**MECE E6106 – Fall 2024**

**Homework # 2**

**(due Nov 8, 2021, 8pm)**

1. We showed that the streamwise upwind Petrov-Galerkin (SUPG) method was developed to overcome the issues of applying the Galerkin finite element method (FEM) to the exact artificial diffusion (EAD) method. **[10 points]**
    a. Starting from the weak form of the SUPG method applied to the advection-diffusion equation (ADE), construct the local stiffness matrices using piecewise linear finite elements and assemble them to derive the global nodal equation.
    b. Compare this expression against the EAD method based on finite difference approximation and comment on the SUPG limitations.

2. Consider the below steady-state ADE,

$$\frac{du}{dx} = 0.05\frac{d^2u}{dx^2}\ ; 0 \leq x \leq 1,$$

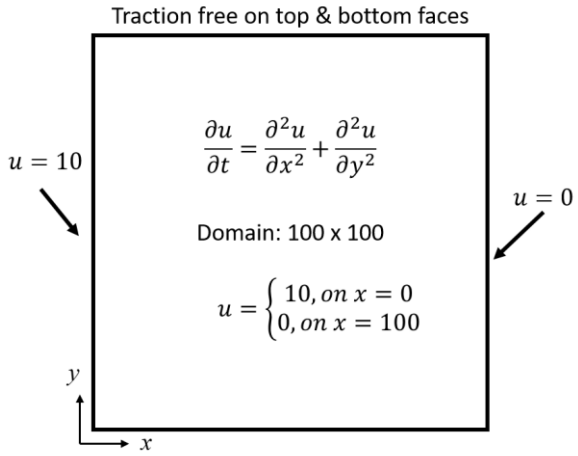
with $u(0) = 0$ and $u(1) = 10$

We have analyzed this problem when applying Galerkin FEM and its strong dependence on the cell Peclet number. Here, we will explore the performance of EAD and SUPG methods. **[50 points]**

Modify the code from Homework 1 to solve the above problem using EAD and SUPG methods. Choose the same grids that you used in Homework 1 for the range of cell Peclet numbers ($Pe_{\Delta x}$) and compare the results against the exact solution. Evaluate the effect of using higher-order (quadratic) elements.

Use element point-of-view to construct the stiffness matrix and load vector, and write an assembly routine to perform the assembly to the global stiffness matrix and load vector. It is also strongly recommended to use numerical integration at the element level (e.g., Gauss quadrature-based integration).

3. Solve the unsteady diffusion problem below in Fenics. **[40 points]**




Use Galerkin FEM and write up the weak form. You may use any time integration scheme (explicit or implicit) and any element type (linear or quadratic triangles, quadrilaterals with any order, etc.). Use this opportunity to explore Fenics, which will be helpful for the final project.

**General note for all assignments:**

- Please submit reports as PDF files. Please attach your code and instructions for compiling and executing it.
- Use carefully chosen plots to support your analysis and discussion. Plots should be only as big as they need to be and not any bigger.
- A thoughtful exploration of the problem beyond what is asked for is encouraged.
- Report style format to be used for all assignments with the following suggested sections:
  - Objective
  - Methodology
  - Results and Discussion
  - Conclusion
  - References
- While it is preferred that you type your report, you may write by hand as long as you do it neatly and legibly. For typed-up reports, equations and other expressions that are tedious to type can be handwritten.