# EES216: Circuit Analysis
# Final Mock Exam

curated by The Peanuts

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**Conditions:** Semi-Closed Book (A3 Both Sides)

## Directions:

1. This exam has 10 pages (including this page).
2. Calculators (Casio 991 Series) are allowed.
3. Write your name clearly at the top of each page.
4. Please check to make sure that there are 6 problems in your exam paper. (It's a PDF, I know)
5. Red color is reserved for grading. Do not write in red.
6. Do not cheat. Do not panic.
7. Good Luck + Warm Wish for a bright and joyful coming new year.

~~*For solution, click here.*~~

## Problem 1

The voltage $v(t)$ in a network is defined by the equation.

$$\frac{d^2v_1(t)}{dt^2} + 8\frac{dv_1(t)}{dt} + 10v_1(t) = 0.$$

Identify the type of this second-order system: Overdamped #

Compared with $s^2 + 2\alpha s + \omega_o^2$

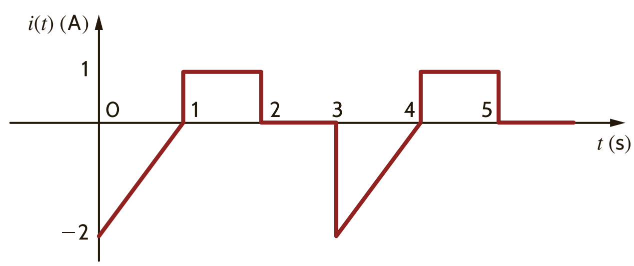
$s^2 + 8s + 10$

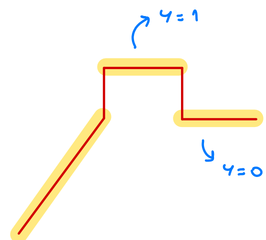
$2\alpha = 8 \qquad \omega_o^2 = 10$

$\alpha = 4 \qquad \omega_o = \sqrt{10}$

$\therefore \alpha > \omega_o$ : Overdamped

Calculate the rms value of the waveform




$y = mx + c$

$m = \frac{\Delta y}{\Delta x}$

$= \frac{0-(-2)}{1-0}$

$m = 2$

$y = 2x + c$

$0 = 2(1) + c$

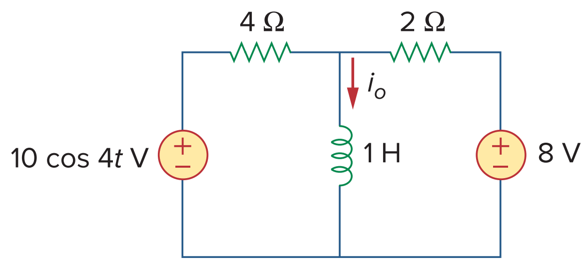
$\therefore c = -2$

$\therefore y = 2x - 2$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

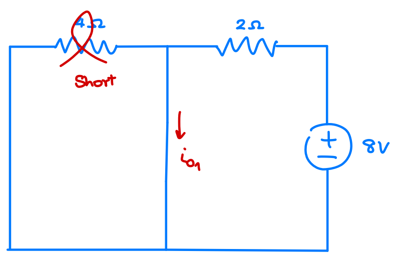
$$\therefore I_{eff} = \left[\frac{1}{3}\left(\int_0^1 (2x-2)^2 + \int_1^2 1^2 dt + \int_2^3 0^2 dt\right)\right]^{\frac{1}{2}} \text{ A}_{rms} \ \#$$

Solve for $i_o(t)$ in the circuit using the superposition principle.



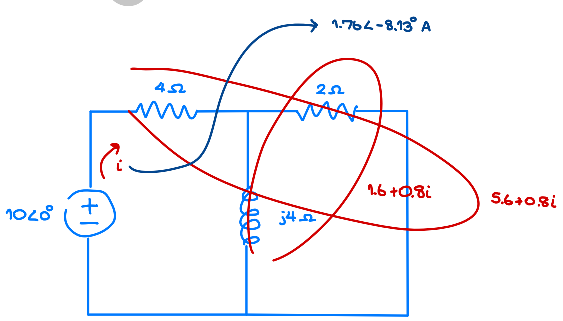

a) Find $i_{o_1}$ due to 8 V voltage source. $i_{o_1}$ = 4 A #

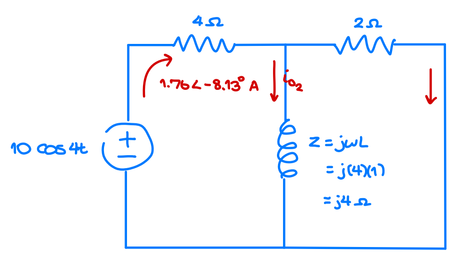
DC




$\therefore i_{o_1} = \frac{8}{2} = 4\ A$

b) Find $i_{o_2}$ due to $10 \cos 4t$ V source. $i_{o_2}$ = $0.79 \angle -71.5°$ A #




$\therefore i_{o_2} = \frac{2}{2+j4}(1.76\angle -8.13°)$

$= 0.79 \angle -71.5°$ A

c) Find $i_o$ = $(0.79 \angle -71.5°) + 4$ A #

In Thailand, the electricity system operates at what standard voltage and frequency?

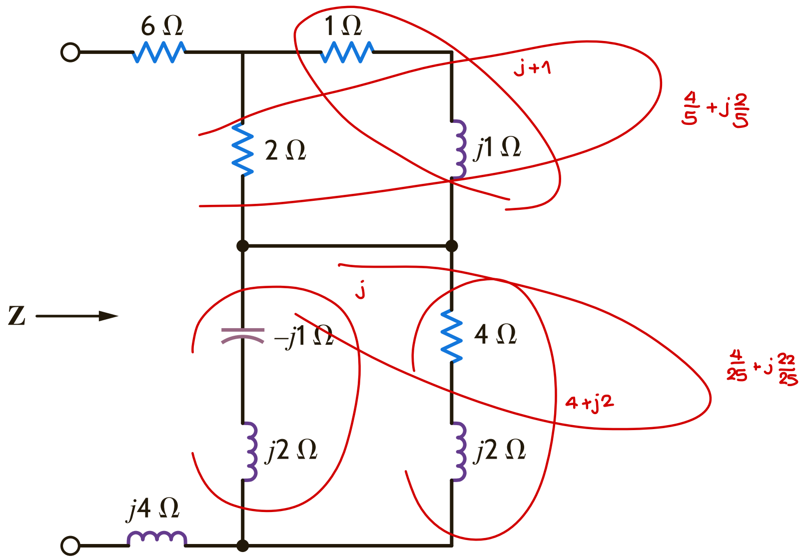
Voltage = 220 volts, Frequency = 50 Hz

State **three** benefits of a three-phase electrical system

1. Nearly all electric power is generated and distributed using three-phase system.
2. It can provide constant (non-pulsating) to load.
3. For the same amount of power, the three-phase system is more economical than single phase.

# Problem 2

Find the equivalent impedance, **Z**, for the circuit.




$\therefore Z_T = 6 + \frac{4}{5} + j\frac{2}{5} + \frac{4}{25} + j\frac{22}{25} + j4$

$= 6.96 + j5.28\ \Omega$ #

→ วิธีการทำเรื่องนี้

$\frac{dv}{dt} \Leftrightarrow j\omega \mathbb{V}$

$\int v\,dt \Leftrightarrow \frac{\mathbb{V}}{j\omega}$

แปลงซะ แล้วจัดรูปให้เหลือ V ตัวเดียว

หรือจะเปลี่ยนเป็นเรื่องกระแส I ก็ได้

$\frac{di}{dt} \Leftrightarrow j\omega \mathbb{I}$

$\int i\,dt \Leftrightarrow \frac{\mathbb{I}}{j\omega}$

เหมือนกันเลย 555

Find $v(t)$ in the following integrodifferential equations using the phasor approach:

$$\frac{dv}{dt} + 5v(t) + 4\int v\,dt = 20\sin(\overset{\omega = 4 \text{ rad/s}}{4}t + 10^\circ)$$

$$j\omega\mathbb{V} + 5\mathbb{V} + 4\frac{\mathbb{V}}{j\omega} = 20\sin(4t + 10^\circ)$$

$$j(4)\mathbb{V} + 5\mathbb{V} + \frac{\cancel{4}}{j\cancel{4}}\mathbb{V} = 20\angle -80^\circ$$

$$\mathbb{V}(j4 + 5 - j) = 20\angle -80^\circ$$

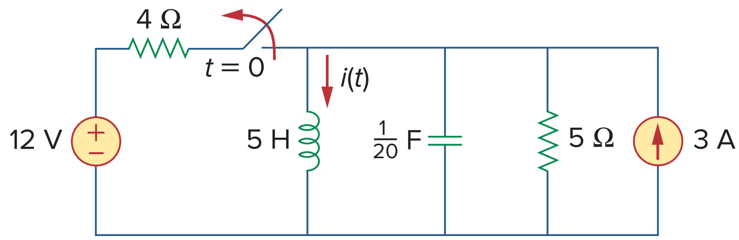
$$\mathbb{V} = \frac{20\angle -80^\circ}{j3 + 5}$$

$$\mathbb{V} = 3.43\angle -111^\circ \text{ Volt } \#$$

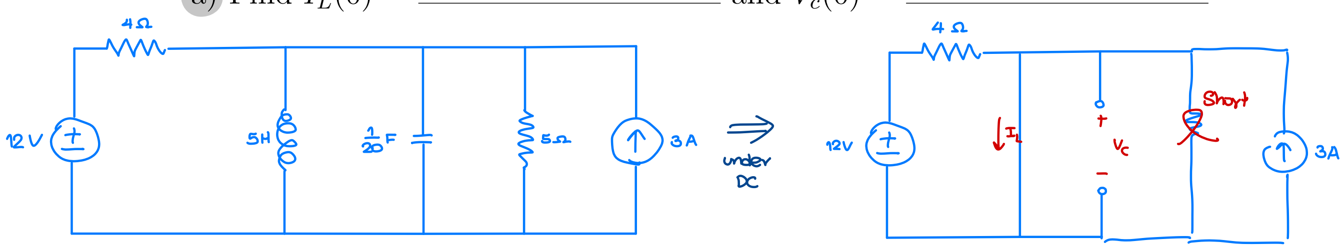
$$= 3.43\cos(t - 111^\circ) \text{ Volt } \#$$

# Problem 3

For the following circuit, the switch opens at $t = 0$:




a) Find $I_L(0) =$ $6\ A$ # and $V_c(0) =$ $0$ Volt #

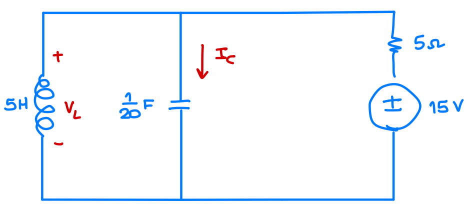



$\therefore I_L = \frac{12}{4} + 3$

$= 6\ A$

b) Find $\frac{dI_L(0)}{dt} =$ $6\ A/s$ # and $\frac{dV_c(0)}{dt} =$ $0\ V/s$ #

$V = IR$




$\frac{dI_L}{dt} = \frac{V_L}{L}$

$= \frac{30}{5}$

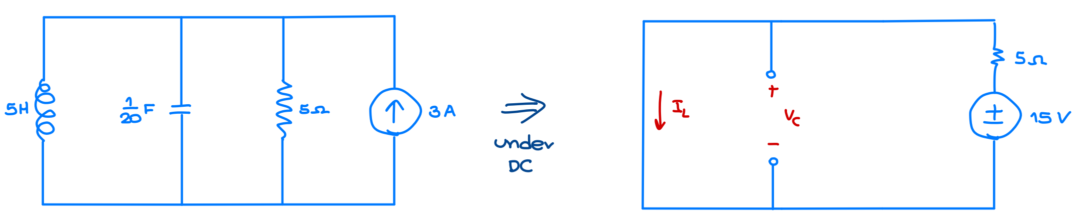
$= 6\ A/s$

$\frac{dV_c}{dt} = \frac{I_c}{C}$

$= \frac{0}{\frac{1}{20}}$

$= 0\ V/s$

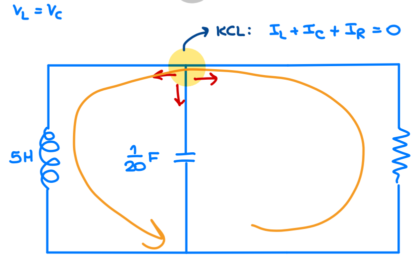
c) Find $I_L(\infty) =$ $3\ A$ # and $V_c(\infty) =$ $0$ Volt #




$I(\infty) = \frac{15}{5} = 3\ A$

d) Find $I_{L_n}(t) =$ $(A_1 + A_2 t)e^{-2t}$ # in term of two unknown constants.

Note $V_L = V_c$




$I_L + I_c + I_R = 0$

$-\frac{V_c}{5} - \frac{1}{20}\frac{dV_c}{dt} = I_L$

$V_c = V_L$

$V_c = 5\frac{dI_L}{dt}$

$V_c = 5\frac{d}{dt}\left(-\frac{V_c}{5} - \frac{1}{20}\frac{dV_c}{dt}\right)$

$V_c = -\frac{5}{5}\frac{dV_c}{dt} - \frac{5}{20}\frac{d^2V_c}{dt^2}$

$\frac{1}{4}\frac{d^2V_c}{dt^2} + \frac{dV_c}{dt} + V_c = 0$

$\frac{d^2V_c}{dt^2} + 4\frac{dV_c}{dt} + 4V_c = 0$

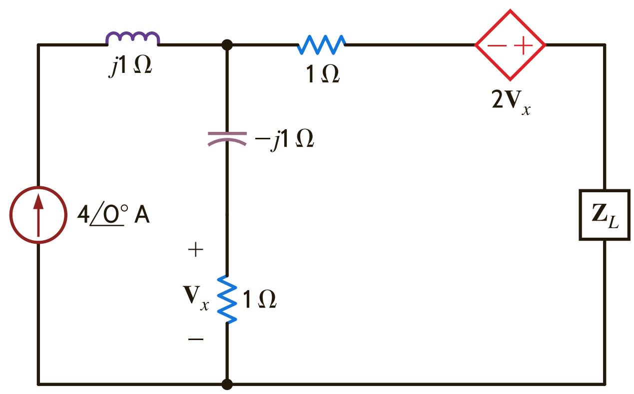
compare with $s^2 + 2\alpha s + \omega_0^2 = 0$

$x^2 + 4x + 4$

$(x+2)(x+2) = 0$

$\therefore x = -2$ : Critically damped

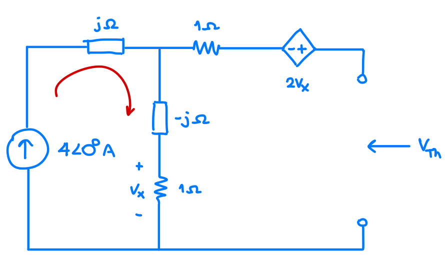
# Problem 4




$Z_L = Z_{Th}^*$

$= 4 + j\ \Omega$

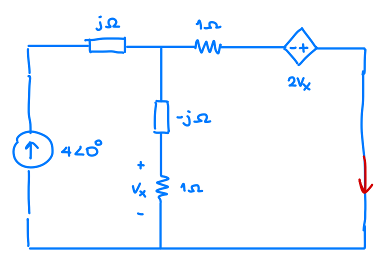
Find the load impedance $\mathbf{Z}_L =$ $4+j\ \Omega$ # for maximum average power transfer




$\therefore V_{Th} = 2V_x + I(1-j)$

$= 2(4) + 4(1-j)$

$= 12.65\angle -18.43^\circ$




$I_1(1-j) = I_{sc}(1) - 2V_x$

$I_1(1-j) = I_{sc} - 2(I_1(1))$

$I_1(1-j+2) = I_{sc}$

$(4 - I_{sc})(3-j) = I_{sc}$

$12 - 4j - 3I_{sc} + jI_{sc} = I_{sc}$

$(-3+j-1)I_{sc} = -12+j4$

$I_{sc} = \frac{-12+j4}{j-4}$

$= 3.06\angle -4.4^\circ$ A

$\therefore Z_{Th} = \frac{V_{Th}}{I} = \frac{12.65\angle -18.43^\circ}{3.06\angle -4.4^\circ} = 4 - j\ \Omega$

Formula for finding the maximum average power is $P_{max} = \frac{|V_{th}|^2}{8R_{th}}$

The value of the maximum average power transferred to $\mathbf{Z}_L$ is $=$ 5 W #

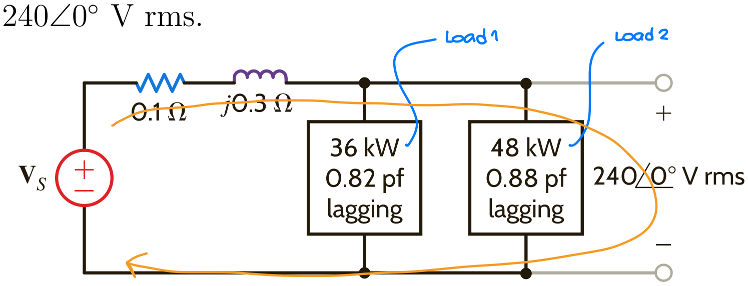
$P_{max} = \frac{|V_{th}|^2}{8R_{th}}$

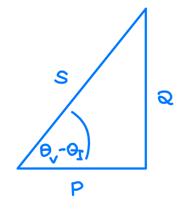
$= \frac{12.65^2}{8(4)}$

$\approx 5$ W

# Problem 5

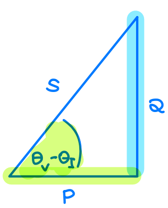
A source supplies power through a line with impedance $0.1 + j0.3\ \Omega$ to two parallel loads. The first load absorbs 36 kW at 0.82 power factor lagging, and the second load absorbs 48 kW at 0.88 power factor lagging. The load voltage is $240\angle 0°$ V rms.




Find the total complex power at the load $\mathbf{S}_L$, the total current $\mathbf{I}_L$, and the equivalent load impedance $\mathbf{Z}_L$.

$\therefore S_L = S_1 + S_2 = 36 + j25.12 + 48 + j25.9$
$= 84 + j51.02$ #




Load 1:

$P_1 = 36$ kW

pf $= 0.82$ (lagging) → positive angle

$\theta = \cos^{-1}(0.82) = 34.91°$

$\tan\theta = \frac{Q}{P} \rightarrow Q_1 = P\tan\theta$
$= 36\tan(34.91°)$
$= 25.12$

$S_1 = P + jQ = 36 + j25.12$

Load 2:

$P_2 = 48$ kW

pf $= 0.88$ (lagging) → positive angle

$\theta = \cos^{-1}(0.88) = 28.35°$

$\tan\theta = \frac{Q}{P} \rightarrow Q_2 = P\tan\theta$
$= 48\tan(28.35°)$
$= 25.9$

$S_2 = P + jQ = 48 + j25.9$

Total current (load)

$S = V_{rms} I^*_{rms}$

$I^*_{rms} = \frac{S}{V} = \frac{84 + j51.02}{240\angle 0°} = 0.35 + j0.2125$

$I_{rms} = 0.35 - j0.2125$ kA #

Equivalent load impedance ($Z_L$)

$Z_L = \frac{V_L}{I_L} = \frac{240\angle 0°}{0.35 - j0.2125} = 501 + j304.2\ \Omega$ #

Determine the complex power at the source $\mathbf{S}_S$.

$S = V_S I^*_{rms}$

$= (338.75 + j83.75)(0.35 + j0.2125) = 100.76 + j101.3$ kVA #

Indicate the source voltage $\mathbf{V}_S$ and the generator power factor.

Kilo, don't forget

$V_S = (1000)(0.35 - j0.2125)(0.1 + j0.3) + 240\angle 0° = 338.75 + j83.75\ V_{rms}$ #

$pf = \frac{P}{|S|} = \frac{100.76}{\sqrt{100.76^2 + 101.3^2}} = 0.705$ (lagging) #

# Problem 6

ก็เหมือนกับ abc นั่นแหละ ระวัง!

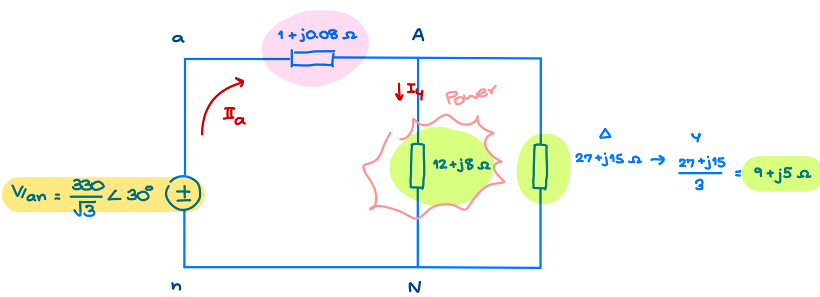
A balanced three-phase delta-connected source supplies power to a load consisting of a balanced delta in parallel with a balanced wye. The phase impedance of the delta is $27 + j15\ \Omega$, and the phase impedance of the wye is $12 + j8\ \Omega$. The *bca*-phase-sequence source voltage is $\mathbf{V}_{ab} = 330\angle 60^\circ\ V_{\text{rms}}$, and the line impedance per phase is $1 + j0.08\ \Omega$. **Find the line currents and the power absorbed by the wye-connected load.**

$\to I_a, I_b, I_c$

แปลงร่าง

$V_{an} = \frac{V_{ab}}{\sqrt{3}}\angle -30^\circ$

$V_{an} = \frac{330}{\sqrt{3}}\angle 30^\circ\ V$




$\Delta$: $27 + j15\ \Omega \to$ Y: $\frac{27 + j15}{3} = 9 + j5\ \Omega$

$$I_a = \frac{\frac{330}{\sqrt{3}}\angle 30^\circ}{1 + j0.08 + (12 + j8) \,//\, (9 + j5)} = 27.51\angle 2.71^\circ\ A_{rms}\ \#$$

$$I_b = 27.51\angle -117.29^\circ\ A_{rms}\ \#$$

$$I_c = 27.51\angle 122.71^\circ\ A_{rms}\ \#$$

Power @Y load

$P = 3 \cdot |I_Y|^2 \cdot 12$

Current divider

$$\frac{27 + j15}{27 + j15 + 12 + j8}(27.51\angle 2.71^\circ) = 18.766\angle 1.234^\circ\ A$$

$P \approx 12{,}677\ W\ \#$