EES216: Circuit Analysis
Quiz 2 — Mock Exam

curated by The Peanuts

Should be number, haha!

Name Nonprawich I. ID 6622772422 Section............. ~~Seat No~~.............

Conditions: Semi-Closed Book

Directions:

1. This exam has 6 pages (including this page).
2. Calculators (Casio 991 Series) are allowed.
3. Write your name clearly at the top of each page.
4. Reading the problem is optional but highly recommended.
5. You may bring one A3 sheet of note, which will magically become illegible the moment the exam begins.
6. Tears shed on your answer sheet may cause short circuits. Please cry responsibly.

For solution, click here.

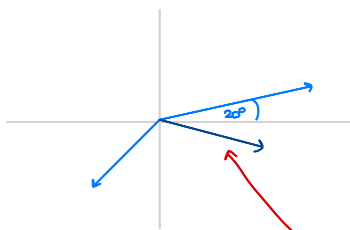
Problem 1.1

If $v(t) = 8\cos(4t + 20°) + 5\sin(4t - 45°) = A\cos(4t + \theta)$. Find A and θ by using phasor diagram:

$= 5\sin(4t - 45 + 135 - 135)$

$= 5\cos(4t - 135°)$

$A =$ 4.06 #, $\theta =$ -11.356 #



$= 8\angle 20° + 5\angle -135°$

$= 7.518 + 2.736j + (-3.535 - 3.535j)$

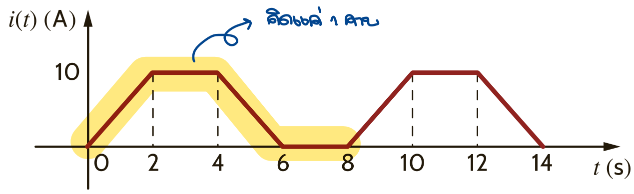
$= (7.518 - 3.535) + (2.736 - 3.535)j$

$= 3.983 - 0.8j$

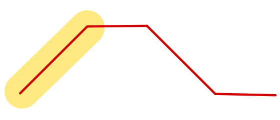
$= 4.06\angle -11.356°$

Problem 1.2

Calculate the rms value of the waveform shown below.



$y = mx + c$



$m = \frac{\Delta y}{\Delta x}$

$= \frac{10-0}{2-0}$

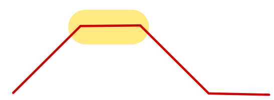
$m = 5$

$y = 5x + c$

$0 = 5 \cdot 0 + c$

$c = 0$

$\therefore\ i(t) = 5t$



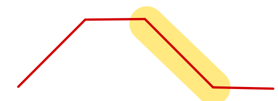
$m = \frac{\Delta y}{\Delta x}$

$= \frac{10-10}{4-2}$

$m = 0$

$y = 0$

$\therefore\ i(t) = 10\ A$



$m = \frac{\Delta y}{\Delta x}$

$= \frac{0-10}{6-4}$

$m = -5$

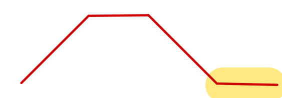
$y = -5x + c$

$10 = -5(4) + c$

$10 = -20 + c$

$c = 30$

$\therefore\ i(t) = -5t + 30$



$m = \frac{\Delta y}{\Delta x}$

$= \frac{0-0}{8-6}$

$m = 0$

$y = 0$

$\therefore\ i(t) = 0\ A$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \left(\frac{1}{8}\left[\int_0^2 (5t)^2 dt + \int_2^4 (10)^2 dt + \int_4^6 (30-5t)^2 dt + \int_6^8 0^2 dt\right]\right)^{\frac{1}{2}} \text{ A} \quad \# \quad = 6.46\text{ A}$$

Problem 1.3

If that current (from 1.2) flows through a $10\,\Omega$ resistor, find the average power absorbed by the resistor.

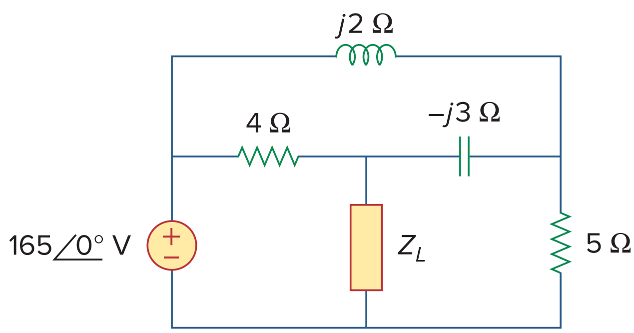
$$P_{avg} = I_{rms}^2 \cdot R$$

$$P_{avg} = \left(\frac{1}{8}\left[\int_0^2 (5t)^2 dt + \int_2^4 (10)^2 dt + \int_4^6 (30-5t)^2 dt + \int_6^8 0^2 dt\right]\right)(10) \text{ W} \quad \#$$

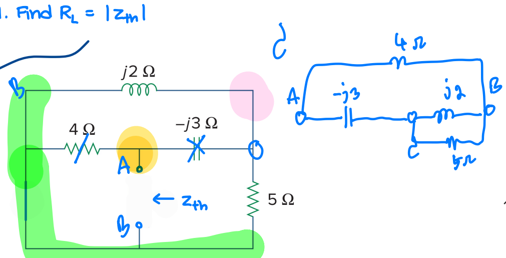
$$= 417.316\text{ W}$$

Problem 1.4

For the circuit shown below, determine the load impedance $\mathbf{Z_L}$ for maximum power transfer (to $\mathbf{Z_L}$). Calculate the maximum power absorbed by the load ($P_{\max}$).



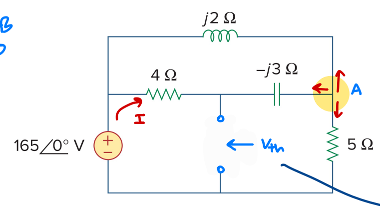
1. Find $R_L = |Z_{th}|$



$$Z_{th} = \frac{\left(-3j + \frac{(5)(2j)}{5+2j}\right)(4)}{\left(-3j + \frac{(5)(2j)}{5+2j}\right) + 4}$$

$$= 0.8234 - 0.8642j \ \Omega$$

2. V_{th}



$$\frac{V_A - 165}{4-3j} + \frac{V_A - 165}{2j} + \frac{V_A}{5} = 0$$

$$V_A = 129.9\angle -20.618° \ V$$

$$I = \frac{165 - V_A}{4-3j}$$

$$= \frac{165 - (129.9\angle -20.618°)}{4-3j}$$

$$I = 1.458 + 12.52j \ A$$

KVL:

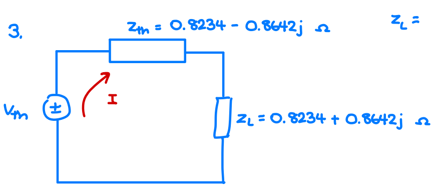
$$-165 + 4I + V_{th} = 0$$

$$V_{th} = 165 - 4I$$

$$= 165 - 4(1.458 + 12.52j)$$

$$= 166.86\angle -17.47°$$

3.



$$Z_L = Z_{th}^*$$

$$P_{max} = \frac{|V_{th}|^2}{8R_{th}}$$

$$= \frac{(166.86)^2}{8(0.8234)}$$

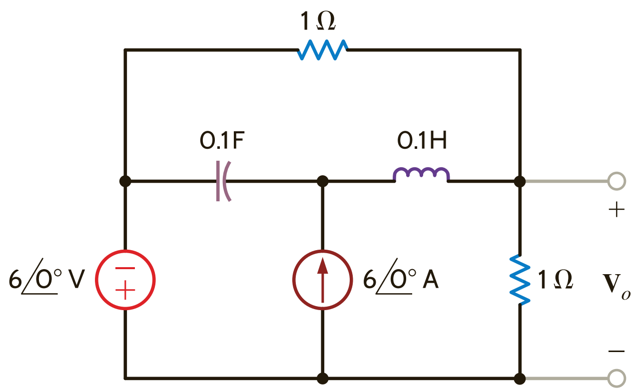
$$= 4226.72 \ W \ \#$$

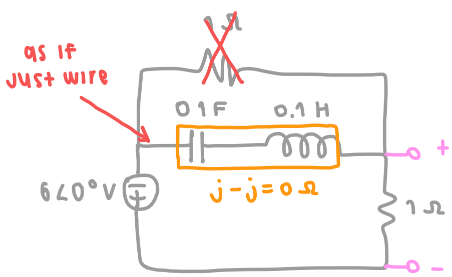
$\omega = 2\pi f$

$\omega = 2\pi\left(\frac{5}{\pi}\right) = 10 \text{ rad/s}$

Problem 2

Use **superposition** to determine $\mathbf{V}_o$ in the circuit. The frequency is $\frac{5}{\pi}$ Hz.



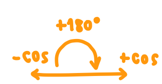


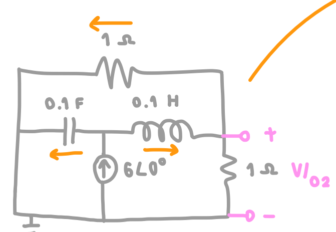
$Z_{C1} = \frac{1}{j(10)(0.1)} = -j\ \Omega$

$Z_{L1} = j(10)(0.1) = j\ \Omega$

$V_{01} = -6\angle 0°\ V$ make positive

$= 6\angle 180°\ V$





$6\angle 0° = \frac{V_A}{-j} + \frac{V_A - V_{02}}{j}$

$6 = jV_A - jV_A + jV_{02}$

$V_{02} = -j6$

$= 6\angle -90°\ V$

$6\angle -90° + 6\angle -90° = I_L$

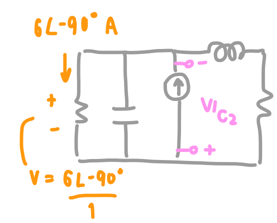
$I_L = 12\angle -90°$

$I_C = 6\angle 0° - 12\angle -90°$

$= 6 + j12$

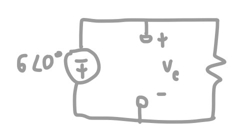
$I_{Ground} = 6\angle -90° + 6 + j12$

$= 6\sqrt{2}\angle 45°$



$V_0 = V_{01} + V_{02}$

$\approx 8.49\angle -135°\ V$



Calculate the average power absorbed by each of the circuit elements.

$P_{\text{Voltage Source}} =$ -36 W, $P_{\text{Current Source}} =$ -18 W,

$P_{\text{Capacitor (0.1F)}} =$ 0, $P_{\text{Inductor (0.1H)}} =$ 0,

$P_{\text{Resistor (1}\Omega\text{, top)}} =$ 18 W, $P_{\text{Resistor (1}\Omega\text{, right)}} =$ 36 W

$I_{Rt} = 0 + \frac{6\angle -90°}{1}$

$|I_{Rt}| = 6\ A$

$P_{RT} = \frac{1}{2}(6^2)(1)$

$= 18\ W$

$I_{Rr} = \frac{6\angle 180°}{1} + \frac{6\angle -90°}{1}$

$|I_{Rr}| = 8.49\ A$

$P_{Rr} = \frac{1}{2}(8.49)^2(1)$

$= 36\ W$

I of Voltage source

$6 + I_{V1} = 0$

$I_{V1} = -6\ A$

$I_{V2} = -6\sqrt{2}\angle 45°$

$I_V = 6\angle 90°$

$P_V = \frac{1}{2}(6)(6\sqrt{5})\cos(0 - -153.4)$

$= -36\ W$

$V_{C1} = -6\angle 180°$

$V_{C2} = -6\angle -90°$

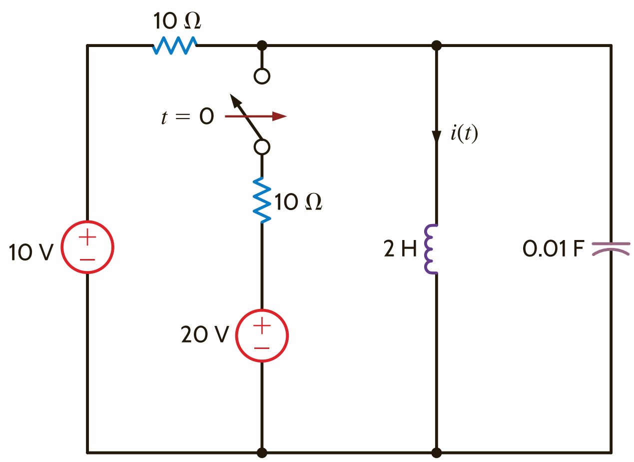
$V_{C2} = 8.49\angle 45°$

$P_C = \frac{1}{2}(-8.49)(6)\cos(0 - 45°)$

$= -18\ W$

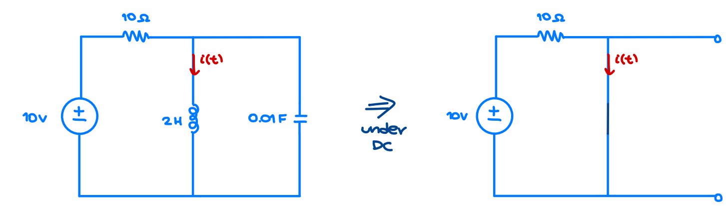
Problem 3

The switch in the circuit has been close for a long time and is opened at $t = 0$. For the following circuit, find $i(t)$ for $t > 0$.



Level 1

① $t<0$



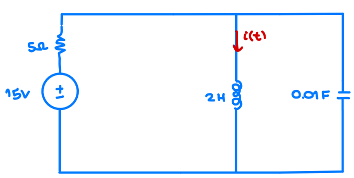
$i(0^-) = \frac{10}{10} = 1\text{ A}$

$v(0^-) = 0$ Volt

Initial and final values:

$i(0^+) = i(0^-) = 1\text{ A}$ #

② $t=0^+$



$I_c = C\frac{dv_c}{dt}$

$V_L = L\frac{di_L}{dt}$

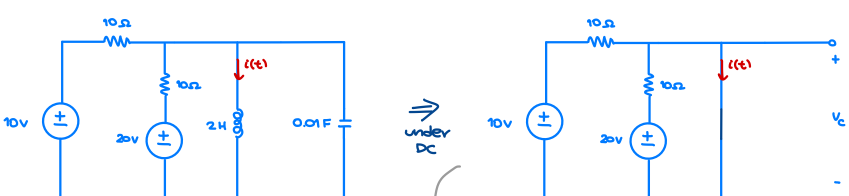
$\frac{dv_c(0^+)}{dt} = \frac{I_c}{C} = \frac{0}{0.01} = 0\text{ V/s}$

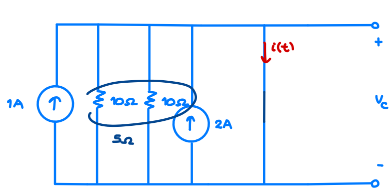
$\frac{di_L(0^+)}{dt} = \frac{V_L}{L} = \frac{0}{2} = 0\text{ A/s}$

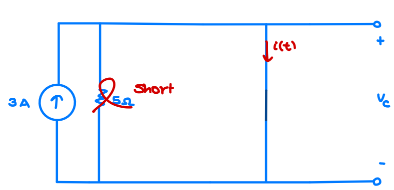
$\frac{dv_c(0^+)}{dt} = 0\text{ V/s}$ #

$\frac{di_L(0^+)}{dt} = 0\text{ A/s}$ #

③ $t>0$



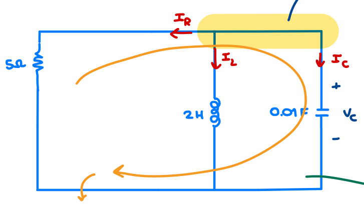




$i(\infty) = 3\text{ A}$

$i(\infty) = 3\text{ A}$ #

Level 2



KCL: $0 = I_L + I_C + I_R$ * Technique ทำให้อยู่ในรูป I_L, V_c ให้ได้

$0 = I_L + \frac{1}{100} \cdot \frac{dV_c}{dt} + \frac{V_c}{5}$

$V_L = V_c$

KVL: $5I_R + V_c = 0$

$\cancel{5}\left(\frac{V_c}{\cancel{5}}\right) + V_c = 0$

$2V_c = 0$

$V_c = 0$

from $V_L = L\frac{di_L}{dt}$, $I_C = C\frac{dV_c}{dt}$

$\frac{dV_c}{dt} = \frac{I_c}{C}$

$\frac{dV_L}{dt} = \frac{I_c}{C}$

$0 = I_L + \frac{1}{100} \cdot \frac{dV_c}{dt} + \frac{V_c}{5}$

$0 = I_L + \frac{1}{100} \cdot \frac{I_c}{C} + \frac{V_L}{5}$

$0 = I_L + \frac{1}{100} \cdot \frac{d}{dt}\left(2\frac{di_L}{dt}\right) + \frac{2}{5}\frac{di_L}{dt}$

$0 = I_L + \frac{1}{100} \cdot \frac{2d^2I_L}{dt} + \frac{2}{5}\frac{dI_L}{dt}$

$\frac{1}{50}\frac{d^2I_L}{dt} + \frac{2}{5}\frac{dI_L}{dt} + I_L = 0$ (Rearranging the term)

$\boxed{s^2 + 2\alpha s + \omega_0^2 = 0}$

$\frac{d^2I_L}{dt} + 20\frac{dI_L}{dt} + 50I_L = 0$

$\omega_0 = 7.07$

$\alpha = 10$

$\alpha > \omega_0$ ∴ $i(t) = A_1e^{-2.928t} + A_2e^{-17.07t}$

$S^2 + 20S + 50 = 0$

$S_1 = -10 + 5\sqrt{2}$

$S_2 = -10 - 5\sqrt{2}$

Level 3: Force response $I(\infty) = 3$ A

Level 4:

1 Complete Response = Natural Response + Transient Response

$I(t) = A_1e^{-2.928t} + A_2e^{-17.07t} + 3$

2 $\frac{dI_L}{dt} = -2.928A_1e^{-2.928t} - 17.07A_2e^{-17.07t}$

$\frac{dI_L(0)}{dt} = -2.928A_1 - 17.07A_2$

$0 = -2.928A_1 - 17.07A_2$

$I(0^+) = A_1e^{-2.928(0)} + A_2e^{-17.07(0)} + 3$

$1 = A_1 + A_2 + 3$

$-2 = A_1 + A_2$ — B

$A_1 = -2.414$

$A_2 = 0.414$

$I(t) = -2.414e^{-2.928t} + 0.414e^{-17.07t} + 3$ A #