KEN4258: Computational Statistics

Assignment 2

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March 2024

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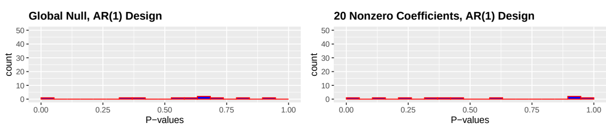
Link to our GitHub repository: https://github.com/nprianikov/compstats

1) Reproduce Figure 1 from (Candès et al. 2018).

```
library(tibble)

set.seed(101)

n <- 500
p <- 200
n_sim <- 10 # TODO: not showing anything with 10000

# Generate X as a single AR(1) time-series of size p
generate_AR1 <- function(n, p) {
  X <- replicate(n, arima.sim(n=p, list(0.5)))
  return(t(X))
}

# Function to generate responses, fit a logistic regression model and return individual p-values
generate_responses <- function(n, p, prob = NULL) {
  X <- generate_AR1(n=n, p=p)
  if (is.null(prob)) {
    prob <- plogis(0.08 * rowSums(X[, 2:22]))
  }
  Y <- rbinom(n, 1, prob)

  fit <- glm(Y ~ X, family = binomial(link="logit"))

  p_values <- summary(fit)$coef[, "Pr(>|z|)"][2] # Take p-values for beta_1 only
  return(p_values)
}

# Combine the lists into a single tibble
df_plot <- tibble(
  p_values_1 = c(replicate(n_sim, generate_responses(n=n, p=p, prob=0.5))),
  p_values_2 = c(replicate(n_sim, generate_responses(n=n, p=p)))
)
```



2) What is the problem that Figure 1 tries to illustrate?

Figure 1 illustrates the p-values for predictors in two different models. The models are given by the following functions:

(1) $Y|X_1,...,X_p \sim \text{Bernoulli}(0.5)$

(2) $Y|X_1,...,X_p \sim \text{Bernoulli}(\text{logit}(0.08(X_2+...+X_{21})))$

Where $(X_1,...,X_p)$ are itself random variables generated by an AR(1) time series with AR coefficient 0.5, and $p = 200$. In other words, the results of the first simulation are completely independent from the 200 values of X_i while the second simulation is only dependent on $(X_2,...,X_{21})$.

3) Propose a solution to address the problem.

```
# draw a new sample from the conditional distribution of Xj | X-j using a random number generator
# Not sure if this is the correct way to sample
simulate_Xj_given_Xj <- function(X, j) {
  Xj <- X[, j]
  X_minus_j <- X[, -j, drop = FALSE]

  # Fit a model to predict Xj from the rest of the data
  model <- glm(Xj ~ X_minus_j - 1, family = "gaussian")
  predicted_Xj <- predict(model, newdata = list(X_minus_j = X_minus_j))

  residuals_sd <- sd(resid(model))
  # Generate new samples for Xj with added randomness
  new_Xj_samples <- predicted_Xj + rnorm(length(predicted_Xj), mean = 0, sd = residuals_sd)

  return(new_Xj_samples)
}



# Feature importance statistic function to test whether Xj and Y are conditionally independent.
compute_Tj <- function(X, y) {
  fit <- glm(y ~ X, family = binomial(link="logit"))
  coef <- coef(fit)
  return(abs(coef))
}





# Conditional Randomization Test for a specific j
conditional_randomization_test <- function(X, y, j, K = 200) {
  original_Tj <- compute_Tj(X, y)[j]
  greater_count <- 0

  for (k in 1:K) {
    # Create a new data matrix by simulating the jth column and keeping the remaining columns the sa
    X_simulated <- X
    X_simulated[, j] <- simulate_Xj_given_Xj(X, j)

    # Compute Tj for the simulated data
    simulated_Tj <- compute_Tj(X_simulated, y)[j]
    # Increment count if simulated Tj is greater than or equal to the original Tj
    if (simulated_Tj >= original_Tj) {
      greater_count <- greater_count + 1
```

```r
    }
  }
  # Calculate p-value
  p_value <- (1 + greater_count) / (K + 1)
  return(p_value)
}

# Conditional Randomization Test for entire matrix X
conditional_randomization_test_all <- function(X, y, K = 1) {
  p_values <- numeric(ncol(X))

  for (j in 1:ncol(X)) {
    p_values[j] <- conditional_randomization_test(X, y, j, K)
  }

  return(p_values)
}

n <- 500
p <- 200
X <- generate_AR1(n=n, p=p)
Y <- rbinom(n, 1, 0.5)
p_values <- conditional_randomization_test_all(X, Y)
print(p_values)
```

```
##   [1] 1.0 0.5 0.5 1.0 0.5 0.5 1.0 0.5 1.0 0.5 0.5 1.0 1.0 1.0 1.0 1.0 1.0 0.5
##  [19] 1.0 1.0 0.5 0.5 0.5 0.5 0.5 1.0 0.5 1.0 1.0 1.0 0.5 0.5 0.5 0.5 0.5 0.5
##  [37] 0.5 0.5 0.5 1.0 1.0 0.5 1.0 0.5 0.5 0.5 0.5 0.5 0.5 1.0 1.0 1.0 0.5 1.0
##  [55] 0.5 0.5 0.5 0.5 0.5 1.0 0.5 0.5 1.0 0.5 0.5 1.0 0.5 0.5 0.5 0.5 0.5 1.0
##  [73] 1.0 0.5 1.0 1.0 1.0 1.0 1.0 0.5 1.0 1.0 0.5 0.5 1.0 0.5 1.0 0.5 1.0 1.0
##  [91] 0.5 0.5 1.0 1.0 0.5 0.5 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 0.5 1.0 1.0 0.5
## [109] 0.5 0.5 1.0 0.5 1.0 0.5 0.5 1.0 0.5 0.5 1.0 1.0 0.5 1.0 0.5 0.5 1.0 1.0
## [127] 0.5 0.5 0.5 0.5 1.0 0.5 0.5 0.5 1.0 0.5 0.5 1.0 1.0 1.0 1.0 1.0 0.5 1.0
## [145] 1.0 0.5 1.0 0.5 0.5 1.0 1.0 1.0 1.0 1.0 1.0 0.5 1.0 0.5 0.5 1.0 1.0 1.0
## [163] 0.5 0.5 0.5 1.0 0.5 1.0 1.0 1.0 1.0 0.5 0.5 1.0 1.0 1.0 0.5 1.0 0.5 1.0
## [181] 1.0 1.0 1.0 1.0 0.5 0.5 0.5 1.0 0.5 1.0 1.0 0.5 0.5 0.5 1.0 0.5 1.0 0.5
## [199] 1.0 0.5
```

4) Show that your solution fixes the problem.

5) Find a real dataset and apply your method.