Robust Bayes factors for evaluating informative hypotheses in the context of ANOVA

Thesis Proposal

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Introduction

Analysis of variance (ANOVA) is a statistical approach for comparing means that is used by many researchers. Important assumptions of ANOVA are normally distributed errors within each group and absence of outliers (Field, 2013). When this is the case, ANOVA can be validly applied. In reality, however, researchers commonly have to face situations in which the normality assumption is not met (Hill & Dixon, 1982; Micceri, 1989). For example, distributions might be skewed or have thicker tails and random samples may contain outliers. In theses situations, problems arise such as biased estimates of the mean, inflated standard errors, substantially lower power and inaccurate confidence intervals (Wilcox, 2017). It has been shown that, even when only minor violations occur, standard statistical techniques may perform very poorly (Ruckstuhl, 2014; Wilcox, 2017, chapter 2).

A solution for these problems is robust inference. Robust inference is based on measures of location and scale that are relatively unaffected by non-normality or outliers (Wilcox, 2017, chapter 2). While non-normality and outliers can be detrimental for estimation and inference when data is analysed with standard statistical techniques, accurate results are achieved with robust inference (Ruckstuhl, 2014; Wilcox, 2017). A simple example of a robust measure of location is the median. Unlike the mean, the value of the median gives an accurate estimate of the location of the central tendency of a non-normal distribution and is unaffected by outliers.

Robust methods are mostly discussed in the context of estimation and null hypothesis significance testing (NHST). Another approach for evaluating hypotheses is a Bayesian model selection approach (Klugkist, Laudy, & Hoijtink, 2005). This approach uses a Bayes factor (BF) to directly evaluate specific scientific expectations, stated as informative hypotheses. In the context of ANOVA, an informative hypothesis can be used to state an expected ordering of means, for example, $H_1 : \mu_1 > \mu_2 > \mu_3$. With the Bayes factor, the relative support in the data can be calculated for an informative hypothesis, H_i, compared with an unconstrained hypothesis, H_u, another informative hypothesis, $H_{i'}$, or it's complement, H_c (Van Rossum, Van De Schoot, & Hoijtink, 2013). For example, H_1 can be compared with $H_2 : \mu_1 > \mu_2 = \mu_3$. Finding a BF_{12} of 5 indicates that the support in the data for hypothesis H_1 is five times larger than the support for hypothesis H_2.

Recently Gu, Mulder, & Hoijtink (2017) developed the approximate adjusted fractional Bayes factor. With the approximate fractional Bayes factor, informative hypotheses can be evaluated for virtually any statistical model.

Additionally, the approximate fractional Bayes factor is implemented in a easy-to-use software package, called BAIN. For the calculation, only the parameter estimates and their covariance matrix are needed.

In the current procedure, the estimates and covariance matrix used in BAIN are not robust and the expectation is that the Bayes factor following from these estimates is not robust either. The objective of this project is therefore, in the context of both equal and unequal variances ANOVA, to replace estimates and their covariance matrix by their robust counterparts and thereby develop a robust Bayes factor. The research question is three-fold: 1) To what extent and in which situations is the Bayes factor as implemented in BAIN robust to non-normality and outliers? 2) What are the best robust estimates of the mean and covariance matrix in the ANOVA context? 3) How does the approximate fractional Bayes factor behave in situations of non-normality and outliers when robust estimates and covariance matrix are used?

Approach

1. Investigate with a simulation study to what extent and in which situations the Bayes factor for evaluating informative hypotheses when comparing means, as implemented in BAIN, is sensitive to non-normality and outliers.

2. Investigate which robust measures of location and scale are most suitable to use in the context of both equal and unequal variances ANOVA. This will be investigated by both studying the existing documentation (Ruckstuhl, 2014; Wilcox, 2017) and by experimental simulations in which non-normality and outliers are manipulated.

3. Robustify the approximate adjusted fractional Bayes factor based on the results from 2).

4. Show with a simulation study in the same situations as investigated in 1) that the robust Bayes factor is not sensitive to non-normality and outliers.

5. Illustrate the robust Bayes factor with a number of example studies from the Open Science Foundation Reproducibility Project Psychology.

Journals for possible publication

British Journal of Mathematical and Statistical Psychology, Psychological Methods

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