

2. Problem Statement

Problem Statement 1: [50 marks]

In each of the following situations, state whether it is a correctly stated hypothesis testing problem and why?

1. $H_0: \mu = 25$, $H_1: \mu \neq 25$

Its correctly stated hypothesis as the alternative hypothesis is a negative statement and null hypothesis has equality

2. $H_0: \sigma > 10$, $H_1: \sigma = 10$

$\sigma = 10$

~~Its correctly stated hypothesis as the alternative hypothesis is not a negative statement and null hypothesis doesn't have equality~~
Its not a correctly stated hypothesis as the alternative hypothesis is not a negative statement and null hypothesis doesn't have equality

3. $H_0: \bar{x} = 50$, $H_1: \bar{x} \neq 50$

Its a correctly stated hypothesis as the alternative hypothesis is negative statement and null hypothesis has equality

4. $H_0: p = 0.1$, $H_1: p = 0.5$

$p = 0.5$

Its not correctly stated hypothesis as its alternative hypothesis is not a negative statement and the value is not given

5. $H_0: s = 30$, $H_1: s > 30$

$s > 30$

Its not correctly stated hypothesis as the alternative hypothesis is not a negative statement

Problem Statement 2: [100 marks]

The college bookstore tells prospective students that the average cost of its textbooks is Rs. 52 with a standard deviation of Rs. 4.50. A group of smart statistics students thinks that the average cost is higher. To test the bookstore's claim against their alternative, the students will select a random sample of size 100. Assume that the mean from their random sample is Rs. 52.80. Perform a hypothesis test at the 5% level of significance

$$H_0: \mu \leq 52 \quad H_1$$

$$\mu > 52$$

$$\alpha = 0.05$$

Reject H_0 if $z > 1.645$

$$z = \frac{(52.80 - 52)}{(4.50 / \sqrt{100})} = 1.78$$

Reject H_0 ; conclude that the average cost of textbooks is statistically significantly higher than the bookstore's claim

$$P = P(z > 1.78) = 0.5 - 0.4625 = 0.0375$$

Hypothesis

$$H_0: \mu = 52$$

$$H_1: \mu \neq 52$$

Significance level : 5%

$$\mu = 52$$

$$S = 4.50$$

$$n = 100$$

$$\bar{x} = 52.8$$

$$Z = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}}$$

$$= \frac{52.8 - 52}{\frac{4.50}{\sqrt{100}}}$$

$$= \frac{.8}{.45} = 1.78$$

Critical value of z is -1.96 and $+1.96$

$z = 1.78$ falls in acceptance region, we accept the null hypothesis. Hence, the mean average cost of its textbooks is Rs. 52.

Problem Statement 3: [100 marks]

A certain chemical pollutant in the Genesee River has been constant for several years with mean $\mu = 34$ ppm (parts per million) and standard deviation $\sigma = 8$ ppm. A group of factory representatives whose companies discharge liquids into the river is now claiming that they have lowered the average with improved filtration devices. A group of environmentalists will test to see if this is true at the 1% level of significance. Assume that their sample of size 50 gives a mean of 32.5 ppm. Perform a hypothesis test at the 1% level of significance and state your decision.

Hypothesis

$$H_0: \mu = 34$$

$$H_1: \neq 34$$

Significance level = 1%.

$$\mu = 34$$

$$\sigma = 8$$

$$n = 50$$

$$\bar{x} = 32.5$$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\frac{32.5 - 34}{8/\sqrt{50}}$$

$$= \frac{-1.5}{1.13}$$

$$= -1.33$$

Critical value of Z is -2.58 and $+2.58$.
The critical value is $Z = \pm 2.58$ for a two-tailed test at 1% level of significance. Since, the computed value of $Z = -1.33$ falls in acceptance region, we accept the null hypothesis.

Problem Statement 4: [100 marks]

Based on population figures and other general information on the U.S. population, suppose it has been estimated that, on average, a family of four in the U.S. spends about \$1135 annually on dental expenditures. Suppose further that a regional dental association wants to test to determine if this figure is accurate for their area of country. To test this,

22 families of 4 are randomly selected from the population in that area of the country and a log is kept of the family's dental expenditure for one year. The resulting data are given below. Assuming, that dental expenditure is normally distributed in the population, use the data and an alpha of 0.5 to test the dental association's hypothesis.

1008, 812, 1117, 1323, 1308, 1415, 831, 1021, 1287, 851, 930, 730, 699,
872, 913, 944, 954, 987, 1695, 995, 1003, 994

Hypothesis

$$H_0: \mu = 1135$$

$$H_1: \mu \neq 1135$$

Significance level = 5%

$$\mu = 1135$$

$$s = 240.37$$

$$n = 22$$

$$\bar{x} = 1031.32$$

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{1031.32 - 1135}{240.37/\sqrt{22}} = \frac{-103.68}{51.25} = -2.02$$

The critical value of z is -1.96 and $+1.96$

Since, the computed value of $z = -2.02$ falls in rejection region, we reject the null hypothesis.

Problem Statement 5: [100 marks]

In a report prepared by the Economic Research Department of a major bank the Department manager maintains that the average annual family income on Metropolis is \$48,432. What do you conclude about the validity of the report if a random sample of 400 families shows an average income of \$48,574 with a standard deviation of 2000?

Problem Statement 6: [100 marks]

Suppose that in past years the average price per square foot for warehouses in the United States has been \$32.28. A national real estate investor wants to determine whether that figure has changed now. The investor hires a researcher who randomly samples 19 warehouses that are for sale across the United States and finds that the mean price per square foot is \$31.67, with a standard deviation of \$1.29. Assume that the prices of warehouse footage are normally distributed in population. If the researcher uses a 5% level of significance, what statistical conclusion can be reached? What are the hypotheses?

Hypotheses $\rightarrow H_0 = \mu = 48,432$ Significance level = 10%
 $H_1 = \mu \neq 48,432$

$$\mu = 48,432$$

$$s = 2000$$

$$n = 400$$

$$\bar{x} = 48,574$$

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{48,574 - 48,432}{2000/\sqrt{400}} = \frac{142}{100} = 1.42$$

critical value of Z is -1.645 and $+1.645$

Since, the computed value of $Z = 1.42$ falls in acceptance region, we accept the null hypothesis.

Problem Statement 6: [100 marks]

Suppose that in past years the average price per square foot for warehouses in the United States has been \$32.28. A national real estate investor wants to determine whether that figure has changed now. The investor hires a researcher who randomly samples 19 warehouses that are for sale across the United States and finds that the mean price per square foot is \$31.67, with a standard deviation of \$1.29. Assume that the prices of warehouse footage are normally distributed in population. If the researcher uses a 5% level of significance, what statistical conclusion can be reached? What are the hypotheses?

$$\text{Hypotheses: } H_0: \mu = 32.28 \\ H_1: \mu \neq 32.28$$

$$\text{Significance level} = 5\%$$

$$\mu = 32.28$$

$$s = 1.29$$

$$n = 19$$

$$\bar{x} = 31.67$$

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{31.67 - 32.28}{1.29/\sqrt{19}} = \frac{-0.61}{0.29} = \cancel{-2.1} -2.1$$

Critical value of Z is -1.96 and 1.96
Since the computed value of $Z = -2.1$ falls in rejection region, we reject the null hypothesis

Problem Statement 9: [50 marks]

Find the t-score below which we can expect 99% of sample means will fall if samples of size 16 are taken from a normally distributed population.

$$1 - \alpha = 0.99$$

$$\alpha = 0.01$$

$$df = n - 1$$

$$df = 15$$

$$t_{0.99} = -t_{0.01}$$

$$t_{0.99} = -t_{0.01} = -2.602$$

Problem Statement 11: [100 marks]

In a production process, the target value of μ is 50 and σ is not known. The sample measurement on a day are 45, 54, 51, 47, 52, 50, 41, 51, 43, 53. Test $H_0: \mu = 50$, against $H_1: \mu < 50$ with $\alpha = 0.05$.

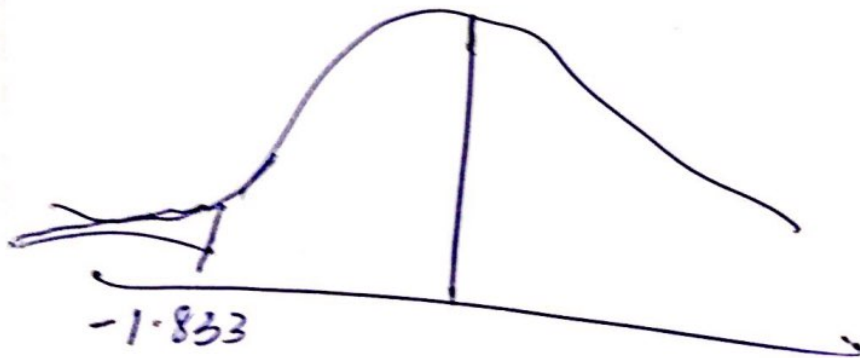
$$t = -0.924$$

$$s = 4.45$$

D.O.F = $n - 1 = 9$ is 1.833 so rejection region is

$R: t < -1.833$. The value of t comes under acceptance region, i.e. H_0 is ~~accepted~~ accepted at 5% level of significance.

t -distribution - one tail test



Problem Statement 12: [100 marks]

A restaurant has been arranging sales of 300 lunch packets per day at Brigade road.

Because of the construction of the new building and other complexes, it expects to increase the sale. During the first 16 days after the occupation of these building, the daily sales were 304, 367, 385, 386, 262, 329, 302, 292, 350, 320, 298, 258, 364, 294, 276, 333. Based on this information will you conclude that the sales have increased?

Note: Solution submitted via github must contain all the detailed steps.

Let μ be the daily sales

H_0 is $\mu \leq 300$ (we considered sales has not increased)

H_1 is $\mu > 300$

$$t = 1.94$$

$$R: t > 1.75 \quad t_{(0.05, 15)}$$

we reject the hypothesis

$n-1 = 15$ Two tail Test

