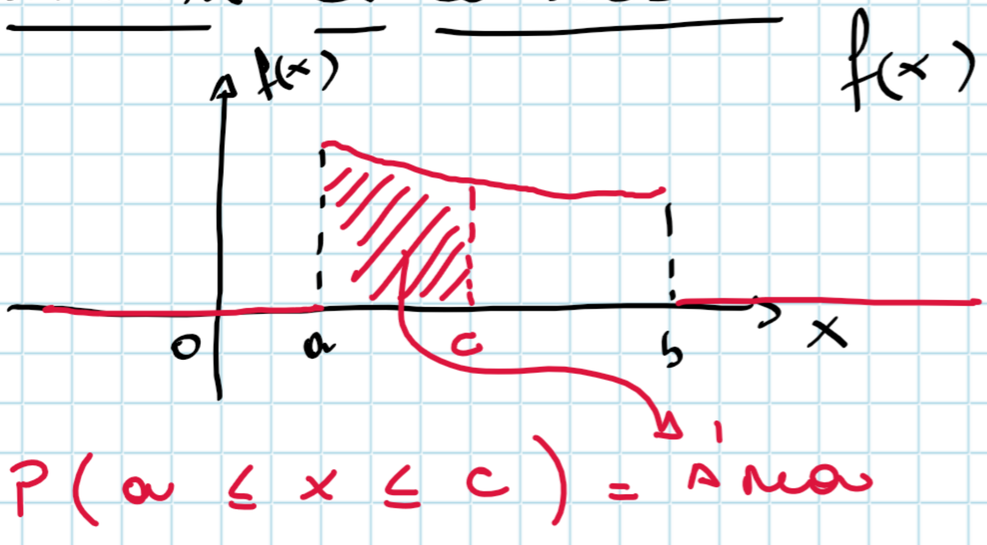
Variables Aleatorias continuas

- Dominio en el espacio muestral
- Recorrido en $\mathbb{R}$

Función de densidad



$P(a \leq x \leq c) =$ Área

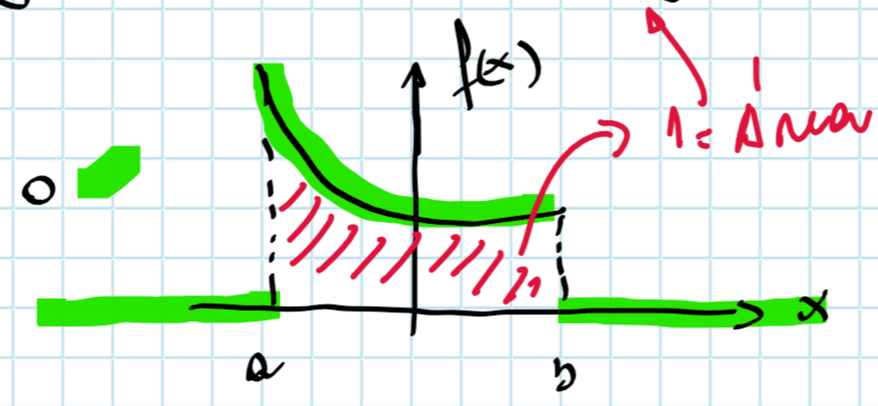
Probabilidad - densidad de puntos (área)

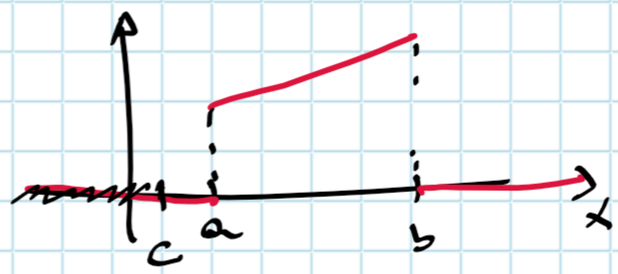
Definimos

$$f(x) = \begin{cases} f(x) & \text{si } a \leq x \leq b \\ 0 & \text{en otro caso} \end{cases}$$

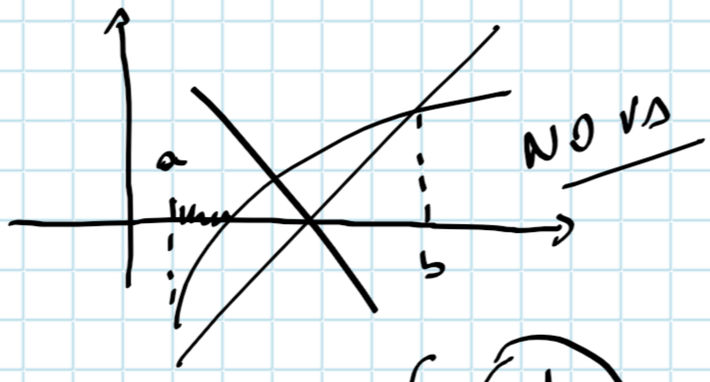
Propiedades de la función de densidad

- Área bajo $f(x)$ entre a y b sea 1
- $f(x) \geq 0$



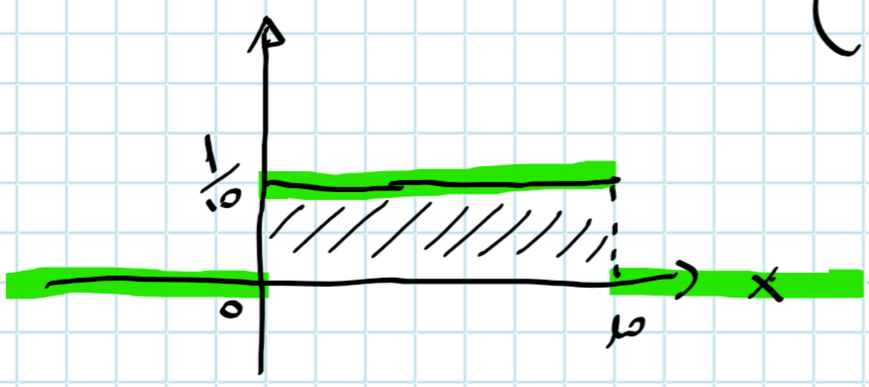


$P(x < c) = 0$



X: U.A.C

$$f(x) = \begin{cases} \frac{1}{10} & \text{si } 0 \leq x \leq 10 \\ 0 & \text{en otro caso} \end{cases}$$



Distribución Uniforme o al azar

Área del rectángulo

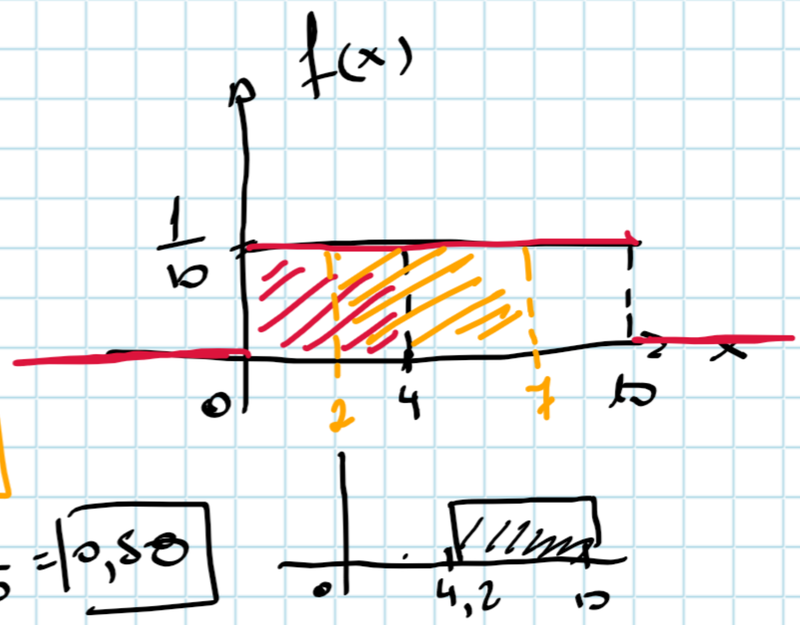
$b \cdot h = (10-0) \cdot \frac{1}{10} = 1$

Probabilidad como área en un intervalo.

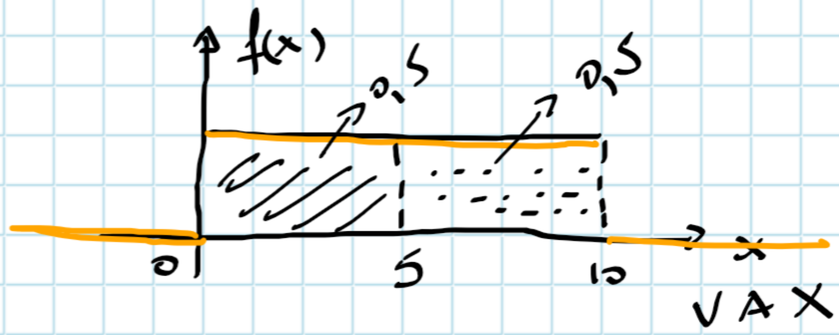
$P(x < 4) = (4-0) \cdot \frac{1}{10} = \frac{4}{10} = \boxed{0,4}$

$P(2 < x \leq 7) = (7-2) \cdot \frac{1}{10} = \boxed{0,5}$

$P(x \geq 4,2) = (10-4,2) \cdot \frac{1}{10} = 5,8 \cdot \frac{1}{10} = \boxed{0,58}$



$$f(x) = \begin{cases} \frac{1}{10} & \text{Si } 0 \le x \le 10 \\ 0 & \text{en otro caso} \end{cases}$$

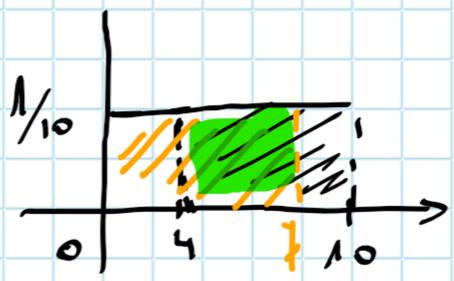


$E(x)$ para una V. A. Uniforme

$$E(x) = \frac{b-a}{2} = \frac{10-0}{2} = \boxed{5}$$

$$P(x < 7 / x \ge 4) = \frac{P(4 < x < 7)}{P(4 \le x \le 10)} \quad \text{(intersección)}$$

$P(x \ge 4)$



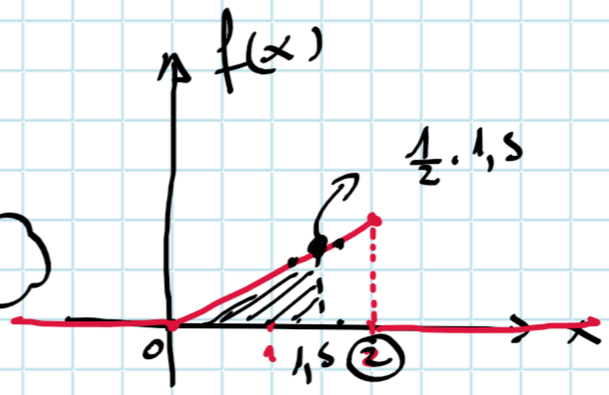
$$= \frac{\overbrace{3}^{\text{base}} \cdot \overbrace{\cancel{1/10}}^{\text{altura}}}{6 \cdot \cancel{1/10}} = \boxed{0,5}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Ejemplo $f(x) = \begin{cases} \frac{1}{2}x & \text{Si } 0 \le x < 2 \\ 0 & \text{en otro caso.} \end{cases}$

Área del Triángulo $\frac{b \cdot h}{2}$

$$A = \frac{(2-0) \cdot \overbrace{\left(\frac{1}{2} \cdot 2\right)}^{\text{altura}}}{2} = \frac{2 \cdot 1}{2} = \boxed{1}$$



a) Probabilidad de que $P(x < 1,5) = \frac{1,5 \cdot 0,75}{2} = \boxed{0,5625}$

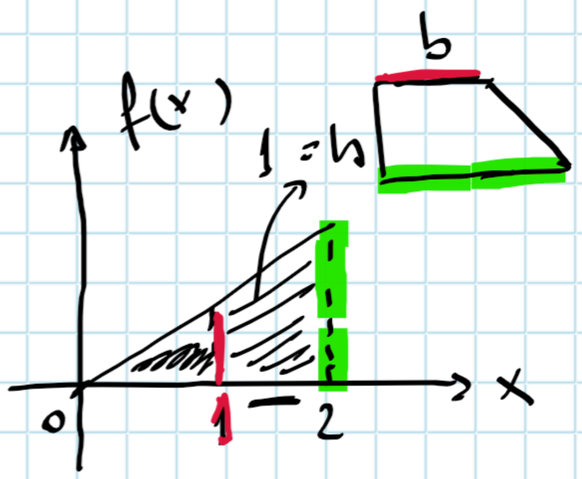
$$P(x > 1,5) = 1 - P(x < 1,5)$$

$$= 1 - 0,5625 = \boxed{0,4375}$$

b) Probabilidad de que:

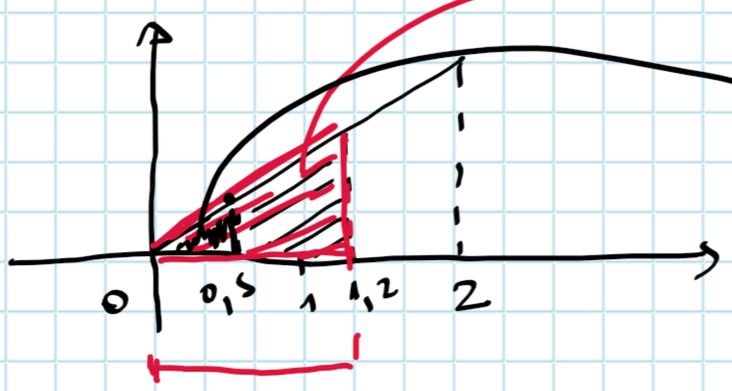
$$P(x > 1) = 1 - P(x < 1)$$

$$= 1 - \frac{1 \cdot \frac{1}{2} \cdot 1}{2} = \boxed{0,75}$$



c) $P(0,5 < x \le 1,2) = \frac{1,2 \cdot \frac{1}{2} \cdot 1,2}{2} - \frac{0,5 \cdot \frac{1}{2} \cdot 0,5}{2} =$

Área del trapecio $\frac{(b+B) \cdot h}{2} =$

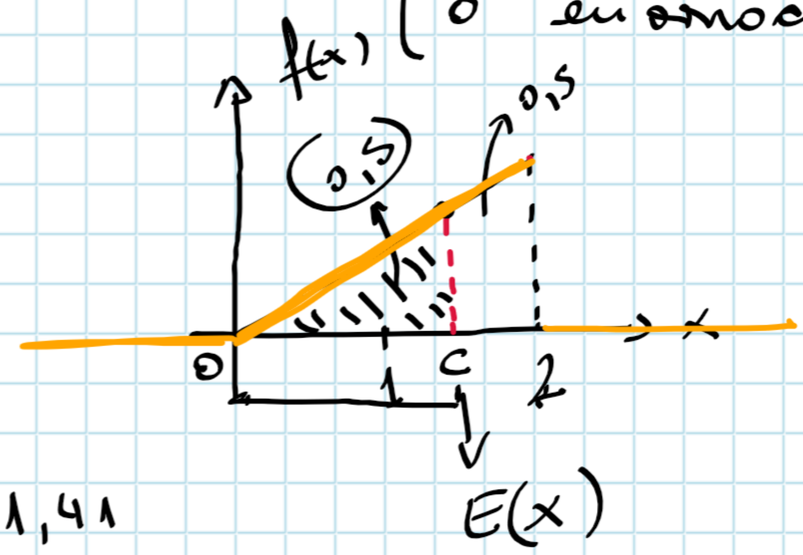


$0,36 - 0,0625 =$

$$= \boxed{0,2975}$$

$E(x) =$

$$f(x) = \begin{cases} \frac{1}{2}x & \text{si } 0 \le x \le 2 \\ 0 & \text{en otro caso} \end{cases}$$



$$\text{Área} = \frac{(c-0)\cdot \frac{1}{2}c}{2} = \frac{1}{4}c^2 = 0{,}5$$

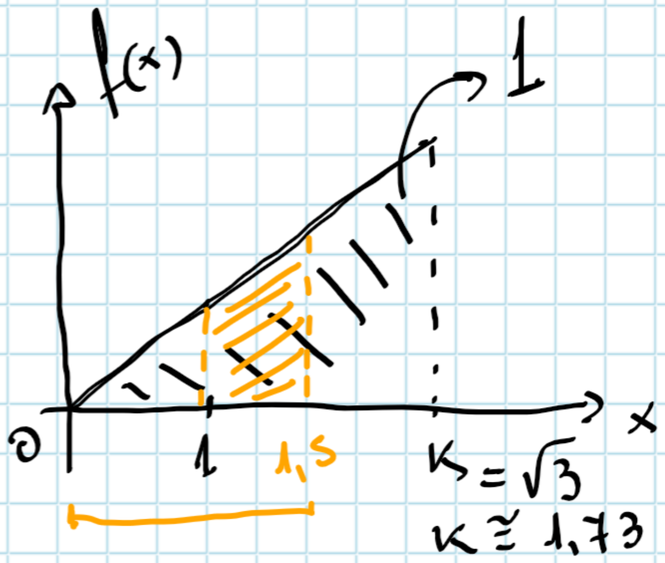
$$c^2 = 0{,}5 : \frac{1}{4}$$

$$c^2 = 2$$

$$c = \sqrt{2} \cong 1{,}41$$

$$E(x) = \sqrt{2} \cong 1{,}4$$

$$f(x) = \begin{cases} \frac{2}{3}x & \text{si } 0 \le x < k \\ 0 & \text{en otro caso} \end{cases}$$



- Hallar el valor de $k \Rightarrow \frac{\overbrace{(k-0)}^{\text{base}} \cdot \overbrace{\frac{2}{3}\cdot k}^{\text{altura}}}{2} = 1$

$$\frac{2}{3}k^2 = 2$$

$$k^2 = 2 : \frac{2}{3}$$

$$k^2 = 3 \longrightarrow \boxed{k = \sqrt{3}}$$

- $P(1 \le x < 1{,}5) = P(x < 1{,}5) - P(x \le 1)$

$$= \frac{(1{,}5-0)\cdot \frac{2}{3}\cdot 1{,}5}{2} - \frac{(1-0)\cdot \frac{2}{3}\cdot 1}{2} = \frac{1{,}5^2}{3} - \frac{1}{3} = \frac{3}{4} - \frac{1}{3} =$$

$$= \boxed{0{,}42}$$

- $P(x < 1{,}5 / x > 1) = \frac{P(1 < x < 1{,}5)}{P(x > 1)} = \frac{0{,}42}{1 - \frac{1}{3}} = \frac{0{,}42}{\frac{2}{3}} = \boxed{0{,}63}$

$P(x > 1) = 1 - P(x < 1)$