ECON532 Applied Econometrics

Lecture 10

ARIMA Models

ARMA(p,q)

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}$$

ARMA(1,1)

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + a_t - \theta_1 a_{t-1}$$

Properties of ARMA(p,q)

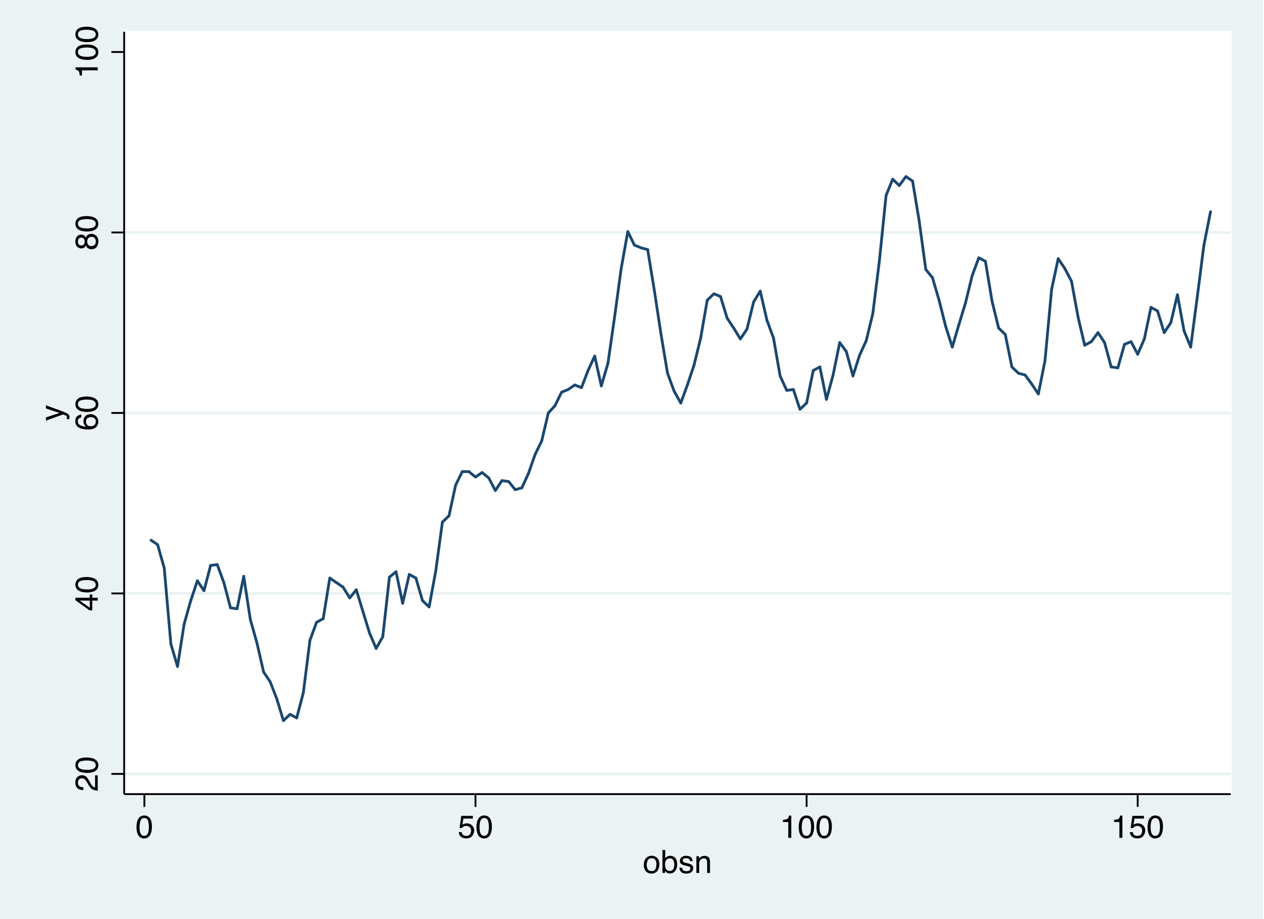
$$E(Y_t) = \frac{\phi_0}{1 - \phi_1 - \cdots - \phi_p} = \mu$$

ρ_k dies down

ρ_{kk} dies down

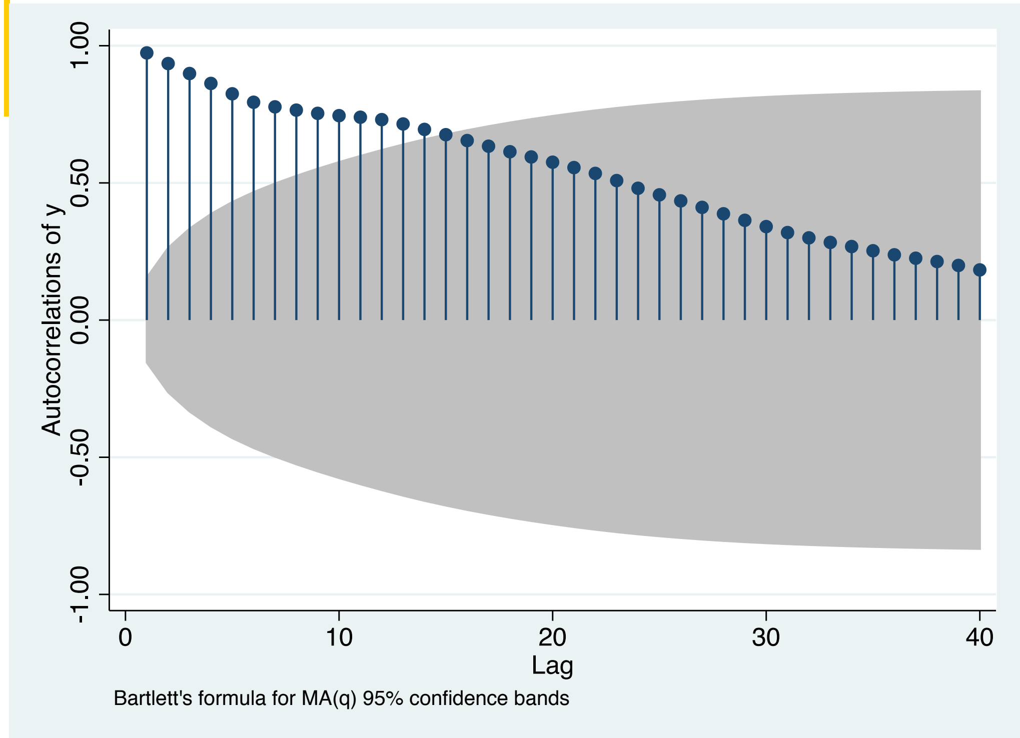
Example: iPad Weekly Sales

Data set iPad.xlsx contains 161 weeks weekly sales of iPad in certain area, the unit is 1,000 pieces.



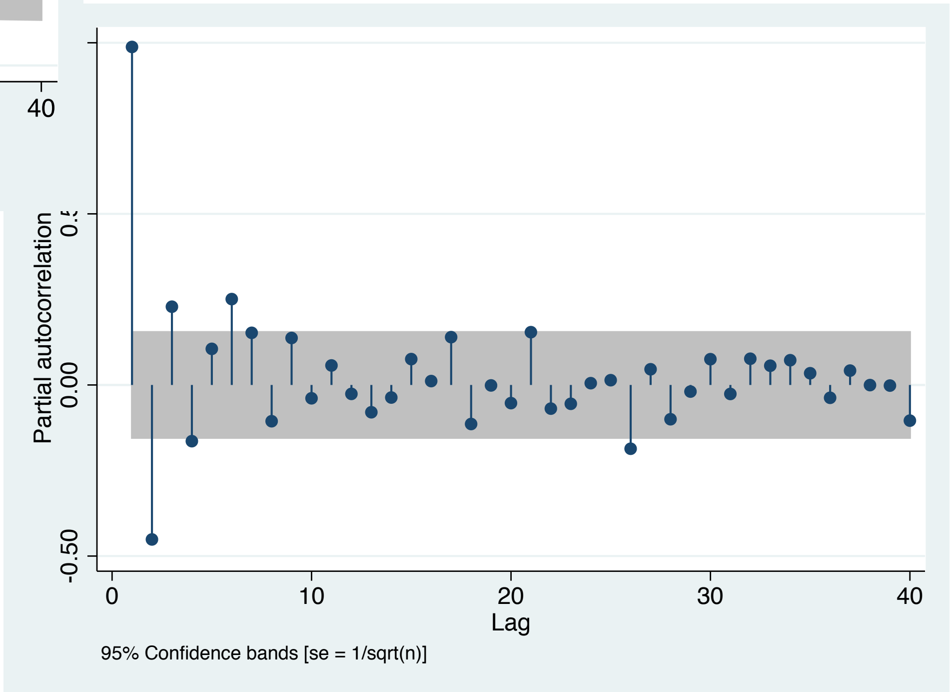

```
. gen obsn=_n
. tsset obsn
.  tsline y
```

Example: iPad Weekly Sales

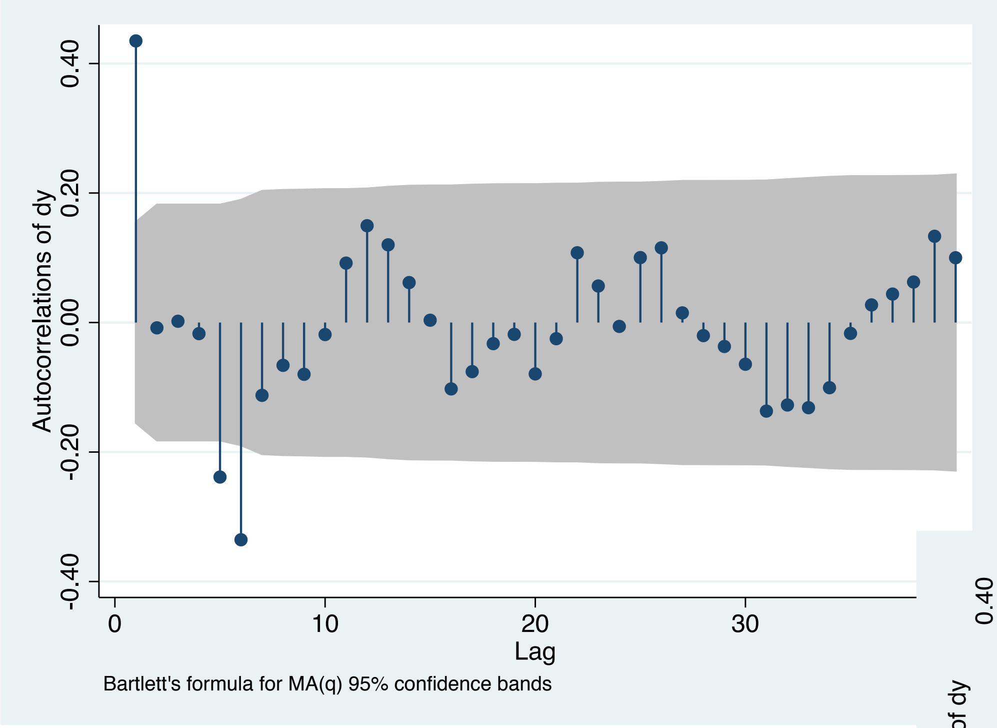


Which ARIMA model shall we fit to the weekly sales data?

ac y
pac y

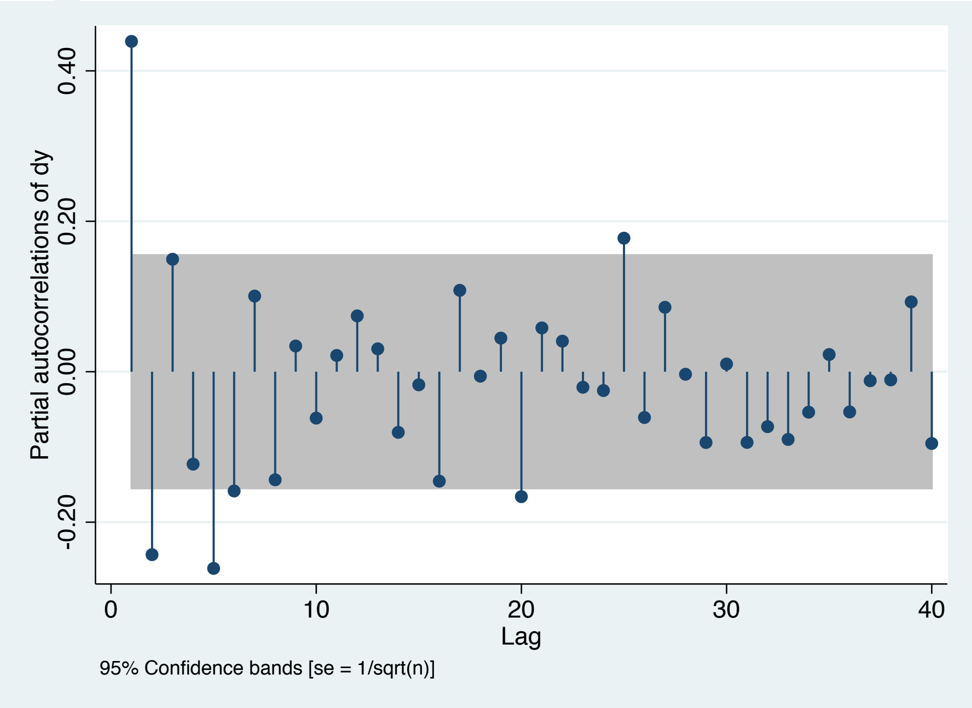


Example: ΔiPad Weekly Sales



Which ARIMA model shall we fit to the changes of weekly sales data?

```
gen dy=d.y
ac dy
pac dy
```



Tentative Model Identification

1. For weekly sales data, ACF dies down, PACF cuts off after lag 6: **AR(6) or ARIMA(6,0,0)**

2. For changes of weekly sales data, ACF dies down, PACF cuts off after lag 2: **ARIMA(2,1,0)**

3. PACF dies down, ACF cuts off after lag 6: **ARIMA(0,1,6)**

4. ACF and PACF dies down: **ARIMA(2,1,6)**

AR(6) for Weekly Sales

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \phi_4 Y_{t-4} + \phi_5 Y_{t-5} + \phi_6 Y_{t-p} + a_t$$

```
. arima y, arima(6,0,0)

ARIMA regression

Sample:   1 - 161                         Number of obs     =        161
                                           Wald chi2(6)      =    7611.67
Log likelihood = -366.4981                Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
       _cons |   61.06177   11.42696     5.34   0.000     38.66535    83.45819
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   1.562099   .0800496    19.51   0.000     1.405205    1.718993
         L2. |  -.8669502   .1488618    -5.82   0.000    -1.158714   -.5751863
         L3. |   .4463048   .1687587     2.64   0.008     .1155439    .7770657
         L4. |   -.119521   .1822403    -0.66   0.512    -.4767055    .2376635
         L5. |  -.2937807   .1640917    -1.79   0.073    -.6153946    .0278332
         L6. |   .2586847   .0864248     2.99   0.003     .0892953    .4280742
-------------+----------------------------------------------------------------
      /sigma |   2.322611   .1365085    17.01   0.000     2.055059    2.590162
------------------------------------------------------------------------------
```

arima in Stata is estimated by MLE

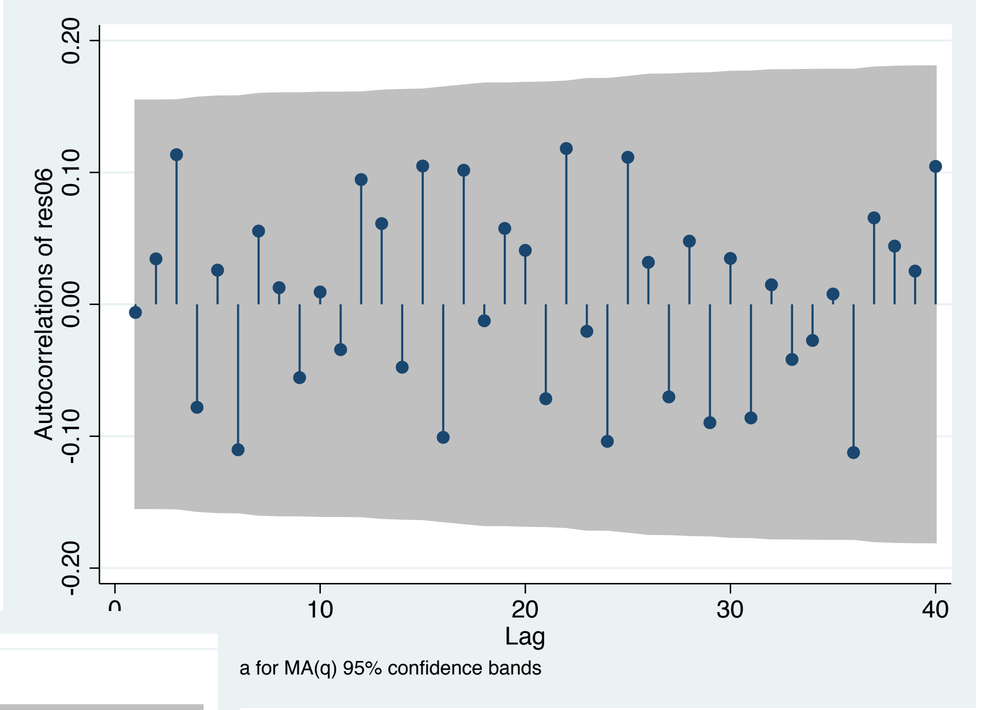
AR(6) for Weekly Sales

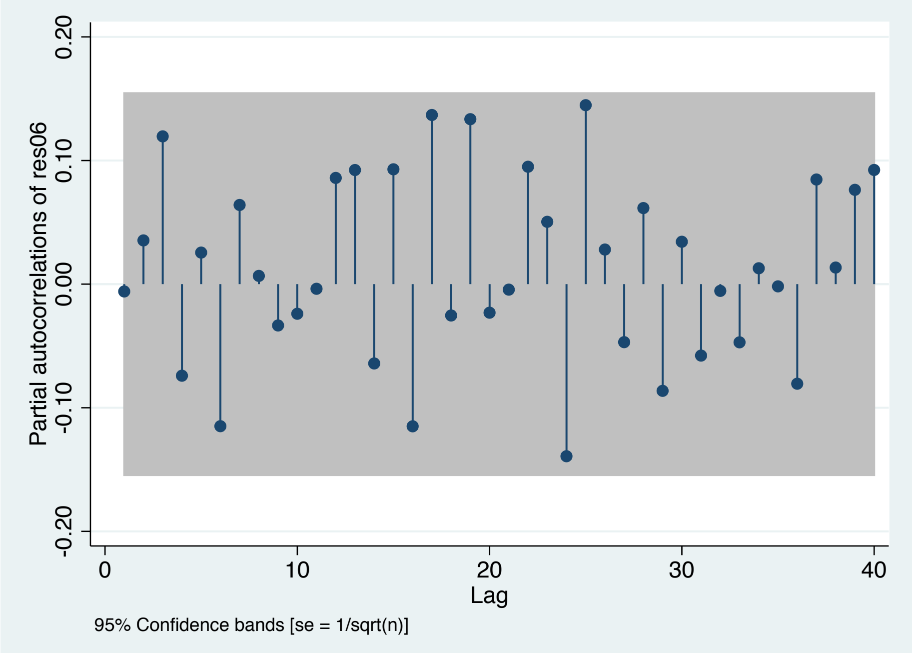
$$
\begin{aligned}
Y_t &= \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} \\
&+ \phi_5 Y_{t-5} + \phi_6 Y_{t-p} + a_t
\end{aligned}
$$

```
. arima y, ar(1/3, 5/6)

ARIMA regression

Sample:   1 - 161                          Number of obs     =        161
                                           Wald chi2(5)      =    7715.78
Log likelihood = -366.7707                 Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
       _cons |   61.04702   11.39256     5.36   0.000     38.71802    83.37602
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   1.546066   .0736121    21.00   0.000     1.401789    1.690343
         L2. |  -.8177858   .1264621    -6.47   0.000    -1.065647   -.5699247
         L3. |   .3528106   .0936637     3.77   0.000     .1692332     .536388
         L5. |  -.3786354   .0810315    -4.67   0.000    -.5374543   -.2198166
         L6. |   .2844281   .0698384     4.07   0.000     .1475474    .4213089
-------------+----------------------------------------------------------------
      /sigma |   2.326695   .1329986    17.49   0.000     2.066023    2.587368
------------------------------------------------------------------------------
```

Residual Test: AR(6) for Weekly Sales

```
. predict res06, resid

. wntestq res06

Portmanteau test for white noise

---------------------------------------

 Portmanteau (Q) statistic =      36.2961

 Prob > chi2(40)            =       0.6377

. ac res06

. pac res06
```





ARIMA(2,1,0) for Weekly Sales

$$Y_t - Y_{t-1} = Z_t = \phi_0 + \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t$$

$$Y_t = \phi_0 + (1 + \phi_1) Y_{t-1} + (\phi_2 - \phi_1) Y_{t-2} - \phi_2 Y_{t-3} + a_t$$

```
. arima y,arima(2,1,0)
ARIMA regression

Sample:   2 - 161                              Number of obs      =        160
                                               Wald chi2(2)       =      50.64
Log likelihood = -370.9592                     Prob > chi2        =     0.0000

------------------------------------------------------------------------------
             |                 OPG
         D.y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
       _cons |   .2289447   .2803981     0.82   0.414    -.3206254    .7785148
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |    .539406   .0759182     7.11   0.000     .3906091     .688203
         L2. |  -.2419928   .0808765    -2.99   0.003    -.4005079   -.0834778
-------------+----------------------------------------------------------------
      /sigma |   2.455985   .1379531    17.80   0.000     2.185602    2.726368
------------------------------------------------------------------------------
```

Residual Test: ARIMA(2,1,0)

. predict res21, resid

. wntestq res21

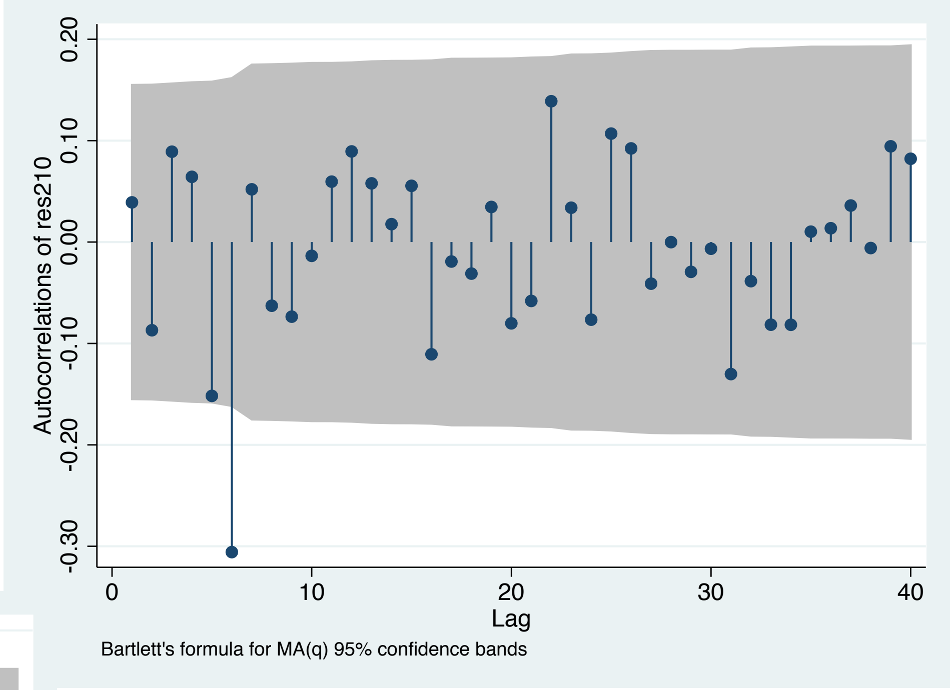
Portmanteau test for white noise

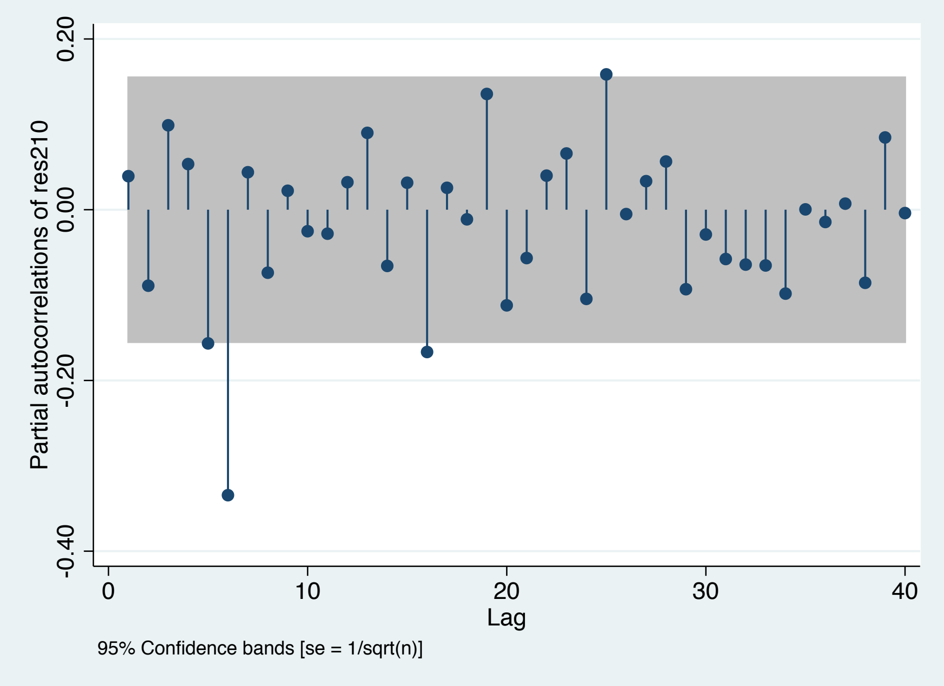
Portmanteau (Q) statistic = 52.3187

Prob > chi2(40) = 0.0918

. ac res21

. pac res21





ARIMA(0,1,6) for Weekly Sales

$$Y_t - Y_{t-1} = Z_t = \theta_0 + a_t - \theta_1 a_{t-1} - \theta_6 a_{t-6}$$

```
. arima d.y,ma(1,6)
ARIMA regression

Sample:    2 - 161                              Number of obs      =        160
                                                Wald chi2(2)       =      39.76
Log likelihood = -356.8053                      Prob > chi2        =     0.0000

------------------------------------------------------------------------------
             |                 OPG
         D.y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
       _cons |    .234198   .2368456     0.99   0.323    -.2300108    .6984068
-------------+----------------------------------------------------------------
ARMA         |
          ma |
         L1. |    .642906   .1118617     5.75   0.000      .423661     .862151
         L6. |  -.3558061   .0870541    -4.09   0.000    -.5264289   -.1851832
-------------+----------------------------------------------------------------
      /sigma |   2.221263   .1483614    14.97   0.000      1.93048    2.512046
------------------------------------------------------------------------------
```

Residual Test: ARIMA(0,1,6)

. predict res16, resid

. wntestq res16

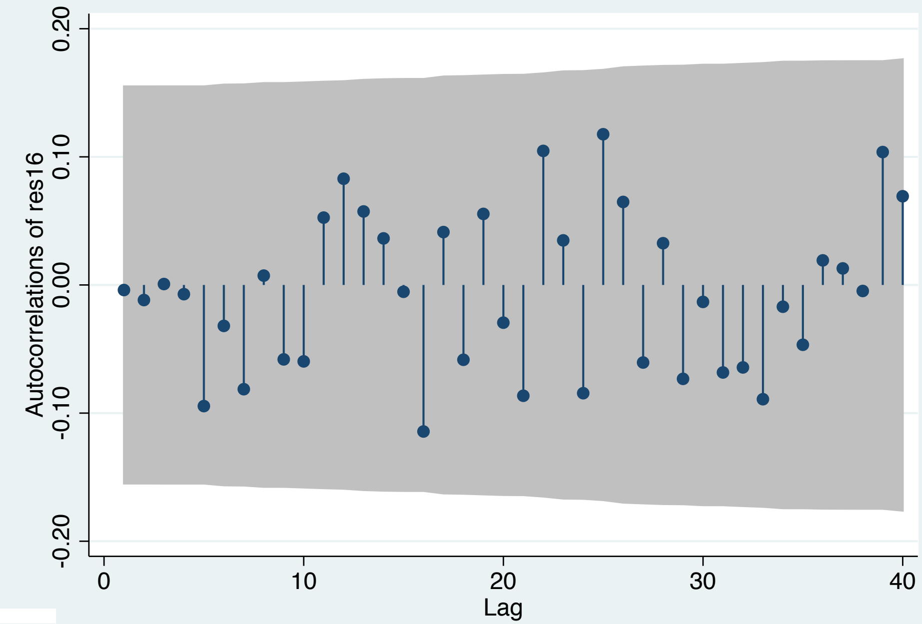
Portmanteau test for white noise

Portmanteau (Q) statistic = 28.4462

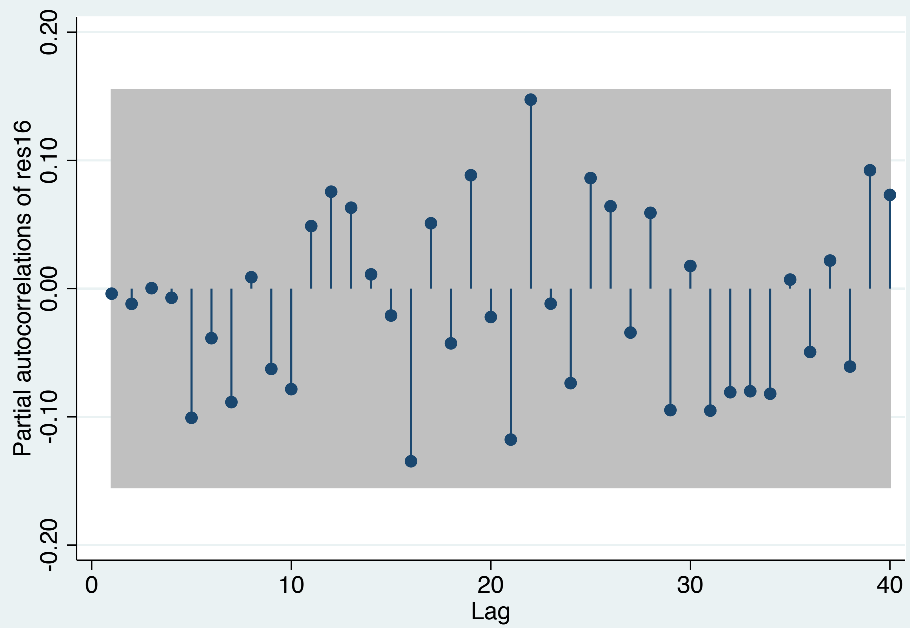
Prob > chi2(40) = 0.9140

. ac res16

. pac res16



Bartlett's formula for MA(q) 95% confidence bands



95% Confidence bands [se = 1/sqrt(n)]

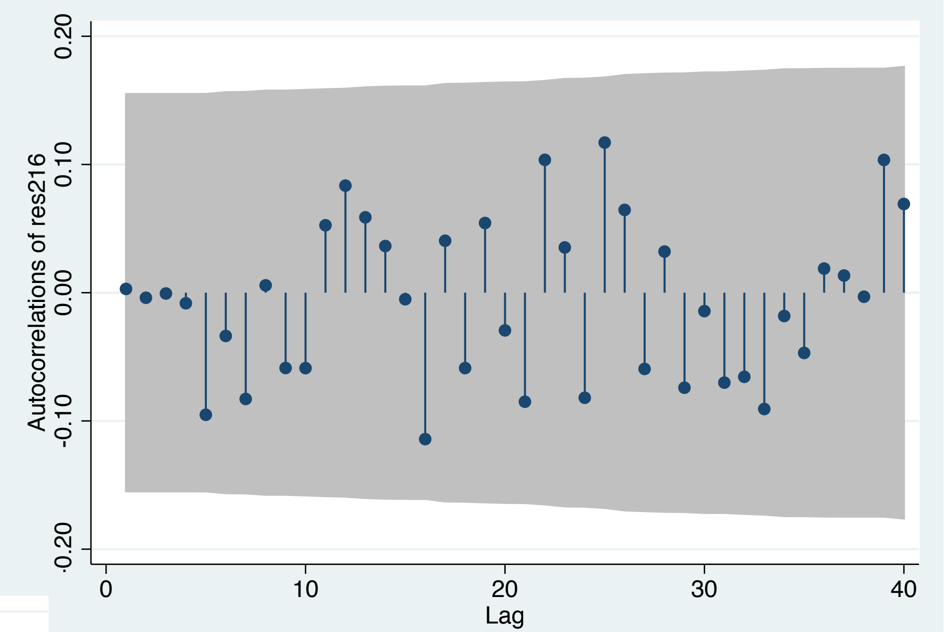
ARIMA(2,1,6) for Weekly Sales

$$Y_t - Y_{t-1} = Z_t$$
$$= \phi_0 + \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t - \theta_1 a_{t-1} - \theta_6 a_{t-6}$$

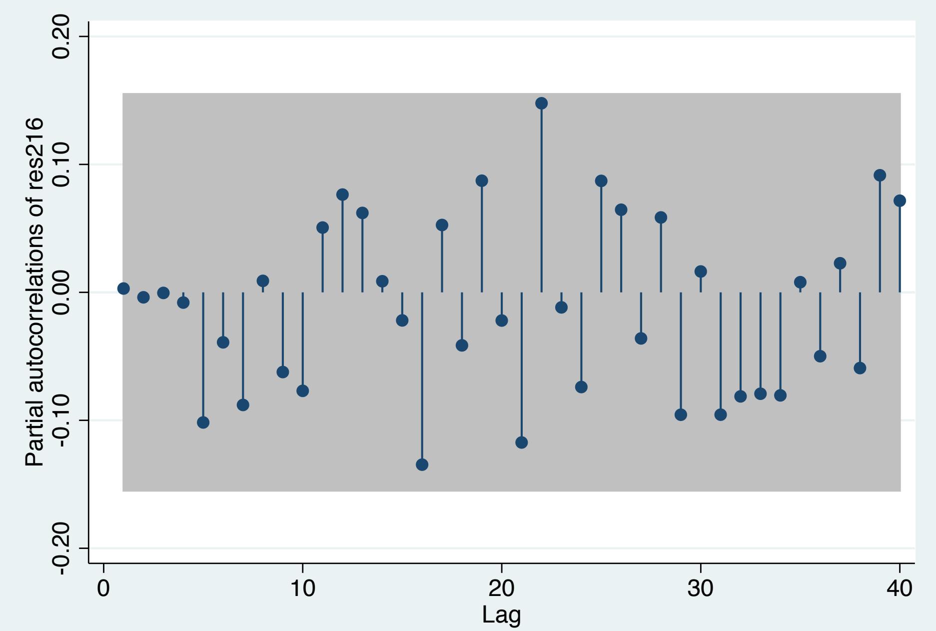
```
. arima d.y, ar(1/2) ma(1,6)
ARIMA regression

Sample:   2 - 161                          Number of obs      =        160
                                            Wald chi2(4)      =      38.25
Log likelihood =   -356.796                Prob > chi2        =     0.0000

------------------------------------------------------------------------------
             |                 OPG
         D.y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
       _cons |    .233622    .2349421     0.99   0.320    -.2268561       .6941
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |  -.0080898    .1360128    -0.06   0.953      -.27467     .2584903
         L2. |  -.0075481    .0994218    -0.08   0.939    -.2024112      .187315
             |
          ma |
         L1. |   .6444355    .2084595     3.09   0.002     .2358625     1.053009
         L6. |  -.3554371    .0879426    -4.04   0.000    -.5278013    -.1830729
-------------+----------------------------------------------------------------
      /sigma |   2.220637    .1650276    13.46   0.000     1.897189     2.544085
------------------------------------------------------------------------------
```

Residual Test: ARIMA(2,1,6)

```
. predict res216, resid

. wntestq res216

Portmanteau test for white noise

Portmanteau (Q) statistic =      28.4673

  Prob > chi2(40)          =        0.9135

. ac res216

. pac res216
```



Bartlett's formula for MA(q) 95% confidence bands



95% Confidence bands [se = 1/sqrt(n)]

iPad Weekly Sales: ARIMA

Which model shall we choose? Why?

ARIMA(0,1,6): $Y_t - Y_{t-1} = Z_t = \theta_0 + a_t - \theta_1 a_{t-1} - \theta_6 a_{t-6}$

$$Y_t = Y_{t-1} + \theta_0 + a_t - \theta_1 a_{t-1} - \theta_6 a_{t-6}$$

A random walk with drift!

ARIMA(0,1,6) without the constant term:

$$Y_t = Y_{t-1} + a_t - \theta_1 a_{t-1} - \theta_6 a_{t-6}$$

A random walk without drift.

ARIMA(0,1,6) for Weekly Sales Without Constant

$$Y_t - Y_{t-1} = a_t - \theta_1 a_{t-1} - \theta_6 a_{t-6}$$

```
. arima d.y,ma(1,6) nocons
ARIMA regression

Sample:    2 - 161                          Number of obs      =        160
                                            Wald chi2(2)       =      40.03
Log likelihood = -357.3297                  Prob > chi2        =     0.0000

------------------------------------------------------------------------------
             |                 OPG
         D.y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ARMA         |
          ma |
         L1. |   .6471472   .1096499     5.90   0.000     .4322374     .862057
         L6. |   -.352221    .086853    -4.06   0.000    -.5224498   -.1819923
-------------+----------------------------------------------------------------
      /sigma |   2.228369    .150964    14.76   0.000     1.932485    2.524253
------------------------------------------------------------------------------
```

Residual Test: ARIMA(0,1,6) without constant

. predict res16noc, resid

. wntestq res16noc

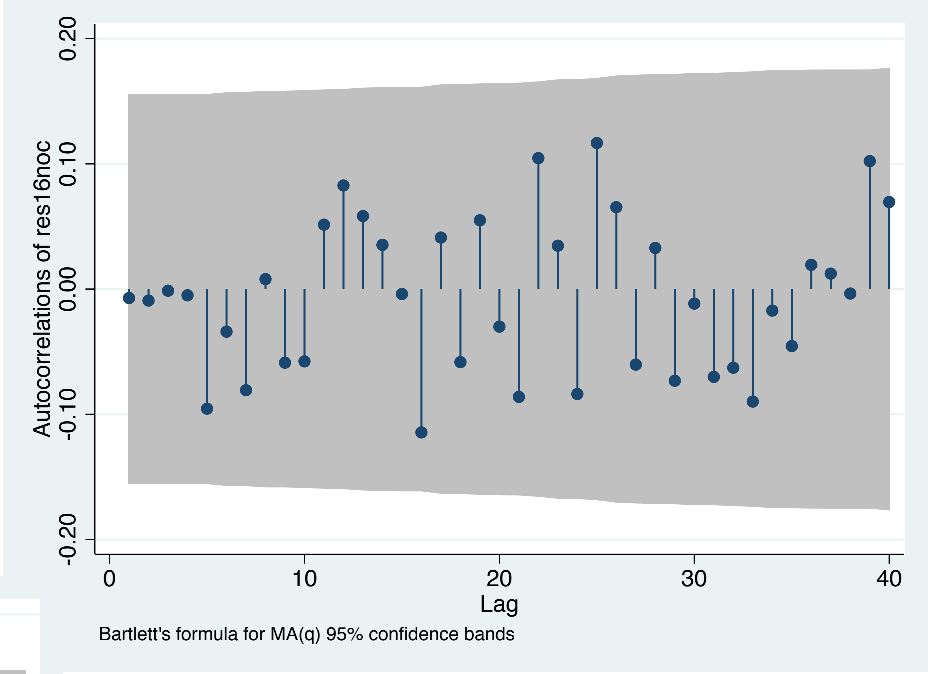
Portmanteau test for white noise

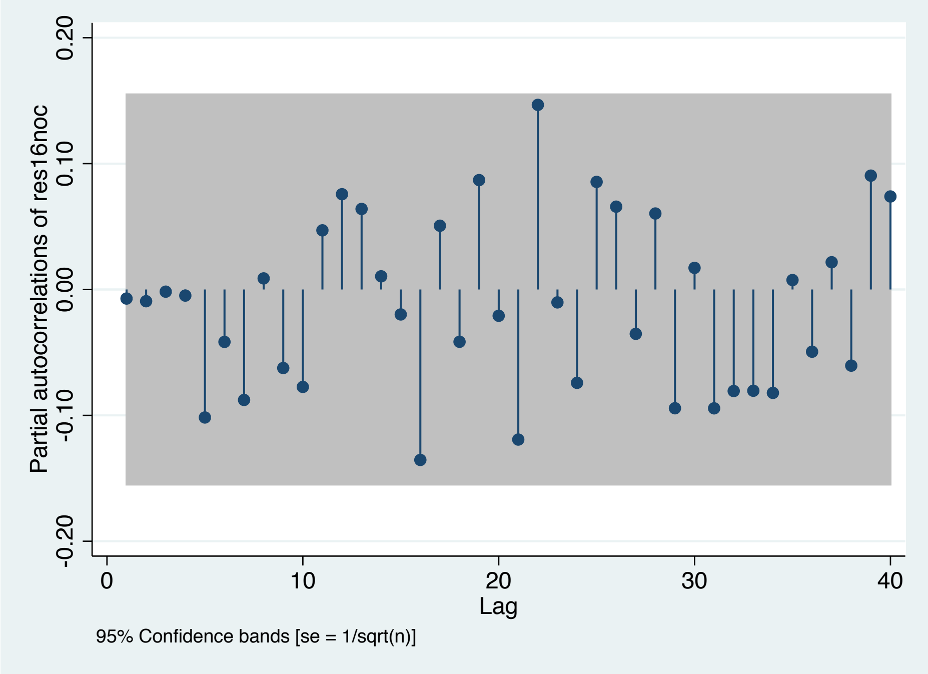
Portmanteau (Q) statistic = 28.2859

Prob > chi2(40) = 0.9175

. ac res16noc

. pac res16noc





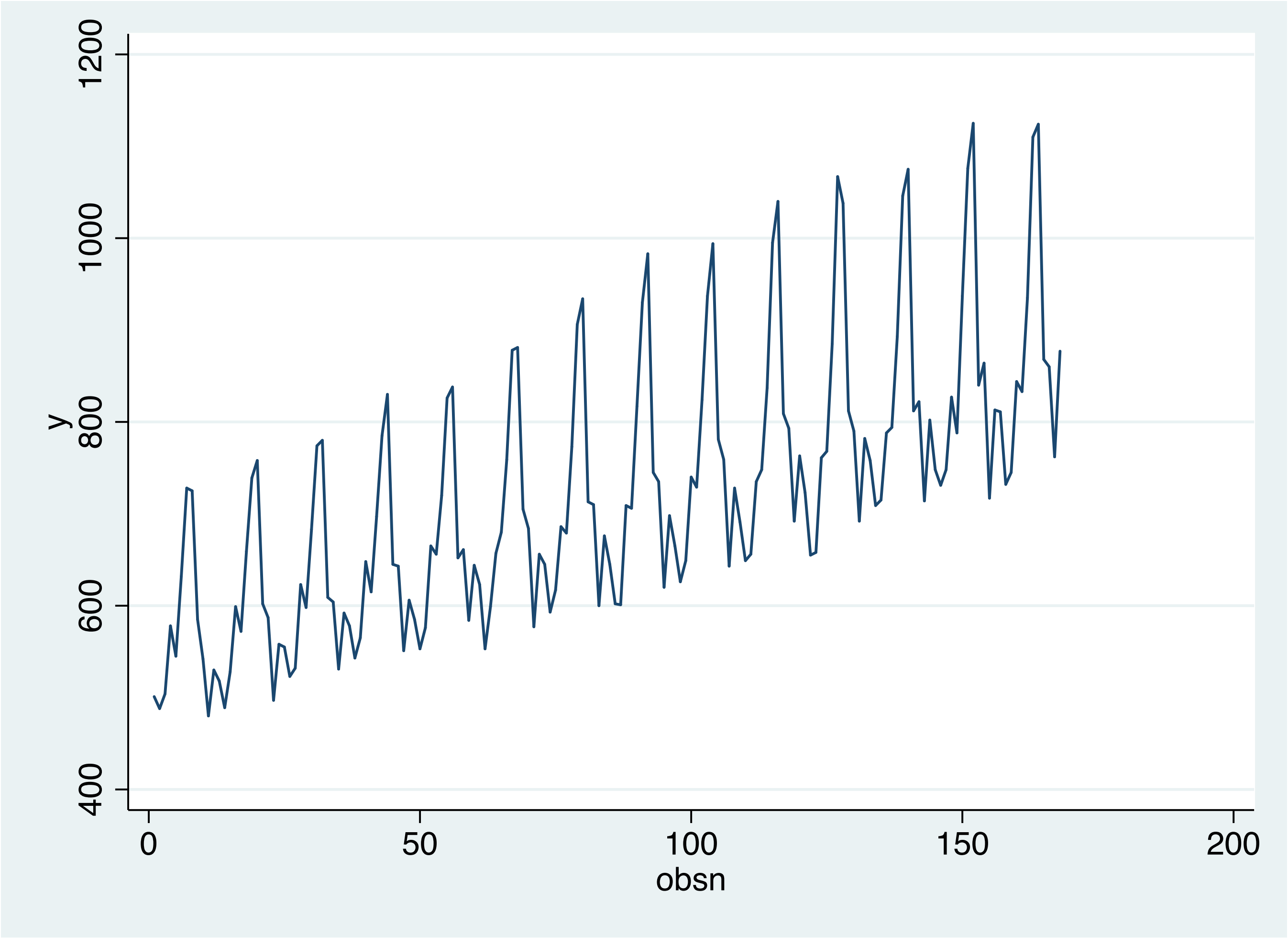
iPad Weekly Sales: ARIMA

Model	AIC	BIC
AR(6)	747.54	769.11
ARIMA(2,1,0)	749.92	762.22
ARIMA(0,1,6)	721.61	733.91
ARIMA(2,1,6)	725.59	744.04
ARIMA(0,1,6) no constant	720.66	729.89

Seasonal ARMA(P,Q)

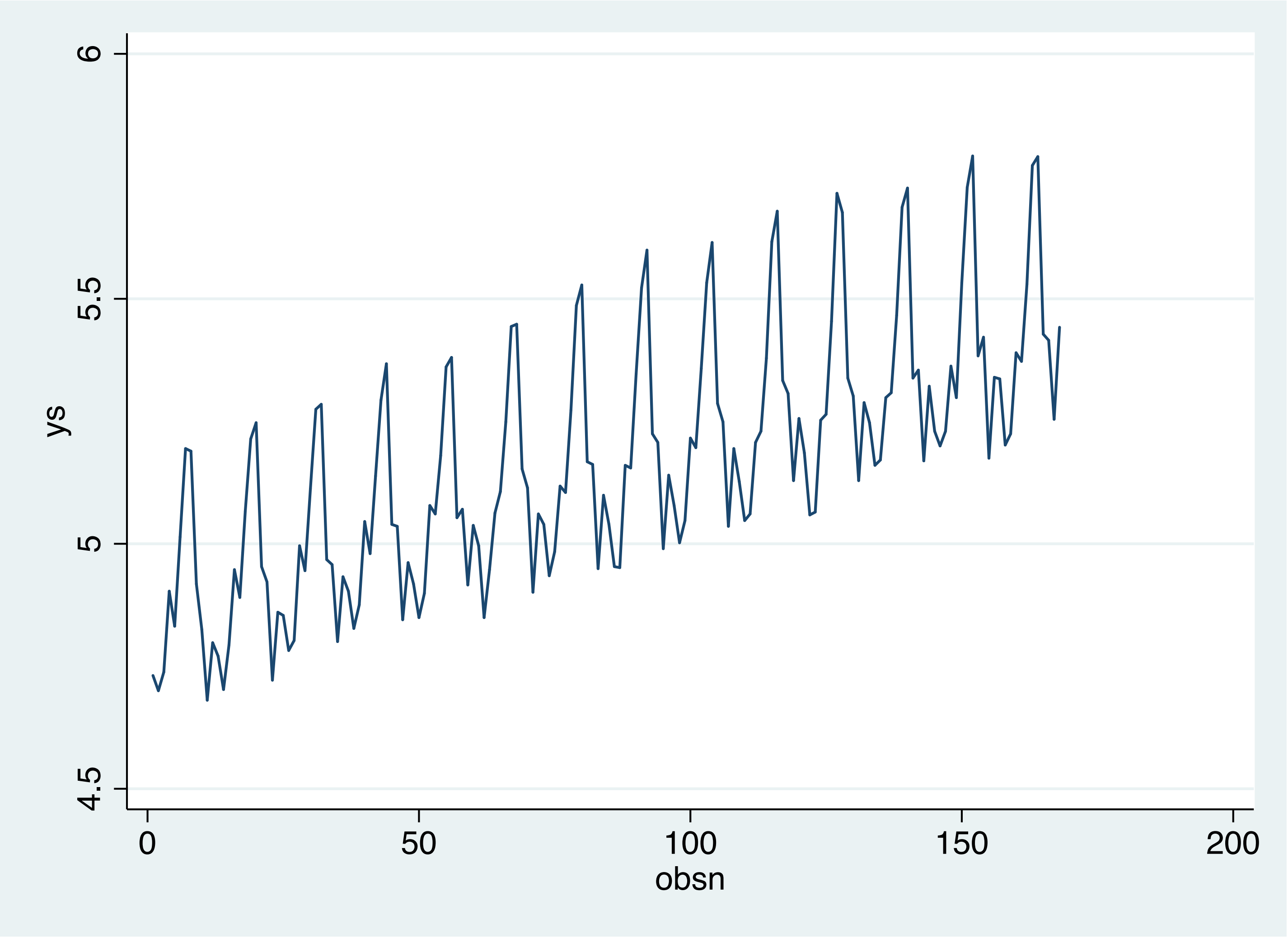
Example: Monthly hotel room occupancy

```
gen obsn=_n.
tsset obsn
tsline y
```



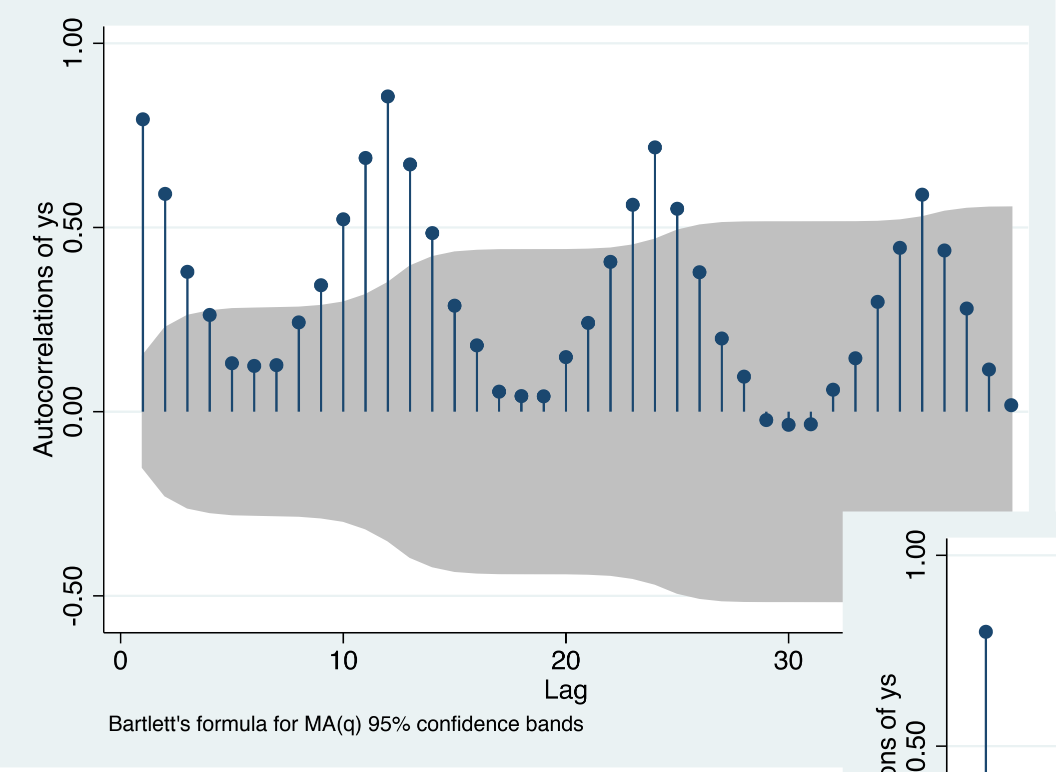
Variance Stabilization: Quartic Root

Y*=Y^.25

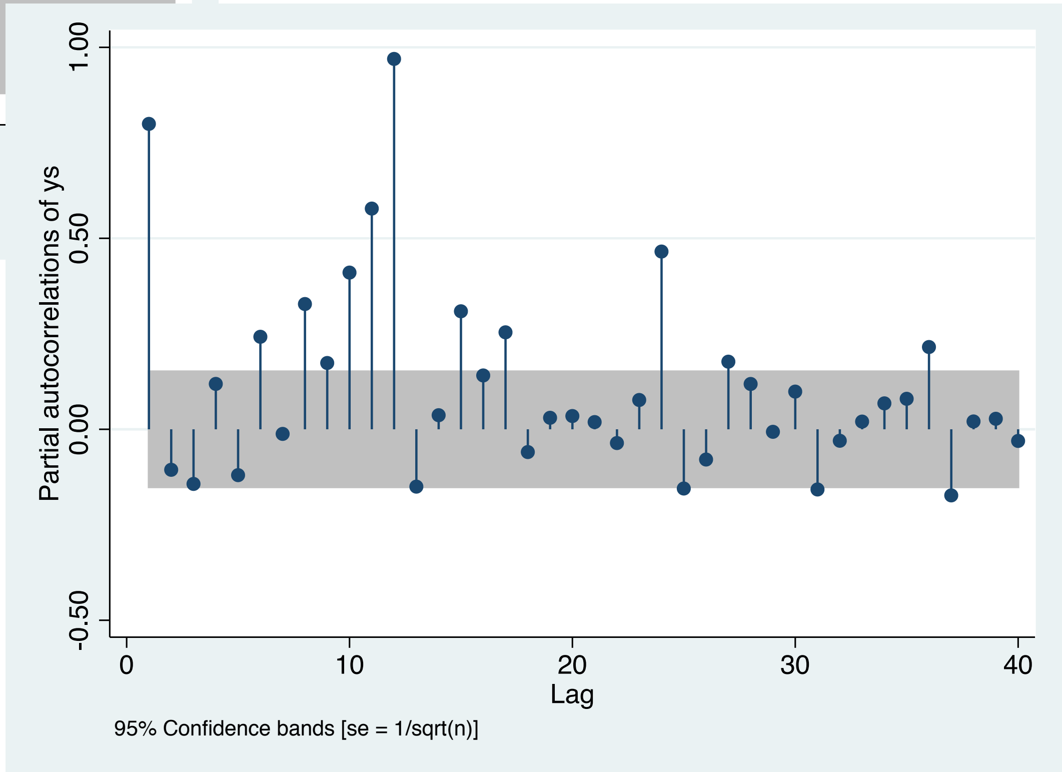



```
gen  ys=y^.25
tsline ys
```

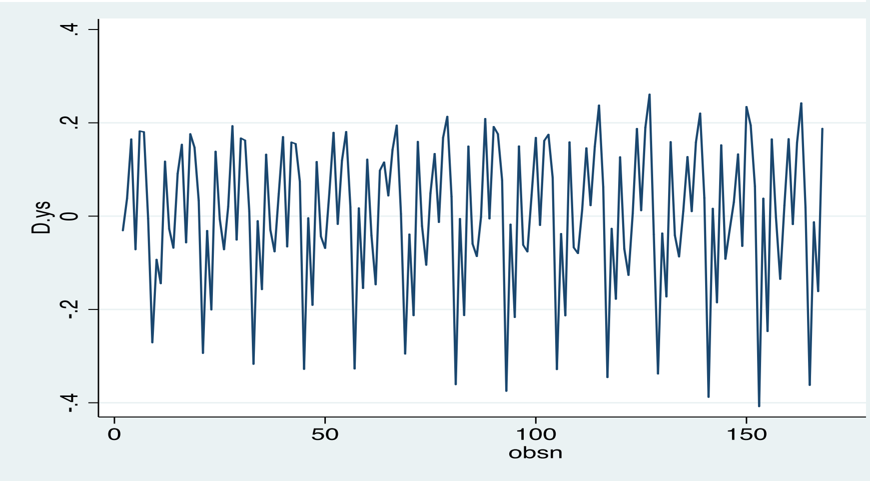
Stationary?

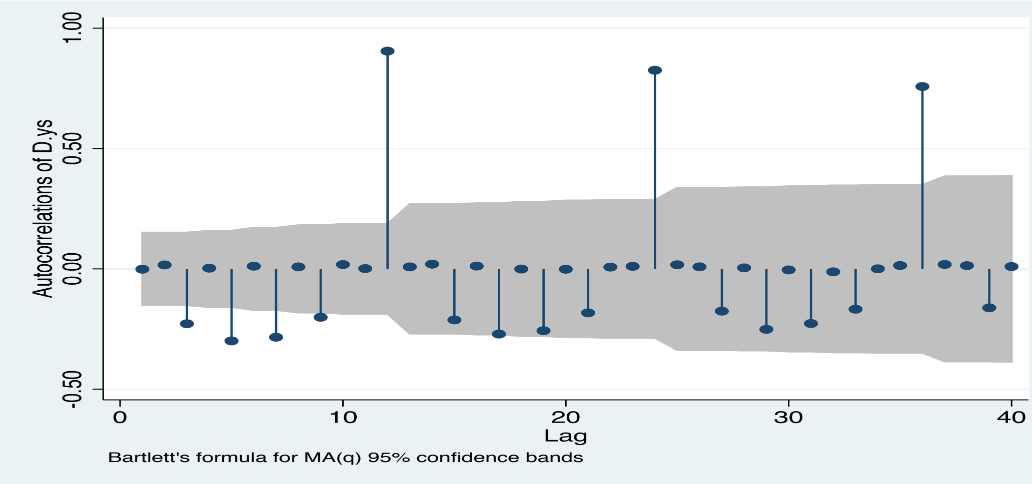


ac ys
pac ys

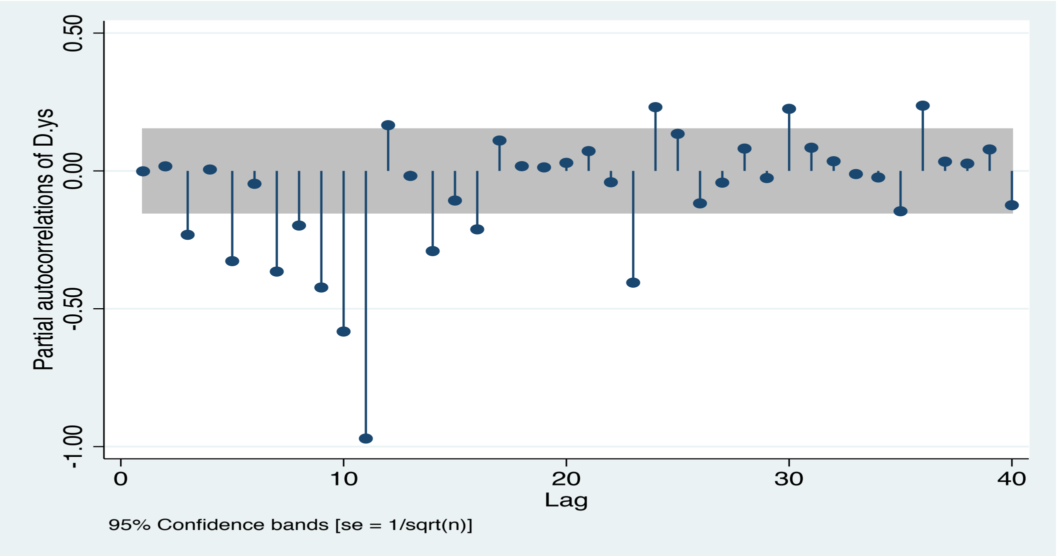


1st order difference





Tsline d.ys
ac d.ys
pac d.ys



Model 1: AR(12) or SAR(1) on Δys

$$Y_t^* - Y_{t-1}^* = \phi_0 + \phi_1(Y_{t-12}^* - Y_{t-13}^*) + a_t$$

arima d.ys, ar(12)

D.ys	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf. Interval]	
ys						
_cons	.0056001	.0447055	0.13	0.900	-.082021	.0932212
ARMA						
ar						
L12.	.968525	.0146012	66.33	0.000	.9399071	.9971429

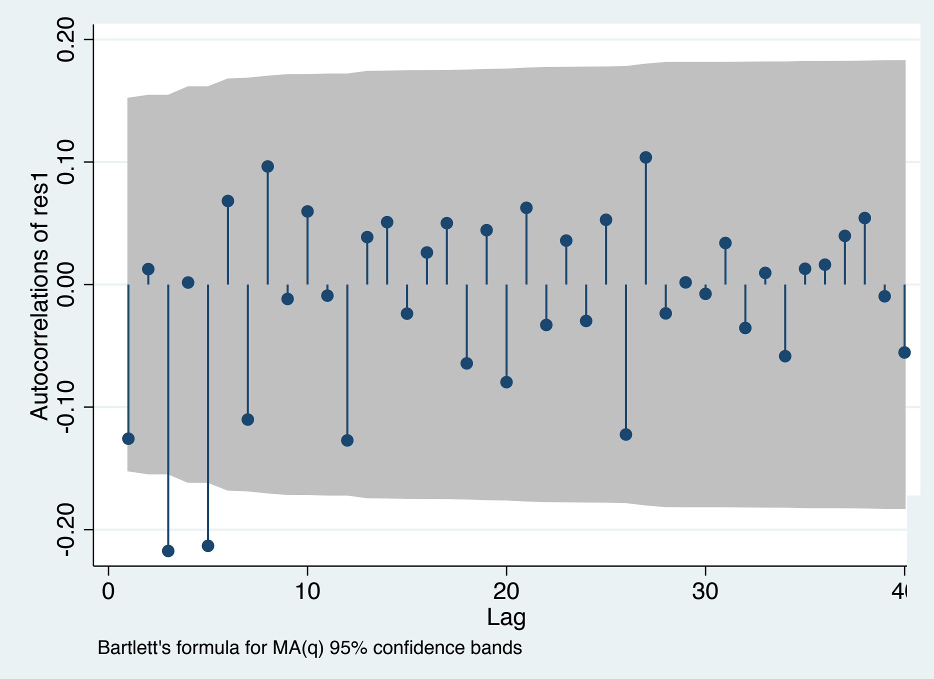
Model 1: AR(12) on Δys

$$Y_t^* - Y_{t-1}^* = \phi_1(Y_{t-12}^* - Y_{t-13}^*) + a_t$$

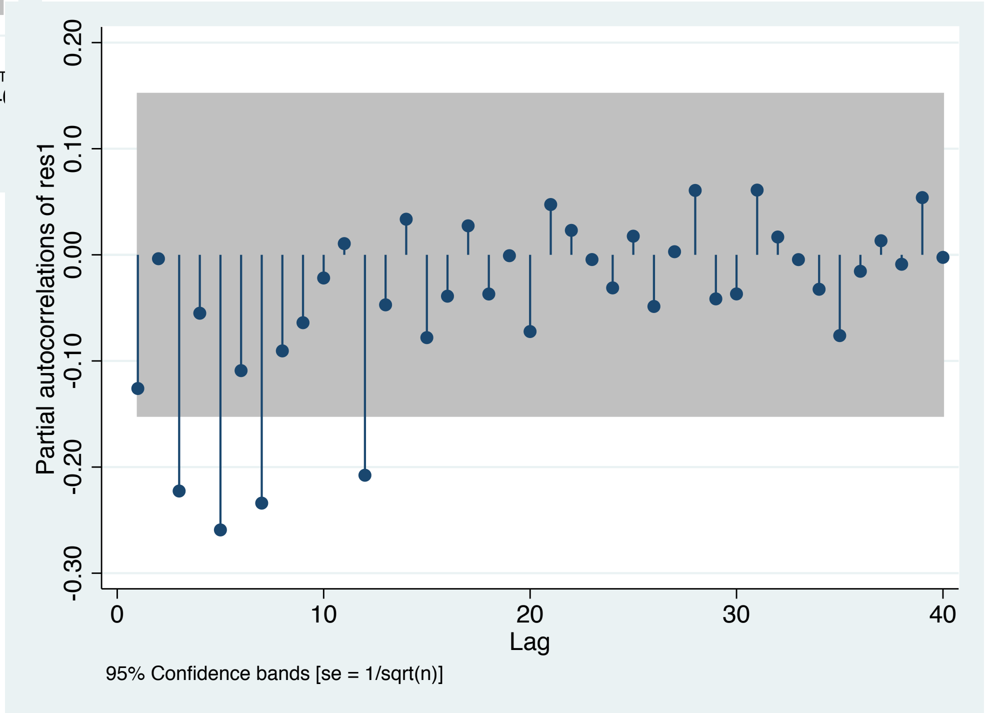
arima d.ys, ar(12) nocons

D.ys	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf. Interval]	
ARMA						
ar						
L12.	.9685633	.0144982	66.81	0.000	.9401474	.9969792

Residuals of Model 1



predict res1, resid
ac res1
pac res1



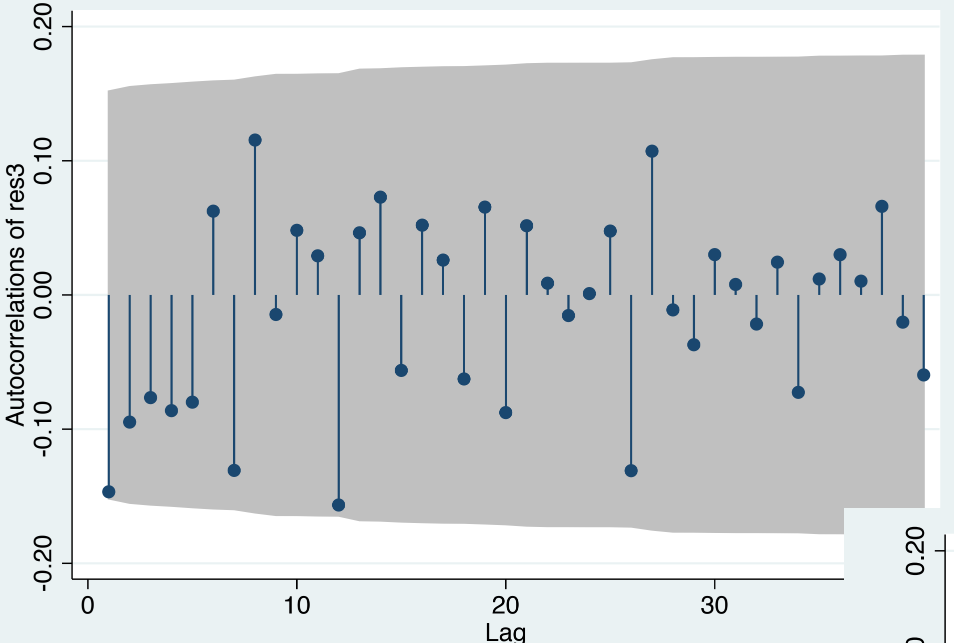
Model 2: MA-Seasonal AR on $S\Delta ys$

$$Y_t^* - Y_{t-1}^* = \phi_1(Y_{t-12}^* - Y_{t-13}^*) + a_t - \theta_3 a_{t-3} - \theta_5 a_{t-5}$$

arima d.ys, ma(3,5) sarima(1,0,0,12) nocons

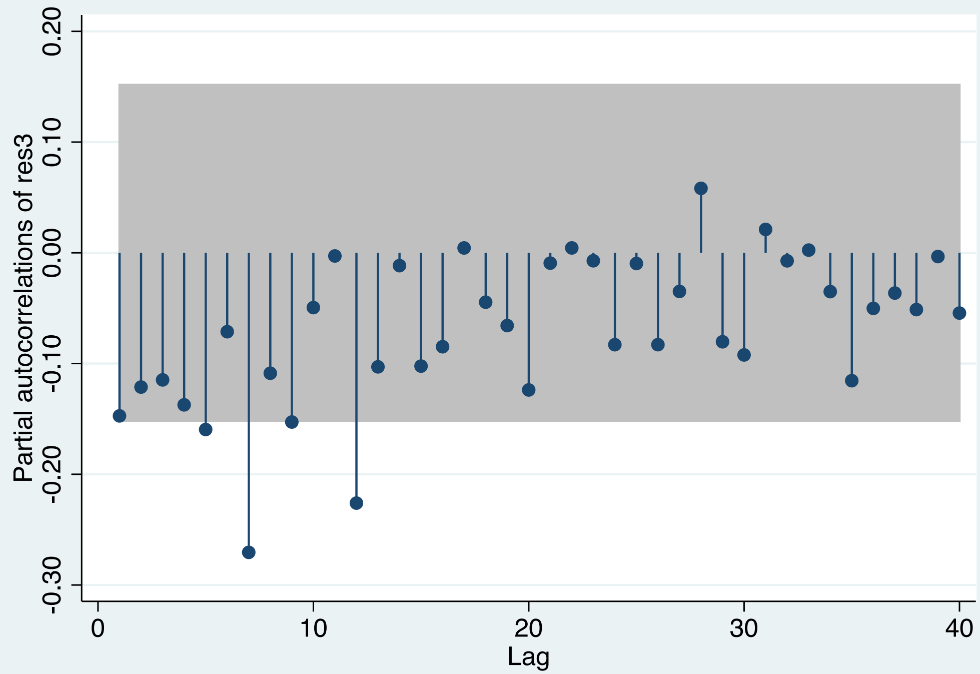
D.ys	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf. Interval]	
ARMA						
ma						
L3.	-.1731372	.0887069	-1.95	0.051	-.3469995	.0007251
L5.	-.2107104	.0854908	-2.46	0.014	-.3782693	-.0431514
ARMA12						
ar						
L1.	.9672734	.0163334	59.22	0.000	.9352605	.9992863

Residuals of Model 2



Bartlett's formula for MA(q) 95% confidence bands

```
predict res2, resid
ac res2
pac res2
```



95% Confidence bands [se = 1/sqrt(n)]

Portmanteau test for white noise

Portmanteau (Q) statistic =	36.0742
Prob > chi2(40) =	0.6476

Identifying Seasonal ARIMA models

1. SAR(P): SACF dies down for lag L,2L,3L,…;SPACF cuts off at lag P*L
2. SMA(Q): SACF cuts off at lag Q*L; SPACF dies down for lag L,2L,3L,…
3. SARIMA(0,D,0) if SACF and SPACF dies down extremely slowly.
4. Usually ONLY ONE SAR or SMA term is needed.
5. SAR terms are often used when lag L SACF are positive
6. SAR terms are often used when lag L SACF are negative
7. Don’t use BOTH SAR and SMA in the same model
8. If a seasonal difference is needed, only seasonal difference ONCE