STAT3119 In class Practice for Chapter 25 (open book) 11/26 15 mins

1. Random effects model for one-factor study vs. Fixed effects study:

$$Y_{ij} = \mu_. + \tau_i + \epsilon_{ij} \qquad (25.38)$$

where

- $\mu_.$ is the constant (population mean)
- τ_i are independent $N(0, \sigma_\mu^2)$ (with a mean of zero), representing the effect of the ith factor levels
- ϵ_{ij} are are independent $N(0, \sigma^2)$
- τ_i and ϵ_{ij} are independent random variables, $i = 1, \ldots, r$, and $j = 1, \ldots, n$.

Parameters	Random effects model	Fixed effects model
Variance of Y_{ij}	$\sigma_\mu^2 + \sigma^2$	σ^2
Observations	Correlated when from same level w. correlation= $\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma^2}$	Independent with each other
Descript H_0 in terms fixed parameter	$H_0 : \sigma_\mu^2 = 0$	$\tau_1+\tau_2=..=\tau_r$

2. Random effects models for 2-factor study vs. Fixed effects study:

$$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \qquad (25.39)$$

where

- $\mu_{..}$ is a constant
- α_i, β_j and $(\alpha\beta)_{ij}$ are **independent** zero-mean normal random variables $N(0, \sigma_\alpha^2)$, $N(0, \sigma_\beta^2)$, and $N(0, \sigma_{\alpha\beta}^2)$
- ϵ_{ijk} are independent $N(0, \sigma^2)$.
- α_i, β_j , $(\alpha\beta)_{ij}$ and ϵ_{ijk} are pairwise independent
- $i = 1, ..., a$, $j = 1, ..., b$ and $k = 1, ..., n$.

Parameters	Random effects model A,B are both random	Fixed effects model
Variance of Y_{ij}	$\sigma_\alpha^2 + \sigma_\beta^2 + \sigma_{\alpha\beta}^2 + \sigma^2$	σ^2
Observations	Independent when they are from different levels	Independent with each other

3. For balanced two factor study with 1-2 factors are random, the ANOVA table (SS, MS, df) are the same but the F-test statistics and their distributions depend on the type of the model. *Need to look Table 25.5 to compare EMS to set the F-test correctly.*

a) ANOVA model II:

Source of variation	**df**	**MS**	***F*-test statistics (in terms of MS)**	**Distribution F(df1, df2)**
Factor A (**random**)	3	MSA	MSA/MSAB	*F*(3, 12)
Factor B (**random**)	4	MSB	MSB/MSAB	*F*(4, 12)
AB interaction	12	MSAB	MSAB/MSE	*F*(12, 60)
Error	60	MSE		

b)ANOVA model III:

Source of variation	df	MS	*F*-test statistics (in terms of MS)	Distribution F(df1, df2)
Factor A (**random**)	3	MSA	MSA/MSE	*F*(3, 60)
Factor B (**fixed**)	4	MSB	MSB/MSAB	*F*(4,12)
AB interaction	12	MSAB	MSAB/MSE	*F*(12, 60)
Error	60	MSE		

4. Balanced Three-factor ANOVA Model II (all three factors are random):

TABLE 25.11 ANOVA Table for Random Three-Factor Study ($a = 3$, $b = 2$, $c = 5$, $n = 3$).

Source of Variation	*SS*	*df*	*MS*
Factor *A* (operators)	17.3	2	8.65
Factor *B* (machines)	4.2	1	4.20
Factor C (batches)	24.8	4	6.20
AB interactions	4.8	2	2.40
AC interactions	31.7	8	3.96
BC interactions	12.5	4	3.13
ABC interactions	11.9	8	1.49
Error	137.7	60	2.30
Total	244.9	89	

Test for the effects	*F*-test statistics (in terms of MS)	Distribution *F*(df1, df2)
ABC interaction	*F*= MSABC/MSE	*F*(8, 60)
AC interaction	*F*= MSAC/MSABC	*F*(8,8)
Factor C	*F*= MSC/MS** *MS**=MSAC+MSBC-MSABC	F(2, df), where $df = \frac{(MS^*)^2}{\frac{MSAC^2}{8} + \frac{MSBC^2}{4} + \frac{MSABC^2}{8}}$