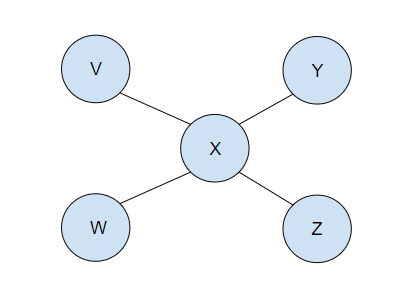
Assignment 3: Graph Theory and Social Networks

1. **a. Draw a diagram of an example of a graph having at least four nodes in which there is a single node X that is pivotal for every pair of nodes (not counting pairs that include X).**

**b. Explain your answer.**

Every pair of nodes is a pivotal pair (except X) for node X. In this graph, Node X is pivotal for each pair of nodes. In this example, Node X is pivotal for Node Y & Z. Node X is also pivotal for Node Y & W. Node X is also pivotal Node V & W, as well as for Node V & Z. This graph makes sure that every pair of nodes passes through Node X and is on the shortest path.

1. **a. Draw a diagram of an example of a graph in which every node is pivotal for at least one pair of nodes.**

Diagram

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**b. Explain your answer.**

In this example, every node is pivotal for at least one pair of nodes, where each node is pivotal for its immediate neighbors. Here, Node A is pivotal for Node E and Node B. Node B is pivotal for Node A and Node C. Node C is pivotal to Node B and Node D. Node D is pivotal to Node C and Node E. Lastly, Node E is pivotal to Node D and Node A.

1. **a. Which node or nodes in Figure 1 are gatekeepers?**

Node B and C are gatekeepers in Figure 1.

**b. Explain your answer**.

Node B is a gatekeeper because it lies on the path between Node A and C, along with Node A and F, Node E and F, & Node E and C. Node C is also a gatekeeper because Node C has neighbors D & G that are not connected by an edge, and it lies on the path between D and the rest of the nodes.

1. **a. Which node or nodes in Figure 1 are NOT local gatekeepers?**

Node A, D, and E are not local gatekeepers.

**b. Explain your answer**

Node A is not a local gatekeeper because for the two neighbors, Node B and E, they are connected by an edge. For Node E, it is not a local gatekeeper because for the two neighbors, Node A and B, they are connected by an edge. Lastly, Node D is not a local gatekeeper because there is only 1 neighbor, and a local gatekeeper needs two neighbors.

1. **a. Which node or nodes in Figure 1 are pivotal?**

Node B and Node C are pivotal in Figure 1.

**b. Explain your answer**

Node B is pivotal for pairs of nodes like Node A & C, and Node E & F. It is also pivotal as it lies on the shortest path for example, from Node A to Node G. Node C is pivotal for pairs of nodes like Node D & B, as it lies on the shortest path from Node D to B.

1. **You will notice that there are no gatekeepers in Figure 2 (Darpanet). Explain why the network was purposely designed that way.**

There are no gatekeepers in Figure 2 because in the case that a node being a gatekeeper no longer functions or is lost, then the entire connection would be gone as well, meaning that these nodes linked with the gatekeeper gets cut off from the rest of the network. In this case, if there was a gatekeeper and something happens to the gatekeeper that’s situated in between the west coast nodes and east coast, then the nodes from the left side (west coast) can no longer connect with the right side (east coast).

1. **a. Which node or nodes in Figure 2 are NOT pivotal?**

Nodes USCB & STAN are not pivotal in Figure 2.

**b. Explain your answer**

A node X is pivotal for a pair of distinct nodes, Y and Z, if it lies on every shortest path between the two distinct nodes, and node X is not equal to either node Y or Z. In this figure, all the nodes are pivotal for a pair of nodes except USCB and STAN. For example, in order for node UCLA to reach node SRI, it does not have to go through USCB or STAN, it can just go straight to SRI, and it is a shorter path.

1. **In your own words: In 2-3 sentences, explain what triadic closure is, and how it plays a role in the formation of social networks.**

Triadic closure is a concept in social network theory such that among 3 nodes, in which 2 nodes have a link to a common node but not to each other, there will be a likelihood of a formation of a link, which makes a triangle. For example, given nodes A,B, and C, if A linked to B and A linked to C exist, there will likely be a new link between B and C, which will create a triangle between the 3 nodes. This plays a role in social networks as it helps create new connections between mutual friends, acquaintances, and so on. By forming more triadic closures, the growth of a social network increases.

1. **Consider the graph to the right, in which every edge but one is labeled as a strong tie (S) or weak tie (W).**
   1. **According to the theory of strong and weak ties, with the strong triadic closure assumption, how must the edge connecting b and c be labeled (W or S)?**

The edge connecting Node B and C would have to be labeled a weak tie (W).

* 1. **Explain your answer.**

For this graph, it would have to be a weak tie because if Node B and C had a strong tie, then it would violate the Strong Triadic Closure Property. According to the textbook, if the node has strong ties to two neighbors, then these neighbors must have at least a weak tie between them (page 52, Ch 3). If Node B and C was a strong tie, then there needs to be an edge from Node E to C as well as Node B to F, which can then only satisfy the Strong Triadic Closure Property.

1. **Consider the graph to the right, in which every edge is labeled as a strong tie (S) or a weak tie (W).**
   1. **According to the theory of strong and weak ties, with the strong triadic closure assumption, which other link or links would you expect to form over time, assuming that the links formed have strong ties? (consider more than one iteration over time and identify all links that should form) b. Explain your answer.**

In one iteration, a link that is expected to form over time is a link between Node B & E. In another iteration, a link that is also expected to form is a link between Node A & E. With the formation of these links, it would completely satisfy the Strong Triadic Closure Property.

1. **a. Write a paragraph addressing the reason “friends of friends” are often a better source of job leads than your closer friends. Describe your reasoning in terms of strong and weak ties and local bridges.**

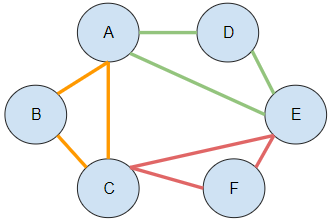
‘Friends of friends’ are often a better source of job leads than closer friends because your closer friends would be sharing the same type of information. The close friends tend to have little new information on their own as you interact with them more often, and it’s likely you would have the same ‘go-to’ contacts as your strong ties. By expanding yourself outside one’s strong ties through a local bridge, weak ties will allow oneself to different opinions and ideas. A new job tip tends to come from ‘friends of friends’ since your close friend would be a local bridge to their network, and it could come from someone that people from your close friend group are not normally close to. That local bridge would be a weak tie.

**b. What has been your experience, or what have you observed in others, regarding the source of job leads?**

In terms of personal experience, I’ve seen people reach out to those they do not normally interact with, and they often learn more about their company, and can also get a referral. I’ve also observed that through these experiences, those interactions help strengthen the relationship between the two people. However, this only applies for if both you and this person have a mutual friend that they share a strong tie with.

1. **Consider the affiliation network to the right, with six people labeled A–F , and three foci labeled X, Y , and Z.**
   1. **Draw the derived network on just the six, joining two people when they share a focus. The derived network should have just the 6 nodes A-F and their interconnections showing shared focus.**

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The second diagram is just a rearrangement of the first diagram.

* 1. **In the resulting network on people, can you identify a sense in which the triangle on the nodes A, C, and E has a qualitatively different meaning than the other triangles that appear in the network? Explain your answer.**

Node A has a connection with Node C and E, but C and E are not linked. Node A and C are connected from 1 focus while A and E are connected from 1 focus as well. Node C and E are linked to a common node and as a result, they form a Triadic Closure.

1. **For the graph to the right, where each edge is labeled as a positive or negative relationship:**
   1. **Identify the four 3-node cliques (triangles) and indicate if each is balanced or unbalanced.**

For Triangle ABC, it is balanced. For Triangle ABX, it is unbalanced. For Triangle AXC, it is unbalanced. For Triangle, BXC, it is balanced.

* 1. **Is the entire graph balanced or unbalanced?**

The entire graph is unbalanced.

* 1. **Explain your answer**

The reason the entire graph is unbalanced is because 2 of the triangles out of all the triangles are unbalanced. In order for the entire graph to be balanced, every one of its triangles has to be balanced, which is part of the Structural Balance Property. According to the Structural Balance Property, “For every set of three nodes, if we consider the three edges connecting them, either all three of these edges are labeled +, or else exactly one of them is labeled +” (Page 122, Ch 5).

1. **A team of anthropologists is studying a set of three small villages that neighbor one another. Each village has 30 people. Everyone in each village knows all the people in their own village, as well as the people in the other villages. When the anthropologists build the social network on the people in all three villages taken together, they find that each person is friends with all the other people in their own village, and enemies with everyone in the two other villages. This gives them a network of 90 people (i.e., 30 in each village), with positive and negative signs on its edges.**
   1. **According to the definitions in Chapter 5, is this network on 90 people balanced?**

Based on this description, the network on 90 people is unbalanced.

* 1. **Explain your answer**

In order for the network to be balanced, all 3 villages have to be friends or 2 of the villages (1 pair) needs to be friends. The problem is that all 3 villages are mutual enemies. Hence, it is an unbalanced network.

* 1. **Is the network weakly balanced?**

Yes, the network is weakly balanced.

* 1. **Explain your answer**

According to the textbook, “A complete graph is weakly balanced network is when it can divided into multiple sets of mutual friends with complete mutual antagonism between each pair of sets”(Page 129, Ch 5). Based on this definition, each set of mutual friends is each village consisting of 30 friends, and each village being enemies with each other, which matches the part with mutual antagonism, satisfies the condition.

* 1. **Would your answer to (a.) change if there were only two villages of 30 each? How?**

If there were only two villages of 30 each, I would change my answer to the network being balanced.

* 1. **Explain your answer**

There are 30 people in the first village that are friends and there are 30 people in the second village that are friends as well. The graph can be divided into 2 sets of mutual friends. With 30 friends in the first village being enemies with a group of 30 friends in the second village, this allows the network to be structurally balanced.