Quiz 1

Graded

Student

PATHE NEVISH ASHOK

Total Points

20 / 20 pts

Question 1

True False **9** / 9 pts

1.1 1 **3** / 3 pts

- ✔ **+ 1 pt** Correct answer is T
- ✔ **+ 2 pts** Valid proof for why the two hyperplanes have the same decision boundary, possibly with minor issues
- **− 1 pt** Minor mistakes in proof -- cannot think of examples of minor mistakes :)
- **+ 0 pts** Totally wrong or else unanswered
- **+ 0 pts** Excessive Scratching / Overwriting

1.2 2 **3** / 3 pts

- ✔ **+ 1 pt** Correct answer is F
- ✔ **+ 2 pts** Valid counter example where value of $\mathbf{w}^\top \mathbf{x} + b$ is strictly positive for both $\mathbf{x}_0$ and $-\mathbf{x}_0$, possibly with minor issues
- **− 1 pt** Minor mistakes in example e.g. value of $\mathbf{w}^\top \mathbf{x} + b$ is zero for one/both the cases.
- **+ 0 pts** Totally wrong or else unanswered

1.3 3 **3** / 3 pts

- ✔ **+ 1 pt** Correct answer is T
- ✔ **+ 2 pts** Valid example where value of x_0, x_1 is given in terms of a, b and universal constants, possibly with minor issues.
- **− 1 pt** Minor issues e.g. examples are not given for general values of a, b, but for specific values of a, b, or if say cases of $a > b$ and $b > a$ are not discussed.
- **+ 0 pts** Totally wrong or else unanswered

Question 2

Sliding Parabolas

7 / 7 pts

+ 0 pts Give marks directly out of 7 using points adjustment. 1 mark per correct answer. No partial marking for incorrect sign etc.

+ 0 pts Completely wrong or else unanswered

✔ **+ 7 pts** All 7 answered corrected.

Question 3

Vector Line-up

4 / 4 pts

✔ **+ 0 pts** Give marks directly out of 4 using points adjustment. 1 mark per correct answer. Deduct 0.5 marks per part for minor mistakes e.g. if answer violates at most one condition e.g. in part a, violating the condition that answer must be a non-zero vector, or if in part b, squared L_2 norm of the vector is an integer but L_2 norm is not an integer, etc. No marks if an answer is outright wrong.

+ 0 pts Completely wrong or else unanswered

+ 4 pts Point adjustment

CS 771A: Intro to Machine Learning, IIT Kanpur			Quiz I (24 Jan 2024)
Name	PATHE NEVISH ASHOK		**20 marks**
Roll No	220757	Dept. CSE	Page **1** of **2**

Instructions:
1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ – such cases may get straight 0 marks.
5. Do not rush to fill in answers. You have enough time to solve this quiz.

Q1. (**True-False**) Write **T** or **F** for True/False (write **only in the box on the right-hand side**). You must also give a brief justification for your reply in the space provided below.(**3 x (1+2) = 9 marks**)

1	The two hyperplane classifiers $\mathbf{a}^\top\mathbf{x}+b$ and $\mathbf{p}^\top\mathbf{x}+q$ with $\mathbf{a},\mathbf{p}\in\mathbb{R}^2, b,q\in\mathbb{R}$ have the same decision boundary if $\mathbf{a}+\mathbf{p}=\mathbf{0}$ and $b+q=0$. Give a brief proof if your answer is T else give a concrete counter example if your answer is F.	T

The hyperplane classifier $a^T x + b$ has the decision boundary $a^T x + b = 0$. Now $a+p=0 \Rightarrow p=-a$, $b+q=0 \Rightarrow q=-b$

$\therefore$ [the decision boundary of $p^T x + q$ is $p^T x + q = 0$] $\Rightarrow$ $p^T x + q = (-a)^T x + (-b)$
$= -(a^T x + b) = 0$

Since the same set of vectors $x \in \mathbb{R}^2$ satisfy both decision boundaries
(although the normals of hyperplane differ)

2	Melbo has learnt a classifier $\mathbf{w}^\top\mathbf{x}+b$ with $\mathbf{w}\in\mathbb{R}^2, b\in\mathbb{R}$. If $\text{sign}(\mathbf{w}^\top\mathbf{x}_0+b)>0$ for some $\mathbf{x}_0\in\mathbb{R}^2$ then it must always be the case that $\text{sign}(\mathbf{w}^\top(-\mathbf{x}_0)+b)<0$. Give brief proof if answer is T else a clear counter example of $\mathbf{w},\mathbf{x}_0,b$ if answer is F.	F

Consider $w = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $x_0 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$, $b = 7$ $\Rightarrow$ $\text{sign}(w^T x_0 + b) = \text{sign}(-1-4+7) > 0$

while $\text{sign}(w^T(-x_0)+b) = \text{sign}(1\cdot1 + 2\cdot2 + 7) > 0$

3	Consider $f,g:\mathbb{R}\to\mathbb{R}$ of the form $f(x)=ax+b, g(x)=bx+a$ with $a,b>0$. If $a\neq b$, there must exist $x_0,x_1\in\mathbb{R}$ such that $f(x_0)<g(x_0)$ and $f(x_1)>g(x_1)$. If your answer is T, give example of x_0,x_1 in terms of a,b, else give a counter example.	T

Consider the function $h(x) = f(x) - g(x) = ax+b-(bx+a)$
$= (a-b)x + (b-a) = (a-b)(x-1)$

$\therefore$ $\because a\neq b$, WLOG assuming $a>b$, $h(x_1) > 0 \;\forall\; x_1 > 1$,
$h(x_0) < 0 \;\forall\; x_0 < 1$

$\therefore$ Such $x_0, x_1 \in \mathbb{R}$ exist. | $h(x_1) \geq 0 \Rightarrow f(x_1) - g(x_1) > 0$
$\Rightarrow f(x_1) > g(x_1)$

and likewise for x_0

Q2. (Sliding parabolas) Consider $f(x) = (x-a)^2 + b, g(x) = -(x-p)^2 + q$ and $h(x) = x^3/2$.

Find values of $a, b \in \mathbb{R}$ such that f and h share a tangent at $x = 1$.	$a =$ 1/4	$b =$ -1/16
Find values of $p, q \in \mathbb{R}$ such that g and h share a tangent at $x = 1$.	$p =$ 7/4	$q =$ 17/16
Find the value of $f + g$ at $x = 1$ i.e., $(f+g)(1)$	$(f+g)(1) =$ 1	
Find the first derivative of $f + g$ at $x = 1$ i.e., $(f+g)'(1)$	$(f+g)'(1) =$ 3	
Find second derivative of $f + g$ at $x = 1$ i.e., $(f+g)''(1)$	$(f+g)''(1) =$ 0	

Write your answers only in the space provided. **(2 + 2 + 1 + 1 + 1 = 7 marks)**

Q4. (Vector line-up) Give examples of 4D vectors (fill-in the 4 boxes) with the following properties. Any example will get full marks so long as it satisfies all the properties mentioned in the question. Your answers to the parts a, b, c, d, e may be same/different. **(4 x 1 = 4 marks)**

a. A vector $\mathbf{v} \in \mathbb{R}^4$ such that $\mathbf{v} \neq \mathbf{0}$ and $\mathbf{v}$ is perpendicular to both the vectors $(1,0,1,1)$ and $(0,1,0,0)$.
b. A vector $\mathbf{v} \in \mathbb{R}^4$ with only integer coordinates (at least 2 non-zero coordinates) whose L_2 norm is also an integer.
c. A vector $\mathbf{v} \in \mathbb{R}^4$ that is perpendicular to its own negative i.e., $\mathbf{v} \perp -\mathbf{v}$.
d. A point $\mathbf{v} \in \mathbb{R}^4$ with equal L_2 distance from the vectors $(1,2,3,4)$ and $(4,3,2,1)$.

1	0	0	-1
0	1	2	2
0	0	0	0
5/2	5/2	5/2	5/2

Anything written here will not be graded

ROUGH WORK

$a^T x + b = 0$
$p^T x + q = 0$

$a + p = 0 \quad b + q = 0$

$\frac{9}{16} - \frac{1}{16} + \frac{12}{16} - \frac{9}{16} = 1$

x_1, x_2 decision boundary

100-1
1011
0100

$a^T x_1 + b = 0$
$p^T x_2 + q = 0$

$(-a)^T x + (-b) = 0 \quad 2[(x-a) - (x-p)]$
$-a^T x - b = 0 \quad = 2(p-a) = 3$

$\cancel{(1-a)^2 + b = q - (1-p)^2}$
$(1-a)^2 + b = 1/2$
$2(1-a) = 3/2$

$b = \frac{9}{16} - \frac{1}{16} = -\frac{1}{16}$
$a = 1/4$

$a = \begin{pmatrix}1\\2\end{pmatrix} \quad x = \begin{pmatrix}2\\1\end{pmatrix} \quad b = -4$
4 on decision bdry

$f'' \; 2 \quad g'' \; -2 \quad 0$

$\begin{pmatrix}-1\\-2\end{pmatrix} x_2 + 4 = 0$

$-5 \quad 7$

$\begin{pmatrix}1\\2\end{pmatrix}\begin{pmatrix}-1\\-2\end{pmatrix} + 7$

$q - (1-p)^2 = 1/2$
$-2(1-p) = 3/2$
$1-p = -3/4$

$q = \frac{9}{16} + \frac{8}{16} = \frac{17}{16}$
$p = 7/4$

$(ax+b) - (bx+a)$
$= (a-b)x + (b-a)$

$= (a-b)(x-1)$