Assignment-1

N Praful Raj

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1 Problem

Find the coordinates of the point where the line through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ crosses the plane

$$\begin{pmatrix} 2 & 1 & 1 \end{pmatrix} x = 7 \tag{1.1}$$

2 Explanation

We know that vector equation of line passing through two points , say A and B is

$$\mathbf{x} = \vec{A} + \lambda \left(\vec{B} - \vec{A} \right) \tag{2.1}$$

We also know that equation of a plane is

$$\mathbf{n}^T \mathbf{x} = c \tag{2.2}$$

Substituting (2.1) in (2.2) as line passes through the plane we can get the point of contact.

3 Solution

Step 1
Let us first findout the equation of line passing through two given points using (2.1)

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2-3 \\ -3+4 \\ 1+5 \end{pmatrix} \tag{3.1}$$

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{3.2}$$

Step 2
Now let us construct the equation of plane from the given data. Using the values we can construct

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \tag{3.3}$$

Step 3
Now using (3.2) , (3.3) in (2.2)

$$\begin{pmatrix} 2 & 1 & 1 \end{pmatrix} \left(\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \right) = 7 \tag{3.4}$$

solving (3.4) we get

$$6 - 4 - 5 - 2\lambda + \lambda + 6\lambda = 7 \tag{3.5}$$

$$5\lambda = 10 \tag{3.6}$$

$$\lambda = 2 \tag{3.7}$$

Now substituting the value of λ in (3.2) we get the point of contact of line on plane

$$\mathbf{x} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} \tag{3.8}$$