DIGITAL LOGIC

Chapter 2:
Introduction to logic circuits

Ru Han

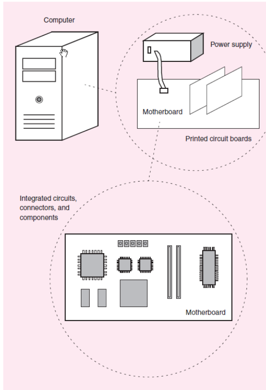


Figure 1.4 A digital hardware system (Part a).

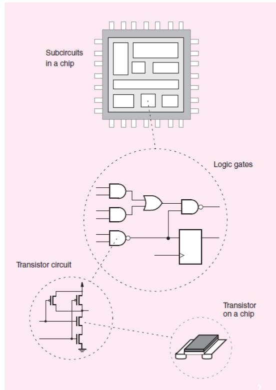


Figure 1.4 A digital hardware system (Part b).

OUTLINE

Chapter 2:

Introduction to logic circuits

- Logic function and circuits
- Boolean algebra for dealing with logic function
- Logic gates and synthesis of simple circuits
- CAD tools and the Verilog hardware description language
- Minimization of functions and Karnaugh maps

2.1 VARIABLES AND FUNCTIONS

<u>Switch</u>: Basic element of every digital (computer) system.

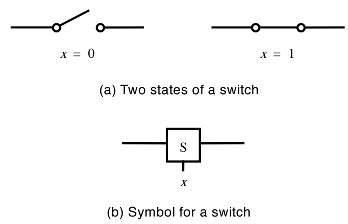
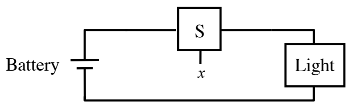


Figure 2.1. A binary switch.

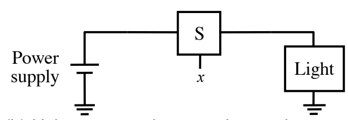
<u>Switch application</u>



(a) Simple connection to a battery

Light on: L=1
Light off: L=0

x=1 L=1
x=0 L=0

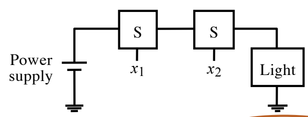


(b) Using a ground connection as the return path

<u>A function of the input variable x:</u>
$L(x)=x$

Figure 2.2. A light controlled by a switch.

<u>Logic Operation: AND</u>



(a) The logical AND function (series connection)

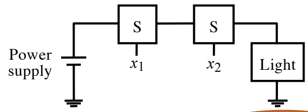
Figure 2.3. Two basic functions.

Logic Abstraction:

- Switch x1, x2:
  0 = Open, 1 = Closed
- Bulb L:
  0 = Off, 1 = On

x1	x2	L
0	0	
0	1	
1	0	
1	1	

Logic Operation: AND



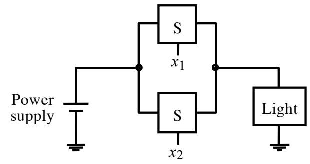
(a) The logical AND function (series connection)

Figure 2.3. Two basic functions.

AND: L(x1,x2)=x1·x2

Design a 2-switch light circuit, such that the light is on only when both switches are closed.

Logic Operation: OR



(b) The logical OR function (parallel connection)

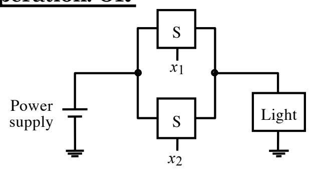
Figure 2.3. Two basic functions.

Logic Abstraction:

- Switch x1, x2:
  0 = Open, 1 = Closed
- Bulb L:
  0 = Off, 1 = On

x1	x2	L
0	0	
0	1	
1	0	
1	1	

Logic Operation: OR



(b) The logical OR function (parallel connection)

Figure 2.3. Two basic functions.

OR: L(x1,x2)=x1+x2

Design a 2-switch light circuit, such that the light is on if either or both switches are closed.

Example: The referee's circuit.

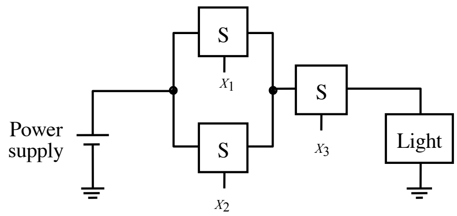


Figure 2.4. A series-parallel connection.

2.2 Inversion

Logic Operation: NOT

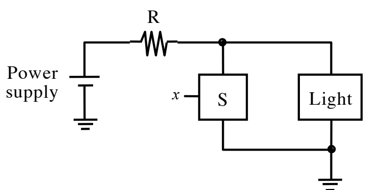


Figure 2.5. An inverting circuit.

Logic Abstraction:

- Switch x:
  0 = Open, 1 = Closed
- Bulb L:
  0 = Off, 1 = On

x	L
0	
1	

Inversion

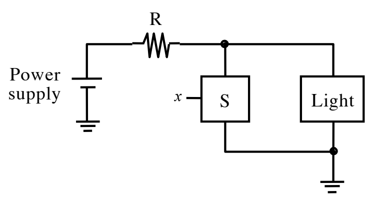


Figure 2.5. An inverting circuit.

NOT: L(x)=x'=!x=~x

Design a 1-switch light circuit, such that the light is on only when the switch is open.

2.3 Truth Tables

x_1	x_2	$x_1 \cdot x_2$	$x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1
		AND	OR

Figure 2.6. A truth table for the AND and OR operations.

x_1	x_2	x_3	$x_1 \cdot x_2 \cdot x_3$	$x_1 + x_2 + x_3$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Figure 2.7. Three-input AND and OR operations.

The AND and OR operations can be extended to n variables.

An AND function of variables $x_1, x_2, \ldots, x_n$ has the value 1 only if all n variables are equal to 1.

An OR function of variables $x_1, x_2, \ldots, x_n$ has the value 1 if one or more of the variables is equal to 1.

2.4 Logic Gates and Networks

Logic Gates

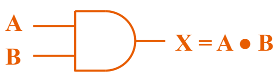
- Switch Model ↔ Logic Gates ↔ Physical Devices
- Symbolic Representations
- Functional Descriptions
- Logic Networks
- Logic Design
- Venn Diagrams

Physical Devices

- Constructed using transistors
- Take one or more electrical input signals, and produce one output signal
- High Voltage (4~5V) = LOGIC 1
  Low Voltage (0~1V) = LOGIC 0
  (Sometime this conversion is reversed)

Symbolic Representations

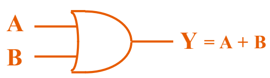
(1) AND Gate



(a) { A, B, X } ∈ { 0, 1 }
(b) may have > 2 inputs
(c) **X = 1 if and only if <u>ALL</u> inputs = 1**

Symbolic Representations

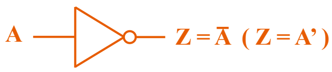
(2) OR Gate



(a) { A, B, X } ∈ { 0, 1 }
(b) may have > 2 inputs
(c) **Y = 0 if and only if <u>ALL</u> inputs = 0**

Symbolic Representations

(3) NOT Gate (Inverter)



(a) { A, Z } ∈ { 0, 1 }

(b) 1 input, 1 output

(c) **Z = 1 if and only if A = 0**

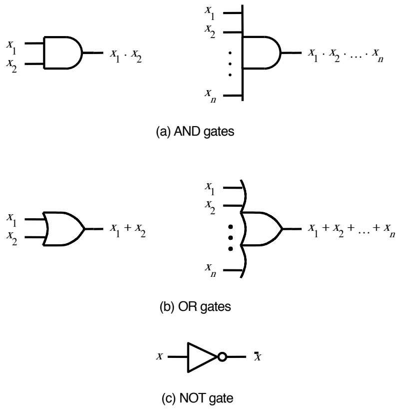


Figure 2.8. The basic gates.

Symbolic Representations

- Example: 3-input network

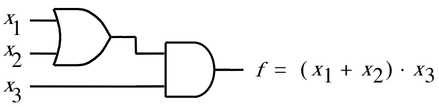


Figure 2.9. The function from Figure 2.4.

Functional Descriptions

Design problem:

- *Analysis* process:

 For an existing logic network, it must be possible to determine the function performed by the network.

- *Synthesis* process:

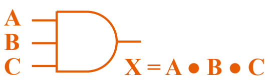
 The reverse task of designing a new network that implements a desired functional behavior.

Functional Descriptions

(1) Logic Function

(2) Truth Table

- Table that lists all possible combinations of inputs, and the corresponding output.
- Example, 3-input AND gate.



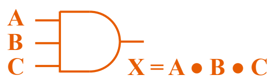
A	B	C	X

Functional Descriptions

(1) Logic Function

(2) Truth Table

- Table that lists all possible combinations of inputs, and the corresponding output.
- Example, 3-input AND gate.



A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Functional Descriptions

(3) Timing Diagram

- Draw waveforms to show inputs and outputs as functions of time.
- Create waveforms so that all possible input combinations are shown, then determine output waveform.
- Gives all information in truth table PLUS implementation details such as propagation delay.

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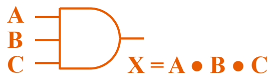
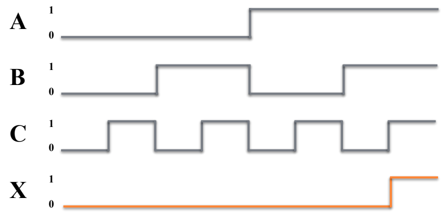
Functional Descriptions

(3) Timing Diagram

- Example, 3-input AND gate





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Logic Network

- Any logic function with any number of input signals can be realized as a network of AND, OR, and NOT gates.
- We can use truth tables and timing diagrams to analyze network.
  - Provide columns in truth table, or waveforms in timing diagram for all inputs, outputs and intermediate signals.
  - Note: **We Will Use Logic Network Or Logic Circuit Interchangeably**

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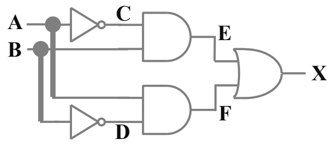
Logic Network

- Example: 2-input network



(a) Algebraic Evaluation

$C = \overline{A}$

$D = \overline{B}$

$E = C \bullet B = \overline{A} \bullet B$

$F = A \bullet D = A \bullet \overline{B}$

$X = E + F = (\overline{A} \bullet B) + (A \bullet \overline{B})$

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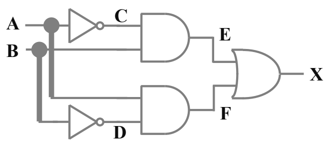
Logic Network

- Example: 2-input network



(B) Truth Table

A	B	C	D	E	F	X
0	0					
0	1					
1	0					
1	1					

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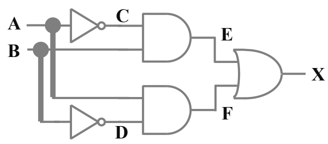
Logic Network

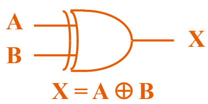
- Example: 2-input network



(B) Truth Table

A	B	C	D	E	F	X
0	0					0
0	1					1
1	0					1
1	1					0

Note: This type of network is known as Exclusive-OR (XOR)



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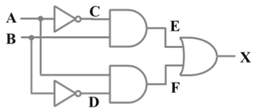
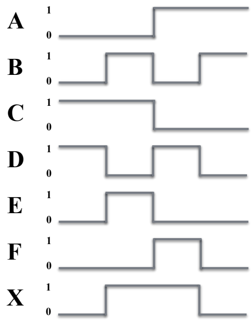
Logic Network

(C) Timing Diagram



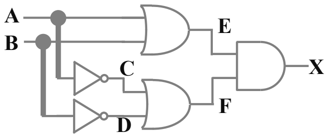


Functional Equivalent Network

- For any logic function, there are ***many different networks*** that could be used to implement the function.
- If two networks have the same output column in their truth tables, then they are ***functional equivalent***.

Functional Equivalent Network

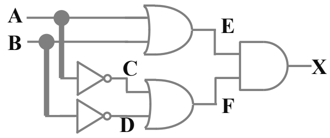
- Example: 2-input network



(a) Algebraic Evaluation

Functional Equivalent Network

- Example: 2-input network



(a) Algebraic Evaluation

$C = \overline{A}$

$D = \overline{B}$

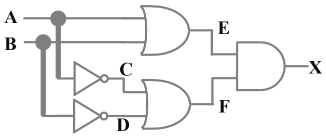
$E = A + B$

$F = C + D = \overline{A} + \overline{B}$

$X = E \bullet F = (A + B) \bullet (\overline{A} + \overline{B})$

Functional Equivalent Network

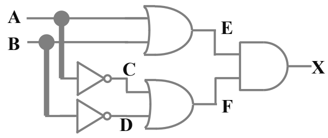
- Example: 2-input network



(B) Truth Table

Functional Equivalent Network

- Example: 2-input network

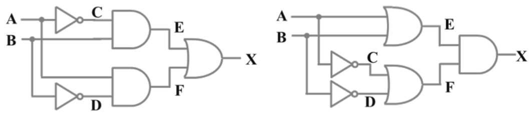


(B) Truth Table

A	B	C	D	E	F	X
0	0	1	1	0	1	0
0	1	1	0	1	1	1
1	0	0	1	1	1	1
1	1	0	0	1	0	0

It is also an XOR network

Functional Equivalent Network



- The two logic networks are used to implement the same function – XOR gate.
- They are functional equivalent networks.

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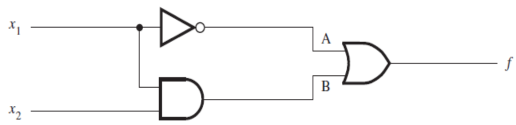
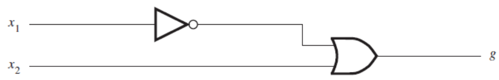
Functional Equivalent Network



(a) Network that implements



(d) Network that implements

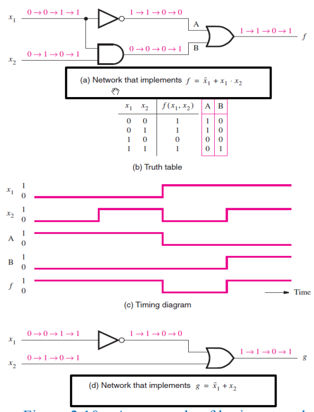


Figure 2.10. An example of logic networks.

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SO:

- More than one logic functions based on One Truth Table
- They are functional equivalent networks
- Is there one network which is the best for cost among them ?

- **Boolean Algebra Will Give You The Answer**

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Conclusion

- Logic variable and functions
- Understand the Truth Table
- Truth table/timing diagram/ Logic function
- XOR

- Text Reading: Chapter: 2.1 ~ 2.4, page22~33
- Assignment: Page 111: 2.8

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2.5 Boolean Algebra

Boolean Algebra

- Huntington's Postulates
- Proofs
- Basic Theorems
- Operator Precedence

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Boolean Algebra

In 1845, George Boole introduced a systematic treatment of logic now called **Boolean algebra**. Logical statements are built up from:

- **Variables**: a, x, etc. Variables can be used to represent propositions (statements that are either true or false) or signals in digital circuits (voltages that are either high or low, representing 0 or 1).
- **Operators**: The two operators (+, •) are used to combine variables to produce more complex statements or logic functions. These operators must satisfy certain properties.

The most well-known form of Boolean algebra is a two-valued system, in which all variables take values on the set {0, 1} and the operators (+,•) correspond to (OR, AND) respectively.

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Huntington's Postulates: In 1904, E. V. Huntington defined Boolean algebra by providing 6 postulates that must be satisfied.

- **Closure** with respect to the operators:
  any logical operation yields a value in the set {0, 1}.
- **Identity** elements with respect to the operators:
  $x + 0 = x$ $\quad$ $x \bullet 1 = x$
- **Commutativity** with respect to the operators:
  $x + y = y + x$ $\quad$ $x \bullet y = y \bullet x$
- **Distributivity**:
  $x \bullet (y + z) = (x \bullet y) + (x \bullet z)$ $\quad$ $x + (y \bullet z) = (x + y) \bullet (x + z)$
- **Complements** exist for all the elements:
  $x + x' = 1$ $\quad$ $x \bullet x' = 0$
- **Distinct elements**:
  $0 \neq 1$

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- In 1938, Claude Shannon showed that a two-valued Boolean Algebra, called **switching algebra**, could be used to describe *digital circuits.*
  - Two elements: {0,1}
  - Two binary operators: + (OR), • (AND)
  - One unary operator: the complement ' ($\bar{\ }$)

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

x	y	$x \bullet y$
0	0	0
0	1	0
1	0	0
1	1	1

x	x'
0	1
1	0

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Is this really a Boolean algebra? Does it satisfy Huntington's Postulates?

- **Closure** with respect to +, •:
  Yes, both operations produce 0 or 1 for all input combinations.
- **Identity** elements with respect to +, •:
  Yes, from truth tables, $x + 0 = x$, $x \bullet 1 = x$.
- **Commutativity** with respect to +, •:
  Yes, if x and y columns are swapped, results are unchanged.

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

x	y	$x \bullet y$
0	0	0
0	1	0
1	0	0
1	1	1

x	x'
0	1
1	0

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Is this really a Boolean algebra? Does it satisfy Huntington's Postulates?

- **Distributivity**:
  Yes (proof to follow).
- **Complements** exist for all the elements:
  Yes, 0' = 1, 1' = 0.
- **Distinct Elements**:
  Yes, by definition: $0 \neq 1$.

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

x	y	$x \bullet y$
0	0	0
0	1	0
1	0	0
1	1	1

x	x'
0	1
1	0

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Is this really a Boolean algebra? Does it satisfy Huntington's Postulates?

- **Closure** with respect to +, •:
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- **Identity** elements with respect to +, •:
  Yes, from truth tables, $x + 0 = x$, $x \bullet 1 = x$.
- **Commutativity** with respect to +, •:
  Yes, if x and y columns are swapped, results are unchanged.
- **Distributivity**:
  Yes (proof to follow).
- **Complements** exist for all the elements:
  Yes, 0' = 1, 1' = 0.
- **Distinct Elements**:
  Yes, by definition: $0 \neq 1$.

Note: In this book, it is called Axioms On pages:33

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Proofs: What are they?

Proofs are a tool for establishing new results or **theorems** in Boolean algebra. The kinds of things we need to prove usually fall into one of several categories.

- Prove that two logical expressions are equivalent.
- Prove that two logic networks are functionally equivalent.
- Prove that one logical expression is the complement of another.

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OK, so prove it! Hmmm, how do we do that?

Here are four ways to prove that two expressions are equivalent.

- **Perfect Induction**: show that the two expressions have identical truth tables.

If there are n input variables, this requires 2^n input combinations. This is a bad idea if there are more than 3 or 4 variables.

- **Axiomatic Proof**: apply Huntington's postulates (or other theorems that have already been proven) to the expressions, until the two expressions are identical.
- **Duality Principle**: every theorem in Boolean algebra remains valid if we interchange all AND's and OR's, and interchange all 0's and 1's.
- **Proof by contradiction**: assume that the hypothesis is ***false*** and then prove that the resulting equation can never be true.

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Duality

- Given a logic expression, its dual can be obtained by
1. Replace all + with . and vice versa
2. Replace all 0 with 1 and vice versa

Pages: 34

- Understand the importance of the duality

How to Understand Duality Principle

The duality principle ensures that "if we exchange ***every*** symbol by its dual in a formula, we get the ***dual*** result".

Examples:

- **0 • 1 = 0** is a true statement asserting that "false and true evaluates to false"

  1 + 0 = 1 is its dual: it is a true statement asserting that "true or false evaluates true."
- **1 • 1 = 1** is a true statement asserting that "true and true evaluates to true".

  0 + 0 = 0 is its dual : it is a true statement asserting, correctly, that "false or false evaluates to false".

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Proof of Distributivity of • over +, by Perfect Induction.

$x \bullet (y + z) = (x \bullet y) + (x \bullet z)$

x	y	z	$y + z$	$x \bullet (y + z)$	$x \bullet y$	$x \bullet z$	$(x \bullet y) + (x \bullet z)$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

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Proof of Distributivity of • over +, by Perfect Induction.

$x \bullet (y + z) = (x \bullet y) + (x \bullet z)$

x	y	z	$y + z$	$x \bullet (y + z)$	$x \bullet y$	$x \bullet z$	$(x \bullet y) + (x \bullet z)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

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Proof of Distributivity of + over •, by Perfect Induction.

$x + (y \bullet z) = (x + y) \bullet (x + z)$

x	y	z	$y \bullet z$	$x + (y \bullet z)$	$x + y$	$x + z$	$(x + y) \bullet (x + z)$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Proof of Distributivity of + over •, by Perfect Induction.

$x + (y \bullet z) = (x + y) \bullet (x + z)$

x	y	z	$y \bullet z$	$x + (y \bullet z)$	$x + y$	$x + z$	$(x + y) \bullet (x + z)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

OK, where are we so far?

- So far, we have only shown (through 1 rigorous proof and several less formal arguments) that Huntington's postulates are satisfied. Therefore, the two valued system with the operations (•, +) as defined above is a Boolean algebra.

Why do we care?

- Armed with these 6 postulates, we can now go on to establish other theorems that will help us analyze logic circuits.

Basic Theorems

These theorems can be verified by examining the truth tables for {+, •, '}.

- ***Theorem 1: Idempotency***
  - a) $x + x = x$
  - b) $x \bullet x = x$

x	$x + x$	$x \bullet x$
0	0	0
1	1	1

- ***Theorem 2: Tautology & Contradiction***
  - a) $x + 1 = 1$
  - b) $x \bullet 0 = 0$

x	$x + 1$	$x \bullet 0$
0	1	0
1	1	0

- ***Theorem 3: Involution***
  - $(x')' = x$

x	x'	$(x')'$
0	1	0
1	0	1

Basic Theorems

- ***Theorem 4: Associativity***
  - a) $(x + y) + z = x + (y + z)$

x	y	z	$(x + y)$	$(y + z)$	$(x + y) + z$	$x + (y + z)$
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

Basic Theorems

- ***Theorem 4: Associativity***
  - a) $(x + y) + z = x + (y + z)$

x	y	z	$(x + y)$	$(y + z)$	$(x + y) + z$	$x + (y + z)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Basic Theorems

- ***Theorem 4:*** ***Associativity***
  a) $(x + y) + z = x + (y + z)$
  b) $(x \bullet y) \bullet z = x \bullet (y \bullet z)$

x	y	z	$(x \bullet y)$	$(y \bullet z)$	$(x \bullet y) \bullet z$	$x \bullet (y \bullet z)$
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

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Basic Theorems

- ***Theorem 4:*** ***Associativity***
  a) $(x + y) + z = x + (y + z)$
  b) $(x \bullet y) \bullet z = x \bullet (y \bullet z)$

x	y	z	$(x \bullet y)$	$(y \bullet z)$	$(x \bullet y) \bullet z$	$x \bullet (y \bullet z)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

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Theorem 5: DeMorgan's Law

(a) $(x + y)' = x' \bullet y'$

Proof: by Perfect Induction.

x	y	$(x + y)$	$(x + y)'$	x'	y'	$x' \bullet y'$
0	0					
0	1					
1	0					
1	1					

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Theorem 5: DeMorgan's Law

(a) $(x + y)' = x' \bullet y'$

Proof: by Perfect Induction.

x	y	$(x + y)$	$(x + y)'$	x'	y'	$x' \bullet y'$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

(b) $(x \bullet y)' = x' + y'$

Proof: apply the duality principle to the expression in part (a). If we interchange all • and + operators, we obtain the expression in part (b).

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Theorem 6: Absorption

(a) $x + (x \bullet y) = x$

Axiomatic Proof:

$x + (x \bullet y) = (x \bullet 1) + (x \bullet y)$	by $x \bullet 1 = x$
$= x \bullet (1 + y)$	by distributivity
$= x \bullet (y + 1)$	by commutativity
$= x \bullet 1$	by $x + 1 = \mathbf{1}$
$= x$	by $x \bullet 1 = x$

(b) $x \bullet (x + y) = x$

Proof: apply the duality principle to the expression in part (a). If we interchange all • and + operators, we obtain the expression in part (b).

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Two More Theorems

- ***Theorem 7:*** ***Common Identities***
  a) $x + (x' \bullet y) = x + y$
  b) $x \bullet (x' + y) = x \bullet y$

- ***Theorem 8:*** ***Consensus***
  a) $(x \bullet y) + (y \bullet z) + (x' \bullet z) = (x \bullet y) + (x' \bullet z)$
  b) $(x + y) \bullet (y + z) \bullet (x' + z) = (x + y) \bullet (x' + z)$

- Try to prove these theorems on your own.
- Note that each pair of equations are related through the duality principle.

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So now we have all these theorems. What good are they?

Example 1: Find the complement of the logical function *f*.

$$f = \bar{x} \bullet (\bar{y} + \bar{z}) \bullet (x + y + \bar{z})$$

Solution: by repeated application of DeMorgan's Law.

$$\bar{f} = \overline{\bar{x} \bullet (\bar{y} + \bar{z}) \bullet (x + y + \bar{z})}$$
$$= \overline{\bar{x} \bullet (\bar{y} + \bar{z})} + \overline{(x + y + \bar{z})}$$
$$= \bar{\bar{x}} + \overline{(\bar{y} + \bar{z})} + \overline{(x + y + \bar{z})}$$
$$= x + (y \bullet z) + (\bar{x} \bullet \bar{y} \bullet z)$$

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So now we have all these theorems. What good are they?

Example 2: Simplify the previous expression.

$$\bar{f} = x + (y \bullet z) + (\bar{x} \bullet \bar{y} \bullet z)$$

Solution: by algebraic manipulation.
Apply the theorems of Boolean algebra.

$\bar{f} = x + (\bar{x} \bullet \bar{y} \bullet z) + (y \bullet z)$	by Commutativity
$= x + (\bar{y} \bullet z) + (y \bullet z)$	by Theorem 7a
$= x + (\bar{y} + y) \bullet z$	by Distributivity
$= x + 1 \bullet z$	by Postulate 5a
$= x + z$	by Postulate 2b

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More Examples

Equivalent Expression	Justification
$\bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + \mathbf{xy\bar{z}} + xyz$	Original Expression
$\boxed{(\mathbf{\bar{x}y\bar{z}} + \mathbf{\bar{x}yz}) + (\mathbf{x\bar{y}\bar{z}} + \mathbf{xy\bar{z}}) + (\mathbf{xyz} + \mathbf{xy\bar{z}})}$	Idempotency
$\bar{x}y(\mathbf{z} + \mathbf{\bar{z}}) + x\bar{z}(\mathbf{y} + \mathbf{\bar{y}}) + xy(\mathbf{z} + \mathbf{\bar{z}})$	**Distributivity**
$\boxed{\bar{x}y(1) + x\bar{z}(1) + xy(1)}$	Complement
$\bar{x}y + x\bar{z} + xy$	**Identity for AND**
$\boxed{y(\mathbf{x} + \mathbf{\bar{x}}) + x\bar{z}}$	Distributivity
$\boxed{y(1) + x\bar{z}}$	Complement
$y + x\bar{z}$	Identity for AND

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Example on pages:36

- Example 2.3
- Example 2.4

Precedence of Boolean Operators

- To avoid confusion or incorrect evaluation, the operators in Boolean expressions are applied according to the following order of precedence:
  1. Expressions in parentheses: ()
  2. NOT: '
  3. AND: •
  4. OR: +
- Also, it is conventional to omit the • symbol for AND where convenient. As a result, many Boolean expressions can be written in a compact form, often eliminating extraneous parentheses.

$$\overline{(x \bullet y)} \Leftrightarrow \overline{xy}$$
$$x \bullet (y + z) \Leftrightarrow x(y + z)$$
$$(\bar{x} \bullet z) + (\bar{y} \bullet \bar{z}) \Leftrightarrow \bar{x}z + \bar{y}\bar{z}$$

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Reference Sheet: Postulates and Theorems

- Postulate 1: Closure
- Postulate 2: Identity
  a) $x + 0 = x$
  b) $x \bullet 1 = x$
- Postulate 3: Commutativity
  a) $x + y = y + x$
  b) $xy = yx$
- Postulate 4: Distributivity
  a) $x(y + z) = xy + xz$
  b) $x + yz = (x + y)(x + z)$
- Postulate 5: Complement
  a) $x + x' = 1$
  b) $x \bullet x' = 0$
- Theorem 1: Idempotency
  a) $x + x = x$
  b) $x \bullet x = x$
- Theorem 2: Tautology and Contradiction
  a) $x + 1 = 1$
  b) $x \bullet 0 = 0$
- Theorem 3: Involution
  $(x')' = x$
- Theorem 4: Associativity
  a) $x + (y + z) = (x + y) + z$
  b) $x(yz) = (xy)z$
- Theorem 5: DeMorgan's Law
  a) $(x + y)' = x'y'$
  b) $(xy)' = x' + y'$
- Theorem 6: Absorption
  a) $x + xy = x$
  b) $x(x + y) = x$
- Theorem 7: Common Identities
  a) $x + x'y = x + y$
  b) $x(x' + y) = xy$
- Theorem 8: Consensus
  a) $xy + yz + x'z = xy + x'z$
  b) $(x + y)(y + z)(x' + z)$
  $= (x + y)(x' + z)$

Notice: 1、 Precedence : “ ( ) ” > “ ’ ” > “ · ” > “+ ”
2、$A \cdot B$ can be written as AB.

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Venn Diagrams

- A graphical technique to illustrate logic functions
- ***Large rectangle*** to represent all possible input combinations
- ***Overlapping bubbles*** to represent each input variable
- Represent logical function by ***sharing regions*** where the function is equal to 1

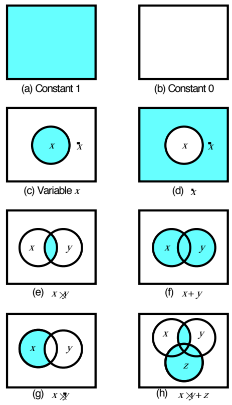
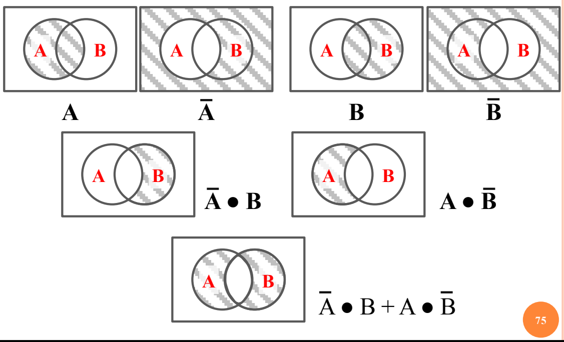


Figure 2.14. The Venn diagram representation.

Venn Diagrams

- Example: $X = \bar{A} \bullet B + A \bullet \bar{B}$



Conclusion

- Axioms of Boolean algebra
- Duality
- Properties (7 kinds and includes 16 identities)
- Can use the Properties freely to simplify or prove or explain logic expressions

- Textbook Reading: Chapter 2.5, Pages:33-43
- Assignment: Page 111: 2.2 , 2.3 and 2.7