$$C = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Assume that:

$$C\vec{x} = \lambda\vec{x}$$

$$\Leftrightarrow (C - \lambda I)\vec{x} = 0 \quad (*)$$

$$\det(C - \lambda I) = \det \begin{bmatrix} 1-\lambda & -2 & 0 \\ -2 & 5-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix}$$

$$= (1-\lambda).(5-\lambda).(2-\lambda) - 4(2-\lambda)$$

$$= (2-\lambda)(\lambda^2 - 6\lambda + 1)$$

let $\det(C - \lambda I) = 0 \Rightarrow \lambda = \{2, 3+2\sqrt{2}, 3-2\sqrt{2}\}$

$$\lambda = 2 \Rightarrow \begin{bmatrix} -1 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vec{x} = 0$$

$$\Rightarrow \vec{x} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda = 3+2\sqrt{2} \Rightarrow \begin{bmatrix} -2-2\sqrt{2} & -2 & 0 \\ -2 & 2-2\sqrt{2} & 0 \\ 0 & 0 & -1-2\sqrt{2} \end{bmatrix} \vec{x} = 0$$

$$\Rightarrow \vec{x} = \begin{pmatrix} 1-\sqrt{2} \\ 1 \\ 0 \end{pmatrix}$$

$$\lambda = 3-2\sqrt{2} \Rightarrow \begin{bmatrix} -2+2\sqrt{2} & -2 & 0 \\ -2 & 2+2\sqrt{2} & 0 \\ 0 & 0 & -1+2\sqrt{2} \end{bmatrix} \vec{x} = 0$$

$$\Rightarrow \vec{x} = \begin{pmatrix} 1+\sqrt{2} \\ 1 \\ 0 \end{pmatrix}$$

$$C = \begin{bmatrix} 0 & 1-\sqrt{2} & 1+\sqrt{2} \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3+2\sqrt{2} & 0 \\ 0 & 0 & 3-2\sqrt{2} \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 \\ 1-\sqrt{2} & 1 & 0 \\ 1+\sqrt{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1-\sqrt{2} \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 0 & 3+\sqrt{2} \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 \\ 1-\sqrt{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times 2 \times \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$