1 Introduction

Analysis of variance (ANOVA) is a statistical approach for comparing means that is used by many researchers. Just as any statistical approach, ANOVA is based on certain assumptions. When the data meet these assumptions, ANOVA can be validly applied. In reality, however, data often violates assumptions. For example, sampling can result in an outlier, i.e. an observation that "deviates so much from other observations as to arouse suspicious that it was generated by a different mechanism" (Hawkins, 1980, p. 1).

Unfortunately, even a small proportion of outliers can seriously affect an ANOVA. Firstly, outliers result in an increased error variance, thereby leading to a reduced power of the statistical test (Wilcox, 2017). Secondly, outliers cause biased parameter estimates (Ruckstuhl, 2014; Wilcox, 2017). Therefore, when an ANOVA is applied to a dataset that contains outliers inferences can be highly inaccurate.

One obvious way of handling the adverse effects of outliers is to remove them from the dataset prior to analysis. However, whether the removal of outliers is advised depends on the source of the outliers. Outliers are often divided in those that are the results of errors in the data and those that come from natural variability (Anscombe, 1960). Outliers that are the result from natural variability can be considered legitimate cases (Osborne & Overbay, 2004). Additionally, sometimes the source of outliers can not be traced back. Ideally, one wants to keep the legitimate cases and outliers for which the source is unknown in the data while at the same time minimizing their influence on estimation and hypothesis testing. One way to achieve that objective is to use robust procedures.

Robust procedures are estimation and hypothesis testing procedures that are relatively unaffected by data imperfections like non-normality and outliers (Wilcox, 2017, p.!). In the presence of outliers, robust procedures result in more accurate inferences compared to classical procedures (Ruckstuhl, 2014; Wilcox, 2017). A simple example of a robust measure is the median. Unlike the mean, the value of the median gives an accurate estimate of the central tendency of the data unaffected by outliers.

Robust procedures are mostly discussed in the context of estimation and null hypothesis significance testing (NHST). Another approach for evaluating hypotheses is a Bayesian model selection approach (Klugkist et al., 2005). This approach uses a Bayes factor (BF) to directly evaluate specific scientific expectations, stated as informative hypotheses. In the context of an ANOVA, an informative hypothesis can be used to state an expected ordering of means, for example,

$$H_1 : \mu_1 < \mu_2 < \mu_3, \tag{1}$$

where μ_g represent the mean of group $g = 1, 2, 3$. With the Bayes factor, the relative support in the data can be calculated for an informative hypothesis,

H_i, compared with it's complement, H_c (van Rossum, van de Schoot, & Hoijtink, 2013), an unconstrained hypothesis, H_u, or another informative hypothesis, H_i'. For example, H_1, as stated in Equation 1, can be compared with

$$H_2 : H_1 : \mu_1 < \mu_2 = \mu_3. \tag{2}$$

Finding a BF_{12} of 5 then indicates that the support in the data for hypothesis H_1 is five times larger than the support for hypothesis H_2.

Recently, Gu, Mulder, & Hoijtink (2017) developed the approximate adjusted fractional Bayes factor (AAFBF). With the AAFBF, informative hypotheses can be evaluated for virtually any statistical model. Additionally, the AAFBF is implemented in an easy-to-use software package called Bain. For the calculation, only the parameter estimates of the statistical model at hand and their covariance matrix are needed.

In the ANOVA context, the parameters estimates are the group means. However, as just explained, when the data contains outliers, the parameter estimates can be seriously biased. Additionally, the covariance matrix of the parameters is based on the estimated variance, which is also seriously affected when the data contains outliers. Hence, the expectation is that the AAFBF resulting from these estimates is negatively affected by outliers. However, to our knowledge, this has never been formally investigated.

This paper investigates to what extent and in which situations the AAFBF is affected by outliers. Firstly, with a simulation study the effect of outliers on the Ordinary Least Squares (OLS) estimator is shown and compared to the effect of outliers on an estimator that is relatively unaffected by outliers, i.e. a robust estimator.

This paper is organized as follows. Section 2 introduces a robust estimator suitable for the ANOVA context. Furthermore, it qualities are discussed and it is proposed to use the robust estimator for estimation of the parameter estimates needed for Bain. In Section 3 ..

2 Robust estimators

3 Methods

References

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