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6 April 2022
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Assignment 6: Game Theory and Social Networks

1. Consider a symmetric game where each player must decide whether to hunt or gather for food. If both players decide to hunt, they successfully capture an animal for meat, and they each get a payoff of 10. If both players gather, they will help each other find food faster and each will get payoff 7. If one player hunts and one gather, the hunter will be injured by the prey and get payoff 0, while the gatherer will get some food and get payoff 5.

a. Draw the matrix representation of this game.

		Player 2	
		Hunt	Gather
Player 1	Hunt	10,10	0,5
	Gather	5,0	7,7

b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.

According to the textbook, "When a player has a strategy that is strictly better than all other options regardless of what the other player does, we will refer to it as a strictly dominant strategy"(pg. 160).

In the case, neither players have a strictly dominant strategy because the higher payoff can only come if both players participate in hunting or if both players participate in gathering. If player 1 chooses hunting or chooses gathering, then the best choice for player 2 would be to choose the same activity as player 1 so both players get a same high payoff. The highest payoff would be if both players choose hunting. If both players choose to gather food, the payoff would be the same for both players.

c. Identify any pure strategy Nash equilibria. Explain your answer.

Key →H-hunt, G-Gather

(H,H) and (G,G) are the two Nash equilibria. If player 1 and player 2 chooses the same activity, which is either hunting or gathering, then both players will get the same payoff, which is higher than if the players split their activities. For instance, If player 1 chooses to hunt for food, then if player 2 chooses hunting as well, they both benefit by getting a high payoff, which is 10. If player 1 chose to hunt and player 2 chose to gather, then player 1 gets payoff 0 due to injury and player 2 would gather some and only get payoff 5.

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2. Two drivers are playing chicken, the game where they drive straight toward each other and see if one or both will swerve to avoid a crash. If one driver swerves and one stays straight, the driver who swerves is declared a "chicken" and gets payoff -10, while the driver who stays straight wins and gets payoff +10. If both drivers swerve, the game is a tie although they are both somewhat "chicken" and each gets payoff -5. If both drivers continue straight, they will crash and both get payoff -50.

   a. **Draw the matrix representation of this game.**

		Driver 2	
		Swerve	Straight
Driver 1	Swerve	-5,-5	-10,+10
	Straight	+10,-10	-50,-50

   b. **Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.**

      In this scenario, neither players have a strictly dominant strategy. Th reason is because the best choice really depends on both players' choices. For example, the best choice in this case is if driver 1 swerves, driver 2 would need to go straight to get the higher payoff which is +10. Vice-versa, if driver 1 drives straight, then it would be best if driver 2 swerves to get the high payoff. If both swerves or both goes straight, both same a low payoff. For either case, there is no strictly dominant strategy for either drivers to choose.

   c. **Identify any pure strategy Nash equilibria. Explain your answer.**

      (Swerve, Straight) and (Straight, Swerve) are the two pure Nash equilibria. For driver 1, it is the best response as driving straight is the high payoff (+10) if driver 2 swerves. For driver 2, it is the best response to drive straight if driver 1 swerves so driver 2 can get the higher payoff(+10).

3. Company A and Company B make competing products. If they both keep production low, the monthly profit for each company will be $16,000. If one company increases production and the other doesn't, the profit for the company that increases production will be $20,000 while the profit for the one that didn't increase production will be $12,000. If both companies increase production, the profit for each will be $14,000.

   a. **Draw the matrix representation of this game.**

		Company B	
		Low	Increased
Company A	Low	$16000,$16000	$12000,$20000
	Increased	$20000,$12000	$14000,$14000

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b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.

Both companies would have a strictly dominant strategy, which is the increased production. If company A chooses increased production, it's a better option for company B to pick increased production as well, since the payoff is still high, \$14,000 is greater than \$12,000. If company A decided on low production, then company B should still go for increased production for the higher payoff (\$20000), and vice-versa. If company B decided on low production, then company A should pick the increased production for a higher payoff. Company A and B have a higher payoff by both choosing increased production.

c. Identify any pure strategy Nash equilibria. Explain your answer.

(Low, Low) and (Increased, Increased) would be the Nash equilibria since increased production is the strictly dominant strategy for both of the companies. Moreover, if company A chose increased production, then company B should pick increased to receive the same payoff (\$146,000), and vice-versa. For low production is if company A choose low production, in order to receive the same payoff, company B would have to choose low production as well.

4. There are 80 cars which begin in city A and must travel to city B. There are two routes between city A and city B:
   Route I begins with a local street leaving city A, which requires a travel time in minutes equal to 10 plus the number of cars which use this street, and ends with a highway into city B which requires one hour of travel time regardless of the number of cars which use this highway.

   Route II: begins with a highway leaving city A. This highway takes one hour of travel time regardless of how many cars use it, and ends with a local street leading into city B. This local street near city B requires a travel time in minutes equal to 10 plus the number of cars which use the street.
   Show your work for all calculations.

   a. Draw the network described above and label the edges with the travel time in minutes needed to move along the edge, in terms of x. Let x be the number of travelers who use Route I. The network should be a directed graph as all roads are one-way.

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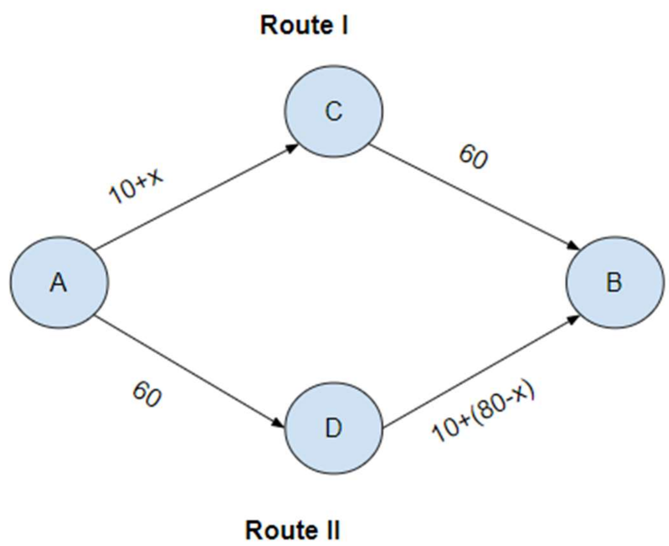

b. **What would be the travel time per car if all cars chose to use Route I? Explain your answer.**
   If all cars chose to use Route I, then x would be 80 cars.
   T = 10 + x + 60 ➔ 10 + 80 + 60 = 150 minutes

c. **Assume that cars simultaneously chose which route to use. Find the Nash equilibrium value of x, where x is the number of cars on each route. Also give the drive time per car and the total drive time for all cars.**
   Total drive time for all cars = 10 + x + 60 → 10 + 40 + 60 = 110 minutes

d. **Explain your answer to part c.**
   The Nash Equilibrium value of x would be 40 cars. With an even balance on both routes, it minimizes the drive time for each route. If x != 40, then the two routes would have unequal travel time and have more congestion. Each route would take 110 minutes of drive time per car, rather than 150 minutes.

Now the government builds a new (two-way) road connecting the nodes where local streets and highways meet. The new road is very short and takes no travel time. This adds two new routes.

Route III: consists of the local street leaving city A (on Route I), the new road, and the local street into city B (on Route II).

Route IV: consists of the highway leaving city A (on Route II), the new road, and the highway leading into city B (on Route I).

e. **What would the travel time be per car if all cars chose Route III?**
   T = (10+ 80) + (10 + 80) = 180 minutes

f. **What would the travel time be per car if all cars chose Route IV?**
   T = 60 + 0 + 60 = 120 minutes

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g. **What happens to total travel time as a result of the availability of the new road? Explain your answer.**

The total travel time is would very likely decrease because with the new road, the cars have more routes they can take which will create less congestion.

h. **If you can assign travelers to routes, it's possible to reduce total travel time relative to what it was before the new road was built. That is, the total travel time of the 80 cars can be reduced (below that in the original Nash equilibrium from part c) by assigning cars to routes. There are many assignments of routes that will accomplish this. Find one (where total travel time for all cars is lower than the Nash equilibrium total travel time). Explain why your reassignment reduces total travel time by giving the number of cars assigned to each of the Routes I, II, III, IV, the travel time per car on each route, and the total travel time of the 80 cars.**

In my example, I would remove Route IV because going on the highway takes up more time. Then I have 26 cars on Route 1, and 27 cars on Route II and Route III.

Travel Time for Route 1 = 10 + 26 + 60 = 116 minutes

Travel Time for Route 2 = 60 + 27 + 10 = 117 minutes

Travel Time for Route 3 = 10 + 27 + 10 + 27 = 74 minutes

In this case, Route I takes 116 minutes and Route II takes 117 minutes, which is greater than Route III, which takes 74 minutes. Hence, 117 minutes is the total travel time for all of the cars to get to City B.

5. There are a number of instances world-wide where elimination of a freeway has been a big improvement for the area. **Write a short (200 word) essay describing, in your own words, one such instance and the reasoning behind and results of the freeway elimination.** *Your essay must describe a specific highway. Give its name and location, and discuss the reasons behind the elimination, and the results of its elimination.*

In Portland, Oregon, there used to be a popular freeway called Harbor Drive. The six-lane parkway was opened in 1943 and removed in May 1974. In the history of American freeways, this highway was the first major highway to be intentionally removed, and it was Portland's first limited-access highway(CNU). Harbor Drive was once the original route of US 99W from the south into downtown Portland(OregonLive). For a decade, Harbor Drive was the only north-south freeway. However, as years went on, more freeways were being built within the city, and Harbor Drive became less important and used less. Former Governor Tom McCall during the 1960s, decided to have the freeway replaced, which occurred officially in 1974. It was decided that it would be better to replace the freeway with parkland. In 1978, Harbor Drive was later converted into a 37-acre park, now known as Tom McCall Waterfront Park, which was a good solution. Not all of the freeway was actually removed; the southernmost segment of Harbor Drive still remains, which can be seen as a default route for drivers that travel on I-5.The removal of the road was viewed as a milestone in urban planning. The park continues to attract the public, and due to the success of it, the Waterfront Park remains here today.

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6. Consider an auction in which there is one seller who wants to sell one unit of a good and a group of bidders who are each interested in purchasing the good. The seller will run a sealed-bid, second-price auction. There will be around a dozen other bidders in addition to your firm. All bidders have independent, private values for the good. Your firm's value for the good is $5,000.
   a. **What bid should your firm submit? Explain your answer.**
      Since this is a sealed-bid second-price auction, my firm would submit $5,000. The reason being is that if my firm does not win, the payoff is 0. If my firm bids $5,000 and wins, then my firm would be paying the value of the second-highest bid, which would result in a positive payoff. Moreover, if my firm decides to bid higher, it will result in overpaying, and if my firm decides to bid lower, then there is the risk of losing the item.
   b. **How does your bid depend on the number of other bidders who show up?**
      In a second-price auction, my bid does not depend on the number of other bidders that show up, considering that the bidders' values are private until the winner is revealed.
7. A seller wants to sell one unit of a good to some bidders, by running a sealed-bid second-price auction. Assume that there are two bidders who have independent, private values v which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 1/2. (If there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x.)
   a. **Show that the seller's expected revenue is 6/4.**
      Since this is sealed-bid second-price auction,
      (1/2)*(1/2) = ¼ -- probability since two bidders
      If bidder1 =1, bidder2 =1, then the expected revenue is 1.
      If bidder1=3,bidder2 =3, then the expected revenue is 3.
      If bidder1=1, bidder2=3, then the expected revenue is 1. (second-highest)
      If bidder1= 3, bidder2 = 1, then the expected revenue is 1. (second-highest)
      → (¼*1)+(1/4*1)+(1/4*1)+(1/4*3) = 6/4
   b. **Suppose that there are three bidders who have independent, private values v which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 1/2. What is the seller's expected revenue in this case? Explain your answer.**
      Since this is a sealed-bid second-price auction,
      (1/2)*(1/2)*(1/2) = 1/8 – probability since three bidders
      If bidder1=1, bidder2=1, bidder3=1, then the expected revenue is 1.
      If bidder1=1, bidder2=1, bidder3=3, then the expected revenue is 1.
      If bidder1=1, bidder2=3, bidder3=3, then the expected revenue is 3 (second highest)
      If bidder1=3, bidder2=3, bidder3=3, then the expected revenue is 3 (second highest)
      If bidder1=3, bidder2=3, bidder3=1, then the expected revenue is 3(second highest)
      If bidder1=3, bidder2=1, bidder3=3, then the expected revenue is 3(second highest)
      If bidder1=3, bidder2=1, bidder3=1, then the expected revenue is 1
      If bidder1=1, bidder2=3, bidder3=1, then the expected revenue is 1
      → (1/8*1)+(1/8*1)+ (1/8*1)+(1/8*1) +(1/8*3)+(1/8*3) +(1/8*3)+(1/8*3) = 16/8 = 2
   c. **Briefly explain why changing the number of bidders affects the seller's expected revenue. Would the expected revenue increase or decrease? Explain your answer.**

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Changing the number of bidders affects the seller's expected revenue, because the more bidders mean there would be a greater chance of being a tie and the probabilities of winning also increase. Less bidders would mean there is a lower probability of winning.

NOTES: (1) When you have multiple identical top bids (a multi-way tie), then the highest and second highest will be that same bid. That means the winner will pay the top bid, because that is both the first- and second-place bid value. (2) Do not use the formulas in section 9.7 of the text to calculate expected revenue. You should use the probabilities for the revenue for each of the bidding combinations: e.g. in 7a the probability that the two bidders each bid 3 (with a revenue of 3) is 1/4.

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Sources:

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