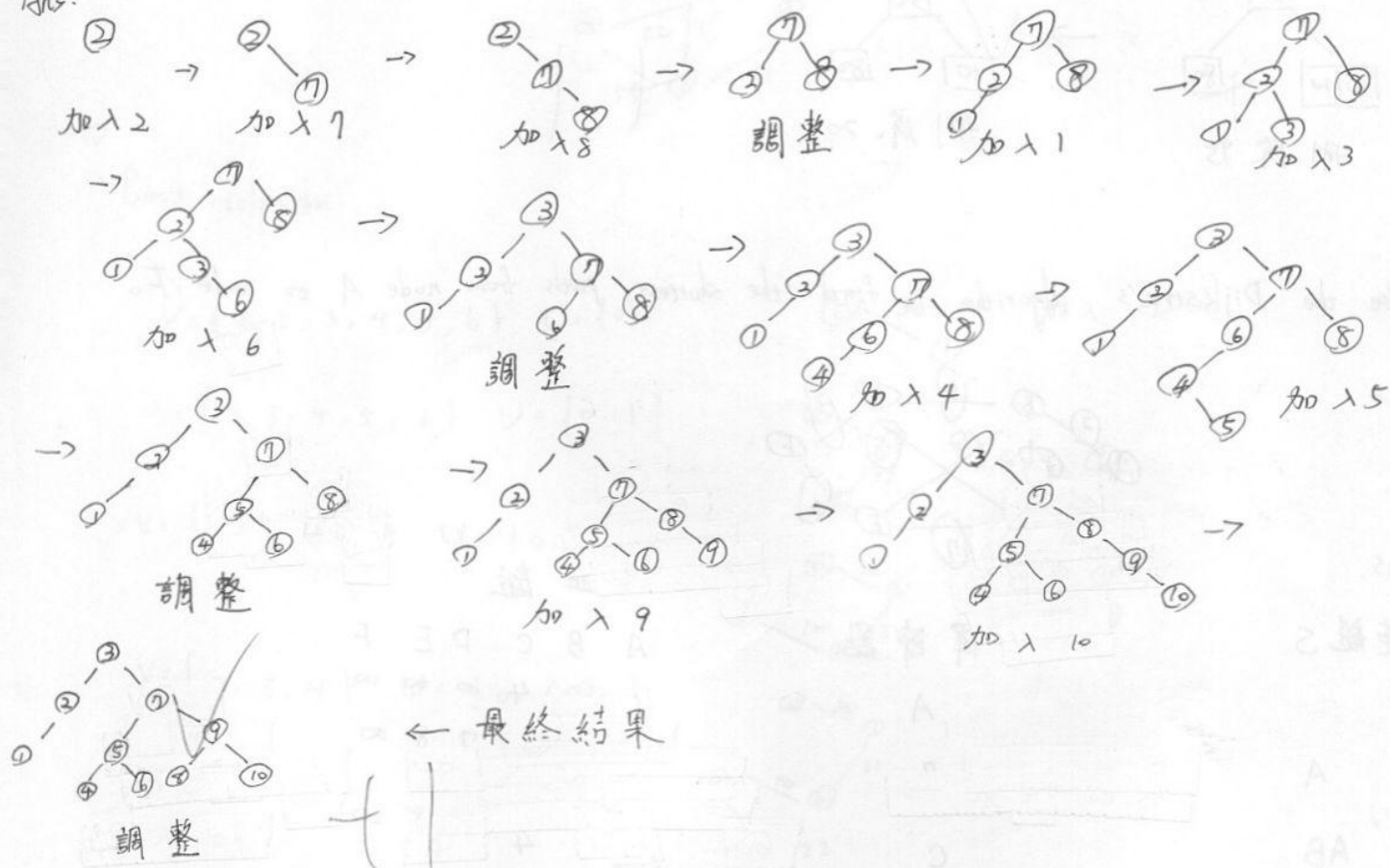


1. (a) Show the final AVL tree after successively inserting the keys

2, 7, 8, 1, 3, 6, 4, 5, 9, 10 into an initially empty AVL tree.

Ans.



b) 若將 98 個不同鍵值存入一個 AVL 樹中，其最大可能高度為何？

Ans. 若 N_h 為高 h 之 AVL 樹之最少節點數。

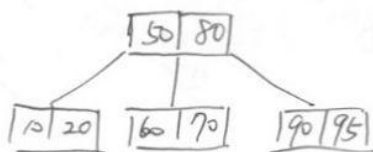
$$N_h = N_{h-1} + N_{h-2} + 1 \quad (N_1 = 1, N_2 = 2)$$

$$\Rightarrow N_1 = 1, N_2 = 2, N_3 = 4, N_4 = 7, N_5 = 12, N_6 = 20$$

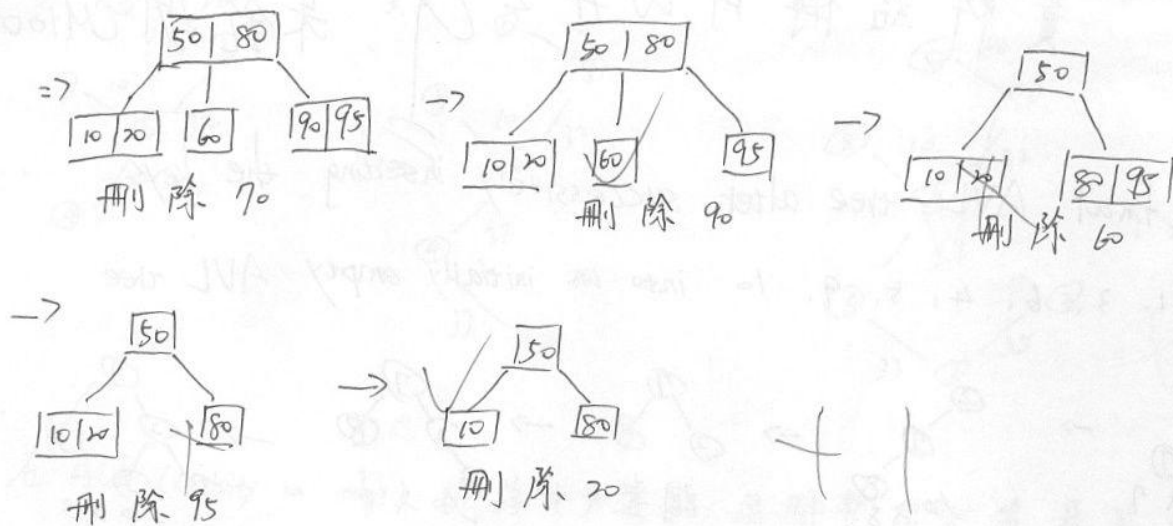
$$N_7 = 33, N_8 = 54, N_9 = 88, N_{10} = 143$$

由以上可知，98 介於 N_9 與 N_{10} 間，為 9 層未滿 10 層。

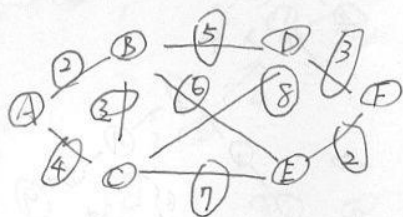
2. Given a 2-3 tree in the following, draw the tree after deleting 70, 90, 60, 95 in the given order.



(背面繼續)



3. Use the Dijkstra's algorithm to find the shortest path from node A to node F.



Ans.

① 距离 S

所擇節點

距離

A

A

A	B	C	D	E	F
0	2	4	∞	∞	∞

AB

B

0	2	4	7	8	∞
---	---	---	---	---	---

ABC

C

0	2	4	7	8	∞
---	---	---	---	---	---

ABCD

D

0	2	4	7	8	10
---	---	---	---	---	----

ABCDE

E

0	2	4	7	8	10
---	---	---	---	---	----

② 路徑

① A B C D E F

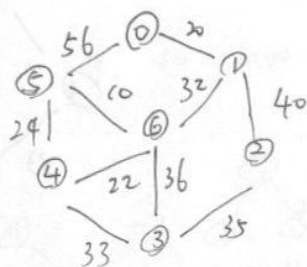
② A A A B B A

③ A A A B B D

由左表知，若要至 F，則需經 D，而至 D 則需先至 B，至 B 則先至 A，故路徑為 A → B → D → F

A: A → B → D → F

Use the Prim's algorithm and Kruskal's algorithm to obtain the minimum cost spanning tree.



① Prim's algorithm

$$V = \{1, 2, 3, 4, 5, 6\} \quad U = \{\emptyset\}$$

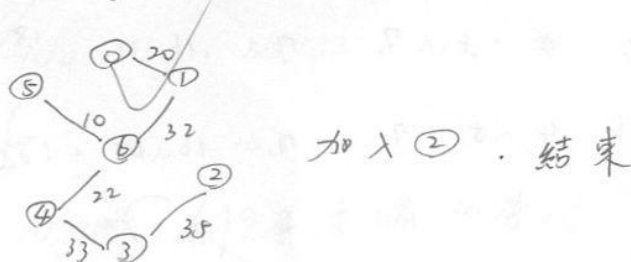
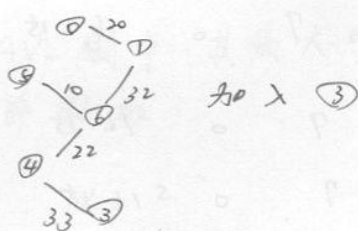
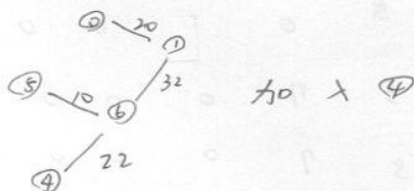
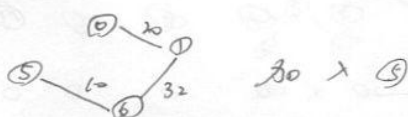
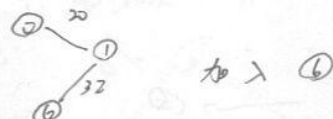
$$V = \{2, 3, 4, 5, 6\} \quad U = \{0, 1\}$$

$$V = \{2, 3, 4, 5\} \quad U = \{0, 1, 6\}$$

$$V = \{2, 3, 4\} \quad U = \{0, 1, 5, 6\}$$

$$V = \{2, 3\} \quad U = \{0, 1, 4, 5, 6\}$$

$$V = \{2\} \quad U = \{0, 1, 3, 4, 5, 6\}$$



② Kruskal's algorithm

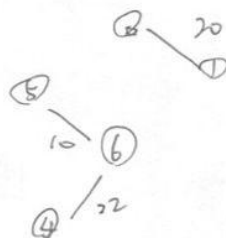
1.



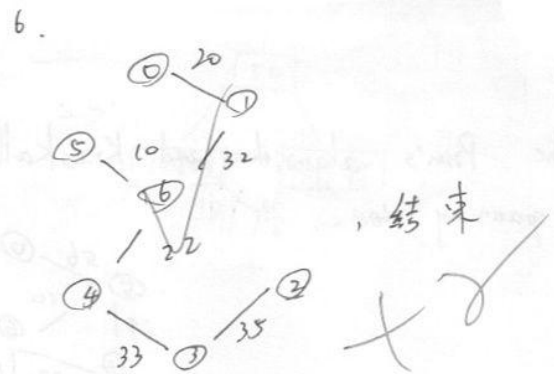
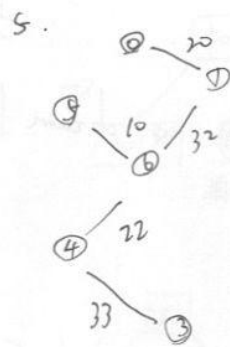
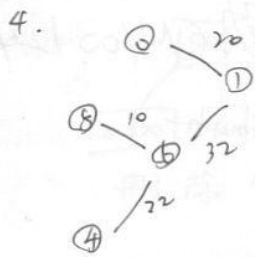
2.



3.

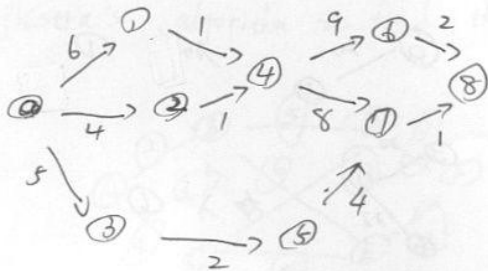


(背面繼續)



5.

在 AOE (Activity on edge) 網路中如圖示，請問整個計畫最早可完成之時間為何？



IS	①	②	③	④	⑤	⑥	⑦	⑧	Stack	
0	0	0	0	0	0	0	0	0	①	
0	6	4	5	0	0	0	0	0	① ② ③	Push 1, 2, 3
0	6	4	5	7	0	0	0	0	② ③	Pop ①
0	6	4	5	7	0	0	0	0	③ ④	Pop ②, push ④
0	6	4	5	7	0	16	15	0	④ ③	Pop ④, push ⑥
0	6	4	5	7	0	16	15	18	③	Pop ⑥
0	6	4	5	7	7	16	15	18	⑤	Pop ③, push ⑤
0	6	4	5	7	7	16	15	18	⑦	Pop ⑤, push ⑦
0	6	4	5	7	7	16	15	18	⑦	Pop ⑦, 結束

A: 計畫最早 18 天完成。