Artificial Neural Networks

- Learning real-valued, discrete-valued, and vector-valued functions from examples.

- Robust to errors in training data.

- Applications: interpreting visual scenes, speech recognition, robot control strategies

Biological Motivation

- Human brain = 10^{11} neurons
- Each connected to 10^{4} others
- Switching time = 10^{-3} seconds
  (Computer switching speed = 10^{-10} seconds)
- It requires 10^{-1} seconds to recognize a human face

$\Rightarrow$ highly parallel and distributed processes

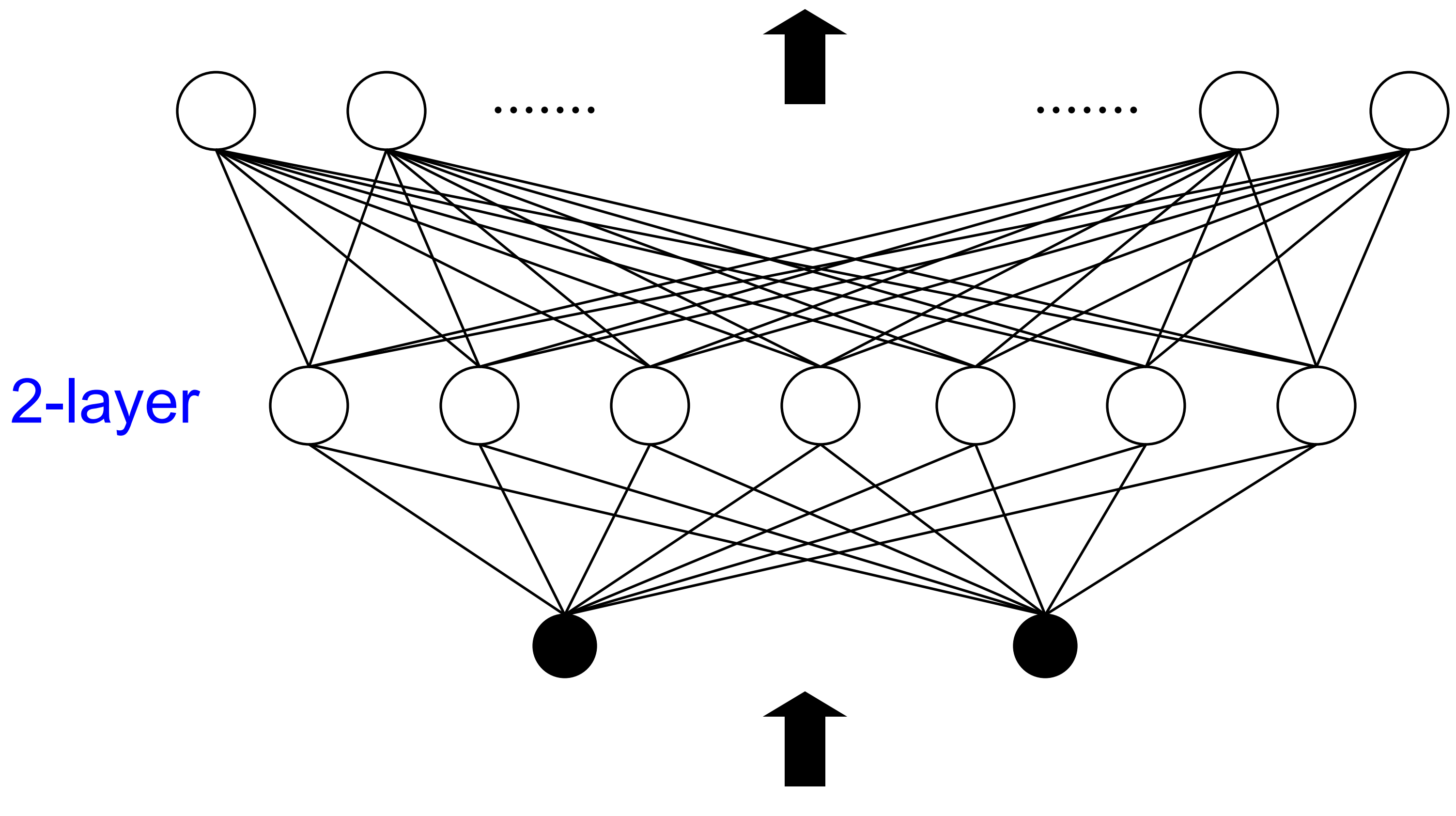
Biological Motivation

- ANN model is not the same as that of biological neural systems

- Using ANNs to study and model biological learning processes

  Obtaining highly effective machine learning algorithms

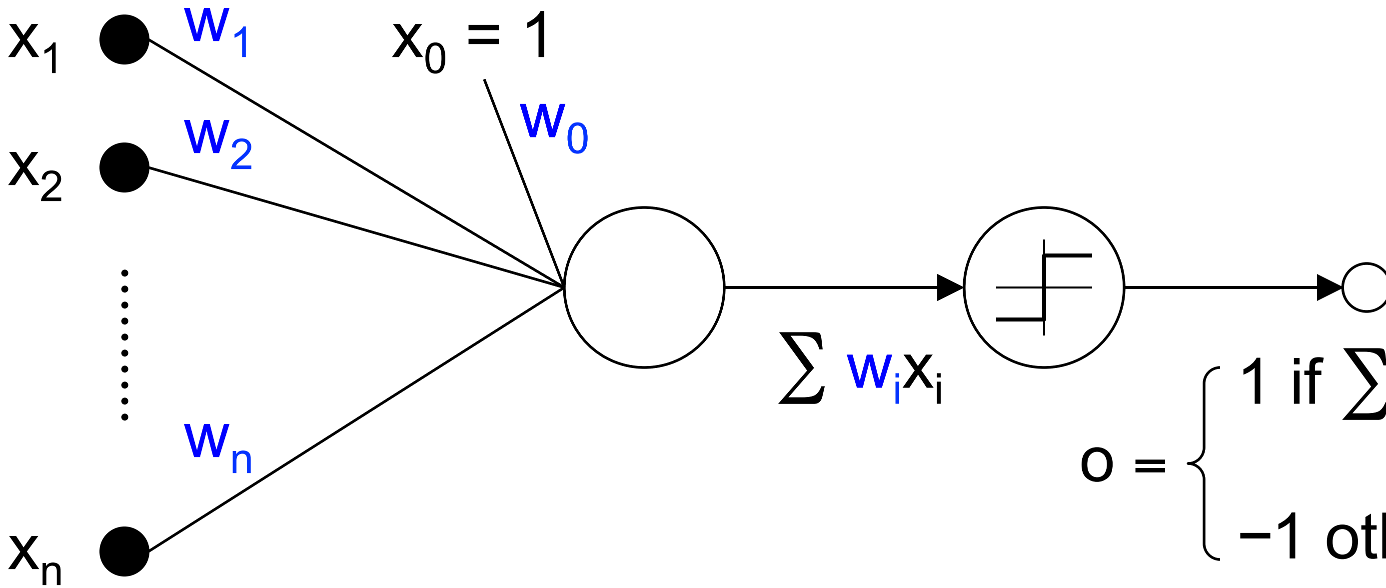
ANN Representation



Appropriate Problems for ANNs

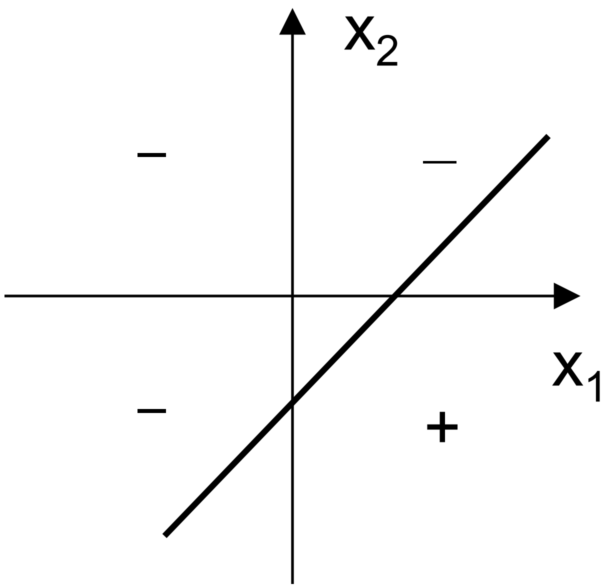
- Instances are represented by many attribute-value pairs
- Target function output may be discrete-valued, real-valued, vector-valued
- Training examples can contain errors
- Long training time is acceptable
- Fast evaluation of the learned target function may be required
- Understanding the learned target concept is not important

Perceptrons

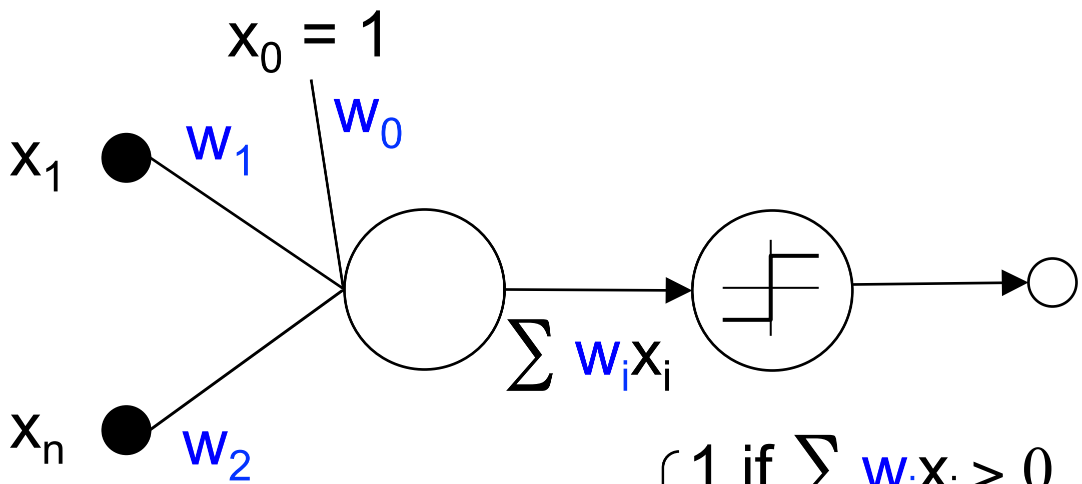


$$o = \begin{cases} 1 \text{ if } \sum w_i x_i > 0 \\ -1 \text{ otherwise} \end{cases}$$

Perceptrons



$A \wedge \neg B$



$$o = \begin{cases} 1 \text{ if } \sum w_i x_i > 0 \\ -1 \text{ otherwise} \end{cases}$$

Perceptron Training Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

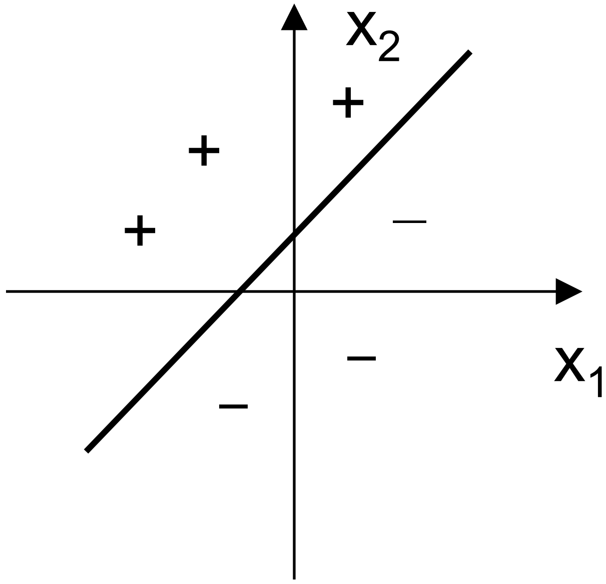
- t target output of the current training example
- o the thresholded output generated by the perceptron
- η learning rate (positive constant)

Perceptron Training Rule

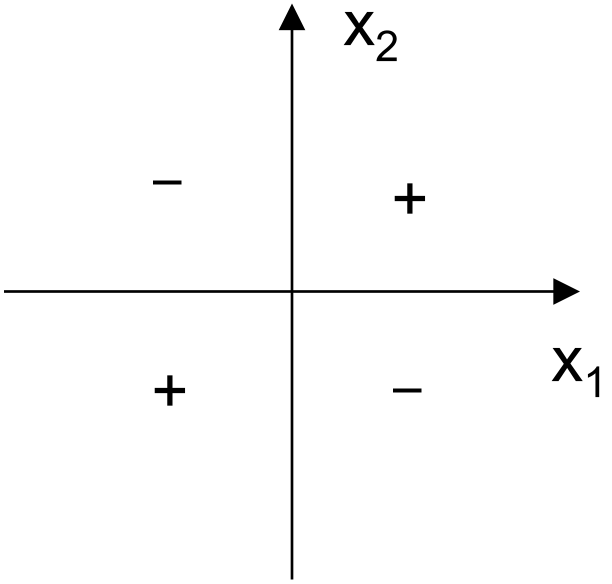
w_0	w_1	w_2	x_1	x_2	$t = x_1 \wedge \neg x_2$	$o = /w_0 + w_1x_1 + w_2x_2/$	Δw_0	Δw_1	Δw_2
1	0	1	1	1	−1	1	−2	−2	−2
−1	−2	−1	1	−1	1	−1	2	2	−2
1	0	−3	−1	1	−1	−1	0	0	0
1	0	−3	−1	−1	−1	1	−2	2	2
−1	2	−1	1	1	−1	−1	0	0	0
−1	2	−1	1	−1	1	1	0	0	0
−1	2	−1	−1	1	−1	−1	0	0	0
−1	2	−1	−1	−1	−1	−1	0	0	0

$$w_i \leftarrow w_i + \Delta w_i$$
$$\Delta w_i = (t - o)x_i$$

Perceptron Training Rule



linearly separable



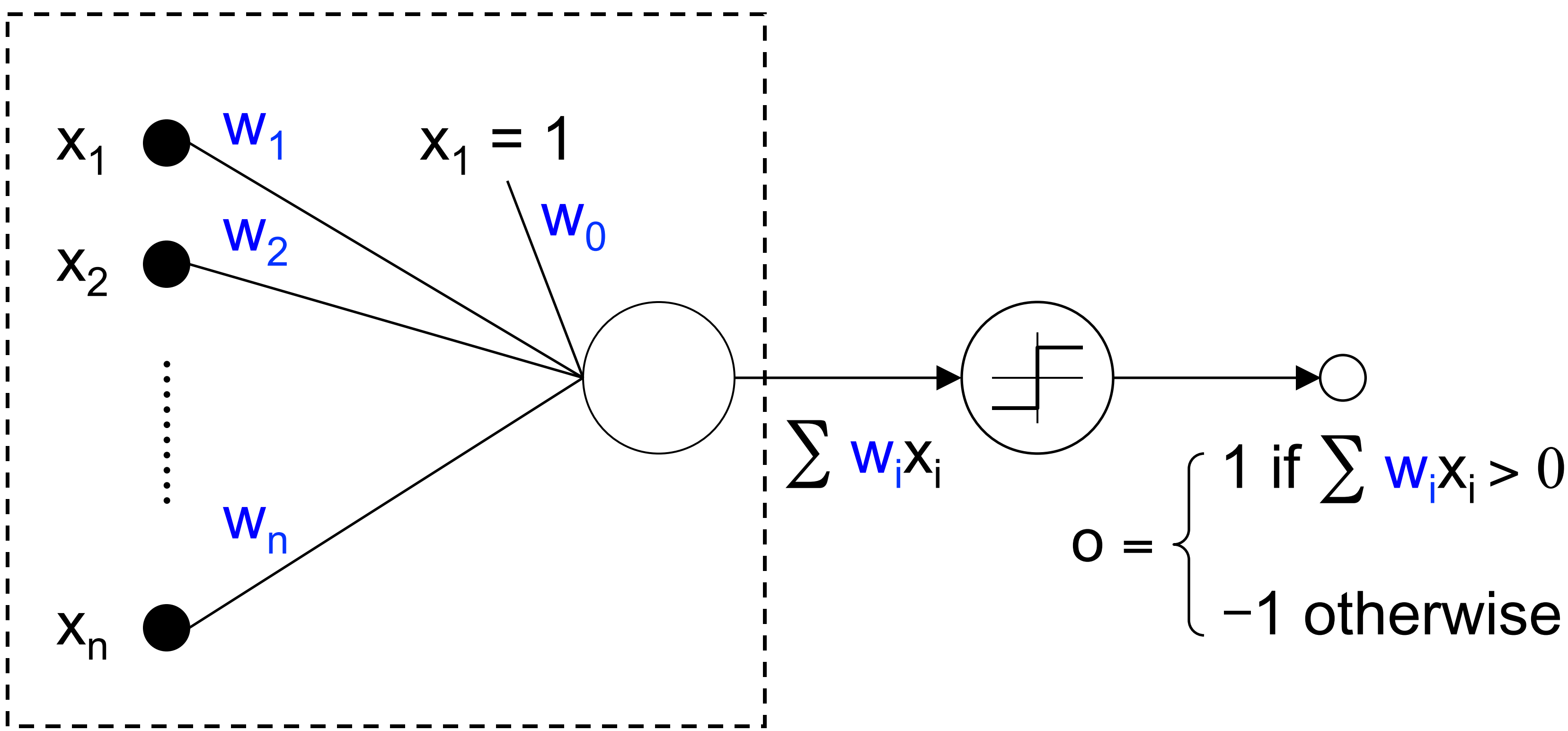
non linearly separable

Perceptron Training Rule

- The learning procedure converges to a weight vector that correctly classifies all linearly separable training examples

- Non-differentiable

Delta Rule (using gradient descent)



linear unit

Delta Rule

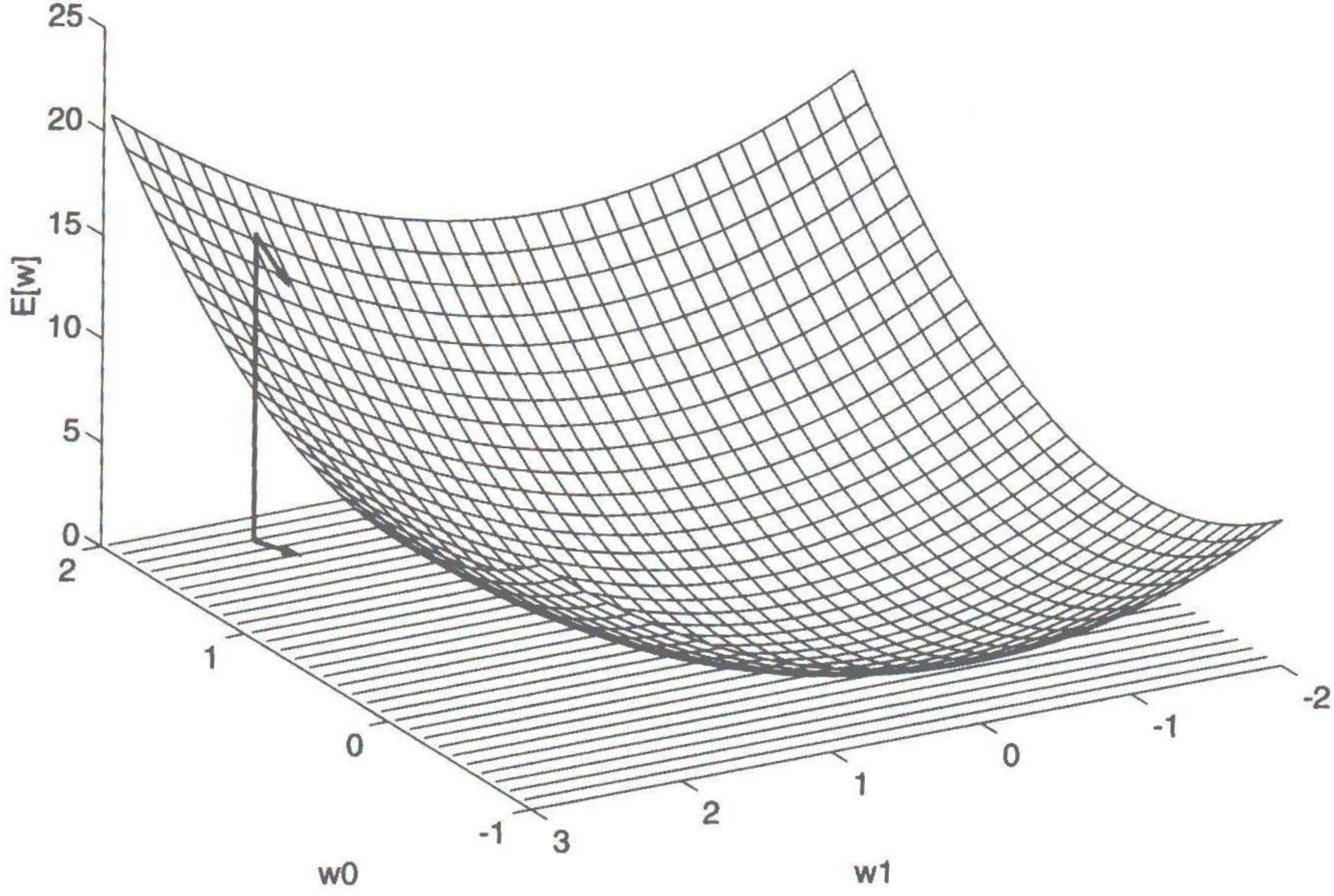
Training error (for the linear unit):

$$E(\vec{w}) = \sum_{d \in D}(t_d - o_d)^2/2$$

t_d target output of training example d

o_d the unthresholded output for d $(= \vec{w}.\vec{x})$

Delta Rule



Delta Rule

Gradient of E (steepest increase direction):

$$\nabla E(\vec{w}) = [\partial E/\partial w_0, \partial E/\partial w_1, \ldots, \partial E/\partial w_n]$$

$$\vec{w} \leftarrow \vec{w} + \Delta\vec{w}$$

$$\vec{w} \leftarrow \vec{w} - \eta \nabla E(\vec{w})$$

Delta Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = -\eta \partial E/\partial w_i$$

$$\partial E/\partial w_i = -\sum_{d \in D}(t_d - o_d)x_{id}$$

$$\Delta w_i = \eta \underbrace{\sum_{d \in D}(t_d - o_d)}_{\delta} x_{id}$$

Delta Rule

- Converging to a local minimum can be quite slow
- No guarantee to converge to the global minimum

Stochastic Approximation

Delta rule:

$$E(\vec{w}) = (t_d - o_d)^2/2$$

$$\Delta w_i = \eta\underbrace{(t_d - o_d)}_{\delta}x_{id}$$

Stochastic Approximation

- Weights are updated upon examining each training example
- Less computation per weight update step is required
- Falling into local minima can be avoided

Stochastic Approximation

- The delta rule converges towards a best-fit approximation to the target concept, regardless of whether the training data are linearly separable

Single Perceptrons

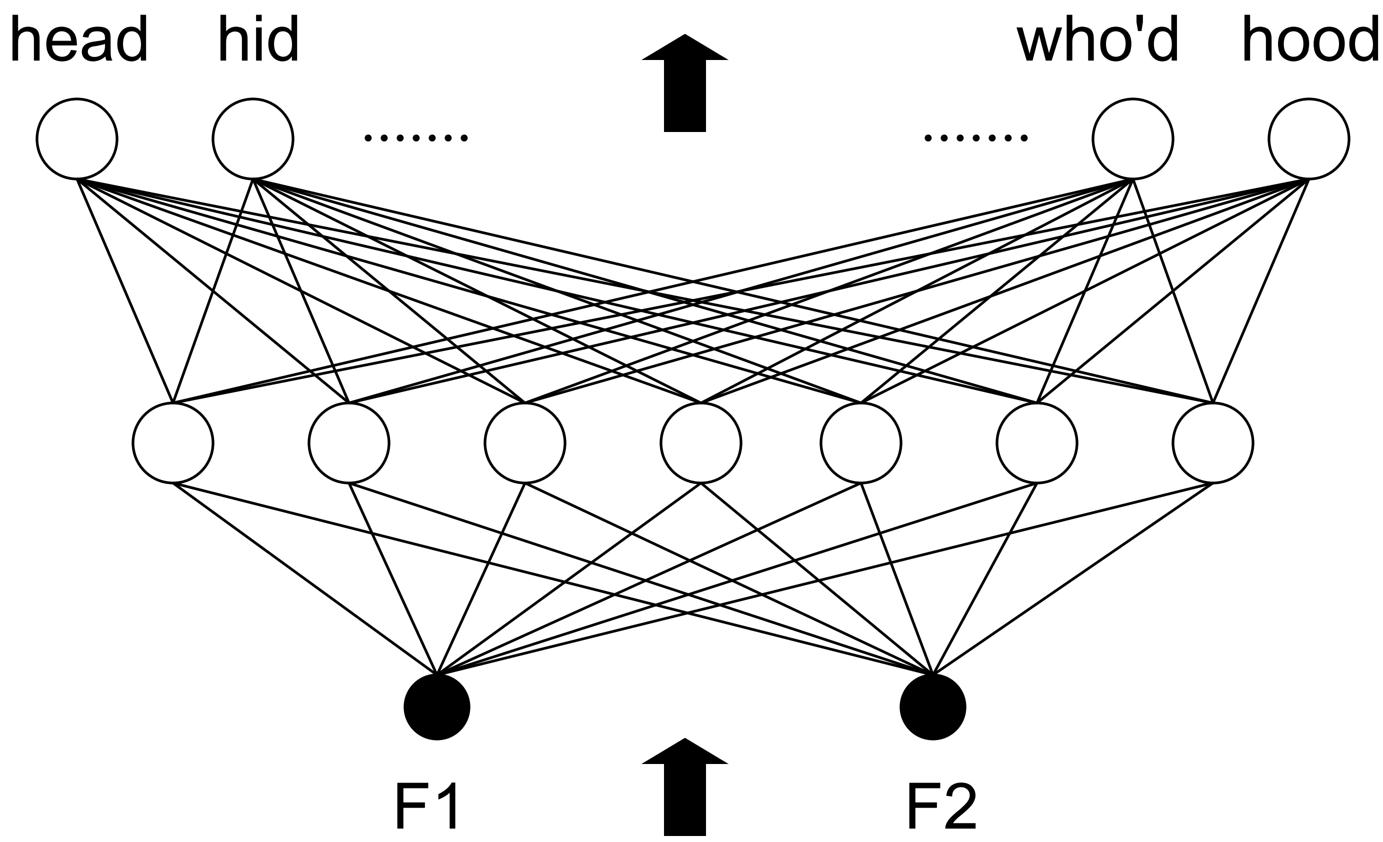
- Single perceptrons can express only linear decision surfaces

- Minsky, M. & Papert, S. (1969). Perceptrons.
  MIT Press.

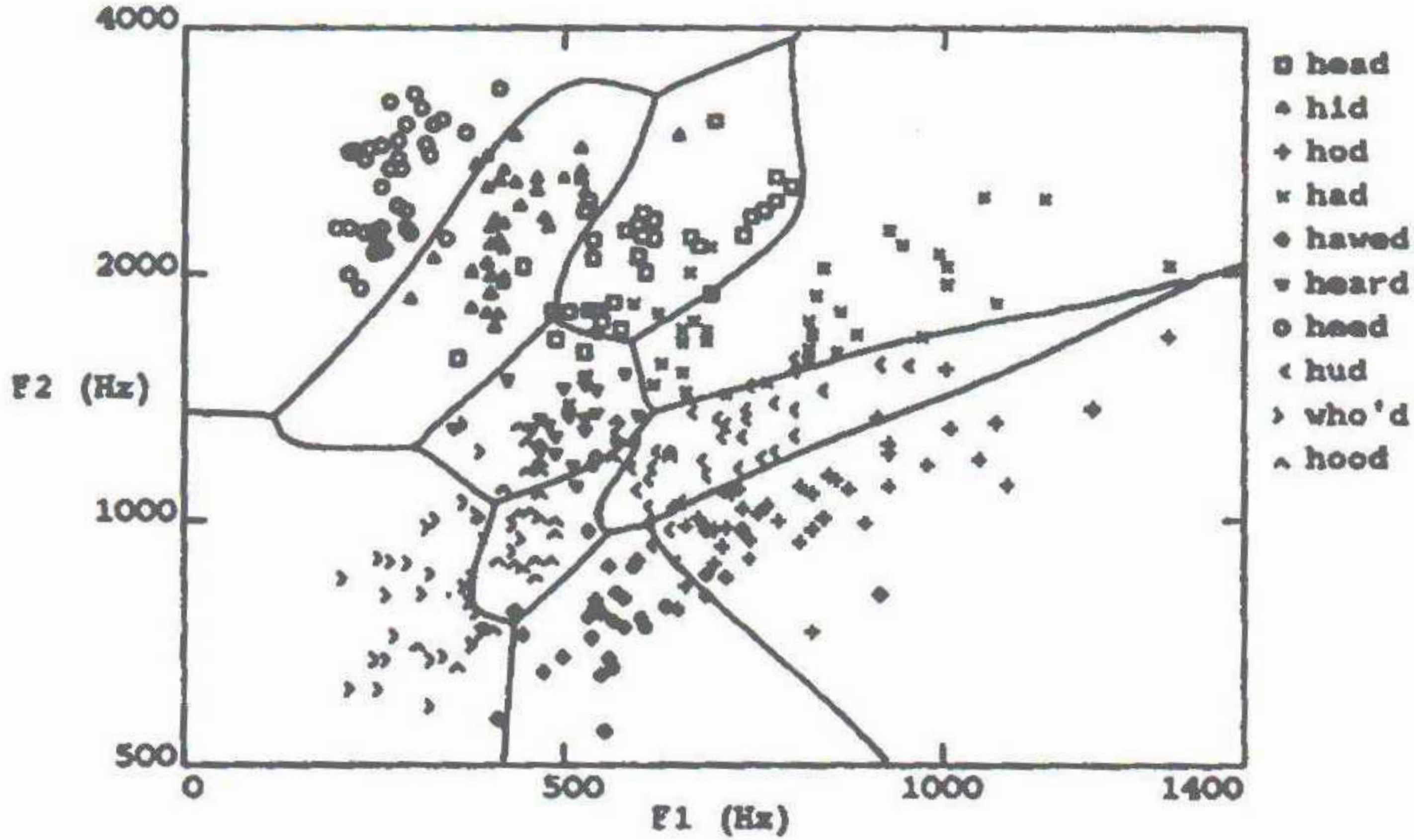
Multilayer Networks

- A multilayer network can represent highly nonlinear decision surfaces

Multilayer Networks



Multilayer Networks

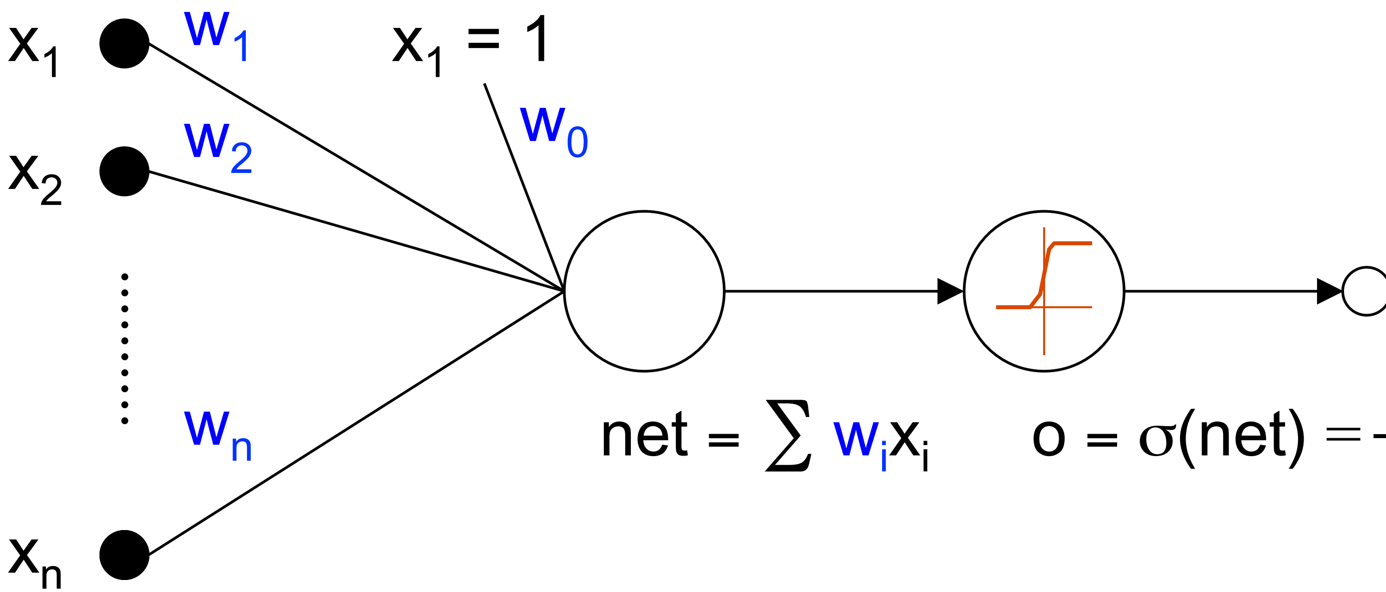


Multilayer Networks

What type of unit ?

- Perceptrons: non-differentiable
- Linear units: only linear functions
- ....

Multilayer Networks



$$net = \sum w_i x_i \qquad o = \sigma(net) = \frac{1}{1 + e^{-net}}$$

sigmoid unit

Multilayer Networks

Sigmoid unit:

$$\partial\sigma(y)/\partial y = \sigma(y).(1 - \sigma(y)) = o(1 - o)$$

Backpropagation Algorithm

Training error:

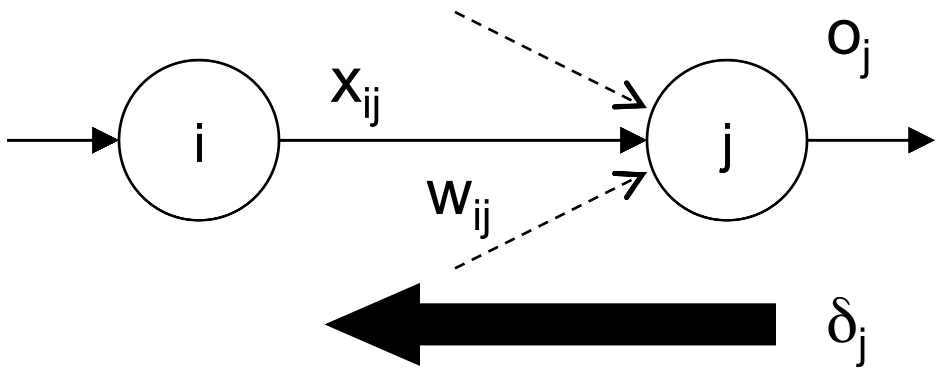
$$E(\vec{w}) = \sum_{d \in D} \sum_{k \in outputs} (t_{kd} - o_{kd})^2/2$$

or stochastic approximation:

$$E(\vec{w}) = \sum_{k \in outputs} (t_{kd} - o_{kd})^2/2$$

Backpropagation Algorithm

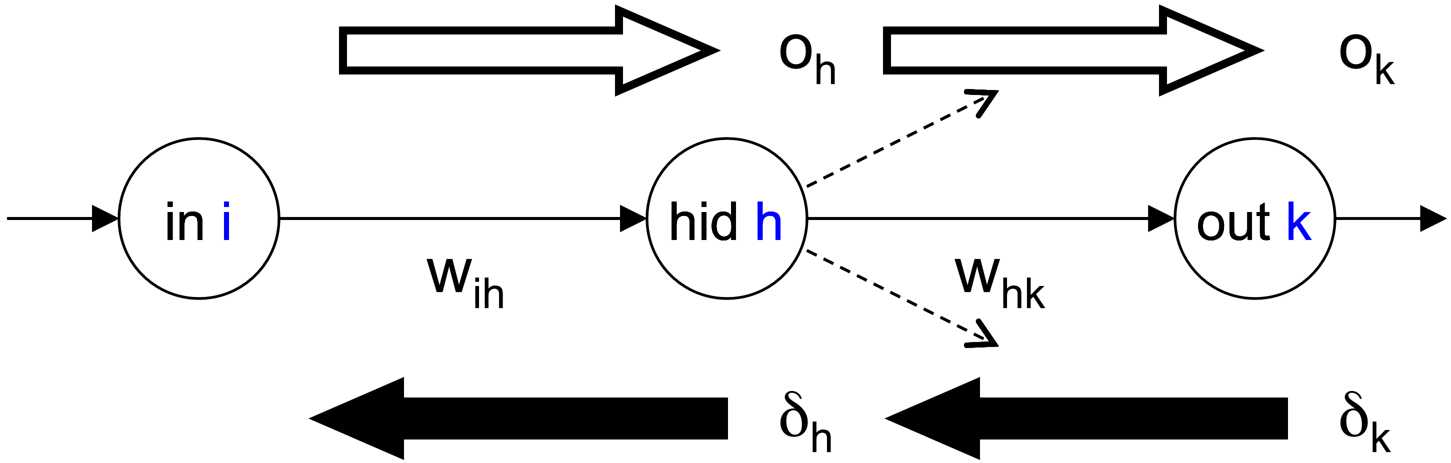
Weight update:



$$\Delta w_{ij} = -\eta \partial E / \partial w_{ij} = \eta \delta_j x_{ij}$$

$$\delta_j = o_j(1 - o_j)(t_j - o_j)$$

Backpropagation Algorithm



$\delta_h = o_h(1 - o_h)\sum_k w_{hk}\delta_k$ $\delta_k = o_k(1 - o_k)(t_k - o_k)$

Backpropagation Algorithm

Adding momentum:

$$\Delta w_{ij}(n) = \eta\delta_j x_{ij} + \alpha\Delta w_{ij}(n-1)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ \text{iteration} & \text{momentum } (0 \le \alpha \le 1) \end{array}$$

- Keeping the search direction $\Rightarrow$ passing small local minima
- Increasing the search step size $\Rightarrow$ speeding convergence

Backpropagation Algorithm

Convergence and local minima:

- Not guaranteed to converge towards the global minimum error, but highly effective in practice

- Approximately linear when the weights are close to 0 (due to sigmoid), hence avoiding local minima of non-linear functions

Backpropagation Algorithm

Heuristics to alleviate the local minima problem:

- Add a momentum term to the weight-update rule
- Use stochastic gradient descent rather than true gradient descent
- Train multiple networks using the same data, but initializing each network with different random weights

Backpropagation Algorithm

Representation power of feedforward networks:

- Boolean functions: any one, using 2-layer (1 hidden + 1 output) networks
- Continuous functions: any bounded one with approximation, using 2-layer networks
- Arbitrary functions: any one with approximation, using 3-layer networks

Backpropagation Algorithm

Hypothesis space and inductive bias:

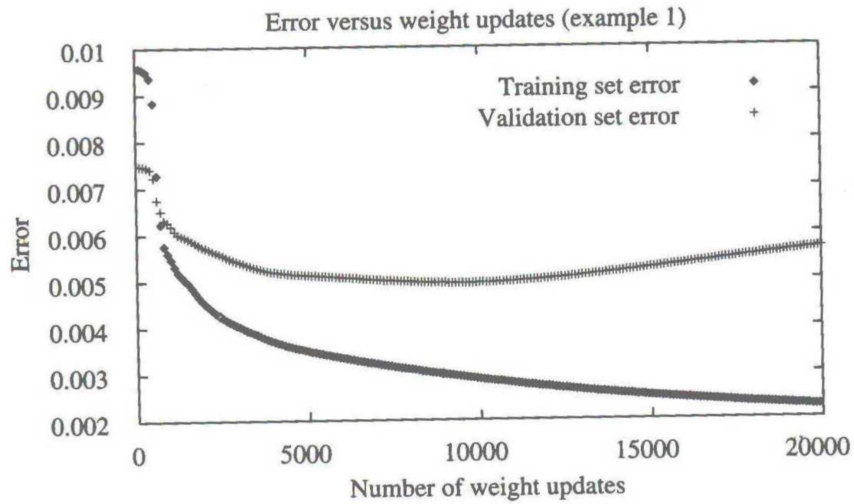
- Hypothesis: every possible assignment of network weights
- Inductive bias: smooth interpolation between data points

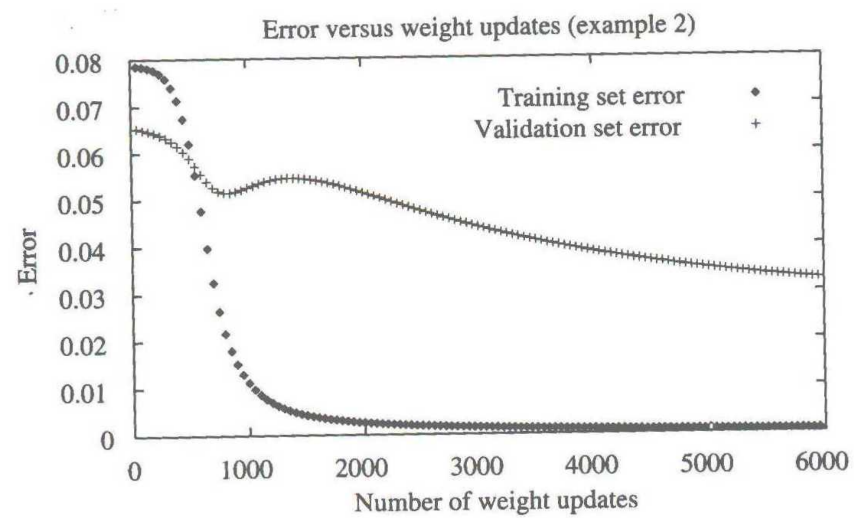
Backpropagation Algorithm

Stopping criterion and overfitting:

- Number of iterations
- Limit of training errors

Backpropagation Algorithm





Applications

To recognize face pose:

- 30×32 resolution input images
- 4 directions: left, straight, right, up

$\Rightarrow 960 \times 3 \times 4$ network

Exercises

- In Mitchell's ML (Chapter 4): 4.1 to 4.10