STAT 3119

Week 14: 11/26/2019 @GWU

Outline

- Quiz #5 Solution
- Note for Final exam
- RCBD: random block effects (Ch 25.5)
- Three factor Studies (Ch 25.6)
- ANOVA II-III with unequal sample size (Ch 25.7)

Final Exam Note :

- Materials Covered before Midterm: (30-40%) Chapter 15-21
- Materials Covered After Midterm: (60-70%) : Chapter 22-25, plus selected design concepts in chapter 26-28
- Final Review session on Dec 3rd (Tues); Last class on Dec 5th (Thurs)
- Test on Dec 12th evening.
- Update: closed book test, you can bring **2 pages** (8.5 x 11 in paper) of notes only. (If you have the one prepared for mid-term and have another one for the later chapters.)

Randomized Complete Block Design: random block (Ch 25.5, p.1060)

We studied RCBD design with *fixed* blocked effects in Chapter 21. However, when blocks are a random sample from a population, the block effects in the RCBD study should be considered to be random variables, as in the following two examples.

Example 1:

- A researcher investigated the improvement in learning in third-grade classes by augmenting the teacher with one or two teaching assistants. Ten schools were selected at random, and three third-grade classes in each school were utilized in the study. In each school,
  - 1 class was randomly chosen to have no teaching assistant,
  - 1 class was randomly chosen to have one teaching assistant,
  - the 3rd class was assigned two teaching assistants.
- The amount of learning by the class at the end of the school year, suitably measured, was the response variable.
- Here the *blocks* are schools, which may be viewed as a random sample from the population of all schools eligible for the study. We don't want to make inference for those 10 schools only.

Example 2:

- In a study of the effectiveness of four different dosages of a drug, 20 litters of mice, each consisting of four mice, were utilized.
- The 20 litters (blocks) here may be viewed as a random sample from the population of all litters that could have been used for the study.

Random Block Effects Model

- For fixed block effects model with no replication (one set of treatments each block), we have no df to estimate interaction effects in the ANOVA model.
- When blocks are considered to be a random sample from a population of blocks, either an additive (i.e., no interaction) or a non-additive (i.e., interaction) model can be employed. We can decide whether the interaction is important using
  - The interaction plot (the treatment*block plot)
  - Tukey 1-df test for interactions
- When the primary emphasis of the analysis is on testing and estimating treatment effects, which is the usual case, the choice between the two models actually is not critical. They both are special cases of two-factor mixed model (25.42).

RCBD with random block: Additive Model

The additive model for random block effects and fixed treatment effects is a special case of mixed two-factor model:with $n = 1$, the interaction term dropped and fixed factor A effects now being the treatment effects denoted by τ_j and random factor B effects now being the block effects denoted by ρ_i:

$$Y_{ij} = \mu_{..} + \rho_i + \tau_j + \varepsilon_{ij} \quad \textbf{(25.67)}$$

where:

$\mu_{..}$ is a constant

ρ_i are independent $N(0, \sigma_\rho^2)$

τ_j are constants subject to the restriction $\sum \tau_j = 0$

ε_{ij} are independent $N(0, \sigma^2)$, and independent of the ρ_i

$i = 1, \ldots, n_b; j = 1, \ldots, r$

Properties of Model. The important properties of mixed two-factor model (25.42) were given in (25.44)–(25.46). These properties for randomized complete block design model (25.67) are:

$$E\{Y_{ij}\} = \mu_{..} + \tau_j \quad \textbf{(25.68a)}$$

$$\sigma^2\{Y_{ij}\} = \sigma_Y^2 = \sigma_\rho^2 + \sigma^2 \quad \textbf{(25.68b)}$$

$$\sigma\{Y_{ij}, Y_{ij'}\} = \sigma_\rho^2 \qquad j \neq j' \quad \textbf{(25.68c)}$$

$$\sigma\{Y_{ij}, Y_{i'j'}\} = 0 \qquad i \neq i' \quad \textbf{(25.68d)}$$

- The coefficient of correlation between any two observations from the same block for model (25.67) is constant for all blocks, denoted by $\omega = \sigma_\rho^2/\sigma_Y^2$. In this setting, the variance-covariance structure of the data is called *compound symmetry*.
- For confidence intervals for treatment contrasts also present no new issues. Again, MSBL.TR will be used as the mean square in the estimated variance of the contrast. We can use the same approach discussed in Chapter 21.

RCBD with random block: Interaction Model

When blocks are a random sample from a population of blocks, the presence of interactions between blocks and treatments can be accommodated by a model including these interaction effects. (*This is because we only need to estimate one parameter σ_ρ to describe the block effects in the random block model. However, in the fixed effects model, we need to estimate $n_b - 1$ fixed block effects ρ_i. We have extra df in the random effects model to estimate the interaction parameters.*)

$$Y_{ij} = \mu_{..} + \rho_i + \tau_j + (\rho\tau)_{ij} + \varepsilon_{ij} \tag{25.74}$$

where:

$\mu_{..}$ is a constant

ρ_i are independent $N(0, \sigma_\rho^2)$

τ_j are constants subject to the restriction $\sum \tau_j = 0$

$(\rho\tau)_{ij}$ are $N\left(0, \frac{r-1}{r}\sigma_{\rho\tau}^2\right)$, subject to the restrictions:

$\sum_j (\rho\tau)_{ij} = 0$ for all i

$\sigma\{(\rho\tau)_{ij}, (\rho\tau)_{ij'}\} = -\frac{1}{r}\sigma_{\rho\tau}^2$ for $j \neq j'$

$(\rho\tau)_{ij}$ are independent of the ρ_i

ε_{ij} are independent $N(0, \sigma^2)$ and independent of the ρ_i and of the $(\rho\tau)_{ij}$

$i = 1, \ldots, n_b; j = 1, \ldots, r$

Properties of Model. The properties of interaction model (25.74) are obtained directly from those in (25.44)–(25.46) for the mixed two-factor model:

$$E\{Y_{ij}\} = \mu_{..} + \tau_j \tag{25.75a}$$

$$\sigma^2\{Y_{ij}\} = \sigma_Y^2 = \sigma_\rho^2 + \frac{r-1}{r}\sigma_{\rho\tau}^2 + \sigma^2 \tag{25.75b}$$

$$\sigma\{Y_{ij}, Y_{ij'}\} = \sigma_\rho^2 - \frac{1}{r}\sigma_{\rho\tau}^2 \qquad j \neq j' \tag{25.75c}$$

$$\sigma\{Y_{ij}, Y_{i'j'}\} = 0 \qquad i \neq i' \tag{25.75d}$$

Note again that the Y_{ij} have constant variance, that observations from different blocks are assumed to be independent, and that any two observations Y_{ij} and $Y_{ij'}$ from the same block are correlated, the covariance being the same for all blocks.

Note: For both additive or interaction model for RCBD with random block design:

- In advance of the experiment, the block is a random variable to be randomly sampled from the given population, and any two observations in a given block are correlated since they share a same block effect and tended to be similar.
- For the given study, once the block level is chosen, the only remaining random variation in an observation is from the residual error terms. Any two observations within the given block is **conditionally independent**.

RCBD with random block: ANOVA table, test and inference

TABLE 25.8 ANOVA for Randomized Complete Block Design—Block Effects Random, Treatment Effects Fixed.

Source of Variation	SS	df	MS	E{MS} Additive Model (25.67)	E{MS} Interaction Model (25.74)
Blocks	$SSBL$	$n_b - 1$	$MSBL$	$\sigma^2 + r\sigma_\rho^2$	$\sigma^2 + r\sigma_\rho^2$
Treatments	$SSTR$	$r - 1$	$MSTR$	$\sigma^2 + n_b \frac{\sum \tau_j^2}{r-1}$	$\sigma^2 + \sigma_{\rho\tau}^2 + n_b \frac{\sum \tau_j^2}{r-1}$
Error	$SSBL.TR$	$(n_b - 1)(r - 1)$	$MSBL.TR$	σ^2	$\sigma^2 + \sigma_{\rho\tau}^2$
Total	$SSTO$	$n_b r - 1$			

ANOVA table terms are the same. Although the EMS are different for fixed, random or mixed effect model. However, the $E(MSTR)$ always has an extra term of treatment effects compared to $E(MSBL.TR)$, therefore the F-test and the distributions for the treatment effects

$$F^* = MSTR/MSBL.TR \sim F(r-1, (n_b-1)(r-1))$$

are the same for ANOVA model I-III.

- We can test the random block effect in additive model by comparing MSBL vs. MSBL.TR , but there is no exact test in the interaction model. However, block is used to reduce errors/confounding and the testing of block difference is not an important object of the study.

Three-factor studies: ANOVA model II-III (Ch 25.6)

- Fixed effects model for 3 factor studies discussed in Chapter 24.
- For balanced three-way ANOVA studies with 1-3 random factors, the ANOVA table is the same but we need to study the **EMS** for different models to construct the correct test.
- For estimation and inference on variance components and fixed effects, we can follow the examples for two-factor studies to specify the fixed and random components in the formula statement of the statistical software, e.g. **R lmer()** function from **lme4** package.

ANOVA Model II—Random Factor Effects

In a study of the effects of operators, machines, and batches of raw material on daily output, all three factors may be considered to have random factor levels. The random ANOVA model for such a three-factor study is:

$$Y_{ijkm} = \mu_{...} + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkm} \quad \textbf{(25.77)}$$

where:

$\mu_{...}$ is a constant

$\alpha_i, \beta_j, \gamma_k, (\alpha\beta)_{ij}, (\alpha\gamma)_{ik}, (\beta\gamma)_{jk}, (\alpha\beta\gamma)_{ijk}, \varepsilon_{ijkm}$ are independent normal random variables with expectations zero and respective variances $\sigma_\alpha^2, \sigma_\beta^2, \sigma_\gamma^2, \sigma_{\alpha\beta}^2, \sigma_{\alpha\gamma}^2, \sigma_{\beta\gamma}^2, \sigma_{\alpha\beta\gamma}^2, \sigma^2$

$i = 1, \ldots, a; j = 1, \ldots, b; k = 1, \ldots, c; m = 1, \ldots, n$

Just as for two-factor random ANOVA model (25.39), the responses Y_{ijkm} for three-factor random ANOVA model (25.77) are normally distributed with constant variance. The expected value and variance of response Y_{ijkm} are:

$$E\{Y_{ijkm}\} = \mu_{...} \quad \textbf{(25.78a)}$$

$$\sigma^2\{Y_{ijkm}\} = \sigma_Y^2 = \sigma_\alpha^2 + \sigma_\beta^2 + \sigma_\gamma^2 + \sigma_{\alpha\beta}^2 + \sigma_{\alpha\gamma}^2 + \sigma_{\beta\gamma}^2 + \sigma_{\alpha\beta\gamma}^2 + \sigma^2 \quad \textbf{(25.78b)}$$

Any two responses are independent except when they have one or more common factor levels; these latter are correlated because they contain some common random terms.

ANOVA Model II: Hypothesis testing

TABLE 25.9 Expected Mean Squares for Balanced Random Three-Factor ANOVA Model (25.77).

Mean Square	df	Expected Mean Square
MSA	$a-1$	$\sigma^2 + nbc\sigma_\alpha^2 + nc\sigma_{\alpha\beta}^2 + nb\sigma_{\alpha\gamma}^2 + n\sigma_{\alpha\beta\gamma}^2$
MSB	$b-1$	$\sigma^2 + nac\sigma_\beta^2 + nc\sigma_{\alpha\beta}^2 + na\sigma_{\beta\gamma}^2 + n\sigma_{\alpha\beta\gamma}^2$
MSC	$c-1$	$\sigma^2 + nab\sigma_\gamma^2 + nb\sigma_{\alpha\gamma}^2 + na\sigma_{\beta\gamma}^2 + n\sigma_{\alpha\beta\gamma}^2$
$MSAB$	$(a-1)(b-1)$	$\sigma^2 + nc\sigma_{\alpha\beta}^2 + n\sigma_{\alpha\beta\gamma}^2$
$MSAC$	$(a-1)(c-1)$	$\sigma^2 + nb\sigma_{\alpha\gamma}^2 + n\sigma_{\alpha\beta\gamma}^2$
$MSBC$	$(b-1)(c-1)$	$\sigma^2 + na\sigma_{\beta\gamma}^2 + n\sigma_{\alpha\beta\gamma}^2$
$MSABC$	$(a-1)(b-1)(c-1)$	$\sigma^2 + n\sigma_{\alpha\beta\gamma}^2$
MSE	$(n-1)abc$	σ^2

We need to examine the EMS column to construct the F-test

Exact F-test:

- For three-way interaction, it is the same F test to compare MSABC vs. MSE.
- For two-way interactions AB, AC and BC, we can test their effects by using their MS divided by MSABC to obtain the F-test with the dfs from the corresponding numerator and denominator MS.
- However, there are no exact F for testing the random main effects for factor A, B, C. Those MS have at least one extra components in addition to the variance components for the specific factor. We have to use Approximate F test.

Approximate F-test (Pseudo F-test):

- We first use a linear combination MS^* of MS to obtain the correct denominator MS that contains the variance component without just an extra term to describe the effects for the given factor to be tested, compared to MSA, MSB, or MSC

$$MS^* = c_1 MS_1 + ... + c_h MS_h$$

If df_i denotes the df associated with MS_i, then the degree of freedom for the MS^* is given by,

$$df = \frac{(c_1 MS_1 + \cdots + c_h MS_h)^2}{\frac{(c_1 MS_1)^2}{df_1} + \cdots + \frac{(c_h MS_h)^2}{df_h}} \qquad \textbf{(25.28)}$$

- Then use MS^* as the denominator to compute the test statistics F^{**} with numerator MSA, MSB, MSC.
- For example, to test for main effects of factor A, $H_0 : \sigma^2_\alpha = 0$ vs. $H_a : \sigma^2_\alpha > 0$. we need to use

$$MS^* = MSAB + MSAC - MSABC$$

because

$$E\{MSAB\} + E\{MSAC\} - E\{MSABC\} = \sigma^2 + nc\sigma^2_{\alpha\beta} + nb\sigma^2_{\alpha\gamma} + n\sigma^2_{\alpha\beta\gamma} \qquad \textbf{(25.82)}$$

This equals precisely $E\{MSA\}$ when $\sigma^2_\alpha = 0$. Hence, the suggested test statistic is:

$$F^{**} = \frac{MSA}{MSAB + MSAC - MSABC} \qquad \textbf{(25.83)}$$

ANOVA Model III— Mixed Effects Model

Consider a three-factor study where factors B and C have random factor levels while factor A has fixed factor levels. The restricted mixed ANOVA model for such a three-factor balanced study is:

$$Y_{ijkm} = \mu_{...} + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkm} \quad \textbf{(25.79)}$$

where:

$\mu_{...}$ is a constant

α_i are constants

$\beta_j, \gamma_k, (\alpha\beta)_{ij}, (\alpha\gamma)_{ik}, (\beta\gamma)_{jk}, (\alpha\beta\gamma)_{ijk}$ are pairwise independent normal random variables with expectations zero and constant variances

ε_{ijkm} are independent $N(0, \sigma^2)$, and are independent of the other random components

$\sum_i \alpha_i = \sum_i (\alpha\beta)_{ij} = \sum_i (\alpha\gamma)_{ik} = \sum_i (\alpha\beta\gamma)_{ijk} = 0$

$i = 1, \dots, a; j = 1, \dots, b; k = 1, \dots, c; m = 1, \dots, n$

Note that all interaction terms in this model are random

And we use the same ANOVA table but with different EMS expression:

TABLE 25.10 **Expected Mean Squares for Balanced Mixed Three-Factor ANOVA Model (25.79) (*A* fixed, *B* and *C* random).**

Mean Square	df	Expected Mean Square
MSA	$a-1$	$\sigma^2 + nbc\frac{\sum \alpha_i^2}{a-1} + nc\sigma_{\alpha\beta}^2 + nb\sigma_{\alpha\gamma}^2 + n\sigma_{\alpha\beta\gamma}^2$
MSB	$b-1$	$\sigma^2 + nac\sigma_\beta^2 + na\sigma_{\beta\gamma}^2$
MSC	$c-1$	$\sigma^2 + nab\sigma_\gamma^2 + na\sigma_{\beta\gamma}^2$
$MSAB$	$(a-1)(b-1)$	$\sigma^2 + nc\sigma_{\alpha\beta}^2 + n\sigma_{\alpha\beta\gamma}^2$
$MSAC$	$(a-1)(c-1)$	$\sigma^2 + nb\sigma_{\alpha\gamma}^2 + n\sigma_{\alpha\beta\gamma}^2$
$MSBC$	$(b-1)(c-1)$	$\sigma^2 + na\sigma_{\beta\gamma}^2$
$MSABC$	$(a-1)(b-1)(c-1)$	$\sigma^2 + n\sigma_{\alpha\beta\gamma}^2$
MSE	$(n-1)abc$	σ^2

We can compare EMS to construct the *exact* or *pseudo F* test as appropriate.

Example: Tests in a balanced three-factor ANOVA

Example is given below and all three factors are random (model II) on page 1069 example and problem 25.23 on page 1082.

TABLE 25.11 ANOVA Table for Random Three-Factor Study ($a = 3$, $b = 2$, $c = 5$, $n = 3$).

Source of Variation	*SS*	*df*	*MS*
Factor *A* (operators)	17.3	2	8.65
Factor *B* (machines)	4.2	1	4.20
Factor *C* (batches)	24.8	4	6.20
AB interactions	4.8	2	2.40
AC interactions	31.7	8	3.96
BC interactions	12.5	4	3.13
ABC interactions	11.9	8	1.49
Error	137.7	60	2.30
Total	244.9	89	

Analysis Results:

1. To test three-way interaction effects

$$H_0 : \sigma_{\alpha\beta\gamma} = 0 \text{ vs. } H_a : \sigma_{\alpha\beta\gamma} = 0 > 0$$

We use the standard $F = MSABC/MSE \sim F(8, 60)$, as shown below, p=0.73 and we cannot reject H_0 (no 3-way interactions).

```
MSAB= 2.4
MSABC= 1.49
MSE =2.30

# F-stat for ABC
(Fstat.ABC= MSABC/MSE)
```

```
## [1] 0.6478261
```

```
# To compute p-value
(p.ABC=  1-pf(Fstat.ABC, 8, 60 ))
```

```
## [1] 0.7344314
```

2. To test two-way interaction effects between factor A and B

$$H_0 : \sigma_{\alpha\beta} = 0 \text{ vs. } H_a : \sigma_{\alpha\beta} = 0 > 0$$

we use MSABC as denominator the $F = MSAB/MSABC \sim F(2, 8)$, as shown below, p=0.21 and we cannot reject H_0 (no 2-way interactions between A and B).

```
# F-stat for AB
(Fstat.AB= MSAB/MSABC)
```

```
## [1] 1.610738
```

```r
# To compute p-value
(p.AB=  1-pf(Fstat.AB, 2, 60 ))
```

```
## [1] 0.2082565
```

3. To test main effects of factor A,

$$H_0 : \sigma_\alpha = 0 \text{ vs. } H_a : \sigma_\alpha = 0 > 0$$

We have to use the approximate (pseudo) F-test.

$$F^{**} = \frac{MSA}{MSAB + MSAC - MSABC}$$

$$F^{**} = \frac{8.65}{2.40 + 3.96 - 1.49} = \frac{8.65}{4.87} = 1.78 \sim F(2, df)$$

The approximate number of degrees of freedom associated with the denominator is, from (25.28):

$$df = \frac{(4.87)^2}{\frac{(2.40)^2}{2} + \frac{(3.96)^2}{8} + \frac{(-1.49)^2}{8}} = 4.63$$

```r
# To compute p-value, df can be non-integer
Fstat.A = 1.78
(p.A=  1-pf(Fstat.A, df1=2, df2 = 4.63 ))
```

```
## [1] 0.2670347
```

As shown above, p=0.27 and we cannot reject H_0: Factor A (operators) do not have a main effect.

ANOVA II and III for unequal sample sizes (Ch 25.7)

For unbalanced studies, we don't have nice ANOVA tables for describe the source of variation. We illustrate an approach based on the method of maximum likelihood. This approach has the advantage of conceptual simplicity and is a general procedure with a number of optimality properties.

We can follow the similar procedure in the last lecture using statistical software to get the estimates using likelihood approach. As long as you know how to implement the random and mixed effect models, you can skip the details in Section 25.7.

Example: A Food Company markets a variety of dairy products. To study the reliability of its own and the government's laboratory methods on testing the milk fat level, a small interlaboratory study was carried out.

- Four testing laboratories were randomly selected from the population of laboratories in the US.

- Each laboratory was sent 12 samples of yogurt, with instructions to evaluate six of the samples using the government's method and six by the company's method.
- In this study, measurement method is a fixed factor with $a = 2$ levels (Government method vs. own method) and laboratories is a random factor with $b = 4$ levels (**mixed effects model**)
- Because of technical difficulties with the Government method, some sample test results were not available, leading to the unbalanced data.

Unbalanced study example: R analysis

1. Read the data

```
Food = read.table(url(
  "https://raw.githubusercontent.com/npmldabook/Stat3119/master/Week-14/CH25TA12.txt"))
names(Food) = c("Response", "Method", "Lab", "units")
Food$Method = as.factor(Food$Method)
Food$Lab = as.factor(Food$Lab)

str(Food)
```

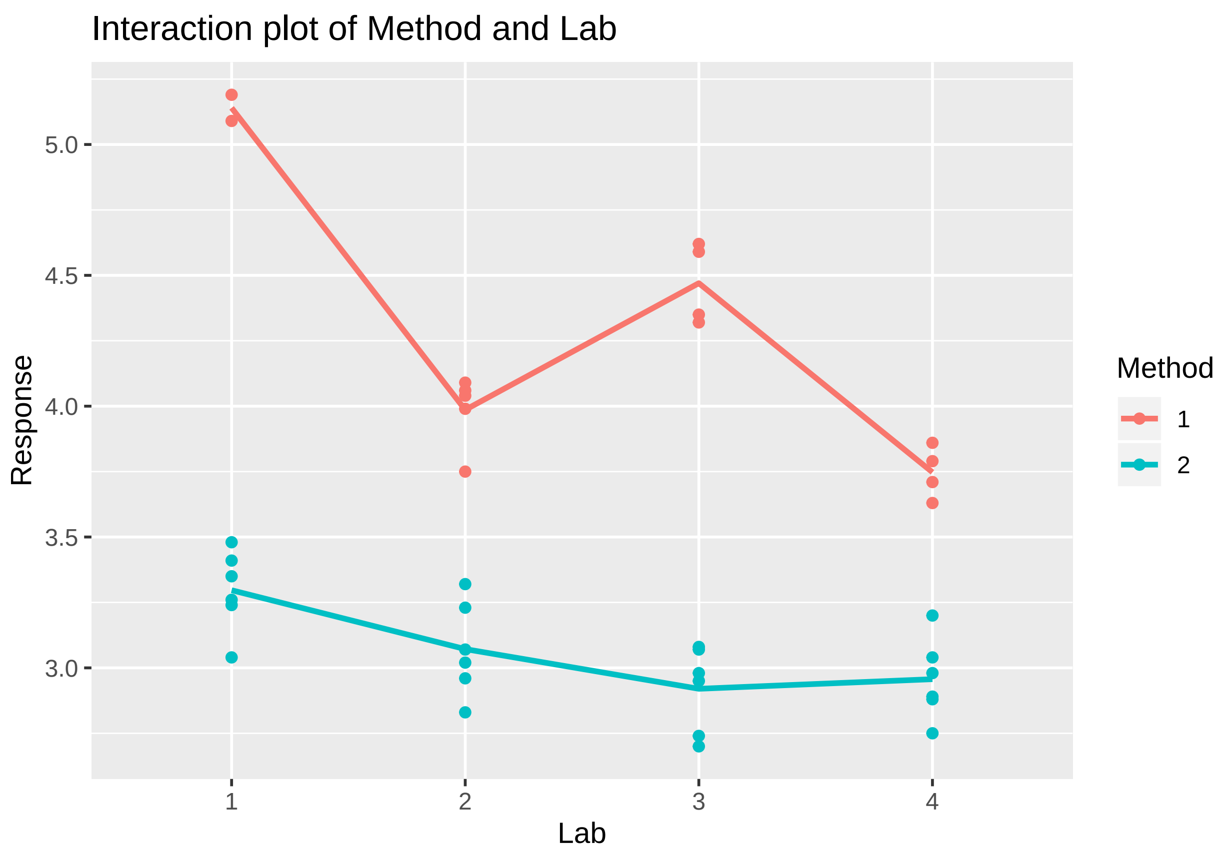
```
## 'data.frame':    39 obs. of  4 variables:
##  $ Response: num  5.19 5.09 4.09 3.99 3.75 4.04 4.06 4.62 4.32 4.35 ...
##  $ Method  : Factor w/ 2 levels "1","2": 1 1 1 1 1 1 1 1 1 1 ...
##  $ Lab     : Factor w/ 4 levels "1","2","3","4": 1 1 2 2 2 2 2 3 3 3 ...
##  $ units   : int  1 2 1 2 3 4 5 1 2 3 ...
```

```
xtabs(~ Method+ Lab, data=Food)
```

```
##       Lab
## Method 1 2 3 4
##      1 2 5 4 4
##      2 6 6 6 6
```

2. plot the data (similar to Fig 25.4)

```
library(ggplot2)
ggplot(Food, aes(x = Lab, y = Response, group = Method  , col = Method  )) +
  geom_point() + stat_summary(fun.y = mean, geom = "line", size=1) +
  labs(title = "Interaction plot of Method and Lab")
```



It suggests a big lab difference and the mean patterns of lab differs by the method, i.e., the model might not be additive with an unimporant interaction.

3. Run mixed effects model and estimation

```
library(lme4)
```

```
## Loading required package: Matrix
```

```
options(contrasts = c("contr.sum", "contr.poly"))
fit.Food <- lmer(Response ~ Method + (1 | Lab) + (1 | Method:Lab), data = Food)
summary(fit.Food )
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Response ~ Method + (1 | Lab) + (1 | Method:Lab)
##    Data: Food
##
## REML criterion at convergence: -5.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.6942 -0.5992  0.1570  0.6313  1.6518
##
## Random effects:
```

```
##  Groups     Name        Variance Std.Dev.
##  Method:Lab (Intercept) 0.11753  0.3428
##  Lab        (Intercept) 0.07347  0.2711
##  Residual               0.02319  0.1523
## Number of obs: 39, groups:  Method:Lab, 8; Lab, 4
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)   3.6953     0.1837  20.120
## Method1       0.6340     0.1240   5.115
##
## Correlation of Fixed Effects:
##         (Intr)
## Method1 0.008
```

Results: This shows for the fixed effect $\hat{\mu}_{..} = 3.6953$ with $SE = 0.1837$, and $\hat{\alpha}_1 = 0.6340$ with $SE = 0.124$. The random components estimate are in the **Random effects** section.

We can also get their confidence intervals easily.

```r
confint(fit.Food, oldNames = FALSE)
```

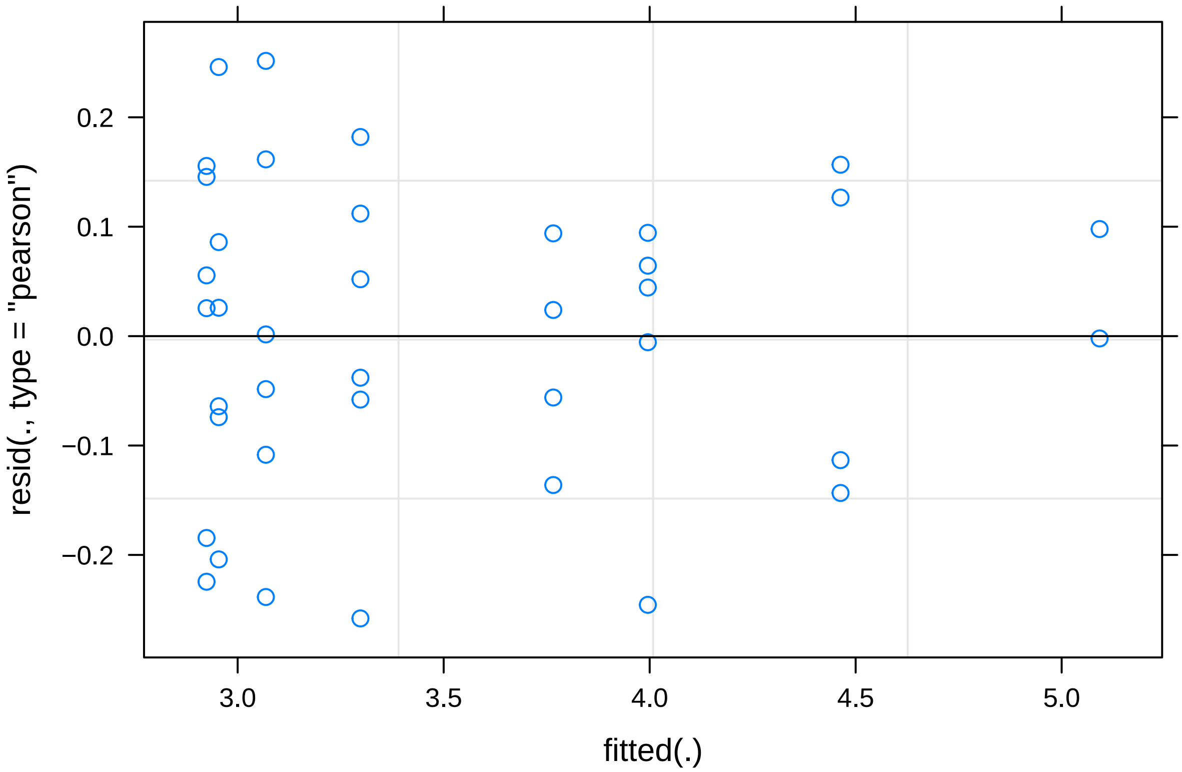
```
## Computing profile confidence intervals ...
```

```
##                                2.5 %    97.5 %
## sd_(Intercept)|Method:Lab 0.1532988 0.6480772
## sd_(Intercept)|Lab        0.0000000 0.7477900
## sigma                     0.1210409 0.2004941
## (Intercept)               3.2929989 4.0977662
## Method1                   0.3646115 0.9043919
```

4. Check model assumptions

1. We can check the constancy of variance:

```r
plot(fit.Food)
```



2. Check the normality of random effects and residuals

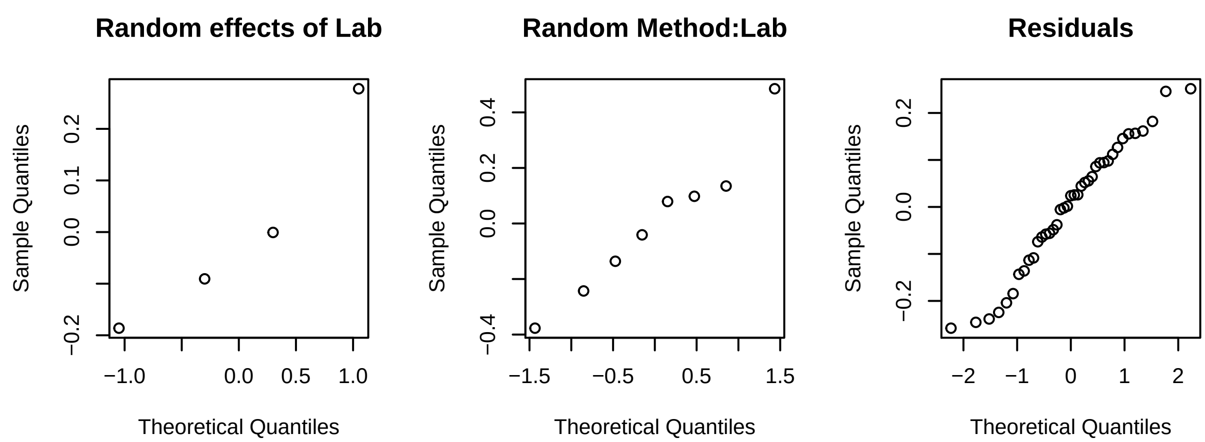
The **ranef()** function will provide the estimated random effects for the data.

```
# check this output to get the name of the random effects
ranef(fit.Food)
```

```
## $`Method:Lab`
##     (Intercept)
## 1:1  0.48514669
## 1:2 -0.24299763
## 1:3  0.13481260
## 1:4 -0.37696166
## 2:1 -0.04093222
## 2:2  0.09790176
## 2:3 -0.13602111
## 2:4  0.07905157
##
## $Lab
##     (Intercept)
## 1  0.2776950438
## 2 -0.0907048443
## 3 -0.0007554848
## 4 -0.1862347147
##
## with conditional variances for "Method:Lab" "Lab"
```

Then we can generate the QQ-plots for these random effects and residuals:

```
par(mfrow = c(1, 3), pty='s')
qqnorm(ranef(fit.Food)$Lab[, 1], main = "Random effects of Lab")
qqnorm(ranef(fit.Food)$'Method:Lab'[, 1], main = "Random Method:Lab")
qqnorm(resid(fit.Food), main = "Residuals")
```



Summary

Reading: Chapter 25 (w. Lecture note 13A/B, 14A)

No homework today, but a pie (chart) for you!

```
library(colorspace); library(plotrix); par(mar=c(0,0,0,0))
pieval= c(30, 20 ,10, 40)
pielabels= c("Excelled in \n Midterm (30%)",
      "You took 26 classes, \n 5 quizzes \n 11 homeworks (20%)",
      "Don't forget \n the project \n (10%) ",
      "Just 2 more classes and \n a final exam to complete \n before the winter break (40%)! ")
pie3D(pieval,radius=0.9, labels=pielabels,explode=0.1,main="Happy Thanksgiving (100%)",
       cex.main=1.5,  col=rainbow_hcl(4),labelcex= 1.2)
```

Happy Thanksgiving (100%)