. Here we discuss the planning of samples sizes for multi-factor studies.

Sample size based on power:

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- When planning sample sizes for three-factor studies with the power approach, one is typically concerned with the power of detecting factor A main effects, the power of detecting factor B main effects, and the power of detecting factor C main effects.

- Most of studies are not designed to test the interactions because: 1) It would need a large sample size to detect a weak interaction; 2) if the interaction effect is known to be very strong between two factors (e.g. $a = b = 2$), the investigator can target to study the four treatment groups, A1B1, A1B2, A2B1, and A2B2, instead and test their differences among the four treatment groups other than A*B interaction.

- Steps for calculate the sample size:

1. One can first specify the minimum range of factor A level means for which it is important to detect factor A main effects and obtain the needed sample sizes as in a one-factor study, with $r = a$. The resulting sample size for each group A is $n_A = bcn_{0A}$ for a balanced factorial design with 3 factors, then from which n_{0A} can be obtained readily for each A_i, B_j, C_k treatment level.

2. In the same way, the values for the minimum range of factor level means for factors B and C can be specified for which it is important to detect the factor main effects, and we can find the needed sample sizes n for each A_i, B_j, C_k treatment level, denoted by n_{0B} and n_{0C}.

3. If the sample sizes obtained from the factor A, B, C, are quite different (e.g. $n_{0A} = 5$, $n_{0B} = 10$ and $n_{0C} = 15$), then a judgment will need to be made as to the final sample sizes.

   - If only one factor is the primary factor to study, we can use the sample size adequate to determine the given factor with the chose significant level α for the study.

   - If 2 factors are co-primary factors to study, we can use the sample size adequate to determine the both factors with the $\alpha_0 = \alpha/2$ for each factor when calculate the sample size, where α is the overall type I error for the study.

   - If all 3 factors are co-primary factors to study, we can use the sample size adequate to determine all 3 factors with the $\alpha_0 = \alpha/3$ for each factor when calculate the sample size, where α is the overall type I error for the study.

Summary this week

1. Reading Chapter 24

2. Homework (due 11/21 before class)

- 24.6, 24.7, 24.8, 24.15, 24.17
- all based on the **Case Hardening** data: CH24PR06.txt in Week-12 folder
- Note: read HW data:

```r
HW6 = read.table(url(
  "https://raw.githubusercontent.com/npmldabook/Stat3119/master/Week-12/CH24PR06.txt"))
names(HW6) = c("response", "factorA", "factorB", "factorC", "Units")

# For 24.15
HW6B = HW6[-(7:8),]
```

3. Quiz on Chapter 23-24 on 11/21 (next Thursday)

   - How to set the regression models to estimate the model parameters
   - testing and inference on factor/treatment means including multiple comparison adjustment

Project discussion and questions

Submit online from blackboard: There is a **Class Project** in the Assignment.

Instruction: https://github.com/npmldabook/Stat3119/tree/master/Project

1. Decide the data set

2. The Introduction/background of the data and research questions

3. The statistical methods

4. Analysis and results