

# EES216: Circuit Analysis

## Midterm Mock Exam

curated by The Peanuts

Name. Nonprawich I. ID. 6622772422 Section.....Seat No.....

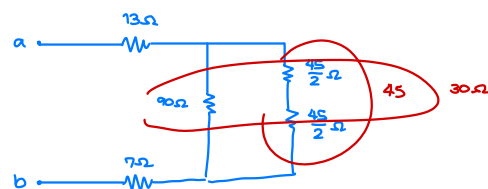
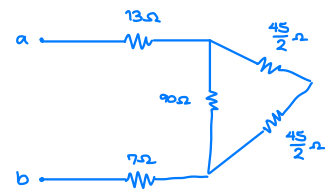
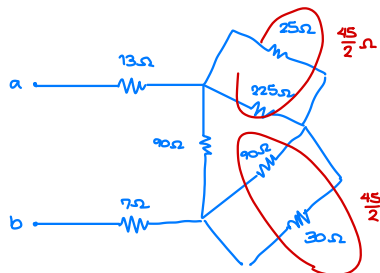
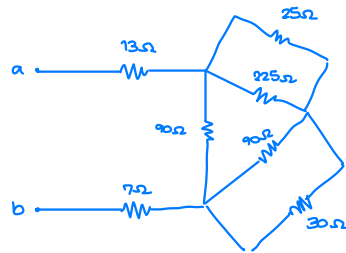
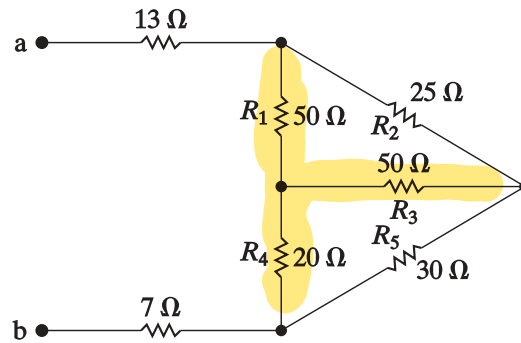
**Conditions:** Semi-Closed Book

**Directions:**

1. This exam has 15 pages (including this page).
2. Calculators are permitted (You may bring 100 of them. Haha)
3. Write your name clearly at the top of each page.
4. Reading the problem is optional but highly recommended.
5. You may bring one A3 sheet of note, which will magically become illegible the moment the exam begins.
6. Tears shed on your answer sheet may cause short circuits. Please cry responsibly.

## Problem 1

Find the equivalent resistance  $R_{ab}$  in the circuit.



$$\therefore R_{ab} = 13 + 30 + 7 = 50 \, \Omega \quad \#$$

## Problem 2

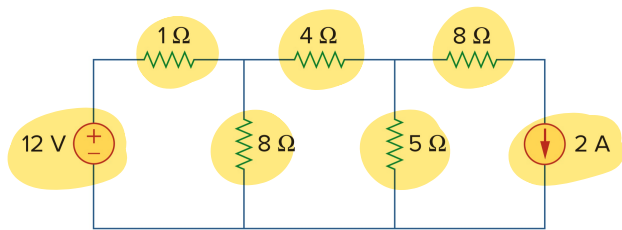
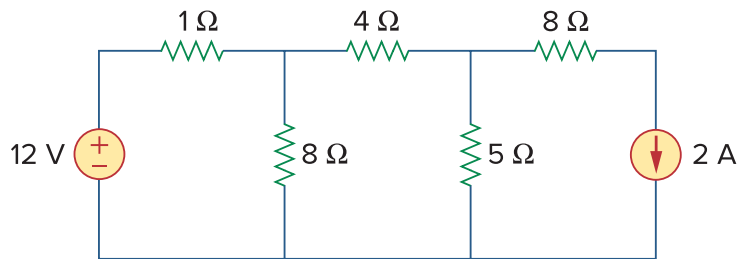
For an ideal operational amplifier (op-amp), state the values of the following characteristics and briefly explain their significance:

1. Open-loop voltage gain ( $A$ )  $\infty \rightarrow$  Voltage difference between output is zero.
2. Input resistance ( $R_i$ )  $\infty \rightarrow$  Current does NOT flow in the op-amp
3. Output resistance ( $R_o$ )  $0 \rightarrow$  Current to the load is 100%. (No drop)

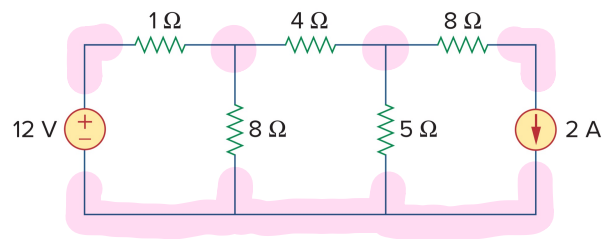
### Problem 3

Let  $a$  be the number of branches in the circuit,  $b$  be the number of nodes, and  $c$  be the number of meshes. Then, compute the value of:

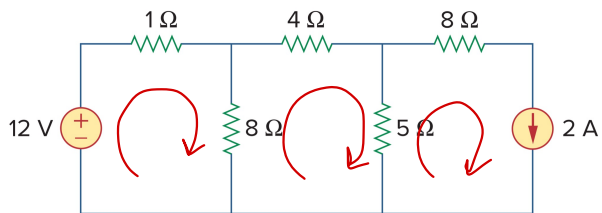
$$a^2 + b^2 - c$$



#Branches =  $a = 7$



#Nodes =  $b = 5$



#Mesh =  $c = 3$

$$\begin{aligned} \therefore a^2 + b^2 - c &= 7^2 + 5^2 - 3 \\ &= 49 + 25 - 3 \\ &= 71 \end{aligned}$$

## Problem 4

Take the last digit of your student ID and compute:

$$(\text{last digit}) \bmod 3$$

Based on the result:

- If the result is 0, find a limitation of **Mesh Analysis**.
- If the result is 1, find a limitation of **Node Analysis**.
- If the result is 2, find a limitation of **Superposition Theorem**.

### Limitation of Mesh Analysis

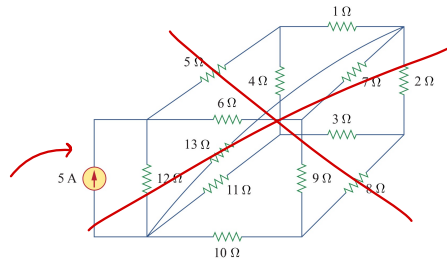
- Not applicable to non-planar circuits
- Becomes complex with many loops
- Difficult with current sources (require transformation) *also* or supermesh

### Limitation of Node Analysis

- Not ideal for voltage sources (require transformation)
- Inefficient for circuits with many nodes or supernode

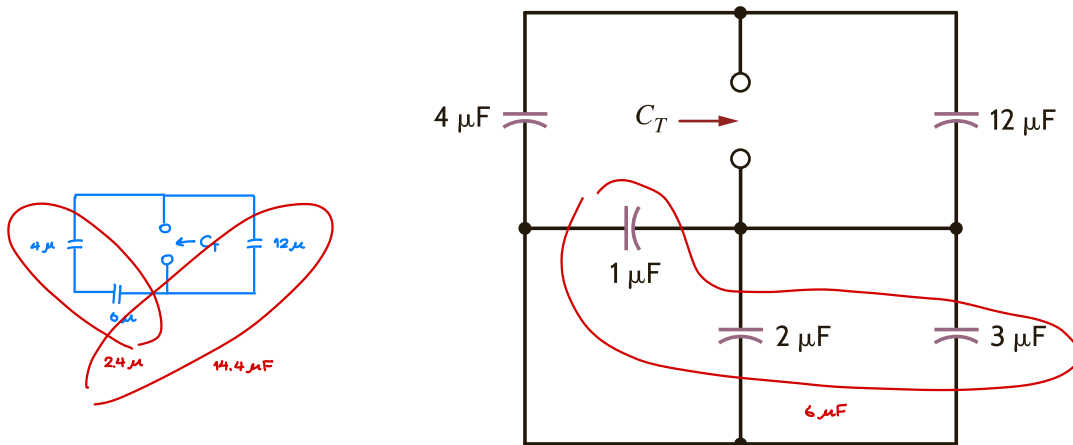
### Limitation of Superposition

- Only works for linear circuits
- Time-consuming for multiple sources
- Does not work for power calculations directly



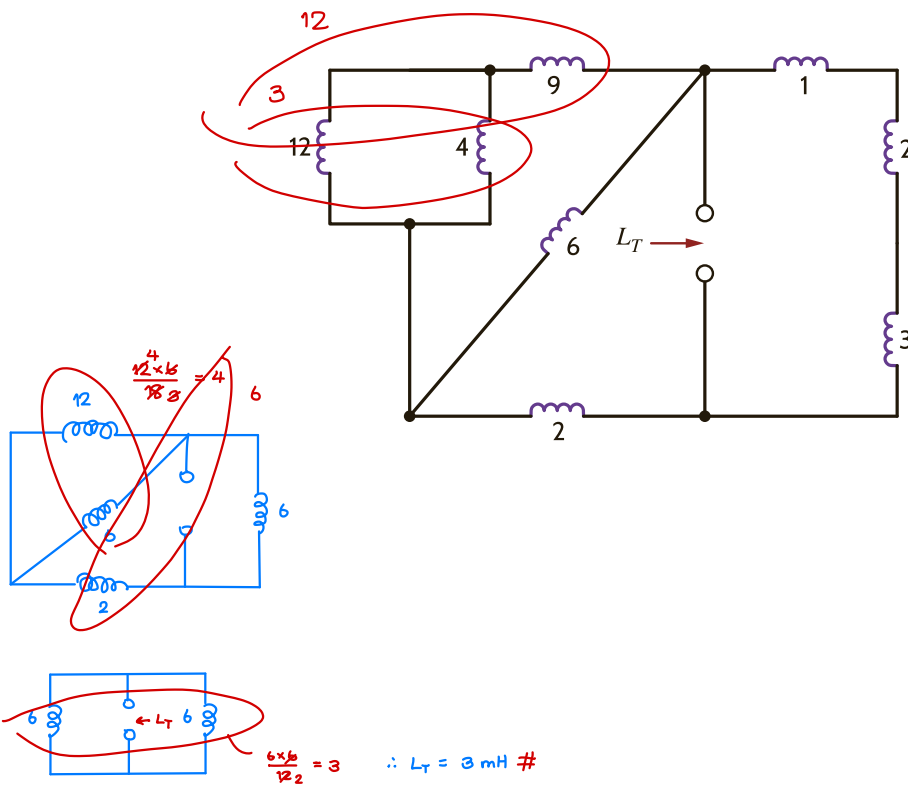
## Problem 5

Determine the value of  $C_T$  in the circuit



$$\therefore C_T = 14.4\ \mu\text{F} \#$$

Find the total inductance  $L_T$  in the circuit, All inductors are in millihenrys.



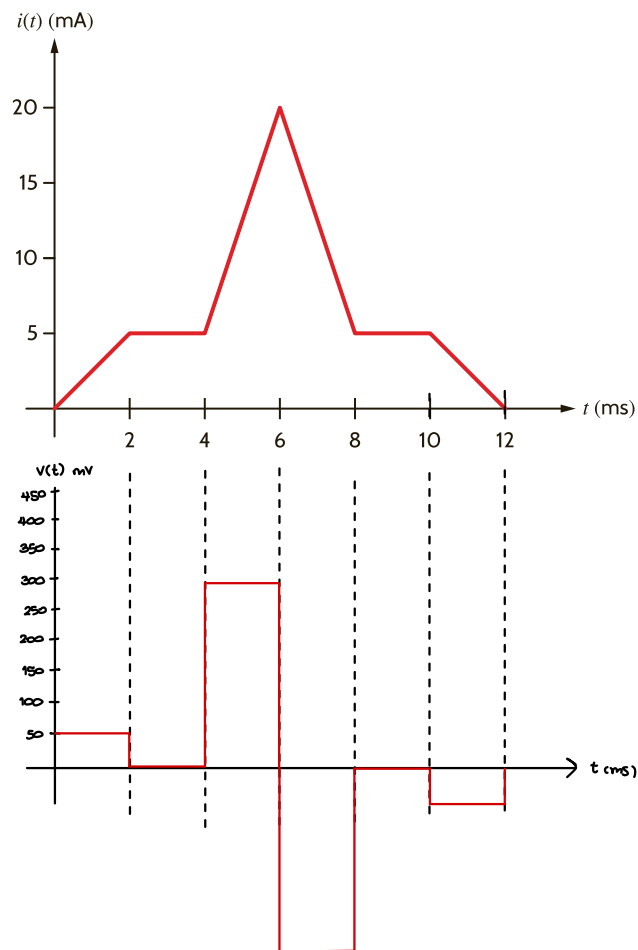
$$\therefore L_T = 3\ \text{mH} \#$$

## Problem 6

The current waveform in a 40-mH inductor is shown below. Derive the waveform for the inductor voltage.

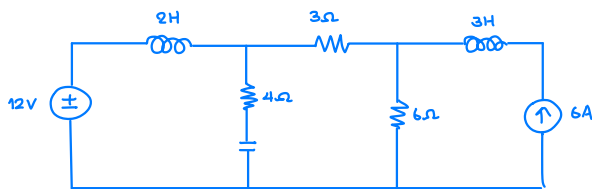
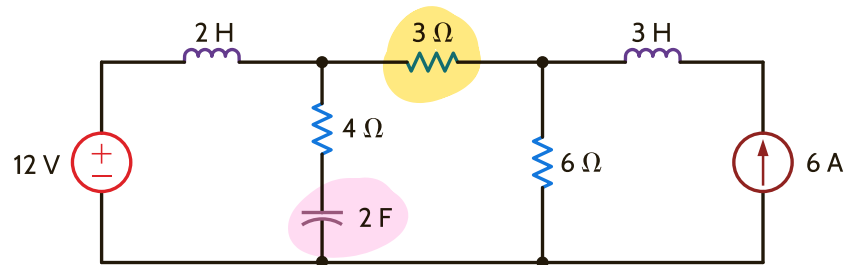
$$v = L \frac{di}{dt}$$

$$v = 40 (\text{slope})$$

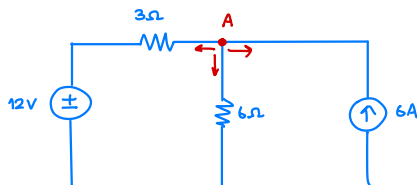
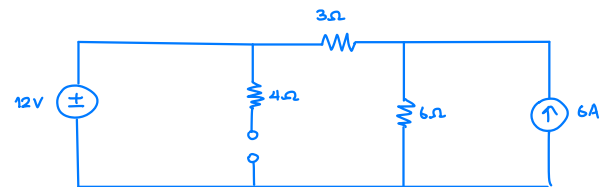


## Problem 7

Given the circuit, find the power dissipated in the  $3\text{-}\Omega$  resistor and the energy stored in the capacitor.



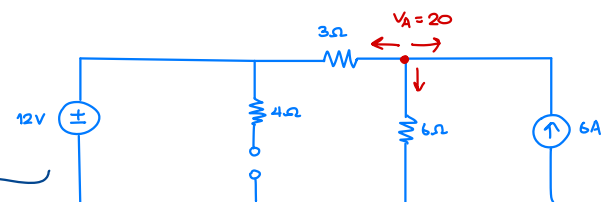
$\Rightarrow$   
under DC  
condition



Find power dissipated in  $R_{3\Omega}$  from  $P = I^2 R$

$$\frac{V_A - 12}{3} + \frac{V_A}{6} - 6 = 0$$

$$V_A = 20 \text{ Volt}$$



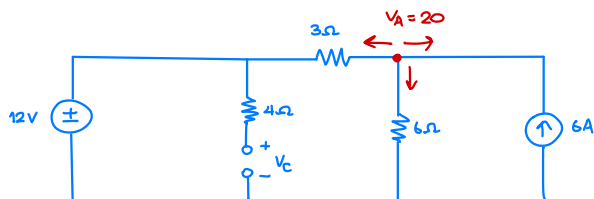
$$I = \frac{V_A - 12}{3} = \frac{20 - 12}{3} = \frac{8}{3} \text{ A}$$

$$\therefore P = I^2 R = \left(\frac{8}{3}\right)^2 (3)$$

$$= \frac{64}{3} \text{ watt } \#$$

$$W = \frac{1}{2} C V^2$$

Find energy stored in the capacitor



$$V_C + V_{4\Omega} = V_{6\Omega} \text{ (Not so sure)}$$

$$V_C + 0 = 20$$

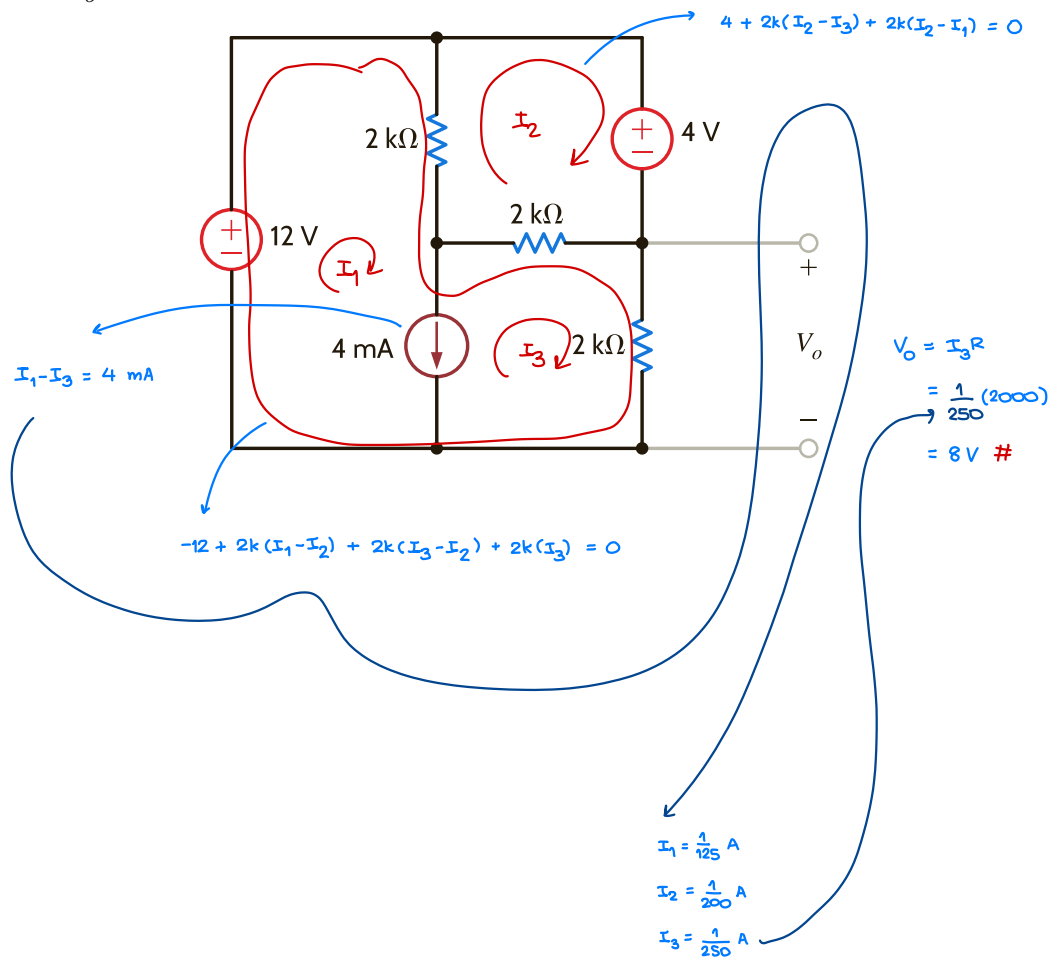
$$V_C = 20$$

$$W = \frac{1}{2} (2) (20)^2 = 400 \text{ Watt } \#$$



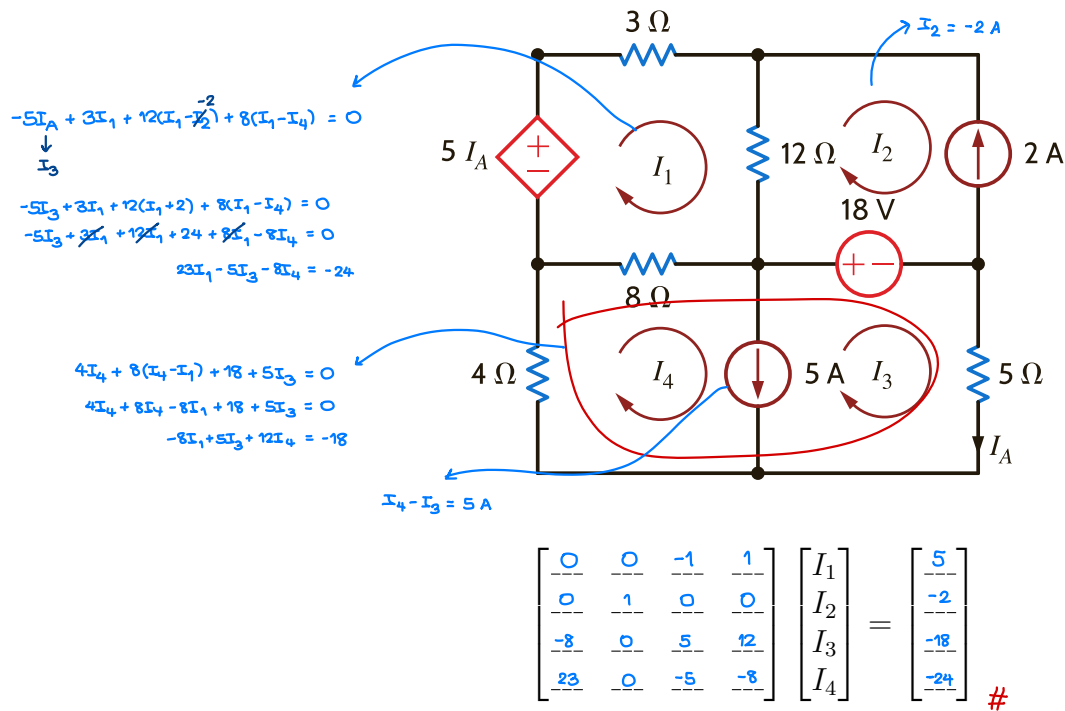
## Problem 8

Find  $V_o$  in the circuit.



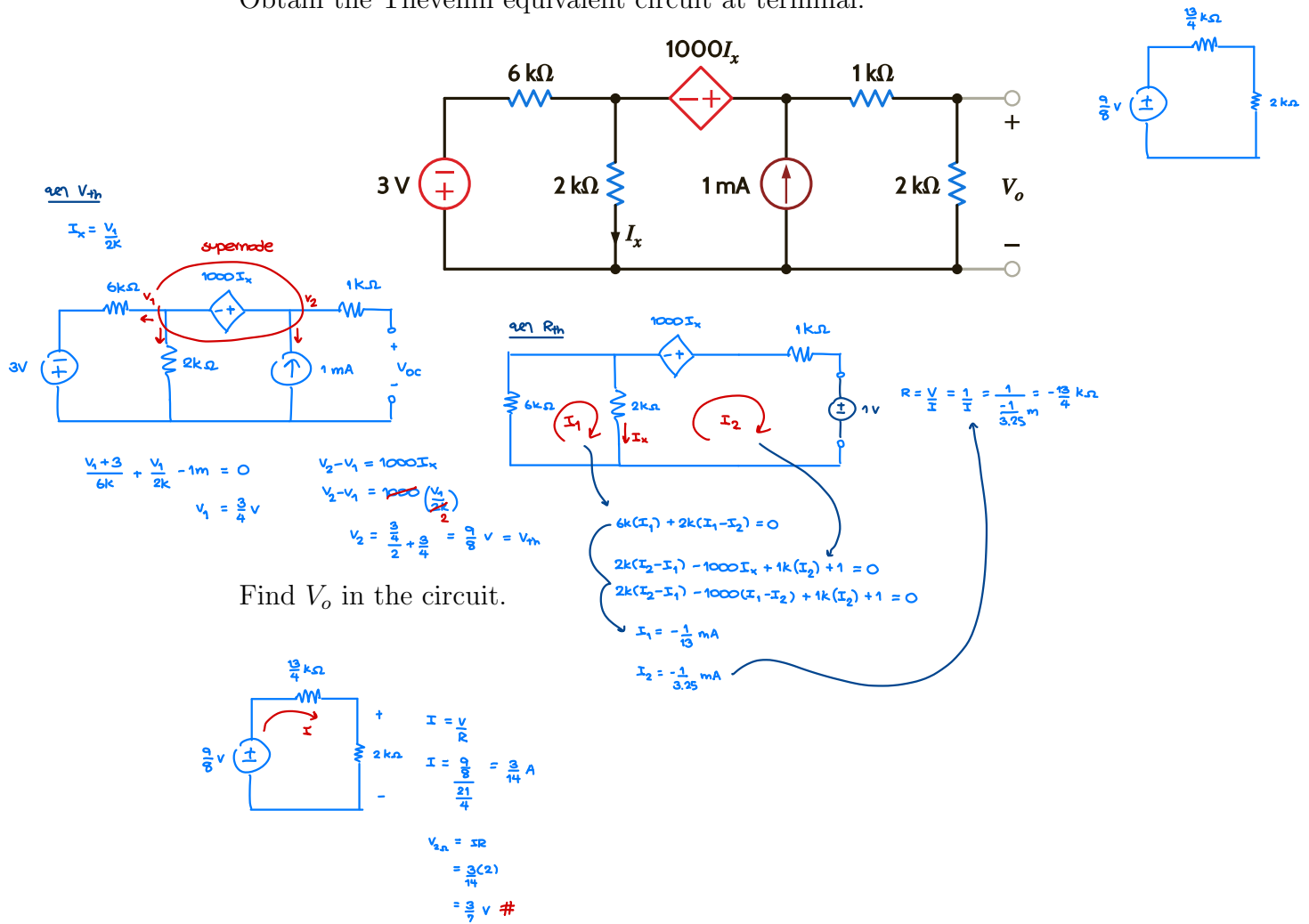
## Problem 9

Determine the loop currents  $I_1, I_2, I_3, I_4$  in the given electrical circuit and express the solution in matrix form.



## Problem 10

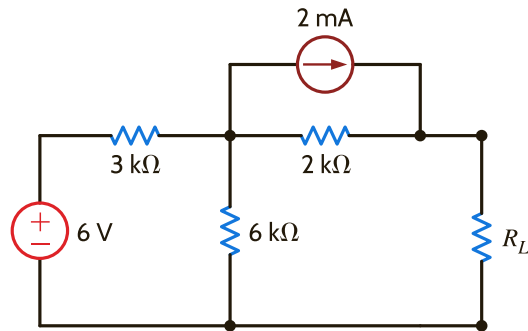
Obtain the Thevenin equivalent circuit at terminal.



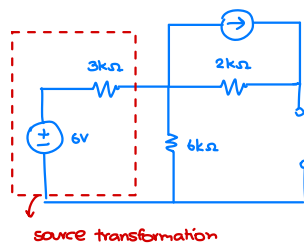
Find Norton equivalent circuit.

## Problem 11

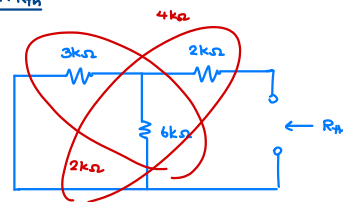
Find  $R_L$  for maximum power transfer and the maximum power that can be transferred to the load.



get  $V_{th}$



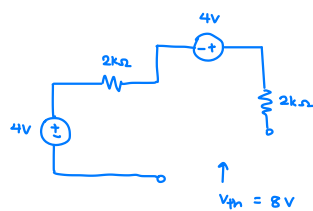
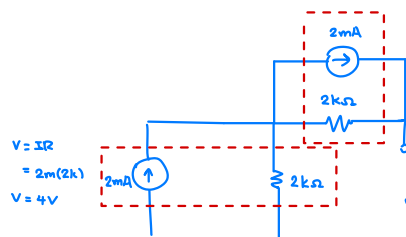
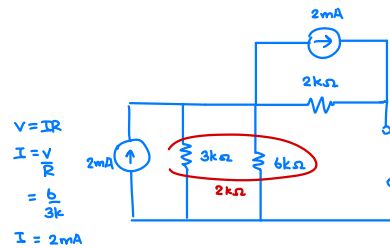
get  $R_{th}$



$$\therefore R_{th} = 4 \text{ k}\Omega$$

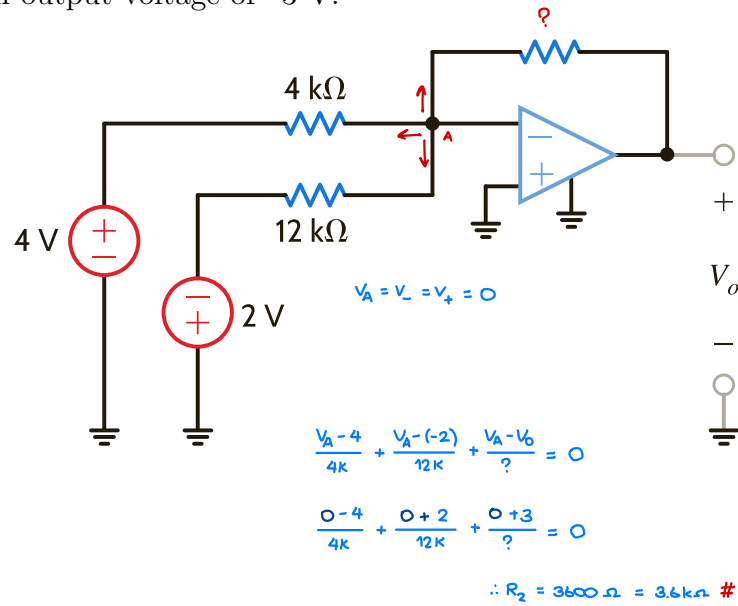
$$\therefore R_L \text{ for } P_{max} = 4 \text{ k}\Omega \#$$

$$\therefore \text{Maximum power} = \frac{V_{th}^2}{4R_{th}} = \frac{8^2}{4(4k)} = 4 \text{ mW} \#$$



## Problem 12

Given the summing amplifier shown below, find the values of  $R_2$  that will produce an output voltage of  $-3$  V.



(စရာအိ 555)

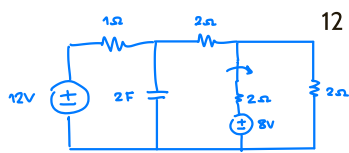
ခိုင်ခံ့သော အားလပ် ကိရိယာတွေမှာ  $\frac{24}{5} + \frac{1}{5}e^{-5t/8}$  V

$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

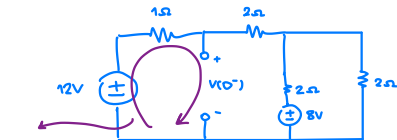
### Problem 13

Consider the circuit. The switch opens at  $t = 0$ . Find  $v_o(t)$  for  $t > 0$ .

$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$



$t < 0$



$$-12 + I(1) + V(0^-) = 0$$

$$-12 + 2 + V(0^-) = 0$$

$$V(0^-) = 10 \text{ V} \#$$

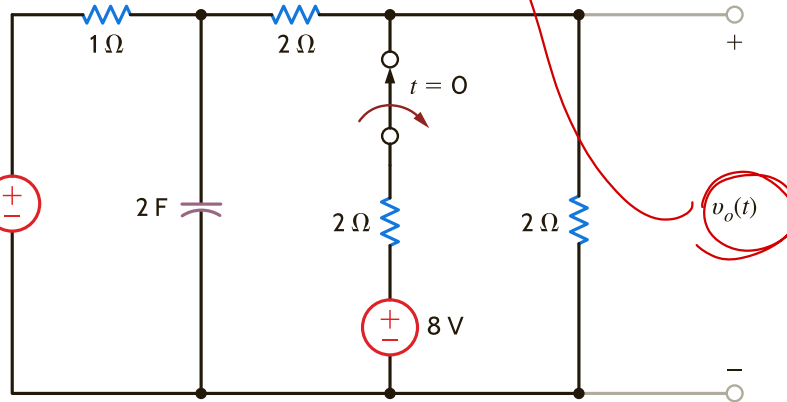


$$-12 + 3I_1 + 2(I_1 - I_2) + 8 = 0$$

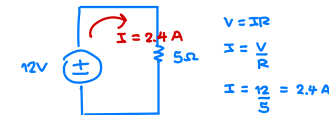
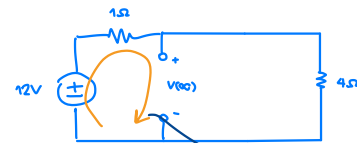
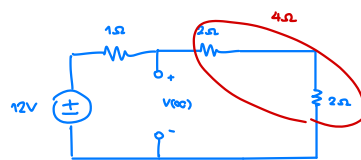
$$2I_2 - 8 + 2(I_2 - I_1) = 0$$

$$I_1 = 2 \text{ A}$$

$$I_2 = 3 \text{ A}$$



$t > 0$



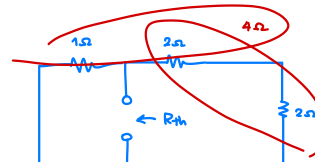
$$V = IR$$

$$I = \frac{V}{R}$$

$$I = \frac{12}{5} = 2.4 \text{ A}$$

Time Constant

$$\tau = R_{th}C = \frac{4(2)}{5} = \frac{8}{5} \text{ s}$$



$$KVL: -12 + 2.4(1) + V(0) = 0$$

$$V(0) = 9.6 \text{ V}$$

$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

$$= 9.6 + [10 - 9.6]e^{-5t/8}$$

$$= 9.6 + 0.4e^{-5t/8}$$

Assume that the circuit reaches steady state after a duration equal to five times the time constant. Calculate the exact time at which the circuit reaches steady state.

$$\tau = \frac{8}{5} \text{ s}$$

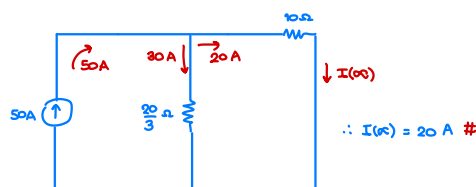
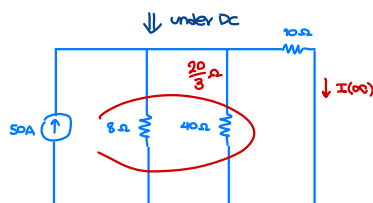
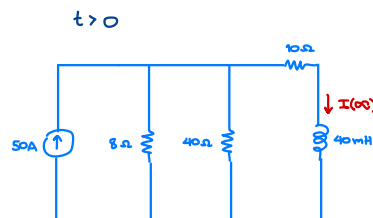
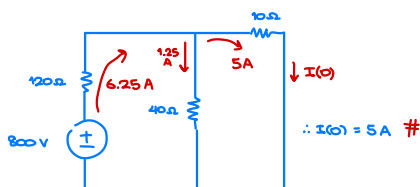
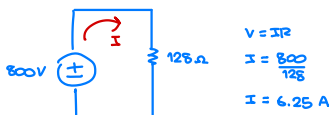
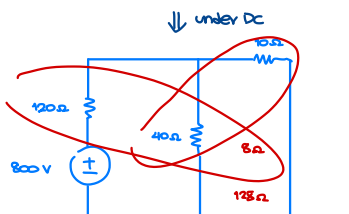
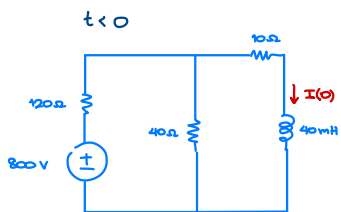
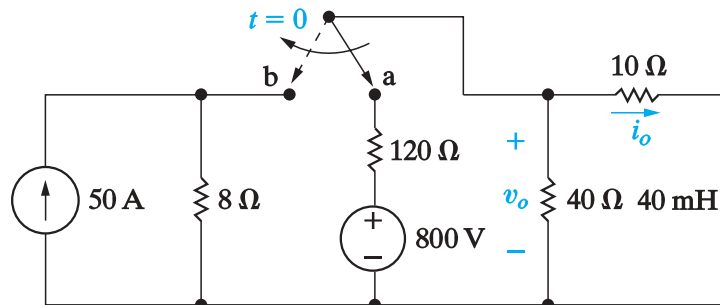
$$\therefore 5\tau = 5\left(\frac{8}{5}\right) = 8 \text{ seconds} \#$$

$$I(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$$

## Problem 14

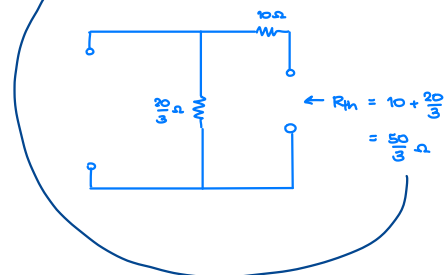
The switch in the circuit shown below has been in position a for a long time. At  $t = 0$ , the switch moves instantaneously to position b.

- ✓ • Find the numerical expression for  $i_o(t)$  when  $t \geq 0$ .
- ✓ • Find the numerical expression for  $v_o(t)$  when  $t \geq 0^+$ .

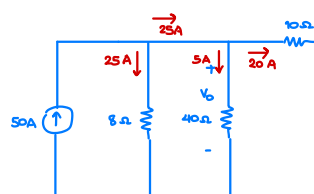


Time Constant

$$\tau = \frac{L}{R_{th}} = \frac{40 \times 10^{-3}}{\frac{50}{3}} = 2.4 \text{ ms} \#$$



$$\begin{aligned} I(t) &= I(\infty) + [I(0) - I(\infty)]e^{-t/\tau} \\ &= 20 + [5 - 20]e^{-t/2.4 \text{ ms}} \\ &= 20 - 15e^{-t/2.4 \text{ ms}} \# \end{aligned}$$

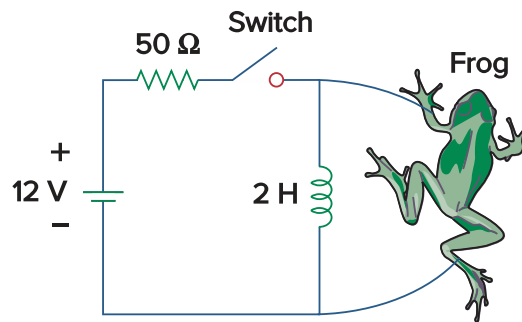


$$\therefore V_o = IR = 5(40) = 200 \text{ Volt} \#$$

answer!

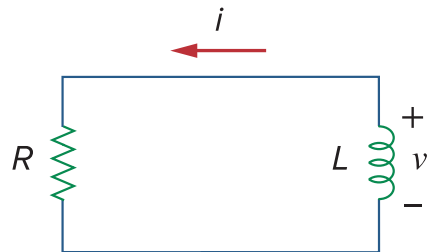
## Practice Problem 1

The circuit shown below is used by a biology student to study “frog kick.” She noticed that the frog kicked a little when the switch was closed but kicked violently for 5 s when the switch was opened. Model the frog as a resistor and calculate its resistance. Assume that it takes 10 mA for the frog to kick violently.





## Practice Problem 2



For the circuit shown above

$$v = 90e^{-50t} \text{ V}$$

and

$$i = 30e^{-50t} \text{ A}, \quad t > 0$$

- a) Find L and R.

$$v = L \frac{di}{dt}$$

$$90e^{-50t} = L(30)(-50)e^{-50t}$$

$$L = 0.06 \text{ H} \quad \#$$

$$R = \frac{V}{I} = \frac{90e^{-50t}}{30e^{-50t}} = 3 \Omega \quad \#$$

- b) Determine the time constant.

$$\tau = \frac{L}{R} = \frac{0.06}{3} = 0.02 \text{ s} \quad \#$$

- c) Calculate the initial energy in the inductor.

$$w = \frac{1}{2} Li^2(0)$$

$$= \frac{1}{2} (0.06)(30)^2$$

$$w = 27 \text{ J} \quad \#$$

- d) What fraction of the initial energy is dissipated in 10 ms?

$$w_{10\text{ms}} = \frac{1}{2} Li^2(0)$$

$$= \frac{1}{2} (0.06)(30e^{-50(0.01)})^2$$

$$= 9.93 \text{ J}$$

$$w_{\text{dissipated}} = w_L(0) - w_L(10\text{ms})$$

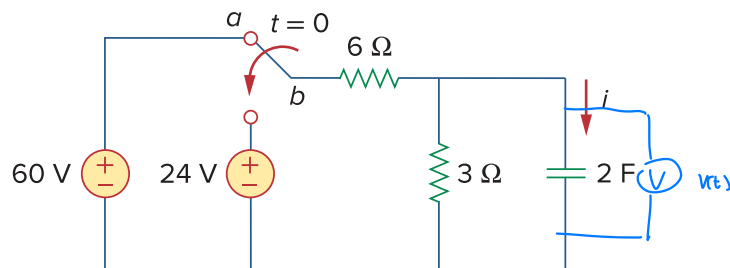
$$= 27 - 9.93$$

$$= 17.07 \text{ J} \quad \#$$

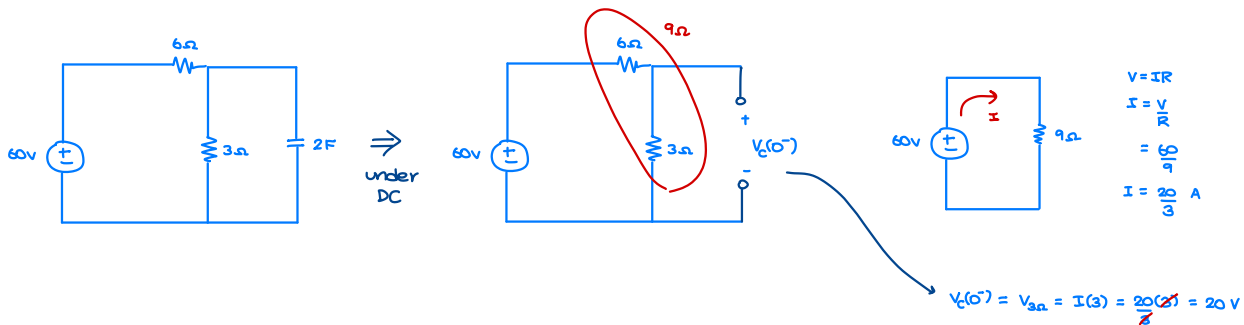
$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

### Practice Problem 3

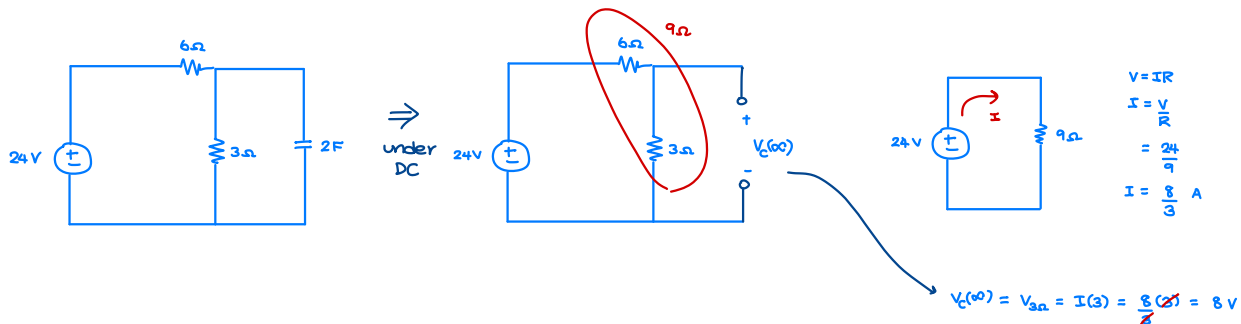
The switch in circuit shown below has been in position a for a long time. At  $t = 0$ , it moves to position b. Calculate  $i(t)$  for all  $t > 0$ .



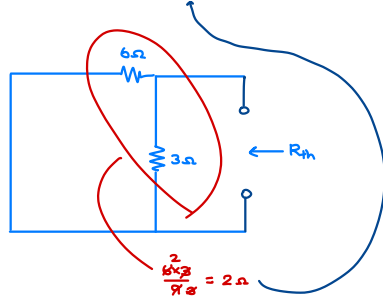
$t < 0$



$t > 0$



Time Constant:  $\tau = RC = 2(2) = 4 s$



$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

$$= 8 + [20 - 8]e^{-t/4}$$

$$V(t) = 8 + 12e^{-t/4}$$

$$I(t) = C \frac{dV}{dt}$$

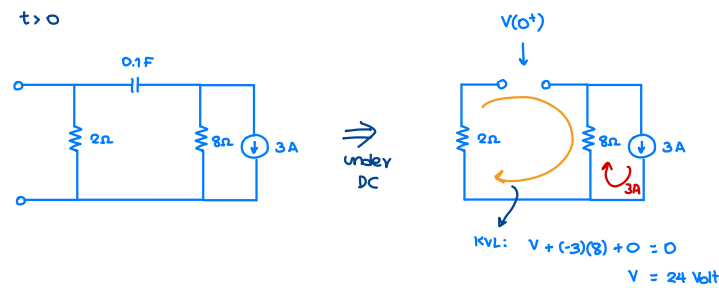
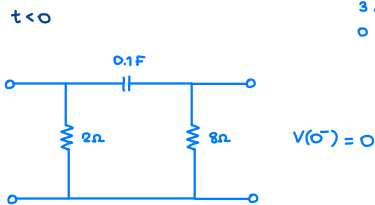
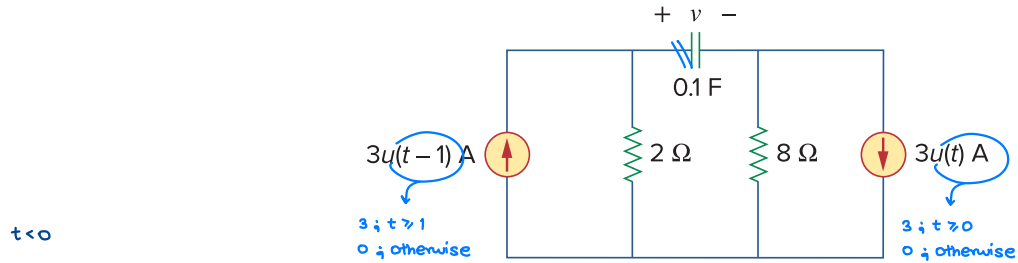
$$= 2 \left( \frac{-1}{4} \right) (12e^{-t/4})$$

$$I(t) = -6e^{-t/4} A$$

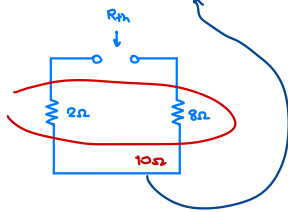
$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

## Practice Problem 4

Determine  $v(t)$  for  $t > 0$  in the circuit shown below if  $v(0) = 0$ .



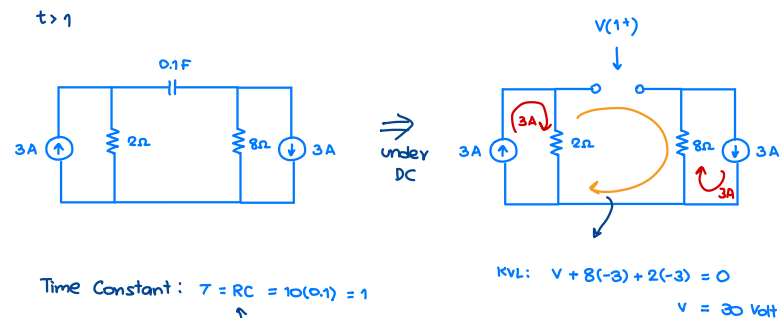
Time Constant:  $\tau = RC = 10(0.1) = 1$



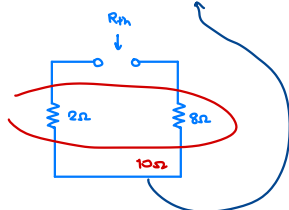
$$\begin{aligned} V(t) &= V(\infty) + [V(0) - V(\infty)]e^{-t/\tau} \\ &= 24 + [0 - 24]e^{-t/1} \\ &= 24 - 24e^{-t} \\ &= 24(1 - e^{-t}) \text{ V} \quad (t > 0) \end{aligned}$$

at  $t = 1$  ( $t < 1$ )

$$\begin{aligned} \therefore V(1) &= 24(1 - e^{-1}) \text{ V} \\ &= 15.17 \text{ V} \end{aligned}$$



Time Constant:  $\tau = RC = 10(0.1) = 1$



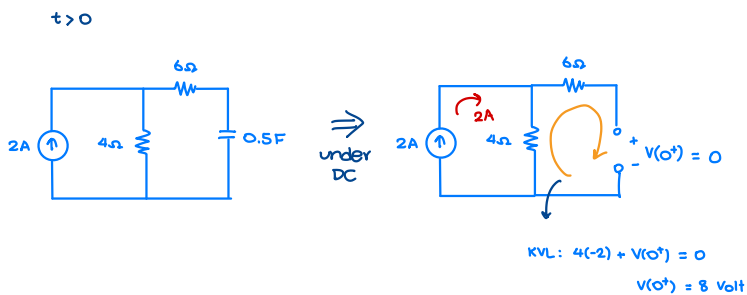
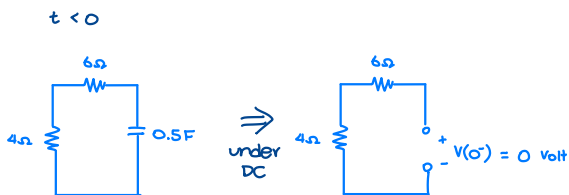
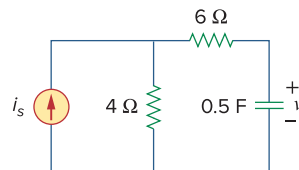
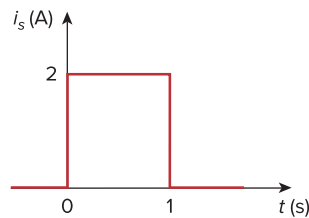
$$\begin{aligned} V(t-1) &= V(\infty) + [V(0) - V(\infty)]e^{-(t-1)/\tau} \\ &= 30 + [15.17 - 30]e^{-(t-1)/1} \\ &= 30 - 14.83e^{-(t-1)} \text{ V} \quad (t > 1) \end{aligned}$$

$$V(t) = \begin{cases} 24(1 - e^{-t}) \text{ V} & ; 0 < t < 1 \\ 30 - 14.83e^{-(t-1)} \text{ V} & ; t > 1 \end{cases} \quad \#$$

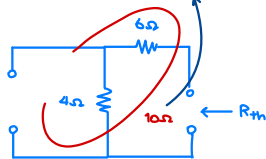
$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

## Practice Problem 5

If the waveform in left is applied to the circuit on the right, find  $v(t)$ . Assume  $v(0) = 0$ .



Time Constant:  $\tau = RC = 10(0.5) = 5 \text{ s}$



$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

$$= 8 + [0 - 8]e^{-t/5}$$

$$= 8 - 8e^{-t/5}$$

$$= 8(1 - e^{-t/5})$$

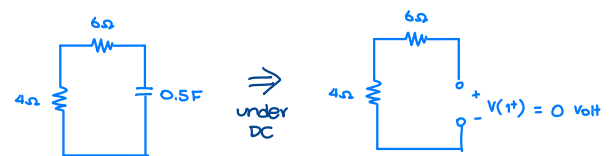
$t \text{ near } t = 1$

$$V(1) = 8(1 - e^{-1/5})$$

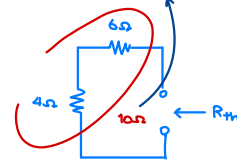
$$= 8(1 - e^{-1/5})$$

$$V(1) = 1.45 \text{ Volt}$$

$t > 1$



Time Constant:  $\tau = RC = 10(0.5) = 5 \text{ s}$



$$V(t-1) = 0 + [1.45 - 0]e^{-(t-1)/5}$$

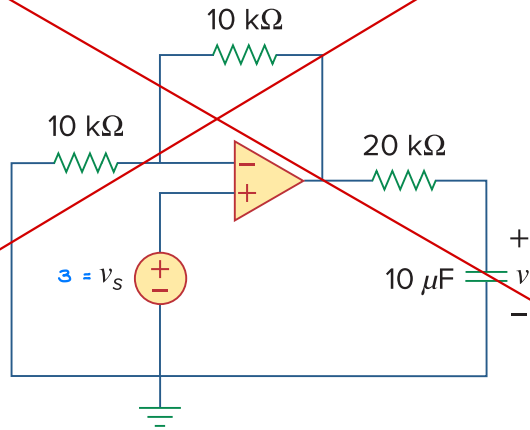
$$V(t-1) = 1.45e^{-(t-1)/5}$$

$$V(t) = \begin{cases} 8(1 - e^{-t/5}) \text{ V} & (0 < t < 1) \\ 1.45e^{-(t-1)/5} \text{ V} & (t > 1) \end{cases} \quad \#$$

balanced?

## Practice Problem 6

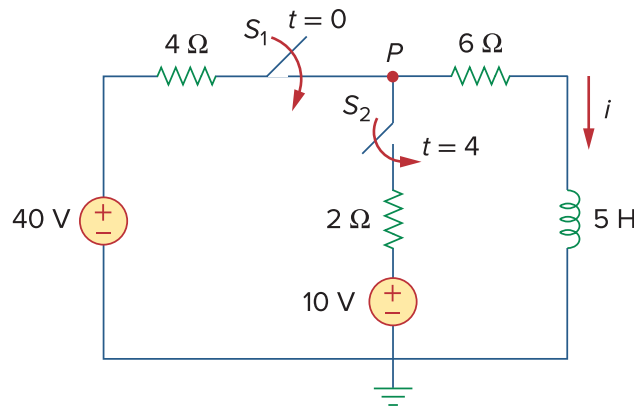
For the op amp circuit, suppose  $v(0) = 0$  and  $v_s = 3 \text{ V}$ . Find  $v(t)$  for  $t > 0$ .



$$I(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$$

## Practice Problem 7

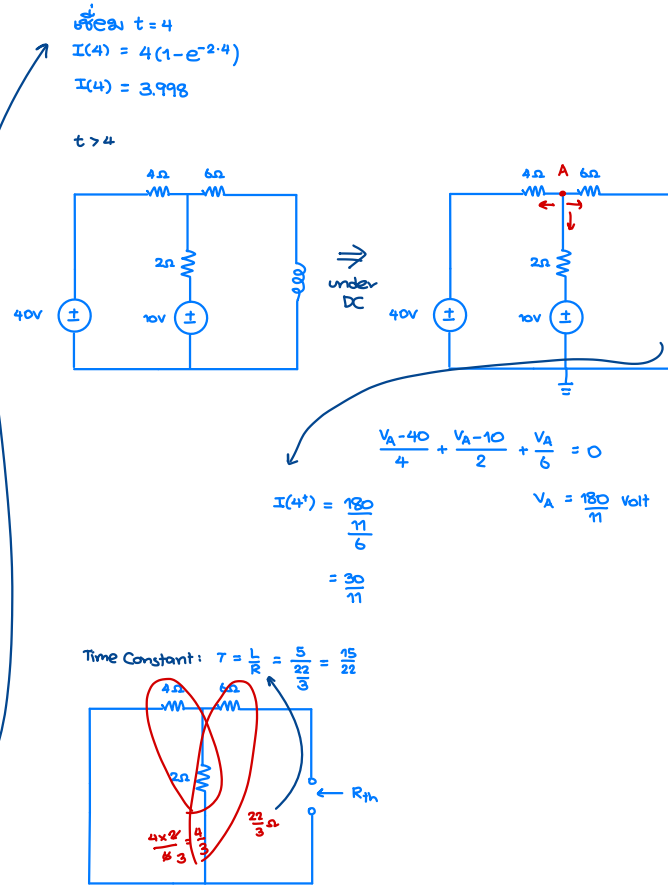
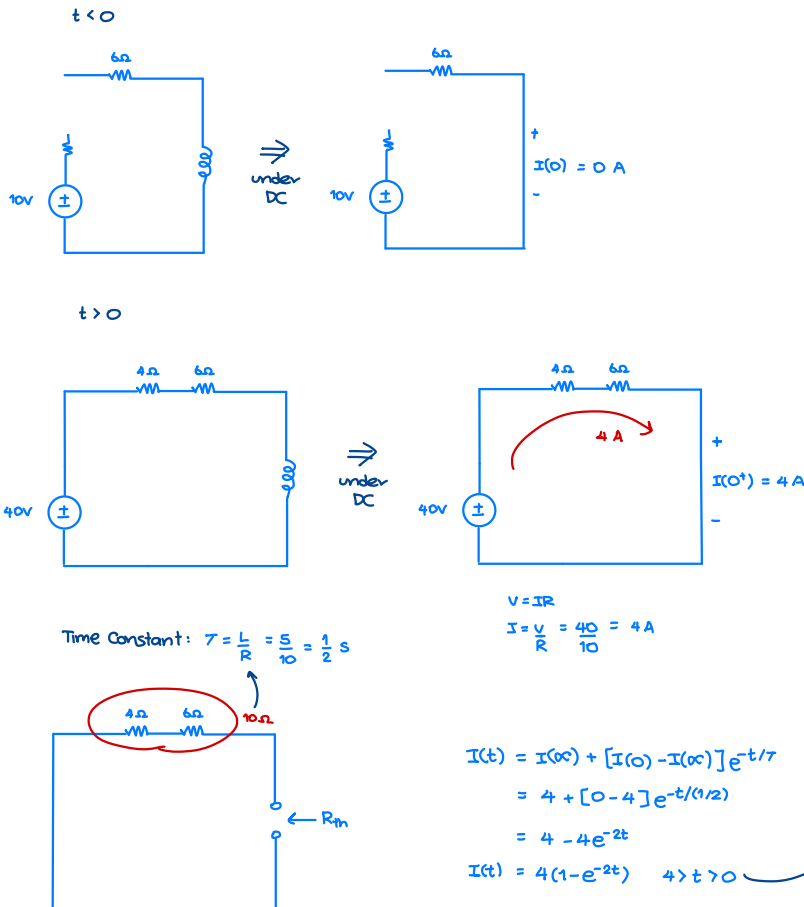
At  $t = 0$ , switch 1 is closed, and switch 2 is closed 4 s later. Find  $i(t)$  for  $t > 0$ . Calculate  $i$  for  $t = 2$  s and  $t = 5$  s.



$$\begin{cases} \frac{30}{11} + 1.2707e^{-22(t-4)/15} & 0 \leq t \leq 4 \\ \frac{30}{11} & t \geq 4 \end{cases}$$

$$I(2) = 4(1 - e^{-2 \cdot 2}) = 3.926 \text{ A} \quad \#$$

$$I(5) = \frac{30}{11} + 1.2707e^{-22(5-4)/15} = 3.02 \text{ A} \quad \#$$



$$I(t-4) = I(\infty) + [I(0) - I(\infty)]e^{-(t-4)/\tau}$$

$$= \frac{30}{11} + \left[ 3.998 - \frac{30}{11} \right]e^{-(t-4)/15/22}$$

$$I(t) = \frac{30}{11} + 1.2707e^{-22(t-4)/15} \quad t > 4$$

$$I(t) = \begin{cases} 4(1 - e^{-2t}) & 0 \leq t \leq 4 \\ \frac{30}{11} + 1.2707e^{-22(t-4)/15} & t \geq 4 \end{cases} \quad \#$$