Combinatorial optimization: Max-Cut, Min UnCut and Sparsest Cut Problems

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Outline

Max-Cut Problem

Given an undirected weighted graph $G = (V, E, W)$, where

$$V = \{1, \dots, n\} \text{ – set of vertices}$$
$$E \subseteq V \times V \text{ – set of edges}$$
$$W : E \to \mathbb{R} \text{ – weights}$$

One wants to find a partition $f : V \to \{0, 1\}$ in order to maximize the sum of edges in the cut:

$$\sum_{(i,j) \in E : f(i) \neq f(j)} w_{ij} \to \max,$$

where w_{ij} stands for weight of the edge (i, j).

Min UnCut Problem

Given an undirected weighted graph $G = (V, E, W)$, one wants to find a partition $f : \{0, 1\}$ in order to minimize the sum of edges out of the cut:

$$\sum_{(i,j)\in E: f(i)=f(j)} w_{ij} \to \min$$

Remark. Let $Opt(MUC)$ and $Opt(MC)$ stand for optimal solutions of Min UnCut and Max-Cut problems respectively. Then it holds

$$Opt(MUC) + Opt(MC) = \sum_{(i,j)\in E} w_{ij}$$

Sparsest Cut Problem

Given an undirected weighted graph $G = (V, E$ and a capacity function $c : E \to \mathbb{R}_+$. Also given a set of demand pairs $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$ and demand values $d_1, d_2, \ldots, d_k$. One wants to find a set $E' \subseteq E$ minimizing

$$\frac{c(E')}{D(E')} \to \min,$$

where

$$c(f) = \sum_{(i,j)\in E'} c_{ij}$$

$$D(f) = \sum_{i:(s_i,t_i) \text{ are separated by } E'} d_i$$

Thank you for attention!