GWID:

Class: 9/19/2019

1. (10 points) You are planning a scientific study based on the required power and a target treatment effect you try to establish using the data that you will collect from a one-factor experiment. Assume the treatment effect is described by  $\mathbf{k} = \Delta/\sigma$ , where  $\Delta = \max(\mu_i)$  -  $\min(\mu_i)$  is minimum range of factor level means for which it is important to detect, and  $\sigma$ = the standard deviation of the response variable Yii.

Choose the **wrong** statement from below (choose only one):

- A. As the required power increases, you need a larger sample size.
- B. As the measurement variability  $\sigma$  increases, you need a larger sample size.
- C. As the effect size  $k = \Delta/\sigma$  increases, you need a larger sample size.
- D. As the desirable type I error increases, you need a smaller sample size.
- 2. If you are given a one-factor ANOVA analysis table to test the effects of several different machines as follows:

Source	SS	$d\!f$	MS
Between machines	2.28935	5	.45787
Error	3.53060	114	.03097
Total	5.81995	119	

- 1) And you know the study is balanced (the same number of experiment units for each machine), then from the ANOVA table, you can calculate
  - (10 points) The number of machines (r) = 6
  - (10 points) The total sample size  $(n_T) = 120$
  - (10 points) The number of experiment units for each machine  $(n_i) = 20$
- 2) (10 points) Define the F-statistic (F\*)= MSTR(between Machine)/MSE, what distribution F\* follows F(5, 114).
- 3) (10 points) Then you calculate  $F^* = .45787/.03097 = 14.78$ , and compare with the underlying distribution to get the P-value < 0.0001, what will be your conclusion based on these results? We reject the null hypothesis that all the machines have similar effects and conclude not all the machines have the same effects.
- 3. In the above study of machines (problem 2), if we conclude that the functions of machines are different, we will proceed with the analysis of factor level means.
  - 1) (10 points) Define a difference between two factor level means as  $D=\mu_i-\mu_{i'}$  , we can get the esimator of D and its estimated variance as follows:  $\hat{D} = \overline{Y}_i - \overline{Y}_{i'}$ , and

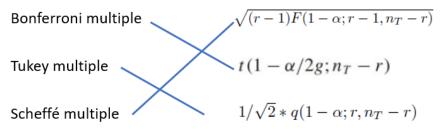
$$s^2\{\hat{D}\} = MSE\left(\frac{1}{n_i} + \frac{1}{n_{i'}}\right)$$
 to make a test statistic  $\frac{\hat{D} - D}{s\{\hat{D}\}}$ , which has the following distribution a. Normal distrubition:  $N(0, 1)$ 

- a. Normal distrubition: N(0, 1)
- b. F-distribution:  $F(r-1, n_T-r)$
- c. t-distribution :  $t(n_T-r)$
- d. studentized range distribution  $q(r, n_T-r)$ .

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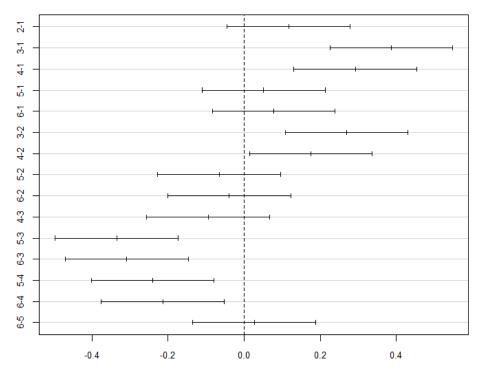
2) (10 points) If we are getting simultaneous confidence intervals for all pairwise comparisons for this study, we can use the Bonferroni, Tukey or Scheffé procedure. They have the similar  $\hat{D} \pm \; {\sf constant*} \; {\sf s} \; (\hat{D}) \; {\sf except} \; {\sf for} \; {\sf the} \; {\sf constant} \; {\sf multiple}.$ 

Please match the simultaneous comparison procedure with the corresponding multiple by drawing a line between them



- 3) (10 points) If all parwise comparison are of interest and we are controlling the overall type I error, which procedure is superior?
  - a. Fisher's LSD procedure
  - b. Bonferroni procedure
  - c. Tukey procedure
  - d. Scheffé procedure
- 4) (10 points) Finally, you are provided with a pairwise comparison plot for the six machines based on the Tukey's HSD procedures:

95% family-wise confidence level



Differences in mean levels of machines

What can you learn from these results about the differences between the six filling machines (e.g. which machine(s) most effect? Which machines differ significantly)?

We can conclude with 95% family confidence that machine 3 and machine 4 are more effective than other machines. However, machines 1,2,5, 6 do not differ significantly.