DES227: Algorithm Design
Midterm Mock Exam

curated by The Peanuts

Name.............ID.............Section.............Seat No.............

Conditions: Semi-Closed Book

Directions:

1. This exam has 14 pages (including this page).
2. Calculators are allowed.
3. Write your name at the top.
4. Reading the problem is optional but highly recommended.
5. Solutions can be written in English or Thai.
6. If you finish early, congratulations!

This mock exam is based on the material covered in DES227, Section 2.

Problem 1.1

For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Prove your assertions.

a. $2n\lg(n+2)^2 + (n+2)^2 \lg \frac{n}{2}$

$= 2n \cdot 2\lg(n+2)$

$= 4n\lg(n+2)$

$= \Theta(n\lg n)$

$= \frac{(n^2+4n+4)\lg n}{2}$

$= \frac{n^2 \lg n}{2} + 2n\lg n + 2\lg n$

$= \Theta(n^2 \lg n)$

$+$

$\therefore \Theta(n^2 \lg n)$ #

$O(n\lg n^2)$

$O(n^2 \lg n)$

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b. $2^{n+1} + 3^{n-1}$

$= 2 \cdot 2^n$

$= \Theta(2^n)$

$= \frac{3^n}{3}$

$= \Theta(3^n)$

$+$

$\therefore \Theta(3^n)$ #

Problem 1.2

List the following functions according to their order of growth from the lowest to the highest:

$$(n-2)!, \quad 5\lg(n+100)^{10}, \quad 2^{2n}, \quad 0.001n^4 + 3n^3 + 1, \quad \ln^2 n, \quad \sqrt[3]{n}, \quad 3^n.$$

$O(n!)$ $O(\lg n)$ $O(4^n)$ $O(n^4)$ $O(\ln^2 n)$ $O(\sqrt{n})$ $O(3^n)$

7 2 6 4 1 3 5

(เร็ว)
น้อย

Class	Name
1	constant
$\log n$	logarithmic
n	linear
$n \log n$	linearithmic
n^2	quadratic
n^3	cubic
2^n	exponential
$n!$	factorial

มาก
(ช้า)

lowest $\longrightarrow$ highest

1 7

$\ln(x) > \log_{10}(x) \quad x > 1$

$\ln(x) < \log_{10}(x) \quad 0 < x < 1$

$\ln^2 n = (\ln n)^2$ faster!

$\log n = \dfrac{\ln n}{\ln 2}$

Problem 2.1

Find the order of growth of the following sums. Use the $\Theta(g(n))$ notation with the simplest function g(n) possible.

a. $\sum_{i=1}^{n}(i+1)2^{i-1}$

$$\sum_{i=1}^{n} i \cdot 2^{i-1} + \sum_{i=1}^{n} 2^{i-1}$$

$$2^{i-1} = 2^0 + 2^1 + 2^2 + \ldots + 2^{n-1}$$

$$= \frac{2^n - 1}{2-1} = 2^n - 1$$

$$= \Theta(2^n)$$

Standard formular : $\approx (n-1)2^n$

$$= \Theta(n2^n)$$

$\Theta(n2^n)$ #

b. $\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j)$

$$\sum_{j=0}^{i-1} i + \sum_{j=0}^{i-1} j$$

$$= i^2 + \frac{(i-1)i}{2}$$

$$\sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} \frac{(i-1)i}{2}$$

formular: $= \dfrac{(n-1)n(2n-1)}{6}$

$$= \Theta(n^3)$$

$$\frac{1}{2}\sum_{i=0}^{n-1}(i^2 - i)$$

$$= \frac{1}{2}\cdot\left[\frac{(n-1)n(2n-1)}{6} - \frac{(n-1)n}{2}\right]$$

$$= \frac{1}{2}\cdot\left[\frac{(n-1)n}{6}((2n-1)-3)\right]$$

$$= \frac{1}{2}\cdot\left[\frac{(n-1)n(2n-4)}{6}\right]$$

$$= \Theta(n^3)$$

$\therefore\ \Theta(n^3)$ #

Problem 2.2

Consider the following algorithm.

ALGORITHM $Secret(A[0..n-1])$
//Input: An array $A[0..n-1]$ of n real numbers
$minval \leftarrow A[0]; maxval \leftarrow A[0]$
for $i \leftarrow 1$ to $n-1$ **do**
if $A[i] < minval$
$minval \leftarrow A[i]$
if $A[i] > maxval$
$maxval \leftarrow A[i]$
return $maxval - minval$

a. What does this algorithm compute?

computes the range of the given array.

b. What is its basic operation?

2 comparision

c. How many times is the basic operation executed?

$$\sum_{i=1}^{n-1} 2$$

$$= 2 \cdot (n-1) \quad \#$$

d. What is the efficiency class of this algorithm?

$$\Theta(n)$$

e. Suggest an improvement, or a better algorithm altogether, and indicate its efficiency class. If you cannot do it, try to prove that, in fact, it cannot be done.

Problem 3

Consider an instance of the stable marriage problem given by the following ranking matrix:

	A	B	C
α	1,3	2,2	3,1
β	3,1	1,3	2,2
γ	2,2	3,1	1,3

For each of its marriage matchings, indicate whether it is stable or not. For the unstable matchings, specify a blocking pair. For the stable matchings, indicate whether they are man-optimal, woman-optimal, or neither. (Assume that the Greek and Roman letters denote the men and women, respectively.)

	men		
α	A	B	C
β	B	C	A
γ	C	A	B

	Woman		
A	β	γ	α
B	γ	α	β
C	α	β	γ

$\therefore \{(\alpha, A), (\beta, B), (\gamma, C)\}$ #

Problem 4

A database has five transactions. Let min sup = 60% and min conf = 80%.

TID	items bought
T100	{M, O, N, K, E, Y}
T200	{D, O, N, K, E, Y}
T300	{M, A, K, E}
T400	{M, U, C, K, Y}
T500	{C, O, O, K, I, E}

$\underbrace{\text{O, O}}_{1}$

min

$\frac{\cancel{60}^{\,3}}{\cancel{100}_{\,20}} \times \cancel{5} = 3$

a. Find all frequent itemsets using Apriori and ~~FP-growth~~, respectively. Compare the efficiency of the two mining processes.

$\frac{5!}{2!(5-2)!} = 10$

1-item	Sup
M	3
O	3
~~N~~	~~2~~
K	5
E	4
Y	3
~~D~~	~~1~~
~~A~~	~~1~~
~~U~~	~~1~~
~~C~~	~~2~~
~~I~~	~~1~~

→

2 item	Sup
~~MO~~	~~1~~
MK	3
~~ME~~	~~2~~
~~MY~~	~~2~~
OK	3
OE	3
~~OY~~	~~2~~
~~EY~~	~~2~~
KY	3
KE	4

→

3-item	Sup
~~MKO~~	~~1~~
OKE	3
~~KEY~~	~~2~~
~~MKE~~	~~2~~
~~MKY~~	~~2~~
~~OKY~~	~~2~~
~~OEY~~	~~2~~

Frequent Itemset = {O, K, E}

{O, K} → {E} // Confidence = 3/3 × 100 = 100% ✓
{O, E} → {K} // C = 3/3 × 100 = 100% ✓
{E, K} → {O} // C = 3/4 × 100 = 75%.
{O} → {K, E} // C = 3/3 × 100 = 100% ✓
{K} → {O, E} // C = 3/5 × 100 = 60%
{E} → {O, K} // C = 3/4 × 100 = 75%.

There are 3 strong results.

b. List all the strong association rules (with support s and confidence c) matching the following metarule, where X is a variable representing customers, and item denotes variables representing items (e.g., "A," "B,"): ∀x ∈ transaction, buys(X,item1) ∧ buys(X,item2) ⇒ buys(X,item3) [s,c]

x → y

freq(O^K)

{O, K} → {E} [S = 3/6 = 0.5 , C = 3/3 × 100 = 100%] ✓
{O, E} → {K} [S = 3/6 = 0.5 , C = 3/3 × 100 = 100%] ✓
{O} → {K, E} [S = 3/6 = 0.5 , C = 3/3 × 100 = 100%] ✓

freq(O^K^E) #tran section freq(O) freq(O^K^E)

Rule: $X \Rightarrow Y$

$Support = \frac{frq(X,Y)}{N}$

$Confidence = \frac{frq(X,Y)}{frq(X)}$

$Lift = \frac{Support}{Supp(X) \times Supp(Y)}$

Problem 4

A database has five transactions. Let min sup = 60% and min conf = 80%.

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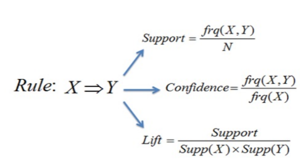
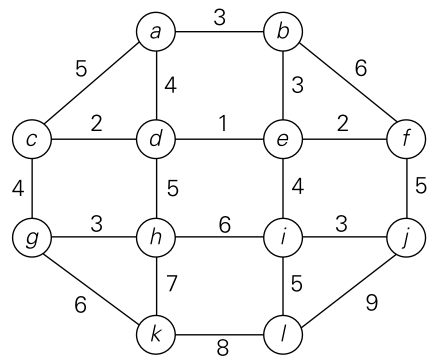
T100	{M, O, N, K, E, Y}
T200	{D, O, N, K, E, Y}
T300	{M, A, K, E}
T400	{M, U, C, K, Y}
T500	{C, O, O, K, I, E}

a. Find all frequent itemsets using Apriori and FP-growth, respectively. Compare the efficiency of the two mining processes.

b. List all the strong association rules (with support s and confidence c) matching the following metarule, where X is a variable representing customers, and item denotes variables representing items (e.g., "A," "B,"): ∀x ∈ transaction, buys(X,item1) ∧ buys(X,item2) ⇒ buys(X,item3) [s,c]

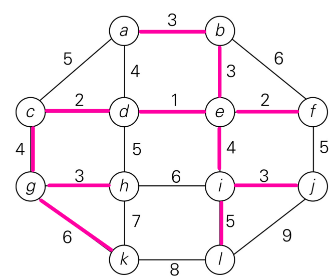


Problem 5

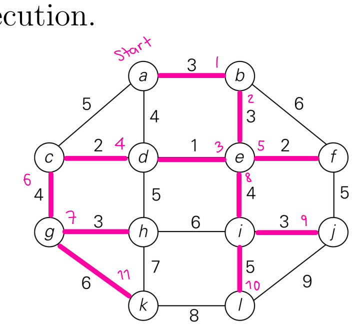


a. Apply Kruskal's algorithm to find a minimum spanning tree of the following graph. Show all steps of the algorithm's execution.

de	ef	cd	ij	gh	ab	be	ad	cg	ei	fj	dh	ca	il	bf	gk	hi	hk	kl	lj
1	2	2	3	3	3	3	4	4	4	5	5	5	5	6	6	6	7	8	9
							X			X	X	X		X		X	X	X	X



b. For the same graph, apply Prim's algorithm to find a minimum spanning tree, starting from vertex A. Show all steps of the algorithm's execution.



c. Compare the two resulting minimum spanning trees. Are they identical? If not, explain why different minimum spanning trees can exist for the same graph.

They are Identical.

Problem 6

Consider the instance of continuous knapsack problem with the knapsack capacity 3000 and the item information as follows:

Item	Weight	Value
1	1400	$8400
2	600	$2400
3	800	$8000
4	1000	$5000

Find the most valuable subset of the items that fits into the knapsack. Identify, a case as general as possible, when Greedy Approach is applied to find the most valuable subset of items that fit into a knapsack will get the same solution for both discrete and continuous knapsacks.

Step 1 V_x / W_x

1) $\frac{8400}{1400} = 6$ ②

2) $\frac{2400}{600} = 4$ ④

3) $\frac{8000}{800} = 10$ ①

4) $\frac{5000}{1000} = 5$ ③

Step 2 มาก → น้อย

Step 3 Delete from 3000

1. $W_3 = 800$, $3000 - 800 = 2200$
2. $W_1 = 1400$, $2200 - 1400 = 800$
3. W_4 = takes 800 from 1000 = 0

1000 → 100%
800 → 80%

Step 4 total Value

item 3 : 8000

item 1 : 8400

item 4 : 80% of 5000 = 4000

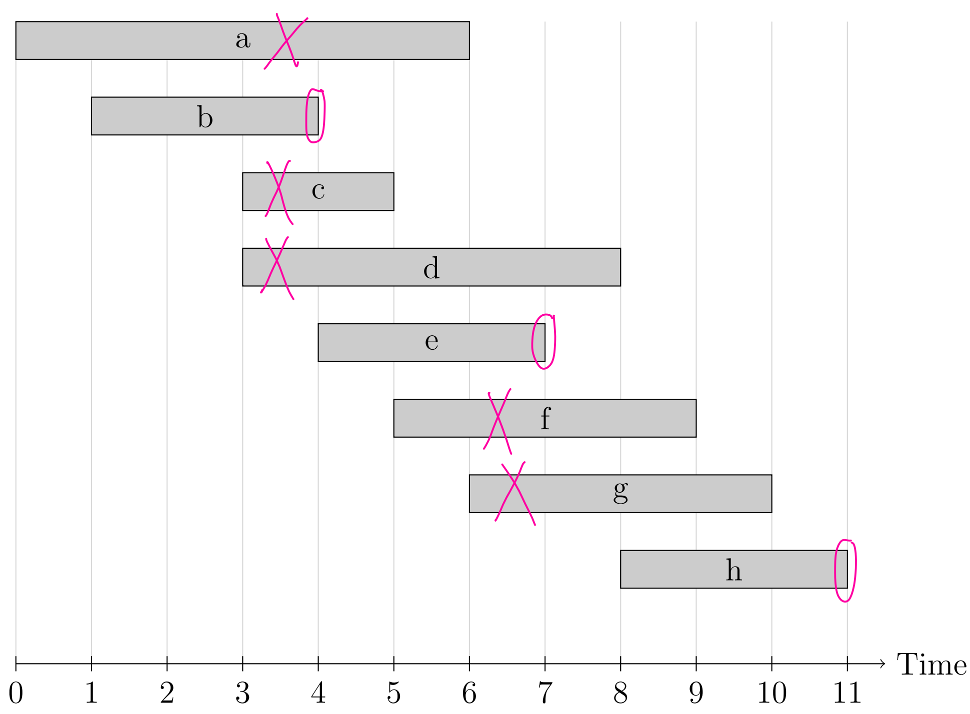
max value = 20,400 #

Problem 7

Interval Scheduling Problem: Consider a set of jobs with specified start and finish times.

- Job j starts at time s_j and finishes at time f_j.
- Two jobs are compatible if they don't overlap.
- **Goal:** Find the maximum subset of mutually compatible jobs.

Given the following jobs with their start and finish times, apply the greedy algorithm to find the maximum subset of mutually compatible jobs. Show all steps of your solution, explaining your selection criteria.



{ b, e, h } #

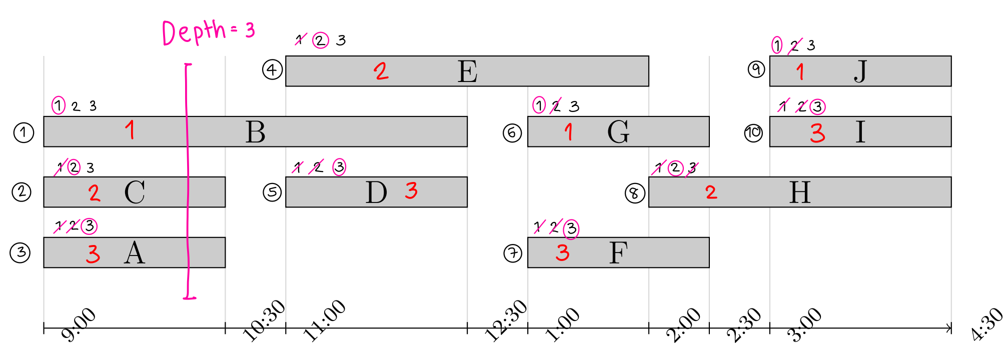
Problem 8

Interval Partitioning Problem: Consider scheduling lectures in classrooms.

- Lecture j starts at time s_j and finishes at time f_j.
- **Goal:** Find the minimum number of classrooms needed to schedule all lectures such that no two lectures occur at the same time in the same room.

Consider the following set of 10 lectures with their start and finish times:

Lecture	Start Time	Finish Time
A	9:00	10:30
B	9:00	12:30
C	9:00	10:30
D	11:00	12:30
E	11:00	2:00
F	1:00	2:30
G	1:00	2:30
H	2:00	4:30
I	3:00	4:30
J	3:00	4:30



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The diagram shows a possible scheduling of all 10 lectures using 4 classrooms.
a. Apply the greedy algorithm for interval partitioning to find the minimum number of classrooms needed. Show your work step by step.

b. Prove that the greedy algorithm produces an optimal solution (minimum number of classrooms) for the interval partitioning problem.

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c. What is the maximum number of lectures that overlap at any point in time in this example? Explain how this relates to the minimum number of classrooms needed.

max 3 Lectures are overlap which is equals to minimum number of classroom.

b/c 1 classroom can not hold overlap lecture .

d. Is the schedule shown in the diagram optimal? If not, provide a scheduling that uses fewer classrooms, or prove that this is the minimum number of classrooms needed.

Depth = 3