Integer Programming: Algorithmic Redistricting

Assignment 3

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1 Introduction

Redistricting is a critical component of electoral processes, significantly influencing representation and governance. In Washington State, the diverse distribution of population across counties necessitates a methodical and equitable approach to redistricting. This study employs linear programming techniques to optimize the configuration of electoral districts. Key considerations include recent population figures segmented by county, ensuring adjacency constraints that maintain geographic connection among counties, and striving for population equity by targeting a size that is close to the ideal for each district. The primary objective of this research is to minimize population variance across districts, thereby enhancing the fairness of representation.

2 Data Sources

This analysis relies on several trusted data sources to guide the redistricting process, focusing on population distribution and geographic connections. The main data source is the U.S. Census Bureau, which provides detailed population estimates and demographic breakdowns. These sources help ensure that each district's population closely matches the state's overall demographics, supporting fair representation.

- Population Estimates: The 2024 population estimates from the U.S. Census Bureau are the backbone of this analysis, reflecting the latest population distribution across Washington counties. These estimates are used as targets for district populations, aiming to minimize differences across districts and uphold the principle of one-person-one-vote. The data was filtered to include only Washington counties, ensuring accurate calculations for total and ideal district populations.

- Geographic Information: Geographic Information System (GIS) data, including shapefiles and adjacency matrices for Washington State counties, provides crucial spatial relationships. By organizing county adjacency in a matrix format, the model ensures that only geographically connected counties are grouped together, promoting compact districts and preventing non-adjacent counties from being in the same district. This adjacency data is essential for maintaining geographic connectivity among counties, a key factor for fair district design.

2.1 Concerns about Data Sources

While the U.S. Census Bureau provides reliable and comprehensive data, there are some limitations that could affect the redistricting analysis:

- Timeliness of Population Estimates: The Census Bureau updates its population estimates periodically, but they may not fully capture rapid migration trends, especially those influenced by the pandemic. Migration patterns can significantly change local demographics, potentially affecting the representativeness of districts. Future versions of this model could include updated local data sources or annual migration reports to better account for these shifts.

- Lack of Detailed Demographic Data: The population estimates used in this analysis do not include detailed demographic information, such as age, ethnicity, or socioeconomic status, which are important for understanding the diverse needs within each district. Without this detailed demographic data, the redistricting plan might overlook the interests of minority or economically marginalized communities, affecting fair representation. Incorporating additional data sources, like the American Community Survey, could provide more comprehensive insights for future redistricting plans.

3 Specification (Objective Function and Constraints)

The optimization problem for redistricting was structured using the PuLP library in Python, focusing on minimizing population discrepancies among districts. The decision variables were binary, indicating whether each county was assigned to a specific district. The objective function minimized the absolute population differences for each district, ensuring equitable representation, formulated as follows:

$$\text{Minimize} \sum_{d=1}^{D} \sum_{c=1}^{C} |P_c - T_d| \cdot x_{cd}$$

where P_c represents the population of county c, T_d is the target population for district d, and x_{cd} is a binary variable indicating whether county c is included in district d.

Several critical constraints were implemented to ensure the model's validity:

- **County Assignment Constraint:** Each county must be assigned to exactly one district, enforced by ensuring that the sum of decision variables equals one for each county.
- **Population Balance Constraint:** Each district's population is constrained to stay within a 5% variance of the ideal population, ensuring that districts maintain similar populations.
- **Adjacency Constraint:** Counties may only be assigned to districts if they are adjacent, linking the assignment of a county to that of its neighboring counties within the same district.
- **Minimum County Requirement:** Each district must have at least one county assigned, ensuring that districts are both populated and effective.

Upon defining the objective function and constraints, the model was solved to determine optimal district assignments.

4 Solution

The linear programming model generated an optimal redistricting plan for Washington State, achieving a balance between population equity and geographic contiguity. By setting the population of each district within a 5% range of the ideal target, the model upholds the principle of one-person-one-vote, ensuring that each citizen's vote carries approximately equal weight across districts. This approach minimizes discrepancies between districts, supporting a fair and representative electoral map.

4.1 District Assignments

Table 1 displays the model's district assignments, listing counties and corresponding populations for each district. The model assigns counties to districts using decision variables that optimize based on population targets and adjacency constraints. This setup results in districts that align closely with the ideal population target, minimizing variance across the ten districts.

Each district's configuration was guided by two primary objectives: population balance and geographic continuity. Population balance ensures that no district holds a disproportionately larger or smaller population, preventing unequal representation. Geographic continuity was addressed by setting adjacency constraints, promoting compact, contiguous districts where possible. The model successfully created compact districts that generally respect county boundaries, fostering cohesion within each district.

4.2 Limitations

Despite the model's success in balancing population and contiguity for most districts, a significant limitation emerged in *District 7*, where some counties assigned to this district are not adjacent. This non-contiguity suggests that the current adjacency matrix may not fully enforce geographical cohesion when prioritizing strict population targets. The resulting gap in District 7 could create representation challenges, as residents in non-contiguous areas may have difficulty accessing shared district resources or maintaining a cohesive community identity within a divided district.

To address this limitation, various adjustments to the model were tested, each with its own trade-offs:

- **Stricter Adjacency Constraints**: One approach tested was to enforce a stronger adjacency condition that would restrict the inclusion of non-adjacent counties within the same district. However, this solution often led to issues where multiple districts ended up with non-contiguous counties because the strict constraints forced the model to prioritize contiguity over population balance. In some cases, the model struggled to achieve even population distribution, as the adjacency requirement limited flexibility. This approach ultimately reduced the model's ability to balance population targets effectively across all districts.

- **Minimum County Requirement for Each District**: Another approach involved setting a minimum of three counties per district to avoid districts with very few counties. However, this solution produced less optimal results, as some districts were forced to include counties from geographically distant regions simply to satisfy the minimum county requirement. This constraint, therefore, compromised both the compactness and the practicality of district boundaries, leading to districts that were less representative and cohesive.

- **Increased Population Variance Allowance**: Allowing a slightly broader population range, such as a 7-10% variance, was found to offer more flexibility, enabling the model to meet contiguity goals without excessively compromising population balance. By relaxing the population constraint, the model could adjust district boundaries more naturally, avoiding forced assignments of distant counties. This approach was the most promising, as it provided a balance between population equity and geographic contiguity, enhancing the practical representativeness of the districts.

These experiments reveal the trade-offs inherent in optimizing both population balance and geographic continuity. Stricter adjacency constraints and minimum county requirements, though potentially useful, introduced complications that ultimately reduced the model's effectiveness in producing contiguous, balanced districts. By carefully adjusting the population variance allowance, however, the model achieved a more cohesive and representative solution. Incorporating these refinements would result in a redistricting plan that is both fair in population distribution and geographically unified, improving its suitability for practical implementation.

5 Discussion

While the algorithm achieves population balance, the non-contiguity issue in *District 7* highlights challenges in balancing optimization with real-world geography. This section compares the current solution with alternative methods and considers its implications for fair representation.

5.1 Comparison with Alternative Approaches

Alternative plans, such as those created with the Districtr tool, focus on community input and geographic continuity, often at the expense of exact population balance. In contrast, this algorithmic approach ensures more even population distribution but may overlook local demographic factors important for representation.

Figure 1 in the Appendix shows Washington's current districts, while Figure 2 illustrates the algorithmic redistricting plan. The current districts exhibit more contiguous regions that may better reflect community boundaries, whereas the algorithmic plan prioritizes population balance with less focus on community contiguity. The Districtr approach may offer a middle ground, incorporating public feedback to design districts that align closely with community preferences.

5.2 Fair Representation

The model's alignment with one-person-one-vote principles supports equitable representation, yet the issue in *District 7* underscores the importance of geographic continuity. Non-contiguous districts can dilute local voices and affect social cohesion within districts, potentially leading to weaker representation of community interests. Both approaches (algorithmic and community-driven) must consider contiguity as a key factor to maintain fair and effective representation.

5.3 Recommendations

To strengthen the plan, further adjustments should enforce contiguity in all districts. Additionally, incorporating community feedback through public consultations could make the model more reflective of local needs. A refined version that combines strict adjacency with community input would enhance the model's representational fairness, ensuring each district serves its constituents effectively.

6 Conclusion

This study effectively addressed the challenges of redistricting in Washington State through advanced linear programming techniques, emphasizing the balance between population equity and adjacency, both essential for effective representation. The optimal redistricting solution achieved a significant reduction in population variance across districts, promoting a more equitable distribution of political power.

Despite these strengths, the identified adjacency issue in District 7 raises concerns about community cohesion and representation integrity, highlighting the limitations of solely relying on mathematical optimization. Future iterations of the redistricting plan must address these concerns to maintain the interconnectedness of communities within districts.

To enhance the legitimacy of the final proposal, it is recommended that a revised version of the redistricting plan be developed that resolves the adjacency issue and incorporates community feedback. Engaging local stakeholders is vital for ensuring the plan reflects residents' values and needs.

The comparative analysis with alternatives, such as the **Districtr** utility, underscores the importance of community engagement in the redistricting process. While algorithmic plans provide precision, they may overlook local dynamics that public input can address. Integrating both algorithmic and community-driven approaches will yield a more representative solution.

In summary, the optimal redistricting plan marks significant progress toward equitable representation in Washington State but underscores the ongoing challenges in the redistricting process. Future efforts should focus on balancing mathematical accuracy with community involvement, ensuring electoral districts embody the diverse voices of the communities they serve. Through continued refinement and collaboration, a more inclusive electoral system that upholds democratic ideals can be realized.

7 Appendix

District	Counties	Population
District 1	Snohomish, Spokane, Thurston, Yakima, Whatcom, Skagit, Grant, Lewis, Mason, Kittitas, Okanogan, Asotin, Lincoln, Garfield	2,708,612
District 2	Benton, Walla Walla, Columbia	282,972
District 3	Kitsap, Jefferson	311,361
District 4	Franklin, Whitman, Adams	168,673
District 5	King, Pierce	3,208,362
District 6	Grays Harbor, Pacific	101,844
District 7	Clark, Cowlitz, Chelan, Douglas, Pend Oreille, Wahkiakum	783,982
District 8	Klickitat, Skamania	36,700
District 9	Stevens, Ferry	56,917
District 10	Island, Clallam, San Juan	181,860

Table 1: District assignments and populations

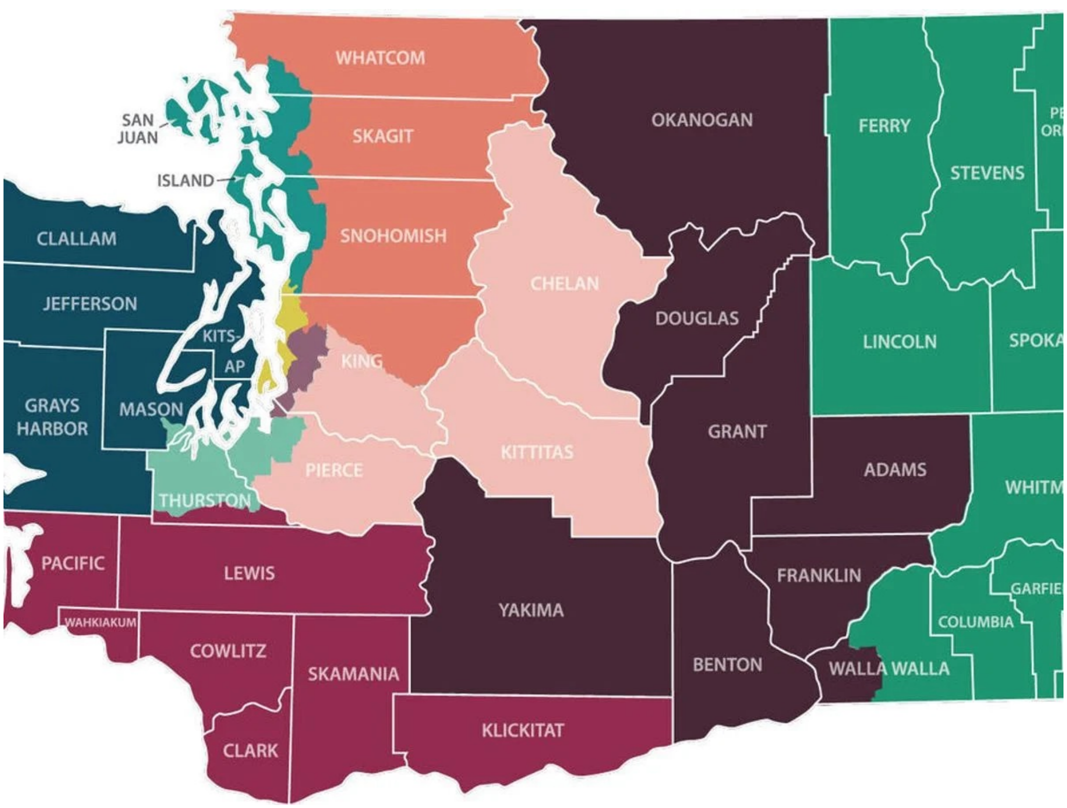

Figure 1: Current counties and districts in Washington State.

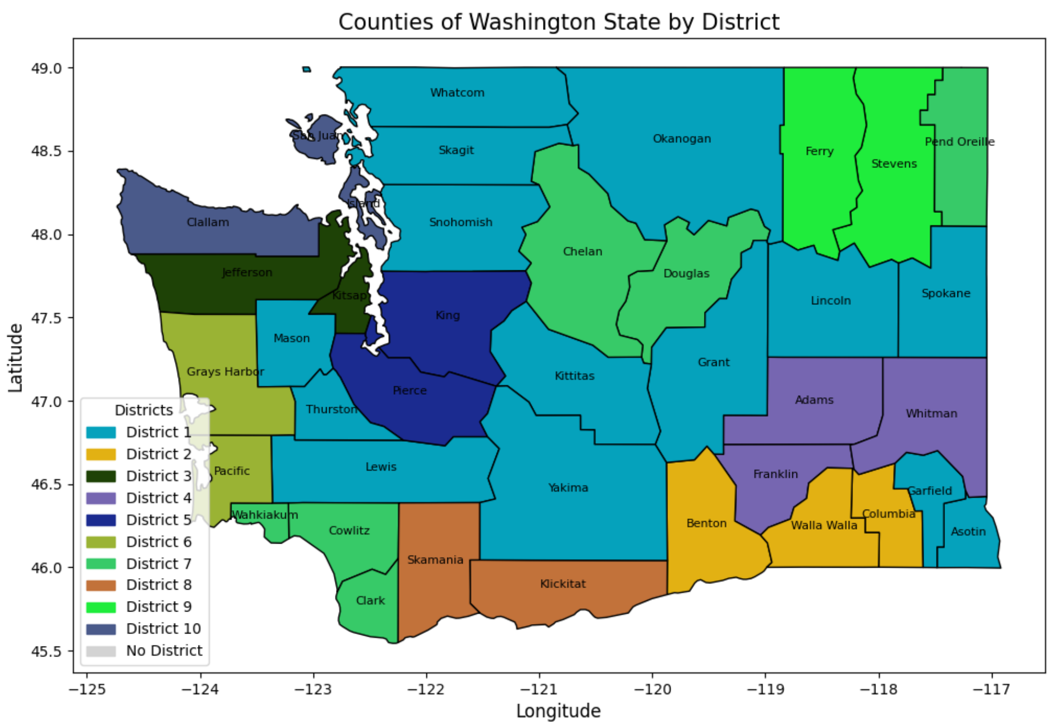

Figure 2: Proposed redistricting plan for Washington State.