

Assignment-1

N Praful Raj

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1 Problem

Find the coordinates of the point where the line through $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ crosses the plane

$$(2 \ 1 \ 1)x = 7 \quad (1.1)$$

2 Explanation

We know that vector equation of line passing through two points, say A and B is

$$\mathbf{x} = \vec{A} + \lambda (\vec{B} - \vec{A}) \quad (2.1)$$

We also know that equation of a plane is

$$\mathbf{n}^T \mathbf{x} = c \quad (2.2)$$

Substituting 2.1 in 2.2 as line passes through the plane we can get the point of contact.

3 Solution

Step 1

Let us first find out the equation of line passing through two given points using 2.1

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2-3 \\ -3+4 \\ 1+5 \end{pmatrix} \quad (3.1)$$

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \quad (3.2)$$

Step 2

Now let us construct the equation of plane from the given data.

Using the values we can construct

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad (3.3)$$

Step 3

Now using 3.2, 3.3 in 2.2

$$(2 \ 1 \ 1) \left(\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \right) = 7 \quad (3.4)$$

solving 3.4 we get

$$6 - 4 - 5 - 2\lambda + \lambda + 6\lambda = 7 \quad (3.5)$$

$$5\lambda = 10 \quad (3.6)$$

$$\lambda = 2 \quad (3.7)$$

Now substituting the value of λ in 3.2 we get the point of contact of line on plane

$$\mathbf{x} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} \quad (3.8)$$

