

Assignment 1

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1 PROBLEM

Find the coordinates of the point where the line through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ crosses the plane

$$(2 \ 1 \ 1)x = 7 \quad (1.0.1)$$

2 EXPLANATION

We know that vector equation of line passing through two points , say A and B is

$$\mathbf{x} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \quad (2.0.1)$$

We also know that equation of a plane is

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.2)$$

Substituting (2.0.1) in (2.0.2) as line passes through the plane we can get the point of contact.

3 SOLUTION

Let us first findout the equation of line passing through two given points using (2.0.1)

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2-3 \\ -3+4 \\ 1+5 \end{pmatrix} \quad (3.0.1)$$

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \quad (3.0.2)$$

Now let us construct the equation of plane from the given data. Using the values we can construct

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad (3.0.3)$$

Now using (3.0.2) , (3.0.3) in (2.0.2)

$$(2 \ 1 \ 1) \left(\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \right) = 7 \quad (3.0.4)$$

solving (3.0.4) we get

$$6 - 4 - 5 - 2\lambda + \lambda + 6\lambda = 7 \quad (3.0.5)$$

$$5\lambda = 10 \quad (3.0.6)$$

$$\lambda = 2 \quad (3.0.7)$$

Now substituting the value of λ in (3.0.2) we get the point of contact of line on plane

$$\mathbf{x} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} \quad (3.0.8)$$

