

EES216: Circuit Analysis

Final Mock Exam

curated by The Peanuts

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Conditions: Semi-Closed Book (A3 Both Sides)

Directions:

1. This exam has 10 pages (including this page).
2. Calculators (Casio 991 Series) are allowed.
3. Write your name clearly at the top of each page.
4. Please check to make sure that there are 6 problems in your exam paper. (It's a PDF, I know)
5. Red color is reserved for grading. Do not write in red.
6. Do not cheat. Do not panic.
7. Good Luck + Warm Wish for a bright and joyful coming new year.

For solution, [click here](#).

Problem 1

The voltage $v(t)$ in a network is defined by the equation.

$$\frac{d^2 v_1(t)}{dt^2} + 8 \frac{dv_1(t)}{dt} + 10v_1(t) = 0.$$

Identify the type of this second-order system: Overdamped #

Compared with $s^2 + 2\alpha s + \omega_o^2$

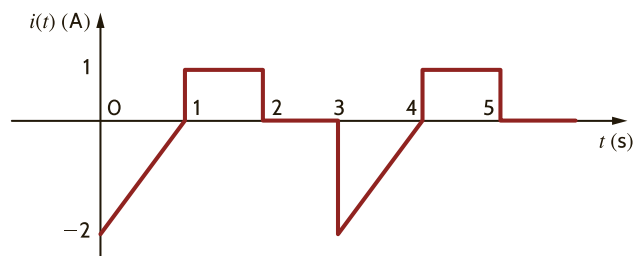
$$s^2 + 8s + 10$$

$$2\alpha = 8 \quad \omega_o^2 = 10$$

$$\alpha = 4 \quad \omega_o = \sqrt{10}$$

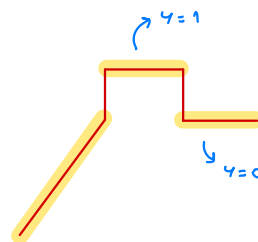
$\therefore \alpha > \omega_o$: Overdamped

Calculate the rms value of the waveform



$$I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$$

$$\therefore I_{\text{eff}} = \left[\frac{1}{3} \left(\int_0^1 (2x-2)^2 + \int_1^2 1^2 dt + \int_2^3 0^2 dt \right) \right]^{\frac{1}{2}} \quad A_{\text{rms}} \quad \#$$



$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{0 - (-2)}{1 - 0}$$

$$m = 2$$

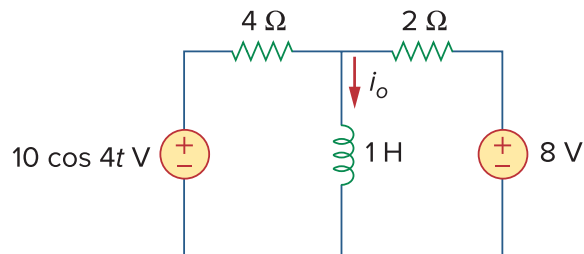
$$y = 2x + C$$

$$0 = 2(1) + C$$

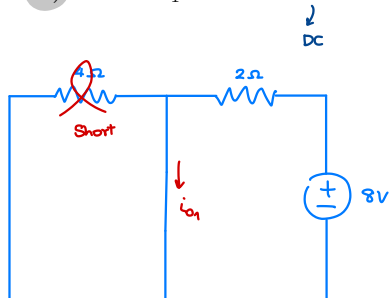
$$\therefore C = -2$$

$$\therefore y = 2x - 2$$

Solve for $i_o(t)$ in the circuit using the superposition principle.

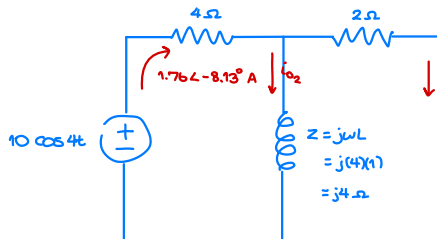
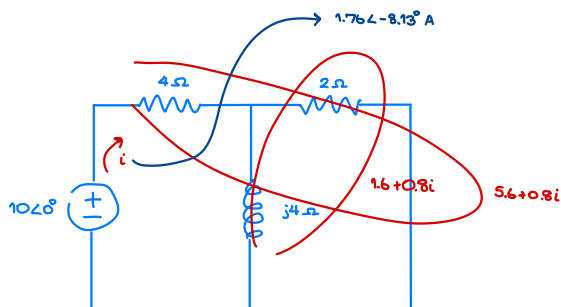


a) Find i_{o1} due to 8 V voltage source. $i_{o1} =$ 4 A #



$$\therefore i_{o1} = \frac{8}{2} = 4 \text{ A}$$

b) Find i_{o2} due to $10 \cos 4t$ V source. $i_{o2} =$ $0.79 \angle -71.5^\circ \text{ A}$ #



$$\begin{aligned} \therefore i_{o2} &= \frac{2}{2 + j4} (1.76 \angle -81.3^\circ) \\ &= 0.79 \angle -71.5^\circ \text{ A} \end{aligned}$$

c) Find $i_o =$ $(0.79 \angle -71.5^\circ) + 4 \text{ A}$ #

In Thailand, the electricity system operates at what standard voltage and frequency?

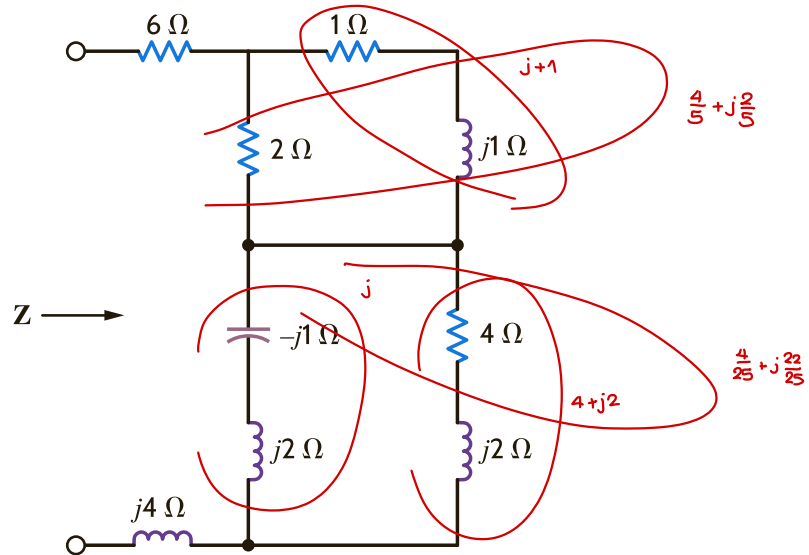
Voltage = 220 volts, Frequency = 50 Hz

State **three** benefits of a three-phase electrical system

1. Nearly all electric power is generated and distributed using three-phase system.
2. It can provide constant (non-pulsating) to load.
3. For the same amount of power, the three-phase system is more economical than single phase.

Problem 2

Find the equivalent impedance, \mathbf{Z} , for the circuit.



$$\begin{aligned} \therefore Z_T &= 6 + \frac{4}{5} + j\frac{2}{5} + \frac{4}{25} + j\frac{22}{25} + j4 \\ &= 6.96 + j5.28 \, \Omega \quad \# \end{aligned}$$

→ ខ្លឹមសារកំណើន

$$\frac{dv}{dt} \Leftrightarrow j\omega V$$

$$\int v dt \Leftrightarrow \frac{V}{j\omega}$$

ប្រភេទ: អនិច្ចកាលនៃអនិច្ចកាល V ត្រូវបាន

អនិច្ចកាលនៃអនិច្ចកាល I ក៏ដោយ

$$\frac{di}{dt} \Leftrightarrow j\omega I$$

$$\int i dt \Leftrightarrow \frac{I}{j\omega}$$

អនិច្ចកាលនៃអនិច្ចកាល 555

Find $v(t)$ in the following integrodifferential equations using the phasor approach:

$$\frac{dv}{dt} + 5v(t) + 4 \int v dt = 20 \sin(4t + 10^\circ) \quad \omega = 4 \text{ rad/s}$$

$$j\omega V + 5V + 4 \frac{V}{j\omega} = 20 \sin(4t + 10^\circ)$$

$$j(4)V + 5V + \frac{4}{j4} V = 20 \angle -80^\circ$$

$$V(j4 + 5 - j) = 20 \angle -80^\circ$$

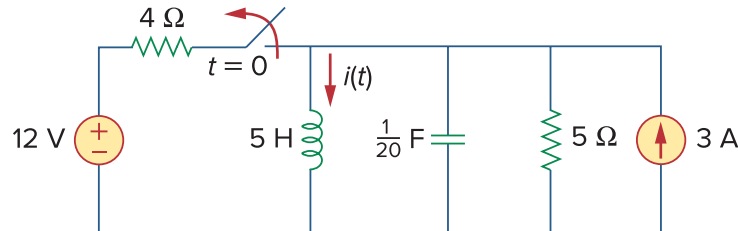
$$V = \frac{20 \angle -80^\circ}{j3 + 5}$$

$$V = 3.43 \angle -111^\circ \text{ Volt } \#$$

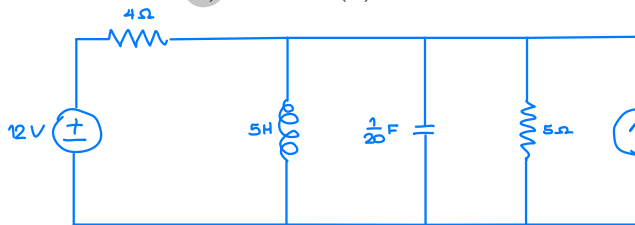
$$= 3.43 \cos(4t - 111^\circ) \text{ Volt } \#$$

Problem 3

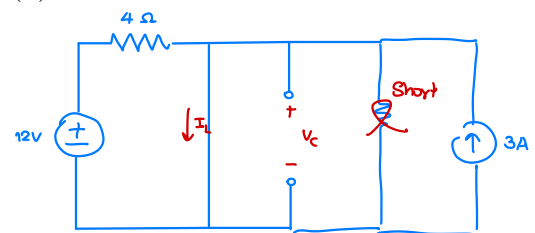
For the following circuit, the switch opens at $t = 0$:



a) Find $I_L(0) = \underline{6 \text{ A} \#}$ and $V_c(0) = \underline{0 \text{ Volt} \#}$



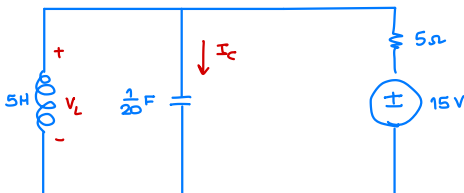
under
DC



$$\therefore I_L = \frac{12}{4} + 3 = 6 \text{ A}$$

b) Find $\frac{dI_L(0)}{dt} = \underline{6 \text{ A/s} \#}$ and $\frac{dV_c(0)}{dt} = \underline{0 \text{ V/s} \#}$

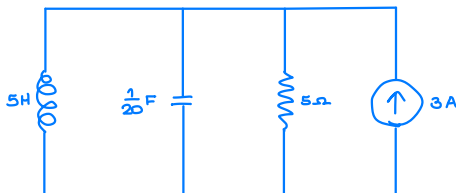
$V = \mathcal{E}$



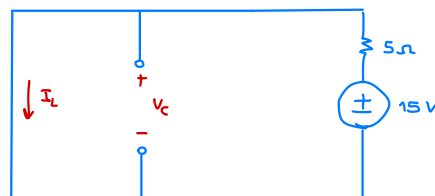
$$\begin{aligned} \frac{dI_L}{dt} &= \frac{V_L}{L} \\ &= \frac{20}{5} \\ &= 6 \text{ A/s} \end{aligned}$$

$$\begin{aligned} \frac{dV_c}{dt} &= \frac{I_c}{C} \\ &= \frac{0}{\frac{1}{20}} \\ &= 0 \text{ V/s} \end{aligned}$$

c) Find $I_L(\infty) = \underline{3 \text{ A} \#}$ and $V_c(\infty) = \underline{0 \text{ Volt} \#}$



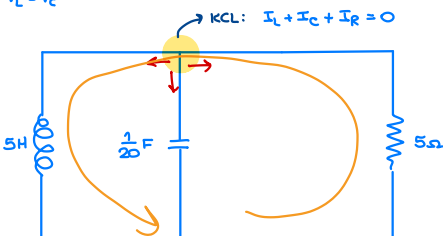
under
DC



$$I(\infty) = \frac{15}{5} = 3 \text{ A}$$

d) Find $I_{L_n}(t) = \underline{(A_1 + A_2 t)e^{-2t} \#}$ in term of two unknown constants.

Note $V_L = V_c$



$$\begin{aligned} V_c &= V_L \\ V_c &= 5 \frac{dI_L}{dt} \end{aligned}$$

$$I_L + I_c + I_R = 0$$

$$-\frac{V_c}{5} - \frac{1}{20} \frac{dV_c}{dt} = I_L$$

$$V_c = 5 \frac{d}{dt} \left(-\frac{V_c}{5} - \frac{1}{20} \frac{dV_c}{dt} \right)$$

$$V_c = -\frac{5}{5} \frac{dV_c}{dt} - \frac{5}{20} \frac{d^2 V_c}{dt^2}$$

$$\frac{1}{4} \frac{d^2 V_c}{dt^2} + \frac{dV_c}{dt} + V_c = 0$$

$$\frac{d^2 V_c}{dt^2} + 4 \frac{dV_c}{dt} + 4V_c = 0$$

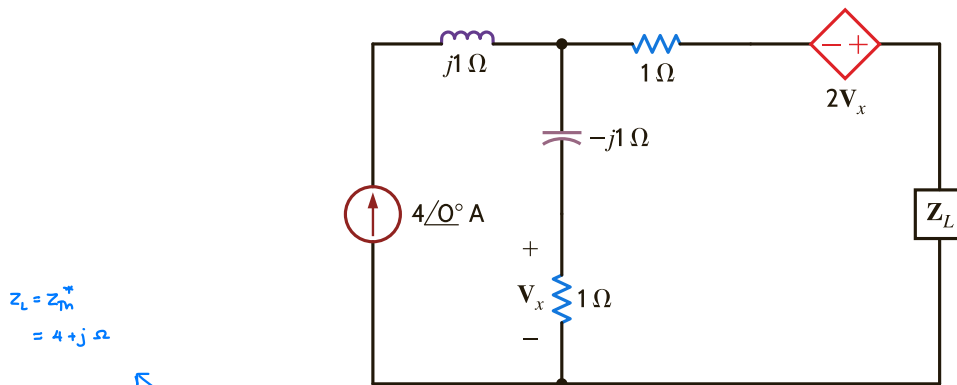
$$\text{Compare with } s^2 + 2\alpha s + \omega_0^2 = 0$$

$$x^2 + 4x + 4$$

$$(x+2)(x+2) = 0$$

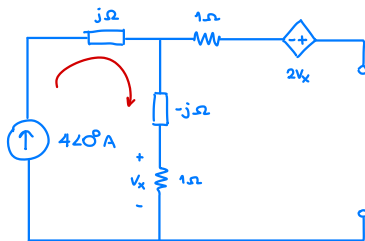
$$\therefore x = -2 \quad \text{: Critically damped}$$

Problem 4



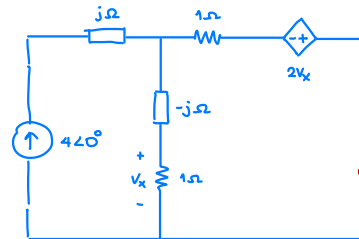
$$Z_L = Z_{in}^* \\ = 4 + j \Omega$$

Find the load impedance $Z_L = \underline{4 + j \Omega}$ # for maximum average power transfer



$$\begin{aligned} \therefore V_{th} &= 2V_x + I(1-j) \\ &= 2(4) + 4(1-j) \\ &= 12.65 \angle -18.43^\circ \end{aligned}$$

$$\therefore Z_{th} = \frac{V_{th}}{I} = \frac{12.65 \angle -18.43^\circ}{3.06 \angle -4.4^\circ} = 4 - j \Omega$$



$$\begin{aligned} I_1(1-j) &= I_{sc}(1) - 2V_x \\ I_1(1-j) &= I_{sc} - 2(I_1(1)) \\ I_1(1-j+2) &= I_{sc} \\ (4 - I_{sc})(3-j) &= I_{sc} \\ 12 - 4j - 3I_{sc} + jI_{sc} &= I_{sc} \\ (-3+j-1)I_{sc} &= -12+j4 \\ I_{sc} &= \frac{-12+j4}{j-4} \\ &= 3.06 \angle -4.4^\circ \text{ A} \end{aligned}$$

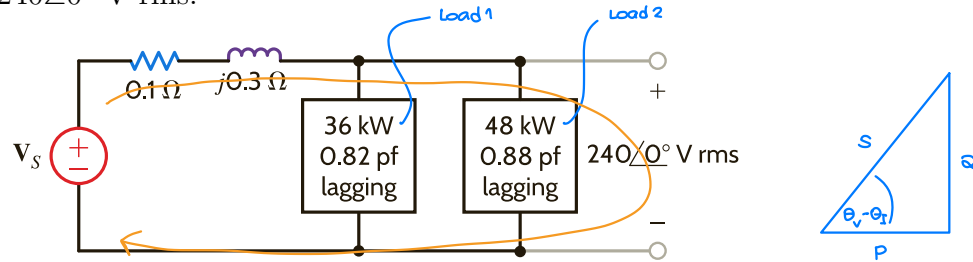
Formula for finding the maximum average power is $P_{max} = \frac{|V_{th}|^2}{8R_{th}}$

The value of the maximum average power transferred to Z_L is $= \underline{5 \text{ W}}$ #

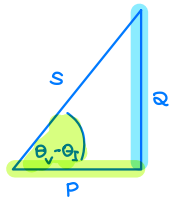
$$\begin{aligned} P_{max} &= \frac{|V_{th}|^2}{8R_{th}} \\ &= \frac{12.65^2}{8(4)} \\ &\approx 5 \text{ W} \end{aligned}$$

Problem 5

A source supplies power through a line with impedance $0.1 + j0.3 \Omega$ to two parallel loads. The first load absorbs 36 kW at 0.82 power factor lagging, and the second load absorbs 48 kW at 0.88 power factor lagging. The load voltage is $240\angle 0^\circ$ V rms.



Find the total complex power at the load S_L , the total current I_L , and the equivalent load impedance Z_L .



Load 1:

$$\begin{aligned} P_1 &= 36 \text{ kW} \\ \text{pf} &= 0.82 \text{ (lagging)} \rightarrow \text{positive angle} \\ \theta &= \cos^{-1}(0.82) = 34.91^\circ \\ \tan \theta &= \frac{Q}{P} \rightarrow Q_1 = P \tan \theta \\ &= 36 \tan(34.91^\circ) \\ &= 25.12 \end{aligned}$$

$$S_1 = P + jQ = 36 + j25.12$$

Load 2:

$$\begin{aligned} P_2 &= 48 \text{ kW} \\ \text{pf} &= 0.88 \text{ (lagging)} \rightarrow \text{positive angle} \\ \theta &= \cos^{-1}(0.88) = 28.35^\circ \\ \tan \theta &= \frac{Q}{P} \rightarrow Q_2 = P \tan \theta \\ &= 48 \tan(28.35^\circ) \\ &= 25.9 \end{aligned}$$

$$S_2 = P + jQ = 48 + j25.9$$

$$\begin{aligned} \therefore S_L &= S_1 + S_2 = 36 + j25.12 + 48 + j25.9 \\ &= 84 + j51.02 \text{ #} \end{aligned}$$

Total current (load)

$$\begin{aligned} S &= V_{\text{rms}} I_{\text{rms}}^* \\ I_{\text{rms}}^* &= \frac{S}{V} = \frac{84 + j51.02}{240\angle 0^\circ} = 0.35 + j0.2125 \\ I_{\text{rms}} &= 0.35 - j0.2125 \text{ kA #} \end{aligned}$$

Equivalent load impedance (Z_L)

$$Z_L = \frac{V_L}{I_L} = \frac{240\angle 0^\circ}{0.35 - j0.2125} = 501 + j304.2 \Omega \text{ #}$$

Determine the complex power at the source S_S .

$$\begin{aligned} S &= V_S I_{\text{rms}}^* \\ &= (338.75 + j83.75)(0.35 + j0.2125) = 100.76 + j101.3 \text{ kVA #} \end{aligned}$$

Indicate the source voltage V_S and the generator power factor.

↖ Kilo, don't forget

$$V_S = (1000)(0.35 - j0.2125)(0.1 + j0.3) + 240\angle 0^\circ = 338.75 + j83.75 \text{ V}_{\text{rms}} \text{ #}$$

$$\text{pf} = \frac{P}{|S|} = \frac{100.76}{\sqrt{100.76^2 + 101.3^2}} = 0.705 \text{ (lagging) #}$$

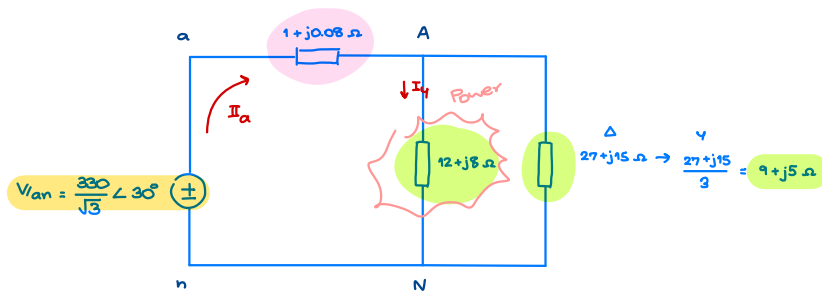
Problem 6

ក៏ទាញយកចំនួន abc
ដំបូងបំផុត៖ ៥៖៦១!

A balanced three-phase delta-connected source supplies power to a load consisting of a balanced delta in parallel with a balanced wye. The phase impedance of the delta is $27 + j15 \Omega$, and the phase impedance of the wye is $12 + j8 \Omega$. The bca -phase-sequence source voltage is $\mathbf{V}_{ab} = 330\angle 60^\circ \text{ V}_{\text{rms}}$, and the line impedance per phase is $1 + j0.08 \Omega$. Find the line currents and the power absorbed by the wye-connected load.

แปลงซ้ำ

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle -30^\circ$$
$$V_{an} = \frac{330}{\sqrt{3}} \angle 30^\circ \text{ V}$$



$$I_a = \frac{\frac{330}{\sqrt{3}} \angle 30^\circ}{1 + j0.08 + (12 + j8) \parallel (9 + j5)} = 27.51 \angle 2.71^\circ \text{ A}_{rms} \#$$

$$I_b = 27.51 \angle -117.29^\circ \text{ A}_{rms} \quad \#$$

$$I_C = 27.51 \angle 122.71^\circ \text{ A}_{rms} \#$$

Power @ 4 load

Current divider

$$P = 3 \cdot |I_y|^2 \cdot 12$$

$$\frac{27 + j15}{27 + j15 + 12 + j8} (27.51 \angle 2.71^\circ) = 18.766 \angle 1.234^\circ \text{ A}$$

$P \approx 12,677$ W #