Part 1

a) 2, 4, 16 ... n

k	2^k	i
0	1	2
1	2	4
2	4	16

$2^{2^k} < n$

$\log 2^{(2^k)} = \log n$

$2^k \cdot \log 2 = \log n$

$\log(2^k \cdot \log 2) = \log(\log(n))$

$k \cdot \log(2 \cdot \log 2) = \log(\log n)$

$\boxed{\Theta k < \Theta \log(\log(n))}$

b) line 2 $\Theta(n)$

line 3 will evaluate true $\sum_{i=1}^{n} \Theta(1) + \sum_{i=1}^{?} \sum_{j=1}^{i^3} \Theta(1)$ times

will evaluate a max of i^3 times

$\sum_{i=1}^{n} \Theta(1) + \sum_{i=1}^{\sqrt{n}} \Theta(1)$

so $\Theta(n^3)$ time

$\Theta(n) + (\Theta(\sqrt{n})^3 + \Theta(2\sqrt{n})^3 + \ldots (\sqrt{n})^3)$ $(i = \sqrt{n}, i < n, i += \sqrt{n})$ $\sqrt{n} \to y$

$\Theta(n) + \Theta(\sqrt{n}^3 + 1^3 + 2^3 + \ldots (\sqrt{n})^3]$ $\sum_{x=1}^{y} x^3 = \frac{1}{4} y^2 (y+1)^2$

$1^3 + 2^3 + \ldots \sqrt{n}^3 = n^{7/2}$

$\Theta(n^{7/2}) = \frac{1}{4} y^4 + \Theta(y^3)$

$= \frac{1}{4} n^{4/2} + \Theta n^{3/2} = n^{7/2}$

$[\Theta = (n^2)]$

$\Theta(n^2)$

d)

line 5 $\Theta(n)$ (for loop)

line 6? if statement will run at most 2 times b/c will pass when i=10 and then again when i=15, but won't pass again b/c there will be an decimal value.

$$\sum_{i=0}^{n=1} (\Theta(1) + \Theta(2 \cdot size))$$

$$= \Theta(n) + \Theta(2n \cdot size)$$

$\Theta(2n \cdot size)$? $\Theta(n)$ $\Rightarrow$ $\Theta = \Theta(n \cdot size)$

c)

line 1 $\Theta(n)$ runtime (for loop)

line 2 $\Theta(n)$ runtime (for loop)

line 3 use Big O b/c we don't know how many iterations it will run

when $m = m + m \Rightarrow$ log run time $\Rightarrow \Theta(\log(n))$

$$\sum_{i=1}^{n} \left(\sum_{k=1}^{n} \left(\Theta(1) + O\sum_{m=1}^{n} \left(\frac{1}{m} \right) \right) \right)$$

At most, will be evaluated every iteration or "n" times

$$= \sum_{i=1}^{n} \left(\Theta(n) + \Sigma(\Theta(\log(n)) \right)$$

$$= \sum_{i=1}^{n} (\Theta(n) + \sum(\Theta(\log(n))$$

$$= \sum_{i=1}^{n} (\Theta(n) + \sum_{i=1}^{n} (\Theta(\log(n))$$

$$= \sum_{i=1}^{n} (\Theta(n) + n\Theta(n\log(n))$$

$$2 \cdot \Theta(n^2) + \Theta(n^2\log(n))$$

$$= \Theta(n^2\log(n)) > \Theta(n^2)$$

$$= \Theta(n^2\log(n))$$