

Name:

ID:

### M5 Advanced Challenge 03

1. Consider the following:



The pink points are a subset of the points that make up one side of the border of a path ( $P$ ), let's call this collection of points  $B_1$ . The yellow points are a subset of the points that make up the other side of the border of  $P$ , called  $B_2$ . Answer the following questions in regards to a robot attempting to traverse  $P$ .

- Group the points in  $B_1$  and  $B_2$  in pairs such that every point in  $B_1$  is paired with exactly one point from  $B_2$ , the difference in the x-values of a pair should be smaller than any other combination.

Ex:

$((5,10), (4.7,15))$  is a pair because  $5 - 4.7$  is the smallest difference available for the point  $(5,10)$

- Compute the midpoint for each pair of points. What is the distance from the midpoint with the smallest x value to the midpoint with the largest x value?

63.21	60
66.11	65.24
70.13	64.45

- Use the [scipy.stats.linregress](#) function to generate a best fitting line for the midpoints. Which of the following is the best fitting line?

$y = 0.23x + 15.24$	$y = 0.25x + 32.26$
$y = 0.17x + 32.26$	$y = 0.17x + 15.24$
$y = 0.55x + 21.25$	$y = 0.25x + 21.25$

- Use [matplotlib.pyplot](#) to plot the best fitting line and the path formed by the midpoints, paste plot below:
- Generate a Lagrange interpolating polynomial from the midpoints that your robot can use to traverse **P**. Use this polynomial to predict  $f(x)$  given  $x$ :

### [Lagrange interpolating polynomials](#)

Given a set of  $k + 1$  data points

$$(x_0, y_0), \dots, (x_j, y_j), \dots, (x_k, y_k)$$

where no two  $x_j$  are the same, the **interpolation polynomial in the Lagrange form** is a [linear combination](#)

$$L(x) := \sum_{j=0}^k y_j \ell_j(x)$$

of Lagrange basis polynomials

$$\ell_j(x) := \prod_{\substack{0 \leq m \leq k \\ m \neq j}} \frac{x - x_m}{x_j - x_m} = \frac{(x - x_0)}{(x_j - x_0)} \dots \frac{(x - x_{j-1})}{(x_j - x_{j-1})} \frac{(x - x_{j+1})}{(x_j - x_{j+1})} \dots \frac{(x - x_k)}{(x_j - x_k)},$$

x	f(x)
21.1	
44.3	