

Assignment 2

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1 PROBLEM

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

2 EXPLANATION

By reading the problem we can understand that the probability of an event is asked to find out based on the occurrence of the other event. Here the Probability of occurrence of six has been asked based on the occurrence of the another event i.e reporting of the occurrence of the six. Such kind of conditional probabilities can be solved using Bayes Theorem.

Theorem 2.1 (Bayes Theorem). *If E_1, E_2, \dots, E_n are n non empty events which constitute a partition of sample space S , i.e. E_1, E_2, \dots, E_n are pairwise disjoint and $E_1 \cup E_2 \cup \dots \cup E_{n-1} \cup E_n = S$ and A is any event of nonzero probability, then*

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{j=1}^n P(E_j)P(A|E_j)} \quad (2.0.1)$$

for any $i=1,2,3,\dots,n$

3 SOLUTION

Let E_1 be the event where six occurs

$$P(E_1) = \frac{1}{6} \quad (3.0.1)$$

Let E_2 be the event where six doesnot occur

$$P(E_2) = \frac{5}{6} \quad (3.0.2)$$

Let A be an event where the man reports the occurrence of six, then the probability that man reports occurrence of six only when six has occurred

$$P(A|E_1) = \frac{3}{4} \quad (3.0.3)$$

Probability that the man reports occurrence of six when six has not occurred

$$P(A|E_2) = \frac{1}{4} \quad (3.0.4)$$

\therefore The probability that man reports six while he is telling truth using (??) is

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} \quad (3.0.5)$$

Using (??),(??),(??),(??) in (??)

$$\frac{\frac{1}{6} * \frac{3}{4}}{\frac{1}{6} * \frac{3}{4} + \frac{5}{6} * \frac{1}{4}} = \frac{3}{8} \quad (3.0.6)$$

$$P(E_1|A) = \frac{3}{8} \quad (3.0.7)$$