Optimization Model

> **Given**
> A directed graph $G=(V,E)$, a type {Method, Entity} $\; c*{i}$ for each node in $V\;$ and a weight $\; w*{ij}$ for each arch in E
>
> **Find**
> A partition of V in k sets ${M_1...M_k} = M $ such that $M_i \cap M_j = \emptyset$ with $i \neq j$ and $\bigcup M_i = V$

Decision Variables

$$x_{ik} = \begin{cases} 1 \text{ if node } i \text{ is in the set } M_k \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

$$y_{ij} = \begin{cases} 1 \text{ if arch } (i,j) \text{ has its endpoints into the same microservice} \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

$$z_{ij}^k = \begin{cases} 1 \text{ if arch } (i,j) \text{ has both its endpoints into the microservice k} \\ 0 \text{ otherwise} \end{cases} \tag{3}$$

Formulation

$$\min \sum_{(i,j)\in E} w_{ij}\,(1-y_{ij}) \qquad \text{(Minimize coupling)} \tag{4}$$

$$\max \sum_k \left(\frac{\sum_{(i,j)\in E} w_{ij} z_{ij}^k}{\sum_{(i,j)\in E} w_{ij} x_{ik}}\right) \qquad \text{(Maximize cohesion)} \tag{5}$$

$$\text{s.t.:} \tag{6}$$

$$\sum_k x_{ik} = 1 \quad \forall i \in V \qquad \text{(Each node belongs to exactly one microservice) (1)} \tag{7}$$

$$\sum_{i|c_i \neq \text{Entity}} x_{ik} \geq 1 \quad \forall M_k \in M \qquad \text{(Each microservice can not contain only nodes of type Entity) (2)} \tag{8}$$

$$\tag{9}$$

$$z_{ij}^k - x_{ik} \leq 0 \quad \forall M_k \in M \forall (i,j) \in E \qquad \text{(Bonding z to x) (3)} \tag{10}$$

$$z_{ij}^k - x_{jk} \leq 0 \quad \forall M_k \in M \forall (i,j) \in E \qquad \text{(Bonding z to x) (4)} \tag{11}$$

$$x_{ik} + x_{jk} - z_{ij}^k \leq 1 \quad \forall M_k \in M \forall (i,j) \in E \qquad \text{(Bonding z to x) (5)} \tag{12}$$

$$\tag{13}$$

$$y_{ij} = \sum_{M_k \in M} z_{ij}^k \quad \forall (i,j) \in E \qquad \text{(Bonding y to z) (6)} \tag{14}$$