Assignment-1

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October 2, 2020

1   Problem

Find the coordinates of the point where the line through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ crosses the plane $\begin{pmatrix} 2 & 1 & 1 \end{pmatrix}$x=7

2   Explanation

We know that vector equation of line passing through two points , say A and B is

$$\mathbf{x} = \vec{A} + \lambda \left(\vec{B} - \vec{A} \right) \tag{2.1}$$

We also know that equation of a plane is

$$\mathbf{n}^T \mathbf{x} = c \tag{2.2}$$

Substituting 2.1 in 2.2 as line passes through the plane we can get the point of contact.

3   Solution

Step 1
Let us first findout the equation of line passing through two given points using 2.1

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2-3 \\ -3+4 \\ 1+5 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} -\lambda + 3 \\ \lambda - 4 \\ 6\lambda - 5 \end{pmatrix} \tag{3.1}$$

Step 2
Now let us construct the equation of plane from the given data.
Using the values we can construct

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \tag{3.2}$$

Step 3
Now using 3.1 3.2 in 2.2

$$\mathbf{n}^T \mathbf{x} = c \tag{3.3}$$

solving 3.3 we get
$-2\lambda + 6 + \lambda - 4 + 6\lambda - 5 = 7$
$5\lambda = 10$

$$\lambda = 2 \tag{3.4}$$

Now substituting the value of λ in 2.1 we get
$$\mathbf{x} = \begin{pmatrix} -2+3 \\ 2-4 \\ 12-5 \end{pmatrix}$$
Therefore the point of contact of line on plane is

$\mathbf{x} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix}$ The figure illustrates the