EES216: Circuit Analysis
Midterm Mock Exam

curated by The Peanuts

Name..Nonprawich I.....ID..6622772422.....Section.............Seat No.............

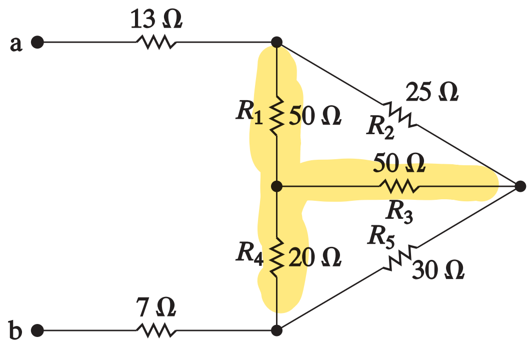
Conditions: Semi-Closed Book

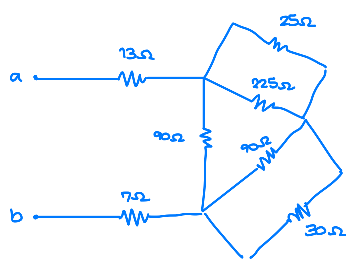
Directions:

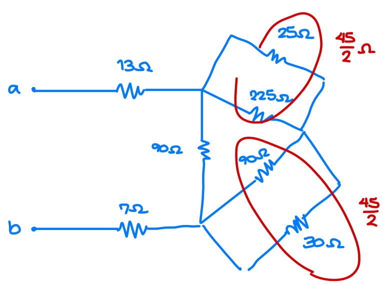
1. This exam has 15 pages (including this page).
2. Calculators are permitted (You may bring 100 of them. Haha)
3. Write your name clearly at the top of each page.
4. Reading the problem is optional but highly recommended.
5. You may bring one A3 sheet of note, which will magically become illegible the moment the exam begins.
6. Tears shed on your answer sheet may cause short circuits. Please cry responsibly.

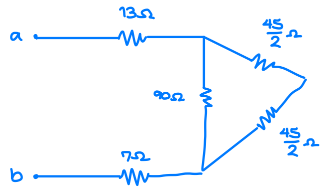
Problem 1

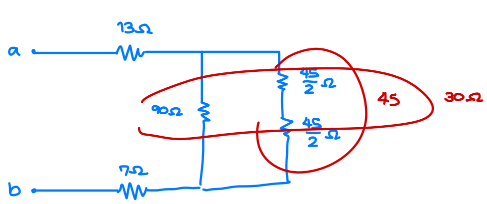
Find the equivalent resistance R_{ab} in the circuit.











$\therefore R_{ab} = 13 + 30 + 7 = 50\ \Omega$ #

Problem 2

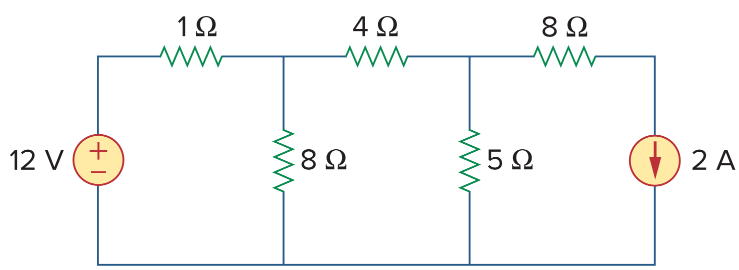
For an ideal operational amplifier (op-amp), state the values of the following characteristics and briefly explain their significance:

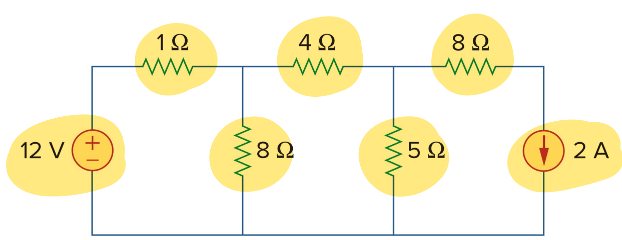
1. Open-loop voltage gain (A) $\infty \rightarrow$ Voltage difference between output is zero.
2. Input resistance (R_i) $\infty \rightarrow$ Current does NOT flow in the op-amp
3. Output resistance (R_o) $0 \rightarrow$ Current to the load is 100%. (No drop)

Problem 3

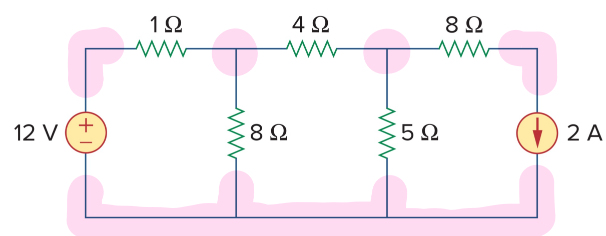
Let a be the number of branches in the circuit, b be the number of nodes, and c be the number of meshes. Then, compute the value of:

$$a^2 + b^2 - c$$

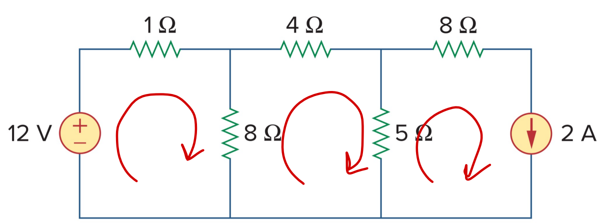




#Branches = a = 7



#Nodes = b = 5



#Mesh = c = 3

$$\therefore a^2 + b^2 - c = 7^2 + 5^2 - 3$$
$$= 49 + 25 - 3$$
$$= 71 \ \#$$

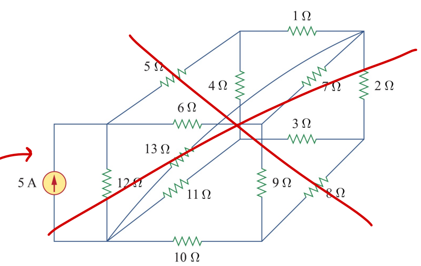
Problem 4

Take the last digit of your student ID and compute:

$$(\text{last digit}) \mod 3$$

Based on the result:

- If the result is 0, find a limitation of **Mesh Analysis**.
- If the result is 1, find a limitation of **Node Analysis**.
- If the result is 2, find a limitation of **Superposition Theorem**.



Limitation of Mesh Analysis

- Not applicable to non-planar circuits
- Becomes complex with many loops
- Difficult with current sources (require transformation) หรือว่า or supermesh

Limitation of Node Analysis

- Not ideal for voltage sources (require transformation) or supernode
- Inefficient for circuits with many nodes

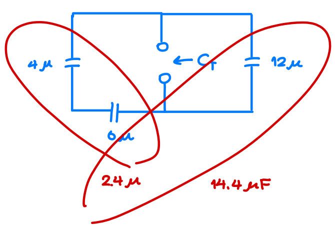
Limitation of Superposition

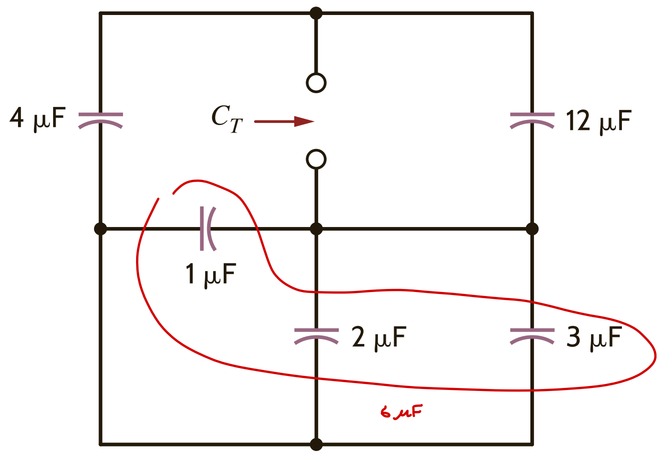
- Only works for linear circuits
- Time-consuming for multiple sources
- Does not work for power calculations directly

Power is not linear

Problem 5

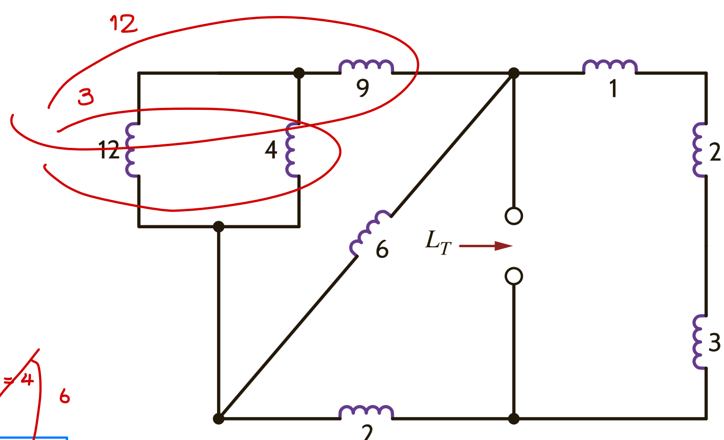
Determine the value of C_T in the circuit

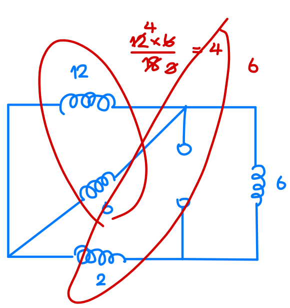


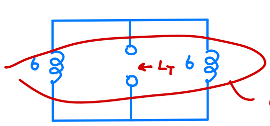


$\therefore C_T = 14.4\ \mu F$ #

Find the total inductance L_T in the circuit, All inductors are in millihenrys.







$\frac{6\times6}{12_2} = 3$

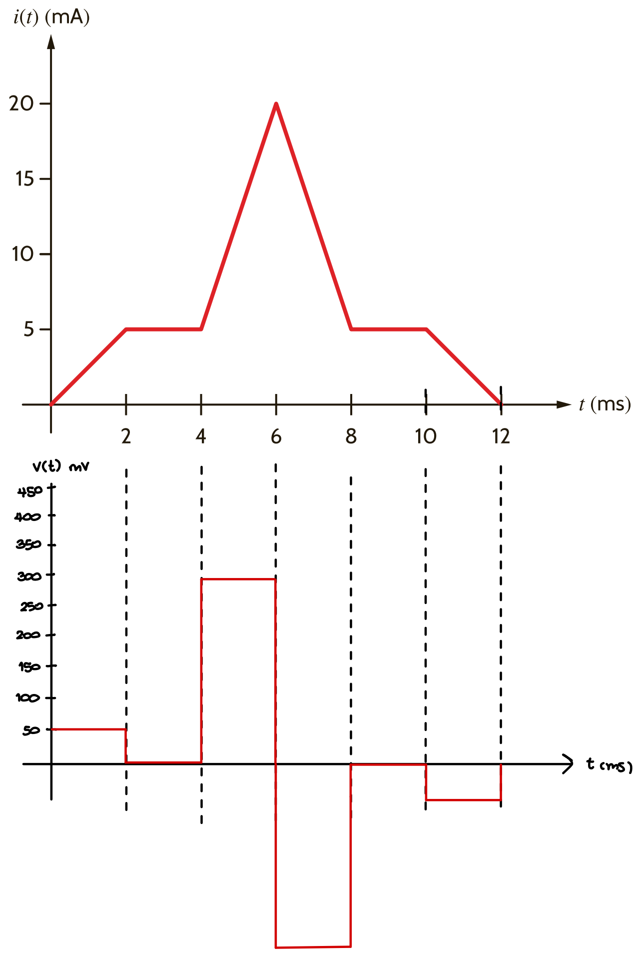
$\therefore L_T = 3\ mH$ #

Problem 6

The current waveform in a 40-mH inductor is shown below. Derive the waveform for the inductor voltage.

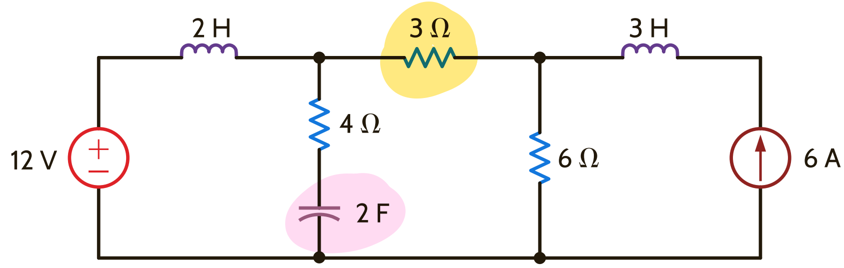
$\boxed{v = L\frac{di}{dt}}$

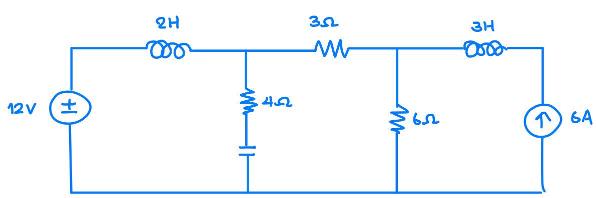
$v = 40\,(\text{Slope})$



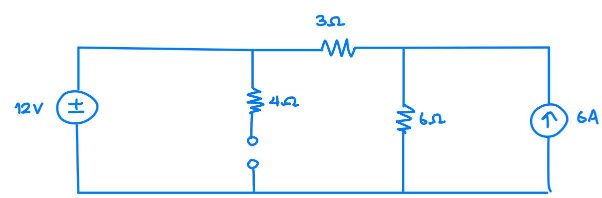
Problem 7

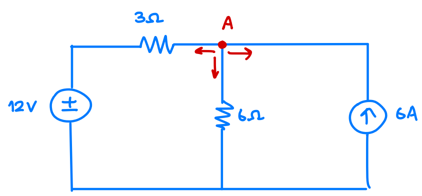
Given the circuit, find the power dissipated in the 3-Ω resistor and the energy stored in the capacitor.





$\Rightarrow$ under DC condition

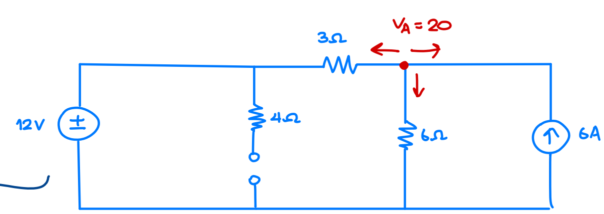




Find power dissipated in $R_{3\Omega}$ from $\boxed{P = I^2R}$

$$\frac{V_A - 12}{3} + \frac{V_A}{6} - 6 = 0$$

$$V_A = 20 \text{ Volt}$$

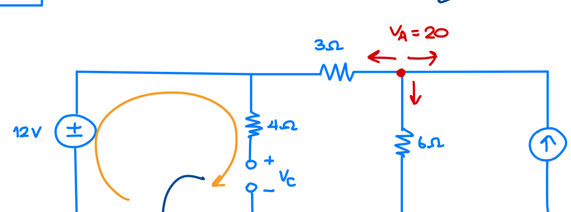


$$I = \frac{V_A - 12}{3} = \frac{20 - 12}{3} = \frac{8}{3} \text{ A}$$

$$\therefore P = I^2R = \left(\frac{8}{3}\right)^2 (3)$$

$$= \frac{64}{3} \text{ watt } \#$$

$\boxed{W = \frac{1}{2}CV^2}$ Find energy stored in the capacitor



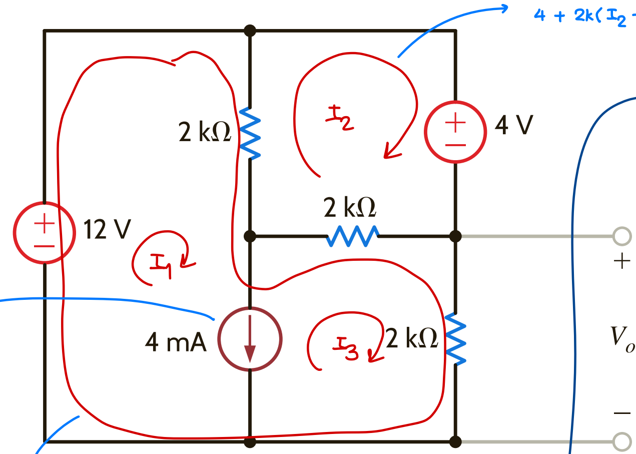
KVL: $-12 + (0)(4) + V_c = 0$

$$V_c = 12 \text{ Volt}$$

$$W = \frac{1}{2}(2)(12)^2 = 144 \text{ Joule } \#$$

Problem 8

Find V_o in the circuit.



$4 + 2k(I_2 - I_3) + 2k(I_2 - I_1) = 0$

$I_1 - I_3 = 4 \text{ mA}$

$-12 + 2k(I_1 - I_2) + 2k(I_3 - I_2) + 2k(I_3) = 0$

$V_o = I_3 R$

$= \frac{1}{250}(2000)$

$= 8 \text{ V}$ #

$I_1 = \frac{1}{125} \text{ A}$

$I_2 = \frac{1}{200} \text{ A}$

$I_3 = \frac{1}{250} \text{ A}$

Problem 9

Determine the loop currents I_1, I_2, I_3, I_4 in the given electrical circuit and express the solution in matrix form.

$$-5I_A + 3I_1 + 12(I_1 - \overset{-2}{\cancel{I_2}}) + 8(I_1 - I_4) = 0$$

$\downarrow$
I_3

$$-5I_3 + 3I_1 + 12(I_1 + 2) + 8(I_1 - I_4) = 0$$

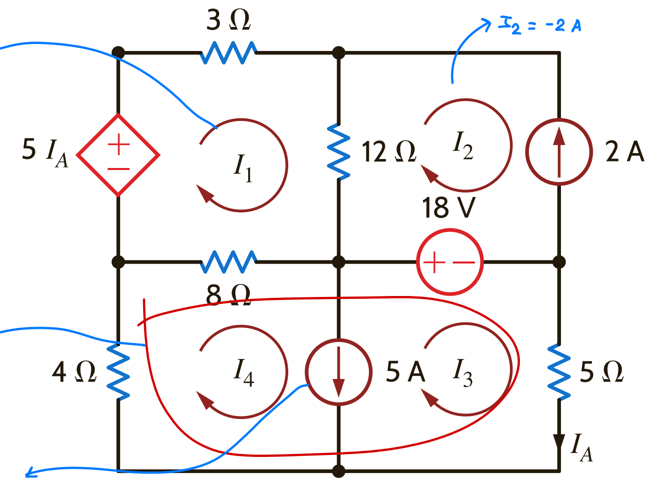
$$-5I_3 + \cancel{3I_1} + \cancel{12I_1} + 24 + \cancel{8I_1} - 8I_4 = 0$$

$$23I_1 - 5I_3 - 8I_4 = -24$$

$$4I_4 + 8(I_4 - I_1) + 18 + 5I_3 = 0$$

$$4I_4 + 8I_4 - 8I_1 + 18 + 5I_3 = 0$$

$$-8I_1 + 5I_3 + 12I_4 = -18$$



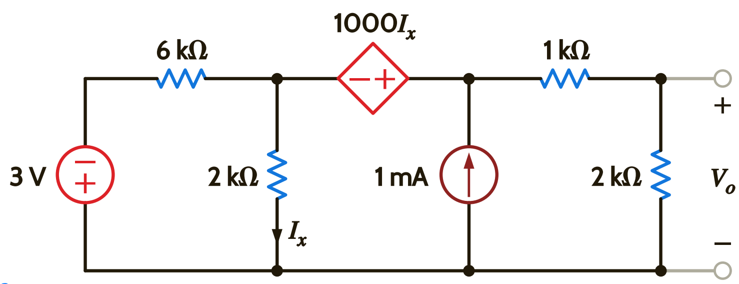
$I_4 - I_3 = 5$ A

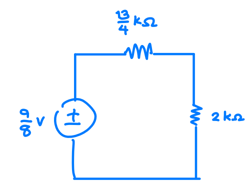
$$\begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ -8 & 0 & 5 & 12 \\ 23 & 0 & -5 & -8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -18 \\ -24 \end{bmatrix}$$

#

Problem 10

Obtain the Thevenin equivalent circuit at terminal.

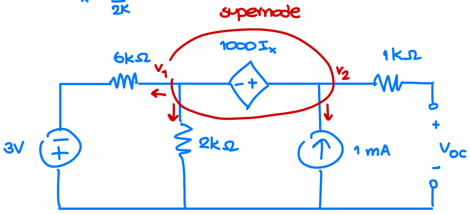




get V_{th}

$I_x = \frac{V_1}{2k}$

supernode



$\frac{V_1+3}{6k} + \frac{V_1}{2k} - 1m = 0$

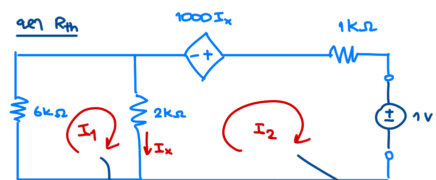
$V_1 = \frac{3}{4} V$

$V_2 - V_1 = 1000 I_x$

$V_2 - V_1 = \cancel{1000}\left(\frac{V_1}{\cancel{2k}}\right)$ 2

$V_2 = \frac{\frac{3}{4}}{2} + \frac{3}{4} = \frac{9}{8} V = V_{th}$

get R_{th}



$6k(I_1) + 2k(I_1 - I_2) = 0$

$2k(I_2 - I_1) - 1000 I_x + 1k(I_2) + 1 = 0$

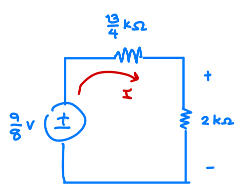
$2k(I_2 - I_1) - 1000(I_1 - I_2) + 1k(I_2) + 1 = 0$

$I_1 = -\frac{1}{13}$ mA

$I_2 = -\frac{1}{3.25}$ mA

$R = \frac{V}{I} = \frac{1}{I} = \frac{1}{\frac{-1}{3.25} m} = -\frac{13}{4} k\Omega$

Find V_o in the circuit.



$I = \frac{V}{R}$

$I = \frac{\frac{9}{8}}{\frac{21}{4}} = \frac{3}{14} A$

$V_{2\Omega} = IR$

$= \frac{3(2)}{14}$

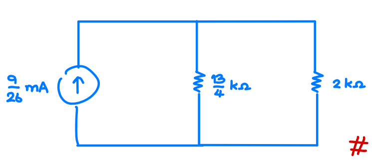
$= \frac{3}{7} V$ #

Find Norton equivalent circuit.

$I = \frac{V}{R}$

$= \frac{\frac{9}{8}}{\frac{13}{4} k}$

$= \frac{9}{26}$ mA



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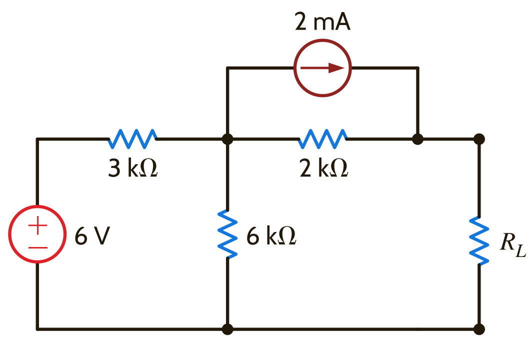
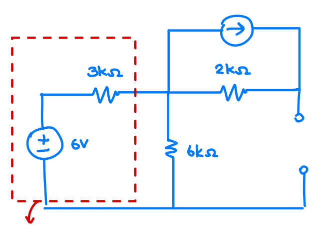
Problem 11

$R_L = R_{th} \to P_{max}$

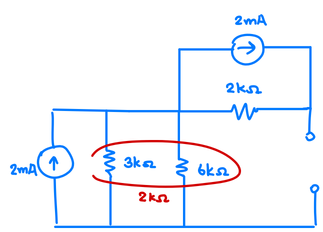
Find R_L for maximum power transfer and the maximum power that can be transferred to the load.



get V_{th}



source transformation

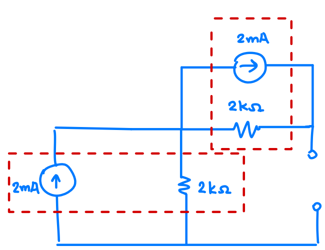


$V = IR$

$I = \frac{V}{R}$

$= \frac{6}{3k}$

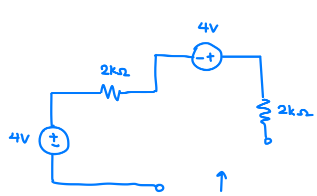
$I = 2mA$



$V = IR$

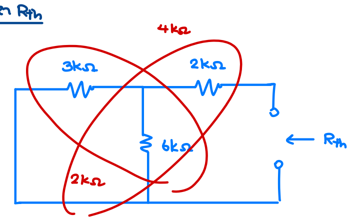
$= 2m(2k)$

$V = 4V$



$V_{th} = 8V$

get R_{th}



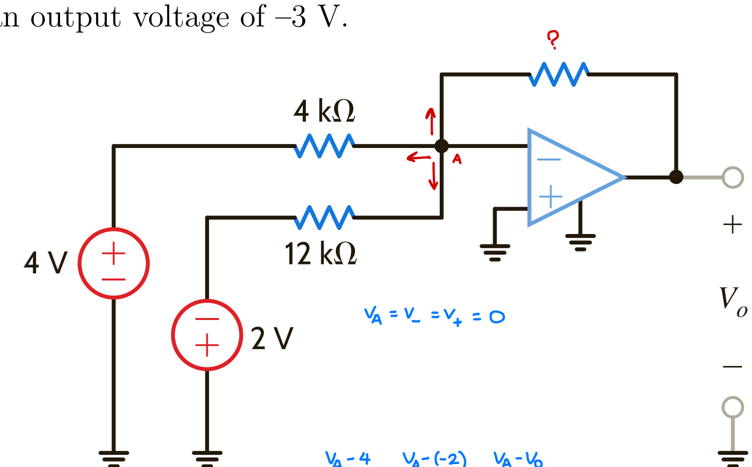
$\therefore R_{th} = 4\,k\Omega$

$\therefore R_L$ for $P_{max} = 4\,k\Omega$ #

$\therefore$ Maximum power $= \frac{V_{th}^2}{4R_{th}} = \frac{8^2}{4(4k)} = 4\,mW$ #

Problem 12

Given the summing amplifier shown below, find the values of R_2 that will produce an output voltage of –3 V.



$V_A = V_- = V_+ = 0$

$$\frac{V_A - 4}{4K} + \frac{V_A - (-2)}{12K} + \frac{V_A - V_o}{?} = 0$$

$$\frac{0 - 4}{4K} + \frac{0 + 2}{12K} + \frac{0 + 3}{?} = 0$$

$\therefore R_2 = 3600\,\Omega = 3.6\text{k}\Omega$ #

(อย่าทัก 555)
ข้อนี้จริงๆ หัวนะ คำตอบต้องเป็น $\frac{24}{5}+\frac{1}{5}e^{-5t/8}$ V

$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$

$v_o + v_c + v_{2\Omega} = 0$

$v_o - (9.6 + 0.4e^{-5t/8}) + (2.4)(2) = 0$

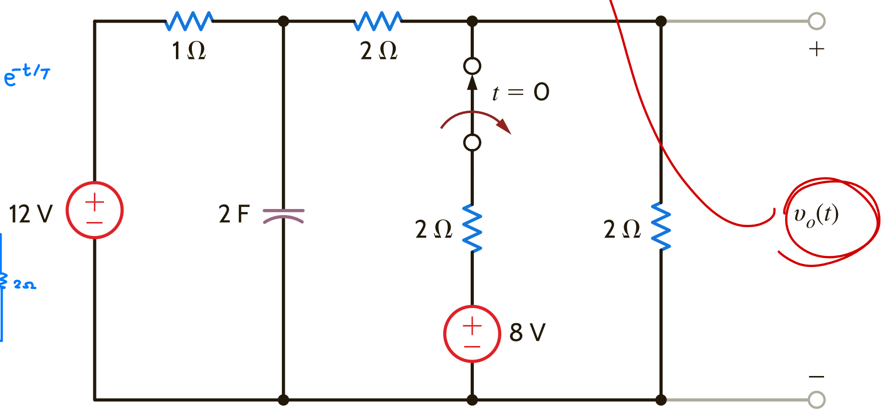
$= 9.6 + 0.4e^{-5t/8} - 4.8$

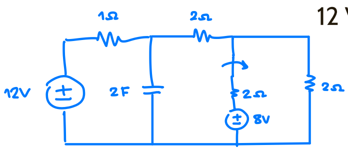
$v_o = 4.8 + 0.4e^{-5t/8}$ #

Problem 13

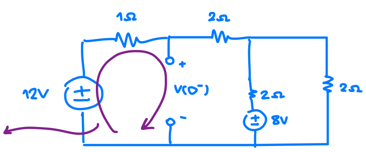
Consider the circuit. The switch opens at t = 0. Find $v_o(t)$ for $t > 0$.

$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$





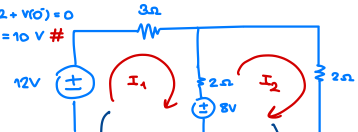
t < 0



$-12 + I(1) + v(0^-) = 0$

$-12 + 2 + v(0^-) = 0$

$v(0^-) = 10$ V #



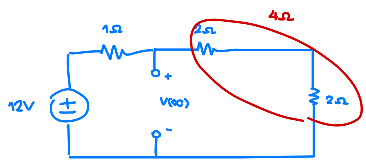
$-12 + 3I_1 + 2(I_1 - I_2) + 8 = 0$

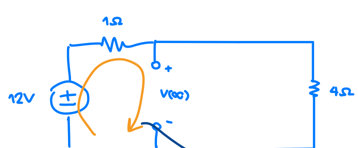
$2I_2 - 8 + 2(I_2 - I_1) = 0$

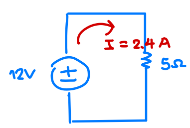
$I_1 = 2$ A

$I_2 = 3$ A

t > 0







$V = IR$

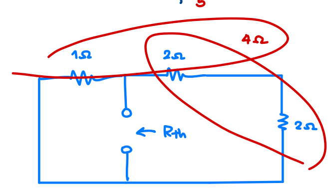
$I = \frac{V}{R}$

$I = \frac{12}{5} = 2.4$ A

Time Constant

$\tau = R_{th}C = \frac{4}{5}(2) = \frac{8}{5}$ s.

$\frac{4}{5}\Omega$



KVL: $-12 + 2.4(1) + v(\infty) = 0$

$v(\infty) = 9.6$ V

$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$

$= 9.6 + [10 - 9.6]e^{-5t/8}$

$= 9.6 + 0.4e^{-5t/8}$

Assume that the circuit reaches steady state after a duration equal to five times the time constant. **Calculate the exact time at which the circuit reaches steady state.**

$\tau = \frac{8}{5}$ s

$\therefore 5\tau = \cancel{5}\left(\frac{8}{\cancel{5}}\right) = 8$ seconds #

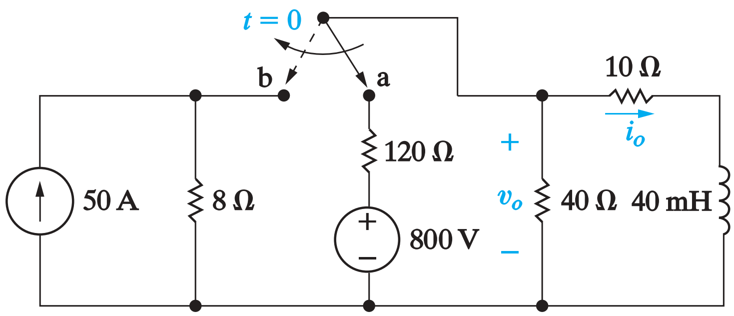
$I(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$

Problem 14

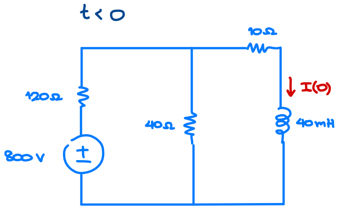
The switch in the circuit shown below has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b.

✓ • Find the numerical expression for $i_o(t)$ when $t \geq 0$.

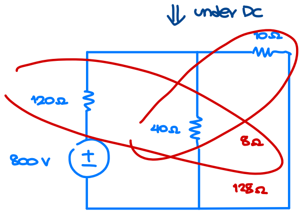
✓ • Find the numerical expression for $v_o(t)$ when $t \geq 0^+$.

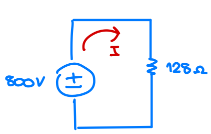


t < 0



⇓ under DC

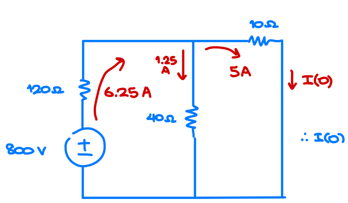




$V = IR$

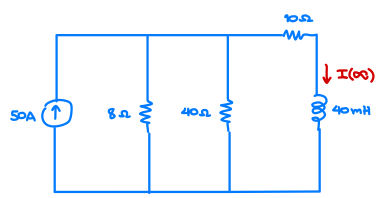
$I = \frac{800}{128}$

$I = 6.25$ A

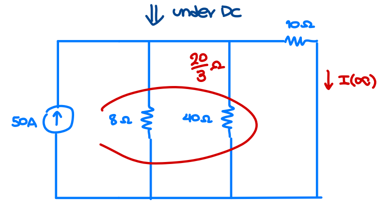


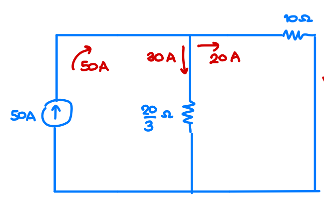
$\therefore I(0) = 5$ A #

t > 0



⇓ under DC

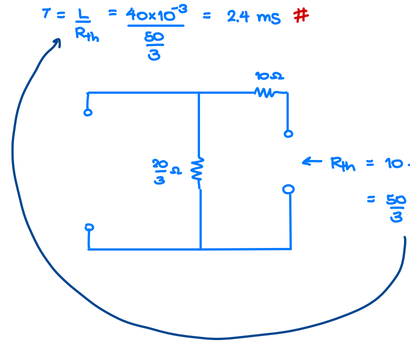




$\therefore I(\infty) = 20$ A #

Time Constant

$\tau = \frac{L}{R_{th}} = \frac{40 \times 10^{-3}}{\frac{50}{3}} = 2.4$ ms #



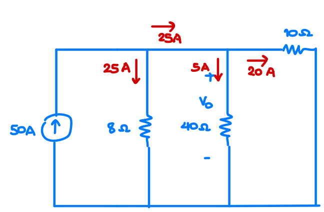
← $R_{th} = 10 + \frac{20}{3}$

$= \frac{50}{3}$ Ω

$I(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$

$= 20 + [5 - 20]e^{-t/2.4m}$

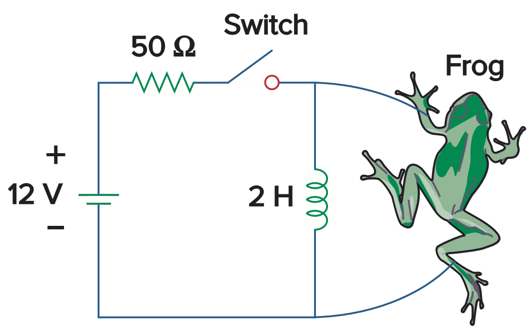
$= 20 - 15e^{-t/2.4m}$ #



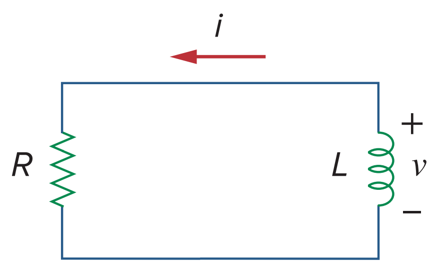
$\therefore V_o = IR = 5(40) = 200$ Volt #

Practice Problem 1

The circuit shown below is used by a biology student to study “frog kick.” She noticed that the frog kicked a little when the switch was closed but kicked violently for 5 s when the switch was opened. Model the frog as a resistor and calculate its resistance. Assume that it takes 10 mA for the frog to kick violently.



Practice Problem 2



For the circuit shown above

$$v = 90e^{-50t}\ V$$

and

$$i = 30e^{-50t}\ A, \qquad t > 0$$

a) Find L and R.

$v = L\frac{di}{dt}$

$\cancel{90e^{-50t}} = L(30)(-50)\cancel{e^{-50t}}$

$L = 0.06\ H$ #

$R = \frac{V}{I} = \frac{\cancel{90}^{3}\cancel{e^{-50t}}}{\cancel{30}\cancel{e^{-50t}}} = 3\,\Omega$ #

b) Determine the time constant.

$\tau = \frac{L}{R} = \frac{0.06}{3} = 0.02\ s$ #

c) Calculate the initial energy in the inductor.

$w = \frac{1}{2}Li^2(0)$

$= \frac{1}{2}(0.06)(30)^2$

$w = 27\ J$ #

d) What fraction of the initial energy is dissipated in 10 ms?

$W_{10ms} = \frac{1}{2}Li^2(0)$

$= \frac{1}{2}(0.06)(30e^{-50(0.01)})^2$

$= 9.93\ J$

$W_{dissipated} = W_L(0) - W_L(10ms)$

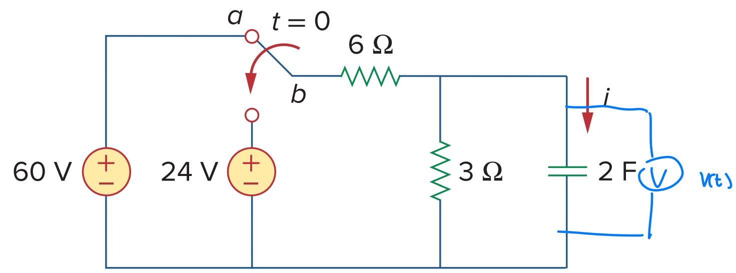
$= 27 - 9.93$

$= 17.07\ J$ #

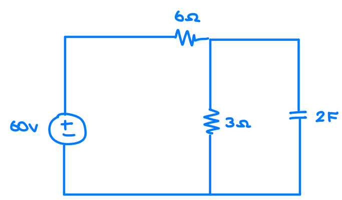
$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$

Practice Problem 3

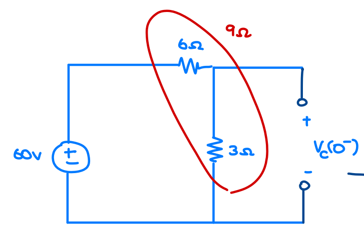
The switch in circuit shown below has been in position a for a long time. At $t = 0$, it moves to position b . Calculate $i(t)$ for all $t > 0$.

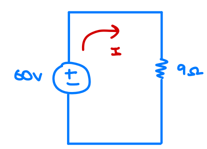


t<0



⇒ under DC





$V = IR$

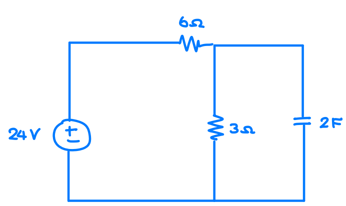
$I = \frac{V}{R}$

$= \frac{60}{9}$

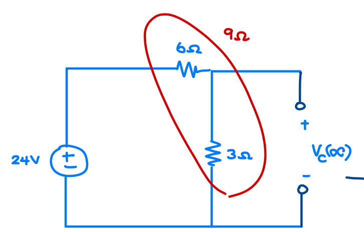
$I = \frac{20}{3}$ A

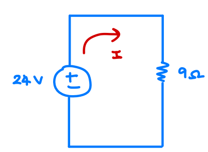
$V_c(0^-) = V_{3\Omega} = I(3) = \frac{20(\cancel{3})}{\cancel{3}} = 20$ V

t>0



⇒ under DC





$V = IR$

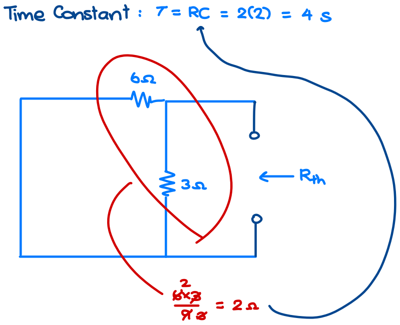
$I = \frac{V}{R}$

$= \frac{24}{9}$

$I = \frac{8}{3}$ A

$V_c(\infty) = V_{3\Omega} = I(3) = \frac{8(\cancel{3})}{\cancel{3}} = 8$ V

Time Constant : $\tau = RC = 2(2) = 4$ s



$\frac{\cancel{6}^{2} \times \cancel{3}}{\cancel{9}\,\cancel{3}} = 2\Omega$

$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$

$= 8 + [20 - 8]e^{-t/4}$

$V(t) = 8 + 12e^{-t/4}$

$I(t) = C\frac{dv}{dt}$

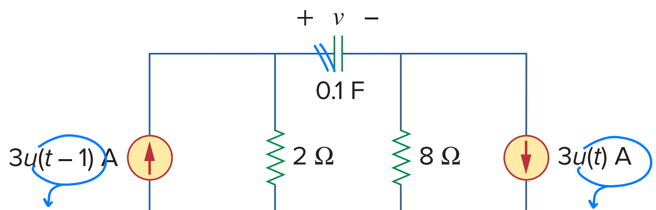
$= \cancel{2}(-\frac{1}{\cancel{4}_{2}})(\cancel{12}^{6}e^{-t/4})$

$I(t) = -6e^{-t/4}$ V #

$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$

Practice Problem 4

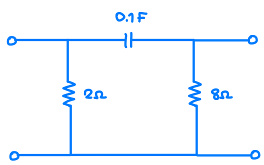
Determine $v(t)$ for $t > 0$ in the circuit shown below if $v(0) = 0$.



3 ; t ≥ 1
0 ; otherwise

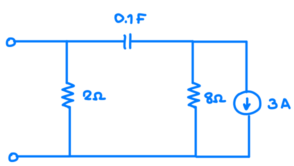
3 ; t ≥ 0
0 ; otherwise

t < 0



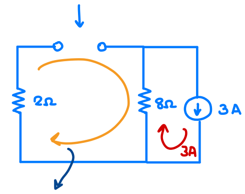
$V(0^-) = 0$

t > 0



⇒ under DC

$V(0^+)$

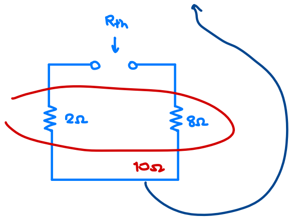


KVL: $V + (-3)(8) + 0 = 0$

$V = 24$ Volt

Time Constant : $\tau = RC = 10(0.1) = 1$

R_{th}



$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$

$= 24 + [0 - 24]e^{-t/1}$

$= 24 - 24e^{-t}$

$= 24(1 - e^{-t})$ V $(1 > t > 0)$

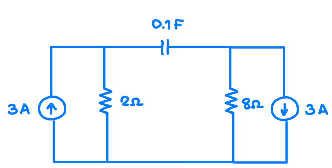
จุดเชื่อม $t = 1$ $(t < 1)$

$\therefore V(1) = 24(1 - e^{-1})$ V

$= 15.17$ V

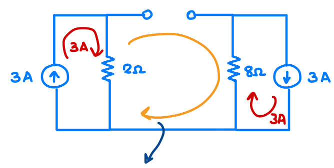
---

t > 1



⇒ under DC

$V(1^+)$

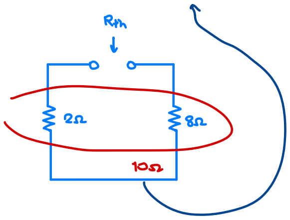


KVL: $V + 8(-3) + 2(-3) = 0$

$V = 30$ Volt

Time Constant : $\tau = RC = 10(0.1) = 1$

R_{th}



$V(t-1) = V(\infty) + [V(0) - V(\infty)]e^{-(t-1)/\tau}$

$= 30 + [15.17 - 30]e^{-(t-1)/\tau}$

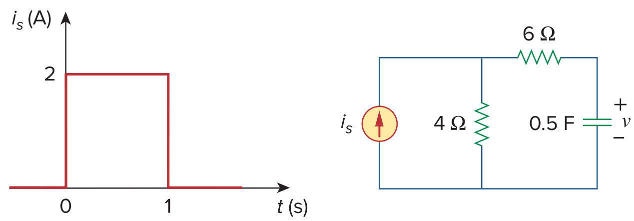
$= 30 - 14.83e^{-(t-1)}$ V $(t > 1)$

$$V(t) = \begin{cases} 24(1 - e^{-t}) \text{ V} & ; 0 < t < 1 \\ 30 - 14.83e^{-(t-1)} \text{ V} & ; t > 1 \end{cases}$$ #

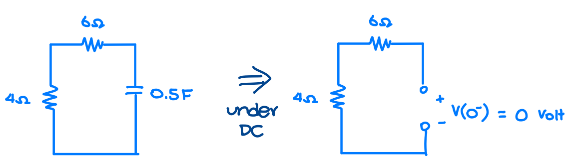
$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$

Practice Problem 5

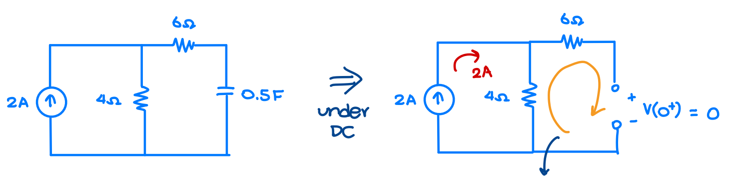
If the waveform in left is applied to the circuit on the right, find $v(t)$. Assume $v(0) = 0$.



$t < 0$



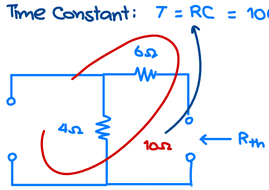
$t > 0$



KVL: $4(-2) + V(0^+) = 0$

$V(0^+) = 8$ Volt

Time Constant: $\tau = RC = 10(0.5) = 5$ s



$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$

$= 8 + [0 - 8]e^{-t/5}$

$= 8 - 8e^{-t/5}$

$= 8(1 - e^{-t/5})$

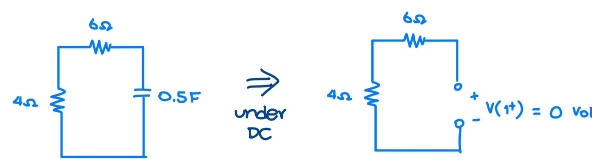
t เชื่อม; $t = 1$

$V(1) = 8(1 - e^{-t/5})$

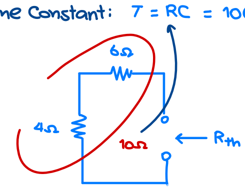
$= 8(1 - e^{-1/5})$

$V(1) = 1.45$ Volt

$t > 1$



Time Constant: $\tau = RC = 10(0.5) = 5$ s



$V(t-1) = 0 + [1.45 - 0]e^{-(t-1)/5}$

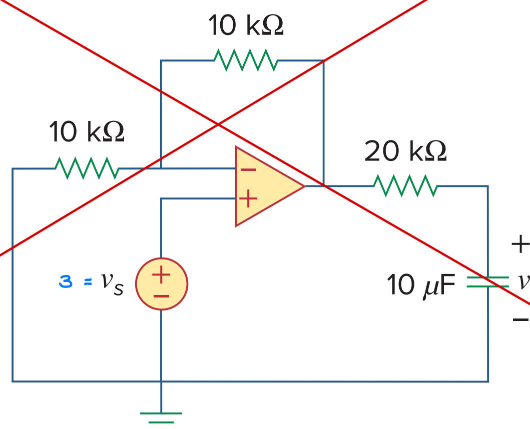
$V(t-1) = 1.45e^{-(t-1)/5}$

$$V(t) = \begin{cases} 8(1 - e^{-t/5})\ \text{V} & (0 < t < 1) \\ 1.45e^{-(t-1)/5}\ \text{V} & (t > 1) \end{cases}$$ #

ไม่น่าออก!

Practice Problem 6

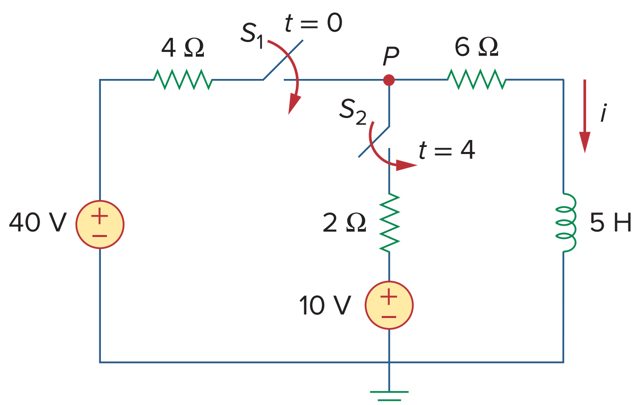
For the op amp circuit, suppose $v(0) = 0$ and $v_s = 3$ V. Find $v(t)$ for $t > 0$.



$I(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$

Practice Problem 7

At $t = 0$, switch 1 is closed, and switch 2 is closed 4 s later. Find $i(t)$ for $t > 0$. Calculate i for $t = 2$ s and $t = 5$ s.



$$I(t) = \begin{cases} & 0 \leq t \leq 4 \\ \frac{30}{11} + 1.2707e^{-22(t-4)/15} & t \geq 4 \end{cases}$$

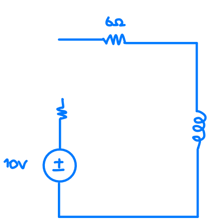
$I(2) = 4(1 - e^{-2\cdot 2})$

$= 3.926$ A #

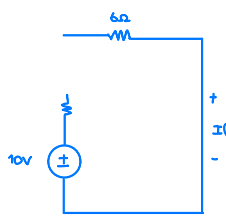
$I(5) = \frac{30}{11} + 1.2707e^{-22(5-4)/15}$

$= 3.02$ A #

$t < 0$

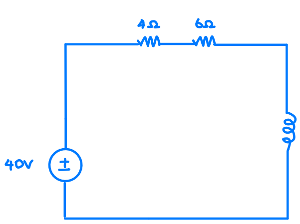


⇒ under DC

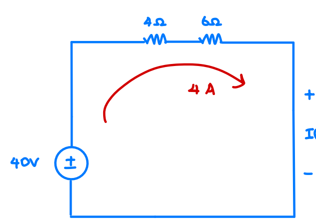


$I(0) = 0$ A

$t > 0$



⇒ under DC

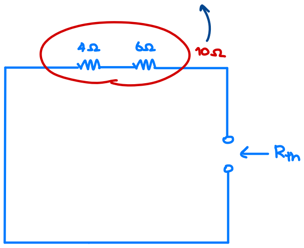


$I(0^+) = 4$ A

$V = IR$

$I = \frac{V}{R} = \frac{40}{10} = 4$ A

Time Constant: $\tau = \frac{L}{R} = \frac{5}{10} = \frac{1}{2}$ s



$I(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$

$= 4 + [0 - 4]e^{-t/(1/2)}$

$= 4 - 4e^{-2t}$

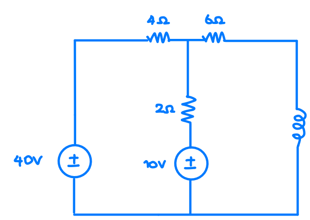
$I(t) = 4(1 - e^{-2t}) \quad 4 > t > 0$

เมื่อ $t = 4$

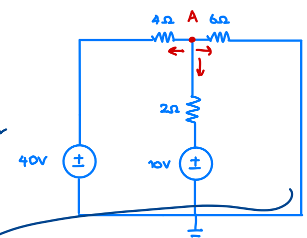
$I(4) = 4(1 - e^{-2\cdot 4})$

$I(4) = 3.998$

$t > 4$



⇒ under DC



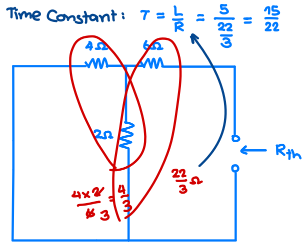
$\frac{V_A - 40}{4} + \frac{V_A - 10}{2} + \frac{V_A}{6} = 0$

$V_A = \frac{180}{11}$ Volt

$I(4^+) = \frac{\frac{180}{11}}{6}$

$= \frac{30}{11}$

Time Constant: $\tau = \frac{L}{R} = \frac{5}{\frac{22}{3}} = \frac{15}{22}$



$I(t-4) = I(\infty) + [I(0) - I(\infty)]e^{-(t-4)/\tau}$

$= \frac{30}{11} + \left[3.998 - \frac{30}{11}\right]e^{-(t-4)/15/22}$

$I(t) = \frac{30}{11} + 1.2707e^{-22(t-4)/15} \quad t \geq 4$

$$I(t) = \begin{cases} 4(1 - e^{-2t}) & 0 \leq t \leq 4 \\ \frac{30}{11} + 1.2707e^{-22(t-4)/15} & t \geq 4 \end{cases}$$ #