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Informational efficiency of Bitcoin—An extension

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HIGHLIGHTS

- We revisit the issue of informational efficiency of Bitcoin.
- We use a battery of robust long-range dependence estimators.
- We establish efficiency of Bitcoin prices.
- Inefficiency exists during April–August, 2013 and August–November, 2016.

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ABSTRACT

We revisit the issue of informational efficiency of Bitcoin using a battery of computationally efficient long-range dependence estimators for a period spanning over July 18, 2010 to June 16, 2017. We report that the market is informational efficient as consistent to recent findings of Urquhart (2016), Nadarajah and Chu (2017) and Bariviera (2017).

1. Introduction

Bitcoin is one of the novel and innovative forms of "digital" currencies endowed with state-of-the-art features, transparency, and simplicity in use (Urquhart, 2016). It has received more attention than the other cryptocurrencies such as Ripple and Litecoin (Dwyer, 2015) and it captures 41% of cryptocurrencies capitalization (Katsiampa, 2017). The popularity in recent years has attracted the interest of researchers and practitioners alike, especially looking for a better understanding of the various characteristics of Bitcoin such as price volatility (Baek and Elbeck, 2015; Katsiampa, 2017), price clustering (Urquhart, 2017), etc.

The recent literature on Bitcoin has received considerable impetus on the testing efficiency of its prices within the meaning of Malkiel and Fama (1970). Urquhart (2016) was the first to test the informational efficiency of Bitcoin prices using six different tests. The author concludes Bitcoin markets to be inefficient, however, they also argue that the market depicts a transitory phase to achieve efficiency, as the markets tend to mature. Subsequently, Nadarajah and Chu (2017) followed up Urquhart (2016) to test the same hypothesis using eight different tests by adding an odd integer power to Bitcoin returns. The authors conclude that Bitcoin returns are market efficient. Recently, Bariviera (2017) revisit the efficient market hypothesis for Bitcoin prices using Range over Standard Deviation or Rescaled Range (R/S) and De-trended Fluctuation Analysis (DFA) method to detect long memory process and variations in informational efficiency respectively. Bariviera (2017) reports consistent results with previous two studies i.e. market is efficient.

We revisit the literature on Bitcoin's price efficiency by using a battery of long-range dependence estimators and allowing for time variation, which are robust and computationally efficient. We further constructed the efficiency index based on used long-range estimates and provided time varying validation of the efficiency hypothesis of Bitcoin's prices.

2. Estimation Methodology

This section briefly mentions various long-range dependence estimators[1] that are used to test the efficient market hypothesis

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[1] Readers are requested to refer the respective references for details about each method.

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of Bitcoin returns. The long-memory dependence is an important property of any time series $\{y_t\}$. It can be mathematically expressed as

$$\log_{s\to\infty} C\lambda_s S^{-\alpha} = 1 \quad (1)$$

where $\lambda_s = cov\,(y_t, y_{t+s})$, $s = 0, \pm1, \pm2, \ldots$, C and $\alpha \in (0, 1)$ are constants. The rate of decrease of the long-memory process is hyperbolic, and hence the sum of the autocovariances tends to infinity. Following Palma (2007), it can be shown that $\sum_{s=-\infty}^{\infty}|\lambda_s| = \infty$. Also, the spectral density of the long-memory process

$$f(\omega) = \frac{1}{2\pi}\sum_{s=-\infty}^{\infty}\lambda_s \exp(-is\omega) \quad (2)$$

can be obtained as the following limiting value of $f(\omega)$ (Beran, 1994).

$$\lim_{\omega\to 0} f(\omega) = C_f\omega^{\alpha-1} \quad (3)$$

where, ω and C_f are a scalar and a positive number, respectively.

Using a fractional integrated process (Granger and Joyeux, 1980), the long-memory process can be written as

$$(y_t - \eta)(1-L)^d = \varepsilon_t \quad (4)$$

where η denotes the expectation of y_t, L denotes the lag operator, d denotes the fractional difference, and ε_t denotes an i.i.d. random variable. The fractional difference d is also expressed as

$$d = H - 0.5 \quad (5)$$

where

$$H = (1 - \alpha/2) \quad (6)$$

is the Hurst exponent (Palma, 2007) and the constant α is given in the Eq. (1). Using (6) in (3), we write the spectral density of the long-memory process near zero as

$$D = C_f\omega^{(1-2H)}. \quad (7)$$

Methods proposed by Taqqu et al. (1995) to estimate H from (7) using ordinary least square (LS) and least absolute deviation (LAD) are known as Periodogram-LS and Periodogram-LAD, respectively. The method proposed by Geweke and Porter-Hudak (1983) to estimate d given in (5) using a linear regression model is known as GPH method as per the names of the three contributors Geweke, Porter and Hudak. The maximum-likelihood estimator (MLE) is due to Haslett and Raftery (1989) utilizes an autoregressive approximation framework to estimate the desired parameter. The de-trended fluctuation analysis proposed by Peng et al. (1994) is a scaling method and is known as the DFA technique. Finally, by using moving average de-trending in DFA, Centered Moving Average-squared absolute fluctuation (CMA-1) and Centered Moving Average-mean absolute fluctuation (CMA-2) were proposed (Bashan et al., 2008).

3. Data

To examine the efficiency hypothesis of Bitcoin, we consider a dataset period spanning over July 18, 2010 to June 16, 2017 (2525 daily observations), which is constrained by availability of Bitcoin price data. The logarithmic transformation of all the series is done taking the first difference as $\boldsymbol{R_t} = (\boldsymbol{In}(\boldsymbol{P_t}) - \boldsymbol{In}(\boldsymbol{P_{t-1}})) \times \mathbf{100}$. The Bitcoin price is obtained from https://www.coindesk.com/price/. Table 1 presents the descriptive statistics of the returns series data and shows that the mean positive returns. Further, the series is negatively skewed, leptokurtic and non-normal. The time-series plot of Bitcoin returns is presented in Fig. 1.

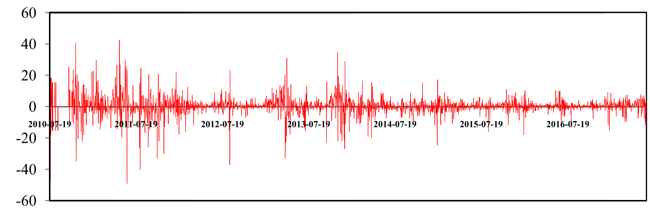

Fig. 1. Bitcoin daily returns.

Table 1
Descriptive statistics.

Statistics	Returns
Mean	0.406
Standard deviation	5.906
Median	0.136
Minimum	−49.153
Maximum	42.458
Skewness	−0.403
Kurtosis	15.621
Jarque–Bera	16 826.593

Table 2
Long-range dependency of estimates.

Estimator	Value	Standard error
CMA-1	0.1044	0.0051
CMA-2	0.3274	0.0180
DFA	0.3414	0.0410
GPH	0.3021	0.0479
MLE	0.2978	0.0156
Periodogram-LAD	0.4068	0.0483
Periodogram-LS	0.3991	0.0636

4. Main results

We investigate the behavior of long-range dependence of daily returns of Bitcoin using the DFA, CMA-1, CMA-2, Periodogram-LAD, Periodogram-LS, GPH, and MLE techniques. We first estimate the value and standard error of each estimator and present them in Table 2. The estimator value is largest for the Periodogram-LAD and smallest for the CMA-1.

We also estimate the value of d and its t-ratio for different descriptive statistics by repeating the process for 2000 Monte Carlo iterations for all the techniques. We present the results in Table 3.

We observe from Table 3 that the quantiles value (5% and 95%) of the t-ratios significantly differs from that of the Gaussian distribution for all techniques except the GPH. The corresponding t-ratios show considerable excess kurtosis. In the GPH, the Jarque–Bera test of normality of the t-ratio cannot be rejected as it lies within the interval of (−1.738, 1.549). The mean value of the t-ratio almost vanishes for the MLE (1.718980e−008) and GPH (1.848022e−015). For all other methods, the values deviate from zero. On the other hand, not a single median value corresponding to the t-ratio is zero. This supports the presence of previously mentioned excess kurtosis. We also find that the dispersion values using the bootstrap-based d-estimates are quite high for all the methods except DFA and MLE. The dispersion values using CMA-1, CMA-2, GPH, Periodogram-LAD and Periodogram-LS are 0.234, 0.232, 0.269, 0.282, and 0.309, respectively. The Periodogram-LS exhibits the maximum dispersion among all the techniques. For such a high value of the dispersion, ‘no long-range dependence in returns’ null hypothesis is not possible to reject even at the 90% confidence level. This indicates that Bitcoin returns are efficient as suggested by CMA-1, CMA-2, GPH, Periodogram-LAD and Periodogram-LS methods. The other two methods DFA and MLE produce dispersion values of 0.112 and 0.104, respectively,

Table 3
Long-range dependency estimates for Bitcoins daily returns.

Statistic	CMA-1		CMA-2		DFA		DPH		MLE		Periodogram-LAD		Periodogram-LS	
	d	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio
Minimum	−0.310	−23.805	−0.523	−21.756	0.053	−11.924	−0.506	−4.435	0.042	−5.079	−0.586	−6.362	−0.567	−9.949
1st quartile	0.273	−4.269	0.170	−3.811	0.259	−1.408	0.261	−0.640	0.221	−1.120	0.257	−0.709	0.231	−1.133
Median	0.396	0.296	0.294	0.614	0.316	−0.029	0.398	0.043	0.274	0.058	0.390	−0.028	0.384	0.025
Mean	0.387	−0.128	0.279	0.402	0.317	−0.089	0.390	0.000	0.272	0.000	0.395	0.013	0.380	0.030
3rd quartile	0.507	4.229	0.402	4.463	0.371	1.395	0.530	0.695	0.325	1.168	0.539	0.768	0.540	1.195
Maximum	0.825	21.392	0.716	27.459	0.552	11.543	1.111	3.573	0.450	3.929	1.127	5.387	1.210	7.598
5% quantile	0.105	−10.772	−0.023	−9.426	0.183	−4.127	0.039	−1.738	0.156	−2.569	0.039	−1.896	−0.019	−2.881
95% quantile	0.650	9.248	0.531	10.007	0.457	3.425	0.702	1.549	0.382	2.426	0.737	2.016	0.756	2.966
Excess kurtosis	0.023	0.279	0.552	0.606	−0.295	1.752	0.257	0.257	−0.468	−0.468	0.407	0.855	0.382	0.975
Skewness	−0.253	−0.288	−0.551	−0.031	0.016	−0.288	−0.277	−0.277	−0.138	−0.138	−0.121	−0.059	−0.172	−0.107
p-value JB	0.000	0.000	0.000	0.000	0.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Iterations	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000

Note: In Table 3, '*p*-value JB' stands for the *p*-value of the Jarque–Bera test of normality, 'CMA-1' stands for Centered Moving Average-squared absolute fluctuation, 'CMA-2' stands for Centered Moving Average-mean absolute fluctuation.

Table 4
Rolling estimates for Bitcoins daily returns.

Statistic	CMA-1		CMA-2		DFA		DPH		MLE		Periodogram-LAD		Periodogram-LS	
	d	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio	*d*	*t*-ratio
Minimum	0.235	−24.971	0.136	−20.065	0.209	−10.706	0.041	−3.331	0.199	−3.552	0.110	−4.071	0.263	−2.656
1st quartile	0.352	−3.424	0.273	−3.416	0.292	−1.874	0.390	−0.791	0.278	−0.384	0.345	−1.432	0.380	−0.905
Median	0.385	−0.980	0.317	−0.459	0.360	0.679	0.512	0.100	0.290	0.127	0.424	−0.227	0.462	−0.090
Mean	0.400	−1.245	0.329	−1.194	0.341	−0.269	0.498	0.000	0.287	0.000	0.443	−0.052	0.472	−0.057
3rd quartile	0.469	2.731	0.403	2.435	0.379	1.403	0.629	0.949	0.316	1.179	0.566	1.645	0.560	0.889
Maximum	0.532	4.700	0.459	4.047	0.442	5.867	0.722	1.627	0.347	2.430	0.766	4.219	0.790	2.685
5% quantile	0.288	−10.451	0.207	−9.411	0.255	−4.879	0.274	−1.632	0.221	−2.682	0.248	−3.414	0.292	−2.167
95% quantile	0.509	4.021	0.440	3.610	0.415	2.790	0.683	1.348	0.324	1.490	0.679	3.278	0.662	1.837
Excess kurtosis	−0.957	1.797	−0.995	1.765	−1.007	0.111	−0.560	−0.560	−0.096	−0.096	−0.404	−0.618	−0.642	−0.447
Skewness	0.066	−1.176	−0.037	−1.219	−0.365	−0.808	−0.496	−0.496	−0.950	−0.950	0.163	0.157	0.185	−0.099
p-value JB	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Iterations	1519	1519	1519	1519	1519	1519	1519	1519	1519	1519	1519	1519	1519	1519

Note: In Table 4, '*p*-value JB' stands for the *p*-value of the Jarque–Bera test of normality, 'CMA-1' stands for Centered Moving Average-squared absolute fluctuation, 'CMA-2' stands for Centered Moving Average-mean absolute fluctuation.

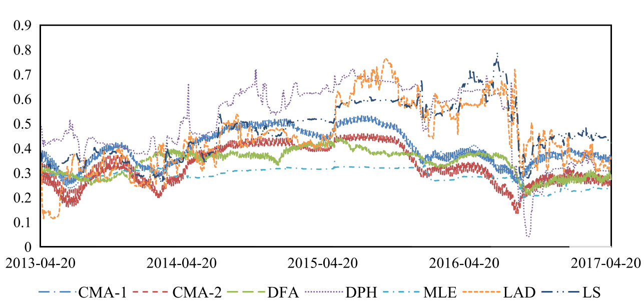

Fig. 2. Rolling estimates of *d* for considered techniques.

that also supports consistency of Bitcoin returns at a little wider confidence interval.

To understand a more precise behavior of Bitcoin returns, rolling estimates of *d* is computed over the period of sample with overlapping windows having 300 observations. The corresponding results are shown in Table 4 and the graph is shown in Fig. 2.

For comparison, values of same statistics are calculated with the assumption of stationarity. It is evidenced from comparing Tables 3 and 4 that mean values of rolling-estimates vary substantially for all methods. This establishes the fact that within a 300 days business cycle, the assumption of stationarity in daily data is not reasonable, and hence the market is efficient. In addition, following Kristoufek and Vosvrda (2013), we calculate the efficiency index (Fig. 3) for overlapping windows. We observe that the market is largely efficient with some exception to the period of April–August, 2013 and August–November, 2016.

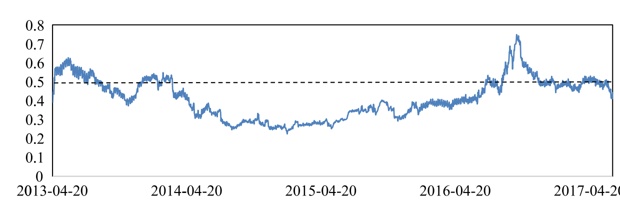

Fig. 3. Efficiency index.

5. Conclusions

The issue of informational efficiency of Bitcoin prices has been a matter of interest since Selgin (2015) and Baek and Elbeck (2015) referred Bitcoins as a speculative commodity. The literature in recent past (Bariviera, 2017; Nadarajah and Chu, 2017; Urquhart, 2016) advocate informational efficiency of Bitcoin prices. We add to the literature by employing a battery of robust estimators with computational efficiency and constructed efficiency index which show that Bitcoin market is efficient with some exception to the period of April–August, 2013 and August–November, 2016.

References

Baek, C., Elbeck, M., 2015. Bitcoins as an investment or speculative vehicle? A first look. Appl. Econ. Lett. 22, 30–34.
Bariviera, A.F., 2017. The inefficiency of Bitcoin revisited: a dynamic approach. Econom. Lett.. http://dx.doi.org/10.1016/j.econlet.2017.09.013.
Bashan, A., Bartsch, R., Kantelhardt, J.W., Havlin, S., 2008. Comparison of detrending methods for fluctuation analysis. Physica A 387, 5080–5090.
Beran, J., 1994. Statistics for Long-Memory Processes. CRC press.

Dwyer, G.P., 2015. The economics of Bitcoin and similar private digital currencies. J. Financ. Stab. 17, 81–91.
Geweke, J., Porter-Hudak, S., 1983. The estimation and application of long memory time series models. J. Time Ser. Anal. 4, 221–238.
Granger, C.W.J., Joyeux, R., 1980. An introduction to long-memory time series models and fractional differencing. J. Time Ser. Anal. 1, 15–29.
Haslett, J., Raftery, A.E., 1989. Space–time modelling with long-memory dependence: Assessing Ireland's wind power resource. Appl. Stat. 1–50.
Katsiampa, P., 2017. Volatility estimation for Bitcoin: A comparison of GARCH models. Econom. Lett. 158, 3–6.
Kristoufek, L., Vosvrda, M., 2013. Measuring capital market efficiency: Global and local correlations structure. Physica A 392, 184–193.
Malkiel, B.G., Fama, E.F., 1970. Efficient capital markets: A review of theory and empirical work. J. Finance 25, 383–417.
Nadarajah, S., Chu, J., 2017. On the inefficiency of Bitcoin. Econom. Lett. 150, 6–9.
Palma, W., 2007. Long-Memory Time Series: Theory and Methods. John Wiley & Sons.
Peng, C.-K., Buldyrev, S.V., Havlin, S., Simons, M., Stanley, H.E., Goldberger, A.L., 1994. Mosaic organization of DNA nucleotides. Phys. Rev. E 49, 1685.
Selgin, G., 2015. Synthetic commodity money. J. Financ. Stab. 17, 92–99.
Taqqu, M.S., Teverovsky, V., Willinger, W., 1995. Estimators for long-range dependence: an empirical study. Fractals 3, 785–798.
Urquhart, A., 2016. The inefficiency of Bitcoin. Econom. Lett. 148, 80–82.
Urquhart, A., 2017. Price clustering in Bitcoin. Econom. Lett. 159, 145–148.