MECE E6106 – Fall 2024
Homework # 3
(due Nov 27, 2024, 8pm)

A damped harmonic oscillator is governed by,

$$m\frac{d^2u}{dt^2} + c\frac{du}{dt} + ku = 0 \qquad (1)$$

where m is the object's mass, c is the damping coefficient, and k is the spring stiffness. Depending on the damping ratio, $\gamma = {}^{c}/_{c_r}$, where $c_r = 2\sqrt{mk}$ is the critical damping, the system behaves as an undamped ($\gamma = 0$), underdamped ($0 < \gamma < 1$), overdamped ($\gamma > 1$), or critically damped ($\gamma = 1$) system.

If $\omega_n = \sqrt{{}^{k}/_{m}}$ denotes the natural frequency of the system, the equation can be rewritten as,

$$\frac{d^2u}{dt^2} + 2\gamma\omega_n\frac{du}{dt} + \omega_n^2 u = 0 \qquad (2)$$

For initial conditions, $u(0) = 1$ and $\frac{du}{dt}(0) = 1$, and a natural frequency, $\omega_n = \pi$, solve equation (2) for all the damping regimes ($\gamma = 0, 0.5, 1, 2$) using the generalized-α time integration method. For each γ, choose at least four values of the spectral radius parameters ($0 \le \rho_\infty \le 1$) and compare your numerical solution against the analytical solution. For each ρ_∞, refine the time step size, and comment on the error convergence and the numerical solution behavior.

General note for all assignments:

- Please submit reports as PDF files. Please attach your code and instructions for compiling and executing it.
- Use carefully chosen plots to support your analysis and discussion. Plots should be only as big as they need to be and not any bigger.
- A thoughtful exploration of the problem beyond what is asked for is encouraged.
- Report style format to be used for all assignments with the following suggested sections:
    - Objective
    - Methodology
    - Results and Discussion
    - Conclusion
    - References
- While it is preferred that you type your report, you may write by hand as long as you do it neatly and legibly. For typed-up reports, equations and other expressions that are tedious to type can be handwritten.