

Scalable Quantum Circuit Simulation with CUDA-Q

Dr. Tai-Yue Li
Assistant Researcher
National Center for High-performance Computing (NCHC)
2503001@narlabs.org.tw

About me



- 李泰岳博士 (Dr. Tai-Yue Li)
- 服務單位：國家高速網路與計算中心 (Since 2025/01~)
- 學歷：國立東華大學物理博士
- 專長：
 - X光影像重建、稀疏視角影像重建
 - 機器學習、深度學習
 - 大規模量子計算、量子機器學習、張量網路

Position



The National Science and Technology Council (NSTC)

is the government ministry of the Republic of China (Taiwan) for the promotion and funding of **academic research**, development of science and **technology** and **science parks**.



The National Institutes of Applied Research (NIAR)

is the institution resulted from the combination of national laboratories into an independent nonprofit institute.

One of 7 national-level research laboratories under NARL



國家高速網路與計算中心

National Center for High-performance Computing

Founded In 1991

National Center for High-performance Computing (NCHC)

Taiwan's only national-level supercomputing center.

The NCHC possesses a large computing and networking platform facilities for use by domestic academia and the general public.

Position

新竹本部



台南分部



台中分部



• Milestones

1991

Taiwan's first National level supercomputer Center

2011

御風者
WINDRIDER
177 TF

2017

台灣杉一號
TAIWANIA 1
1.7 PF

2018

台灣杉二號
TAIWANIA 2
9 PF

2021

台灣杉三號
TAIWANIA 3
2.7 PF

2023

TAIDE Project
Trustworthy AI
Dialogue Engine
3.8 PF
NV_H100*9

2024

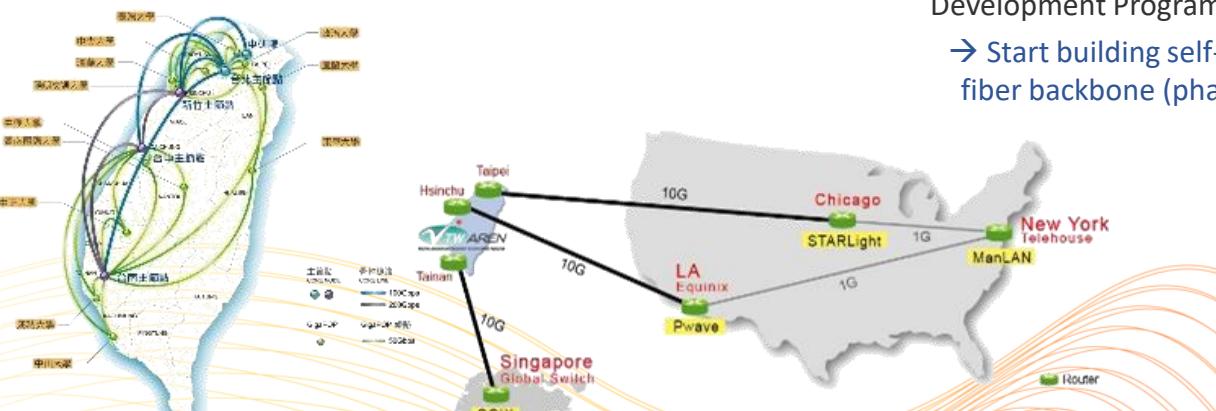
Forerunner 1
is expected to begin services
3.53 PF

2025-2029

Computing power plan
Target 480 PF

2003

NPO under NARLabs



2004

TWAREN Services 10G

2016
100G Network Backbone

Forward-looking Infrastructure Development Program
→ Start building self-build fiber backbone (phase1)

2022

*information security level A
*Formosa Open Internet Exchange activated

2023

self-build fiber backbone (phase1) activated

2025

NCHC IDC (Tainan)
scheduled for opening



• Computing Power Plan

2024-2028

Taiwan CHIP-BASED Industrial Innovation Program

To support the Taiwan Chip-based Industrial Innovation Program, NCHC will develop a shared heterogeneous architecture supercomputer for general-purpose AI (GPU), large-scale scientific research (CPU), and future quantum computing. We aim to create a user-friendly cloud service platform to improve application efficiency.

16 PF_{GPU}

- 400+ GPUs
- 10 PB storage
- InfiniBand 200Gbps
- PUE <1.35

100 PF_{GPU}

- 1680+ GPUs
- 25 PB storage
- InfiniBand 200Gbps
- PUE <1.3

80 PF_{GPU+CPU}

- 1200+ GPUs
- 20 PB storage
- InfiniBand 400Gbps
- PUE <1.27

35 PF_{GPU+CPU}

- 520+ GPUs
- 10 PB storage
- InfiniBand 400Gbps
- PUE <1.25

49 PF_{GPU+CPU+Quantum}

- 700+ GPUs
- 10 PB storage
- InfiniBand 800Gbps
- PUE <1.25

Total
280 PF

2024

2025

2026

2027

2028

2029

Total
200 PF

60 PF_{GPU+CPU}

- 1000+ GPUs
- 15 PB storage
- InfiniBand 400Gbps
- PUE <1.25

75 PF_{GPU+CPU}

- 1260+ GPUs
- 25 PB storage
- InfiniBand 800Gbps
- PUE <1.25

65 PF_{GPU+CPU}

- 1020+ GPUs
- 20 PB storage
- InfiniBand 800Gbps
- PUE <1.25

“New Silicon Valley” in southern Taiwan Project

The National Science and Technology Council announced the launch of a new national program in August 2024, planning to invest in the development of Taiwan's own artificial intelligence platform over the next five years to promote the smart technology industry ecosystem in southern Taiwan. The plan will establish 200 PF of computing power and increase the nation's overall computing power to 480 PF.

2027-2029

National Quantum Team in Taiwan

Phase1 2022~2026 \$250M
Phase2 2026~2030 TBD



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Planning Project for the
Testbed of the Quantum
Computing Subsystems



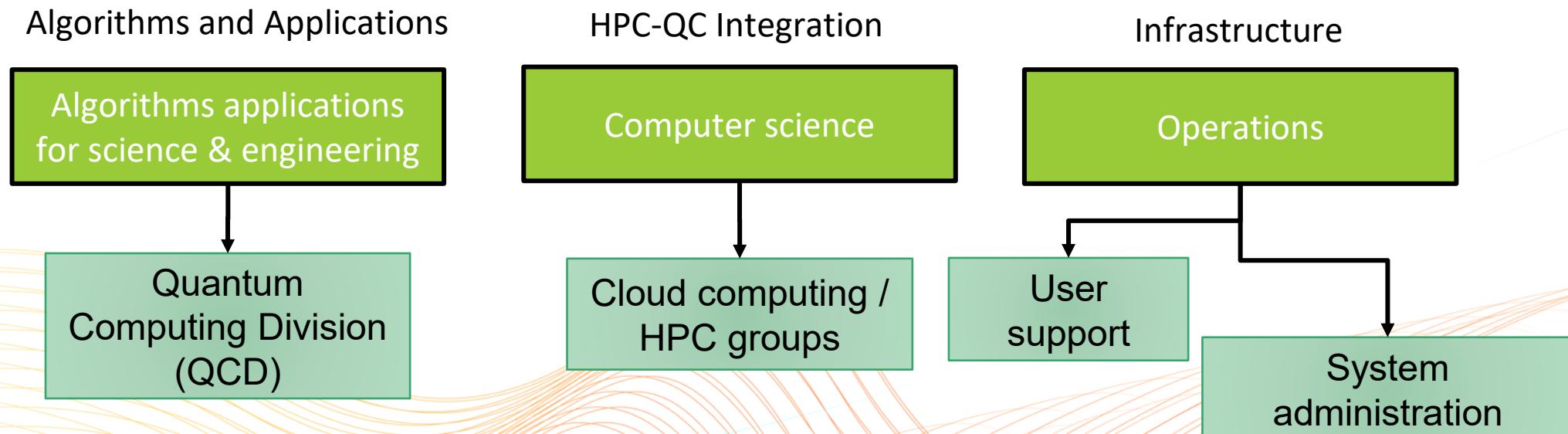
Quantum Simulator
/ QC-HPC

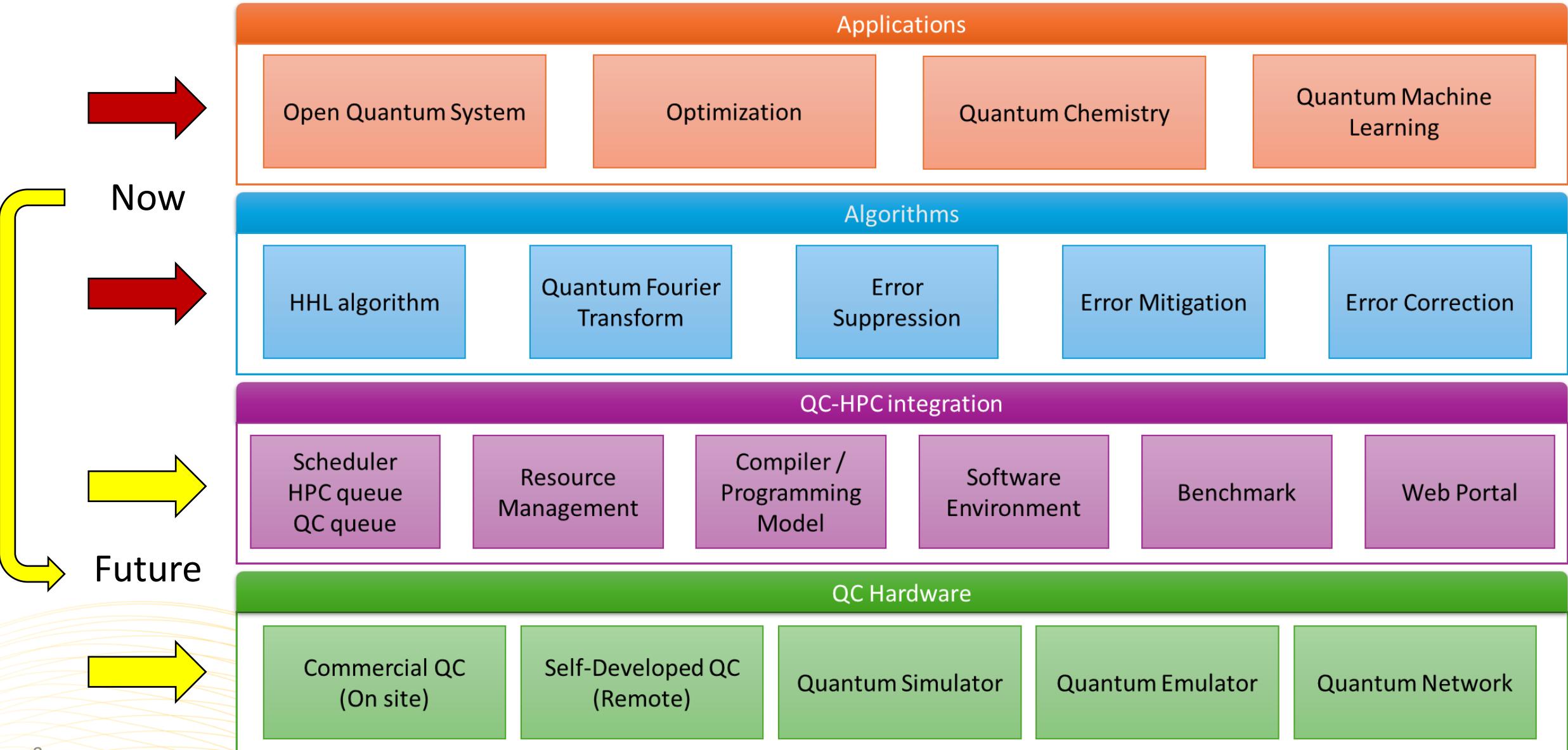
Primary Missions

- Constructing quantum computing systems with practical problem-solving capabilities.
- Developing reliable quantum communication systems.
- Cultivating quantum talent and fostering industrial applications.

Quantum computing in NCHC (planning)

- 負責量子電腦主機的建置，於台灣本土提供量子計算的服務；
- 整合既有HPC資源，提供量子與HPC運算的整合服務；
- 培育新一代量子+HPC計算人才；
- 促進量子計算應用於各種科學研究領域。

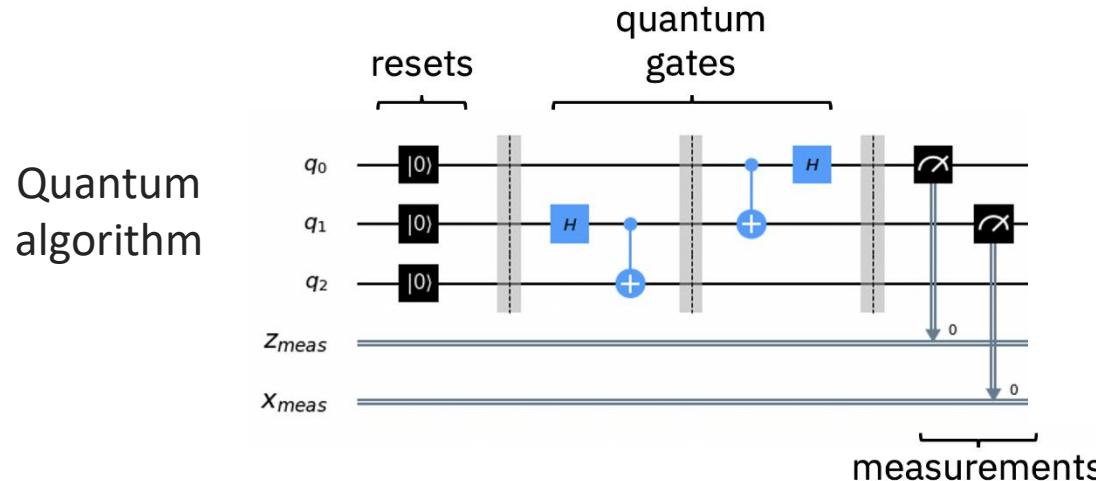




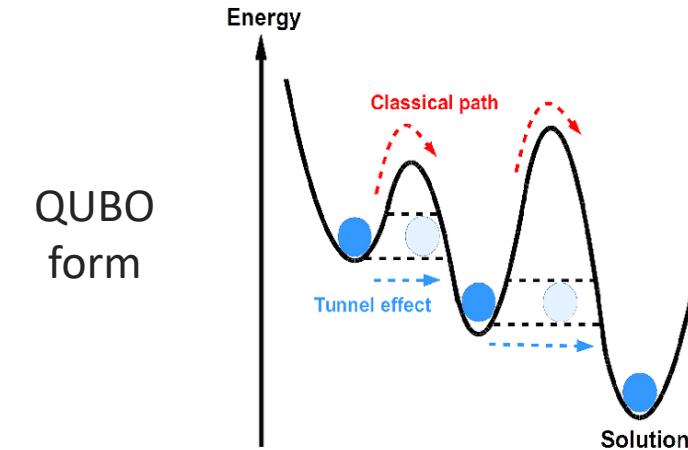
Quantum Computing /Computer Introduction

Gate-based QC vs Annealing-based QC

Gate-based



Annealing-based



Quantum Tunnelling

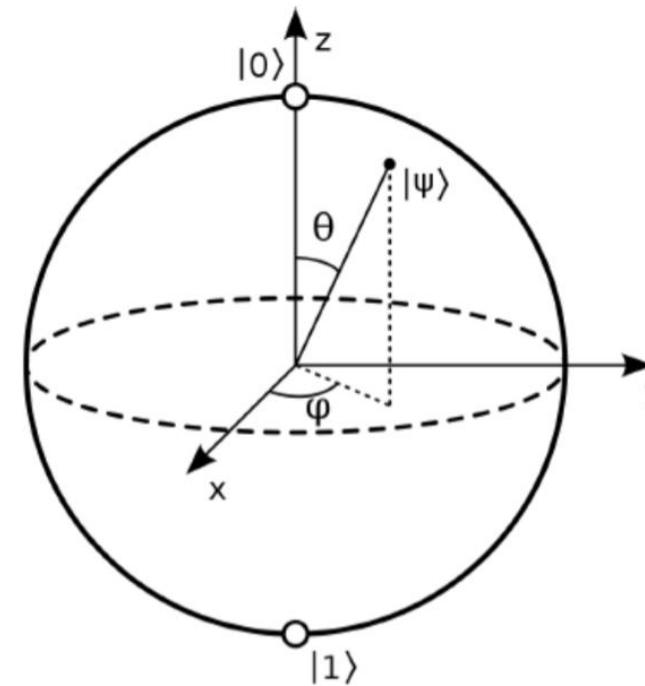


Gate-based QC vs Annealing-based QC

Feature	Gate-Based Method	Annealing-Based Method
Universality	Universal : can implement a wide range of quantum algorithms	Specialized : designed primarily for optimization problems
Algorithm Flexibility	Supports arbitrary quantum circuits (e.g., Shor's, Grover's)	Limited to problems mapped to QUBO/Ising models
Hardware Requirements	Demands precise control and low-noise conditions fewer qubits (~100 qubits)	Can scale to thousands of qubits with simpler control mechanisms D-wave (~5640 qubits)
Robustness to Noise	Highly sensitive to decoherence; requires complex error correction	More robust in its adiabatic evolution against certain noise types
Result Accuracy	Capable of yielding exact results with error correction	Provides approximate, probabilistic solutions

Superposition

Bit



Qubit
(Bloch Sphere)

0

0 1

1 0 1



$$c_0|0\rangle + c_1|1\rangle = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

$$c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle = \begin{bmatrix} c_{00} \\ c_{01} \\ c_{10} \\ c_{11} \end{bmatrix}$$

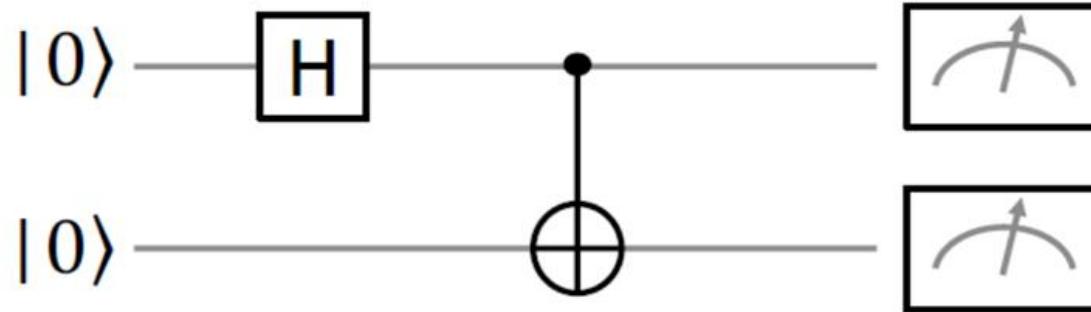
$$c_{000}|000\rangle + c_{001}|001\rangle + c_{010}|010\rangle + c_{011}|011\rangle + c_{100}|100\rangle + c_{101}|101\rangle + c_{110}|110\rangle + c_{111}|111\rangle =$$

$$\begin{bmatrix} c_{000} \\ c_{001} \\ c_{010} \\ c_{011} \\ c_{100} \\ c_{101} \\ c_{110} \\ c_{111} \end{bmatrix}$$

Entanglement

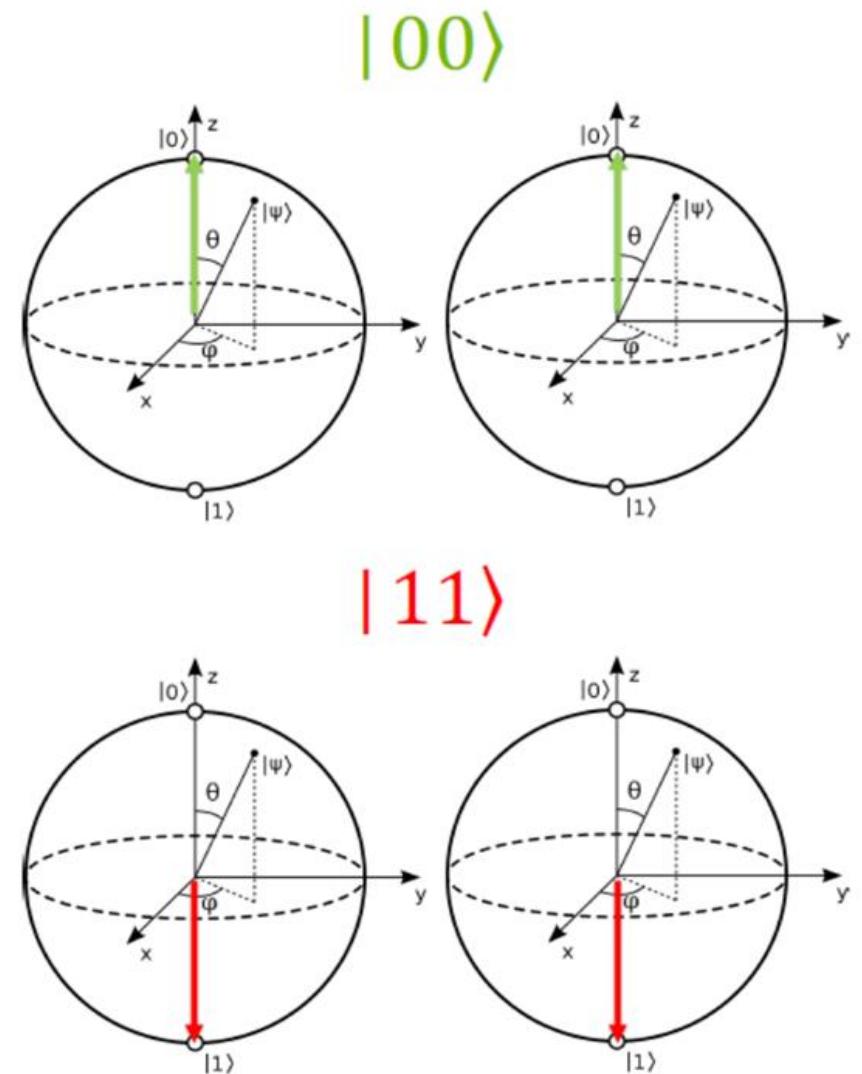
Hadamard Gate:
 $\text{Had}|0\rangle = |0\rangle + |1\rangle$
 $\text{Had}|1\rangle = |0\rangle - |1\rangle$

CNOT Gate:
 $\text{CNOT}|10\rangle = |11\rangle$
 $\text{CNOT}|11\rangle = |10\rangle$



$$|00\rangle \rightarrow |00\rangle + |11\rangle$$

(Bell state)



Single Qubit

二進制： $|0\rangle$ 、 $|1\rangle$

十進制： $|0\rangle$ 、 $|1\rangle$

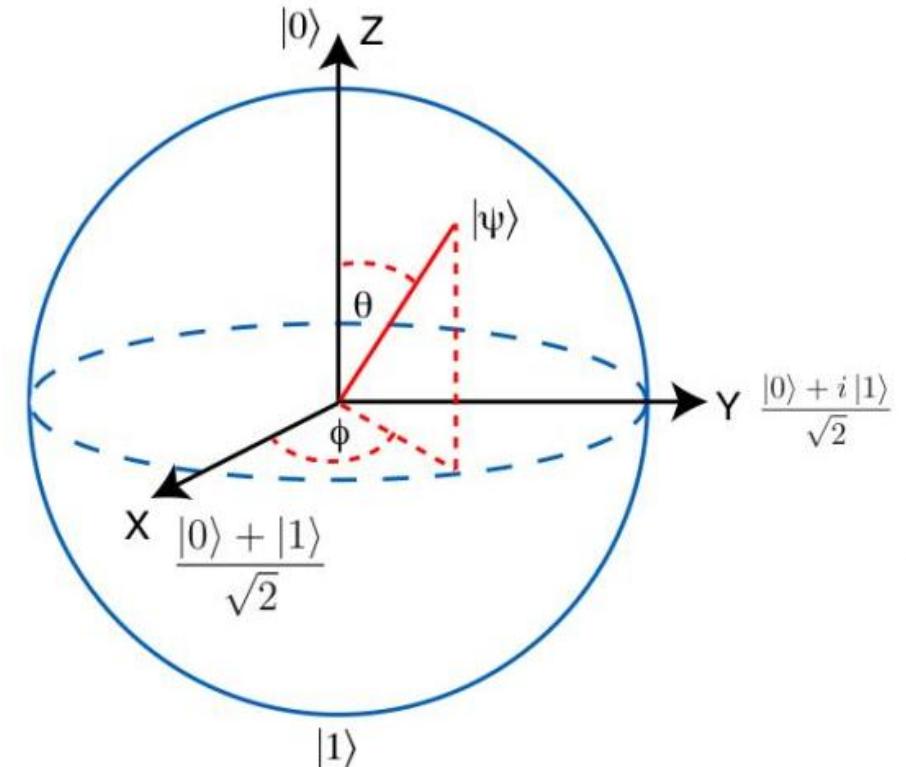
$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$X\text{軸} : \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$Y\text{軸} : \frac{|0\rangle + i|1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$Z\text{軸} : |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Bloch sphere

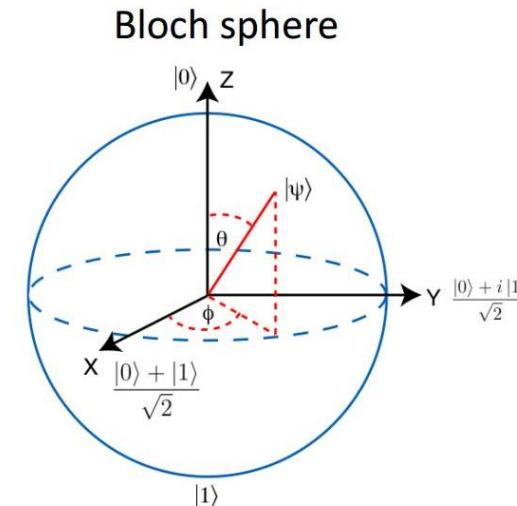


Two Qubits

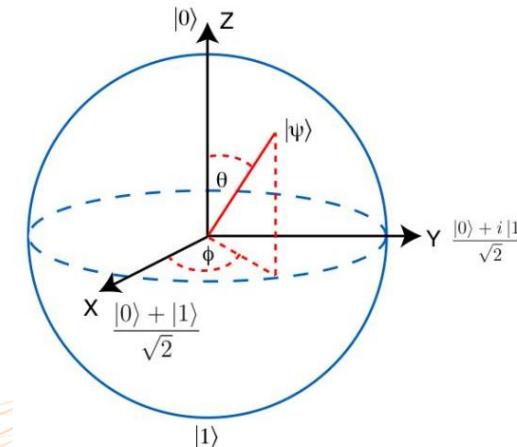
二進制： $|00\rangle$ 、 $|01\rangle$ 、 $|10\rangle$ 、 $|11\rangle$

十進制： $|0\rangle$ 、 $|1\rangle$ 、 $|2\rangle$ 、 $|3\rangle$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$



Bloch sphere



N Qubits

二進制： $|00 \dots 00\rangle$ 、 $|00 \dots 01\rangle$ 、 $|00 \dots 10\rangle$ 、 $|11 \dots 11\rangle$

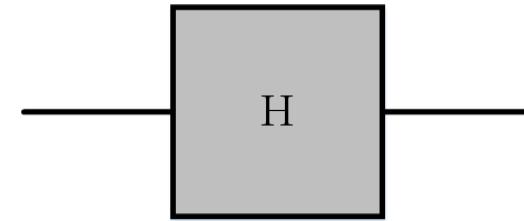
十進制： $|0\rangle$ 、 $|1\rangle$ 、 $|2\rangle$ 、 $|3\rangle$ 、...、 $|2^n - 1\rangle$

$$|00 \dots 00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, |00 \dots 01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, |00 \dots 10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, |11 \dots 11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

Single Qubit Gate

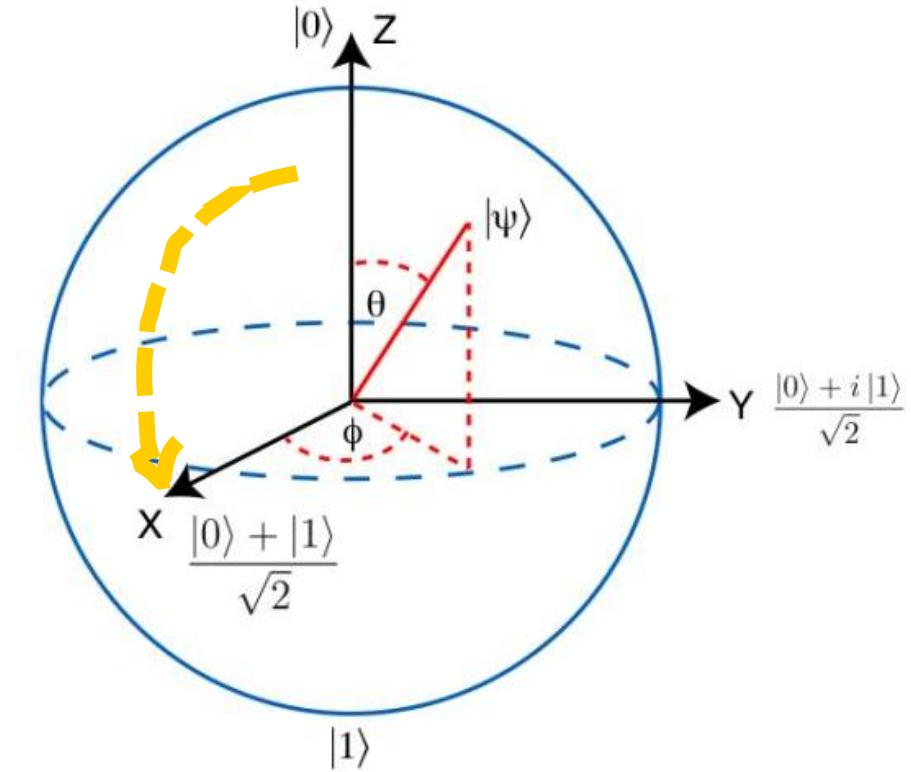
Hadamard Gate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



Input	Output
$ 0\rangle$	$(0\rangle + 1\rangle)/\sqrt{2}$
$ 1\rangle$	$(0\rangle - 1\rangle)/\sqrt{2}$

Bloch sphere



Single Qubit Gate

S Gate

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

S^\dagger Gate

$$S^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

T Gate

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

T^\dagger Gate

$$T^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}$$

U_2 Gate

$$U_2(\varphi, \lambda) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -e^{i\lambda} \\ e^{i\varphi} & e^{i(\lambda+\varphi)} \end{pmatrix}$$

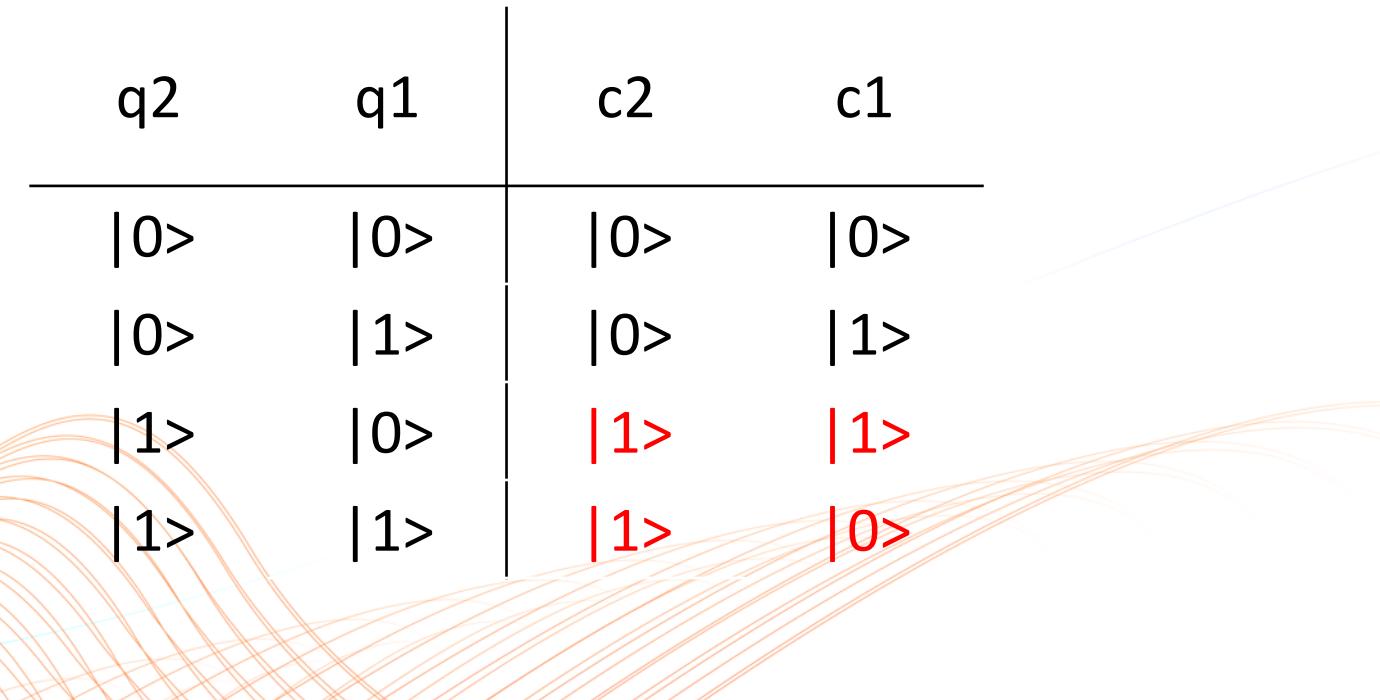
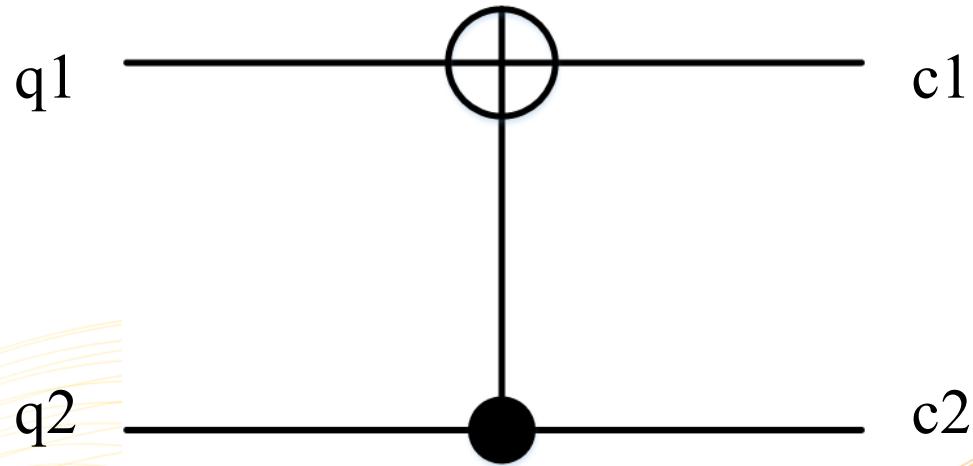
U_3 Gate

$$U_3(\theta, \varphi, \lambda) = \begin{pmatrix} \cos(\theta/2) & -e^{i\lambda} \sin(\theta/2) \\ e^{i\varphi} \sin(\theta/2) & e^{i(\lambda+\varphi)} \cos(\theta/2) \end{pmatrix}$$

Two Qubits Gate

CNOT Gate

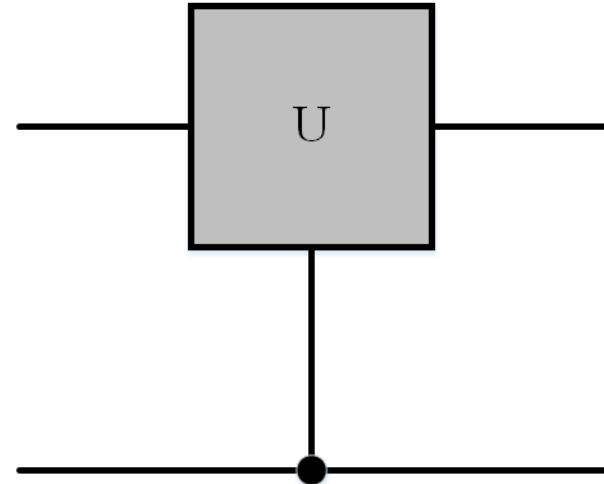
$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



Two Qubits Gate

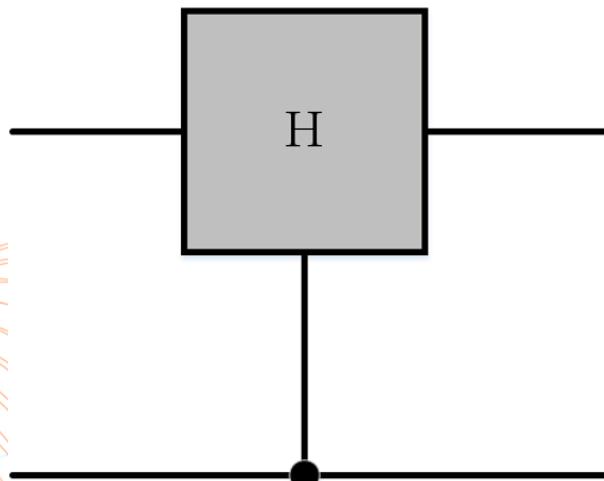
CU Gate

$$CU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & u_{11} & u_{12} \\ 0 & 0 & u_{21} & u_{22} \end{pmatrix}$$



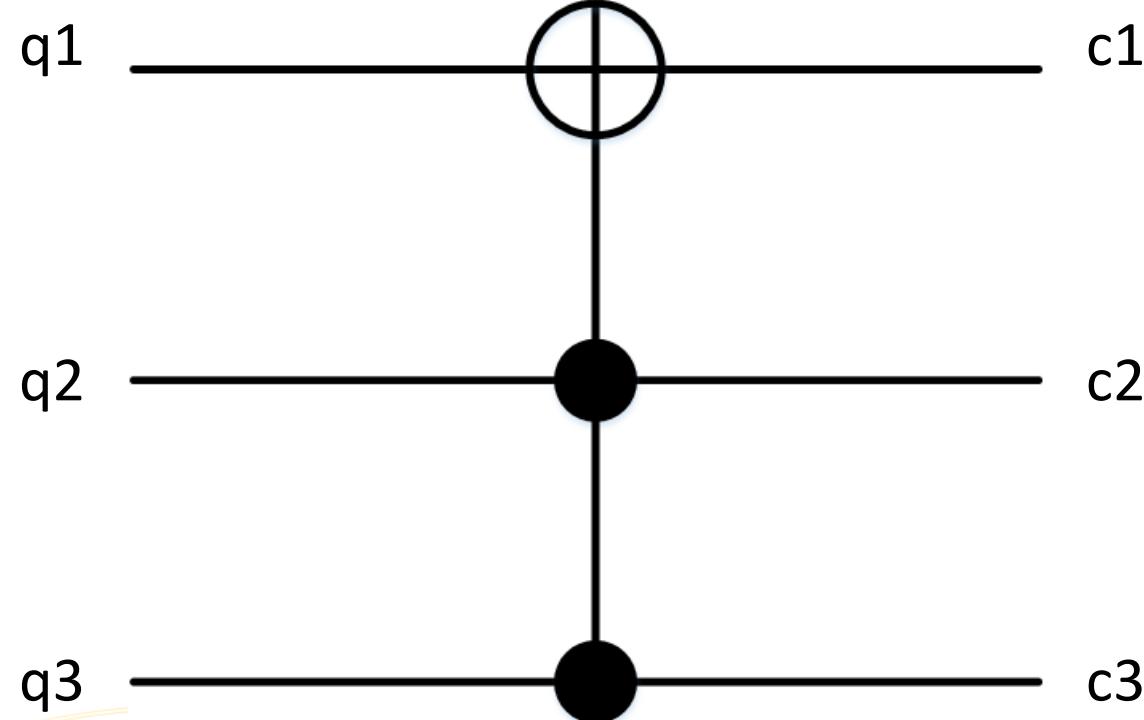
CH Gate

$$CH = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$



Three Qubits Gate

CCNOT Gate

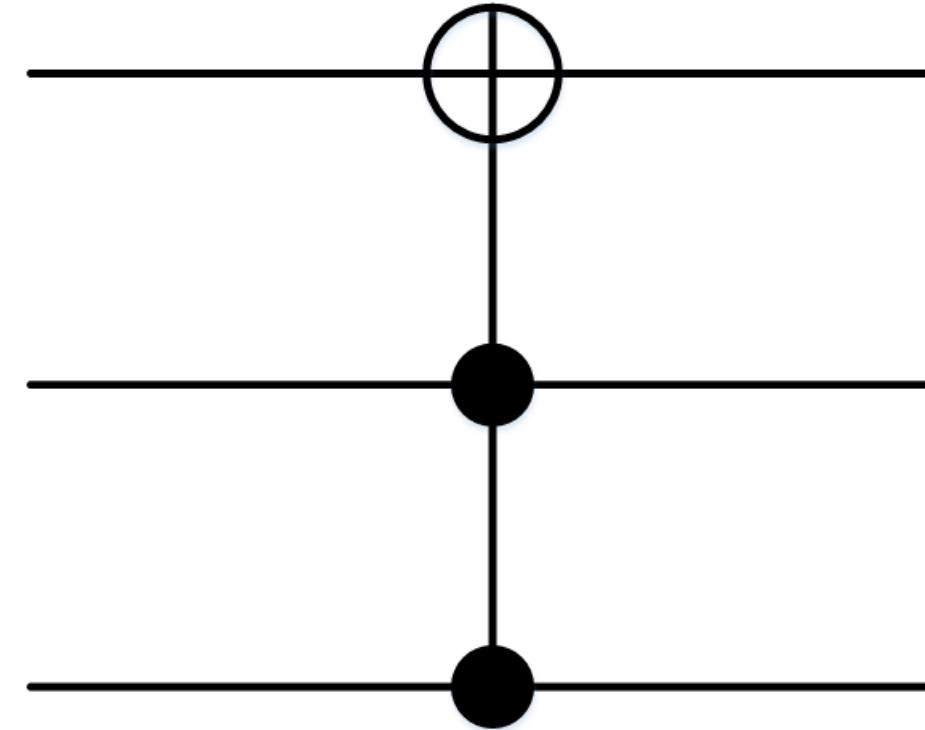


q_3	q_2	q_1	c_3	c_2	c_1
$ 0\rangle$					
$ 0\rangle$	$ 0\rangle$	$ 1\rangle$	$ 0\rangle$	$ 0\rangle$	$ 1\rangle$
$ 0\rangle$	$ 1\rangle$	$ 0\rangle$	$ 0\rangle$	$ 1\rangle$	$ 0\rangle$
$ 0\rangle$	$ 1\rangle$	$ 1\rangle$	$ 0\rangle$	$ 1\rangle$	$ 1\rangle$
$ 1\rangle$	$ 0\rangle$	$ 0\rangle$	$ 1\rangle$	$ 0\rangle$	$ 0\rangle$
$ 1\rangle$	$ 0\rangle$	$ 1\rangle$	$ 1\rangle$	$ 0\rangle$	$ 1\rangle$
$ 1\rangle$	$ 1\rangle$	$ 0\rangle$	$ 1\rangle$	$ 1\rangle$	$ 1\rangle$
$ 1\rangle$	$ 0\rangle$				

Three Qubits Gate

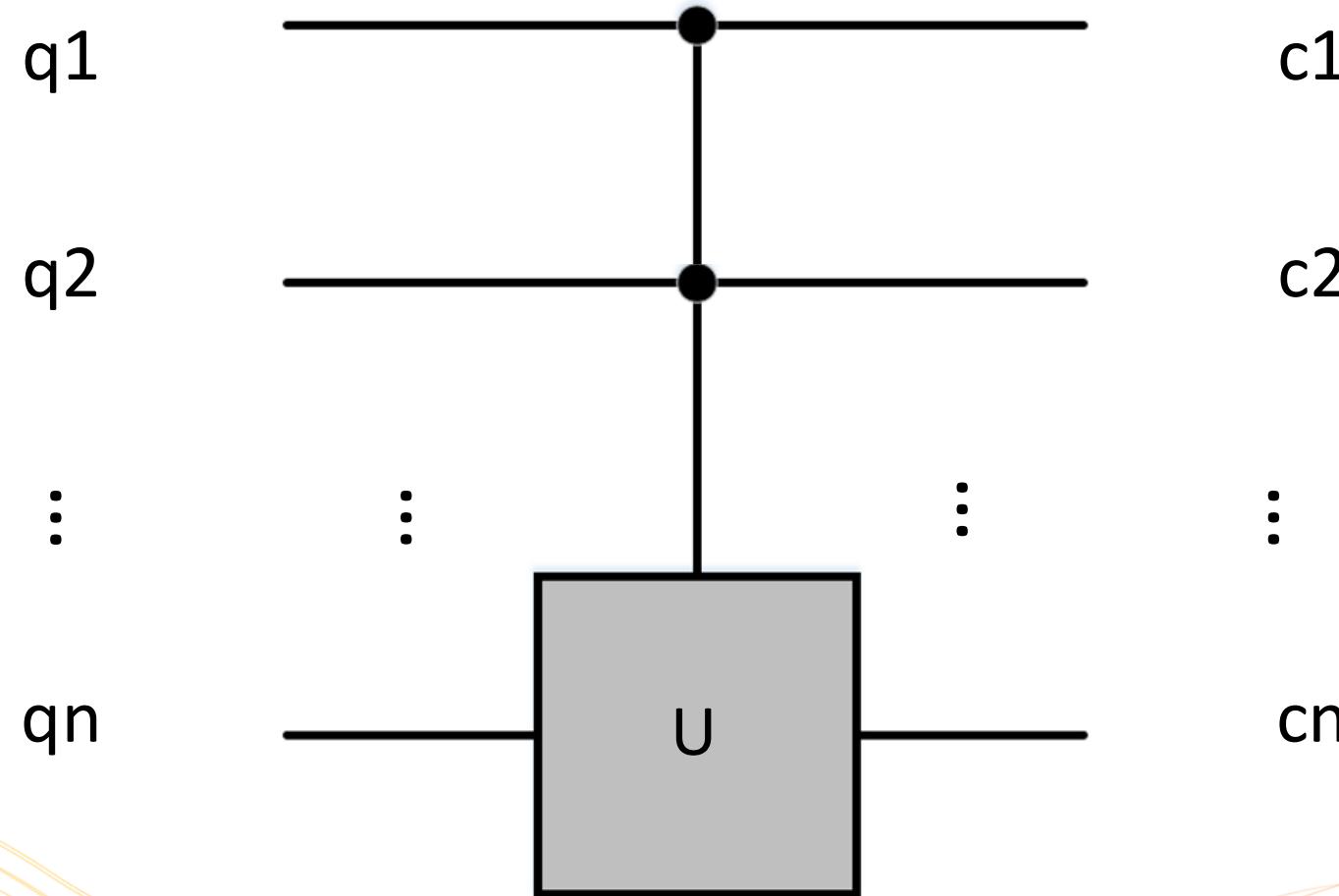
CCNOT Gate

$$CCNOT = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

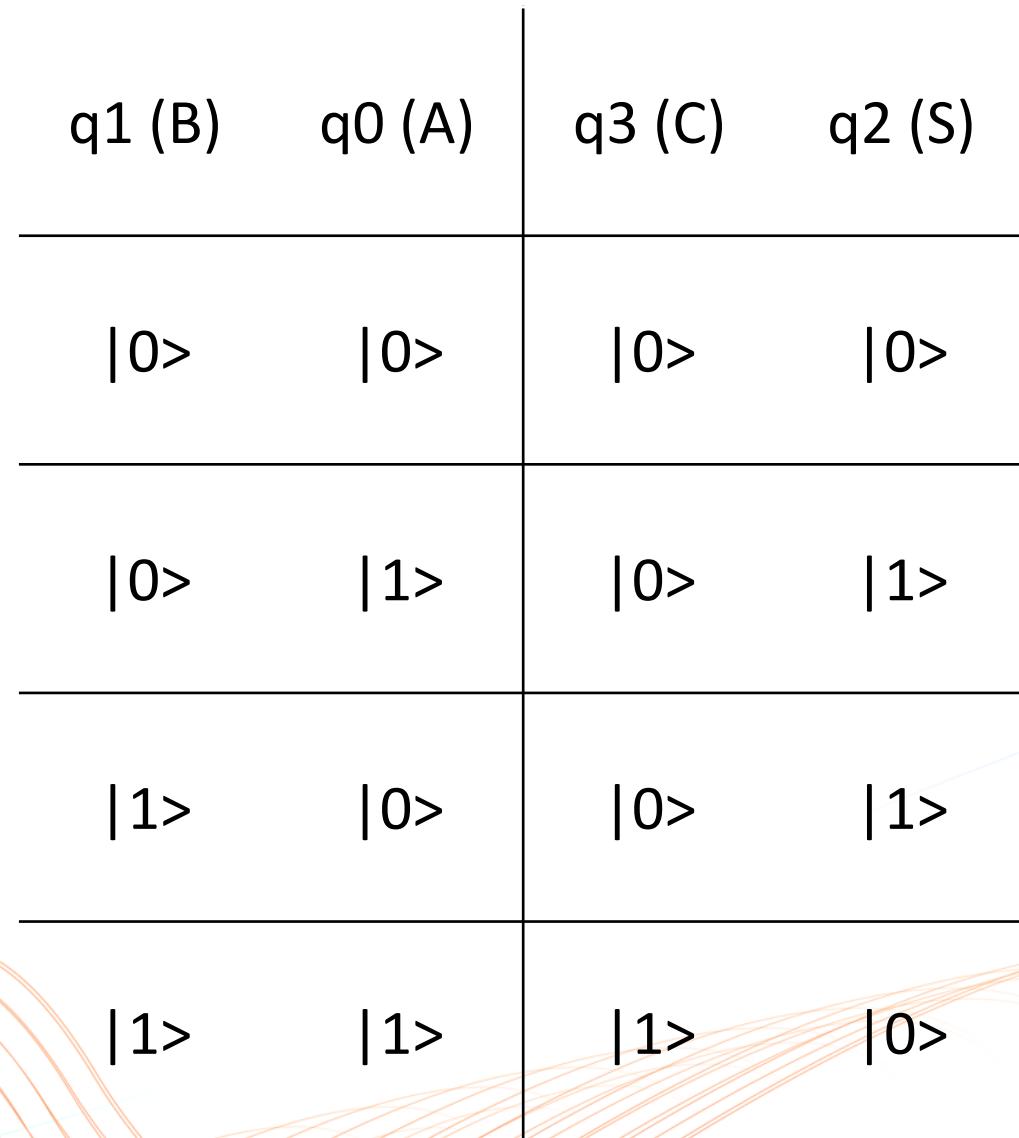
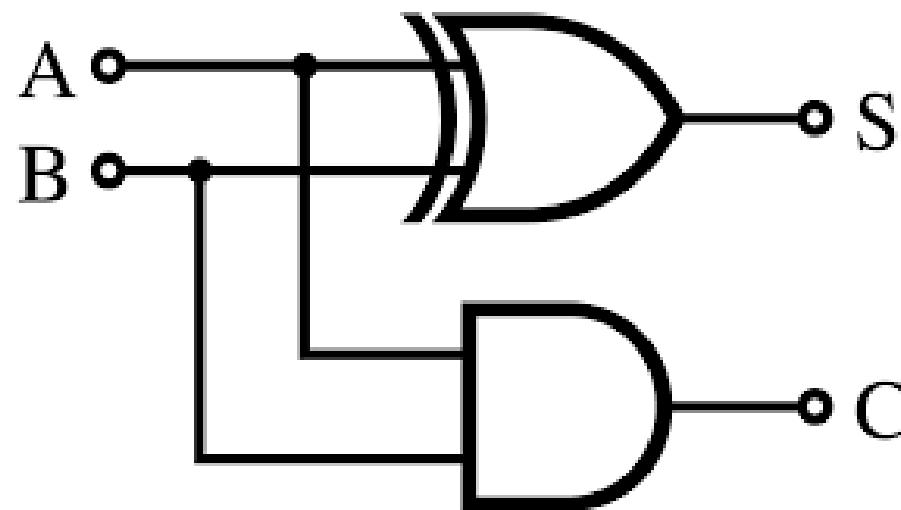
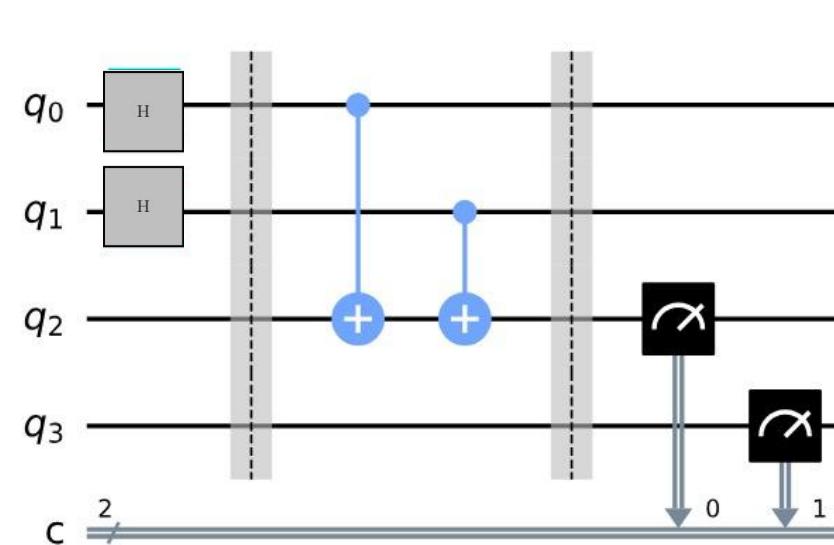


N Qubits Gate

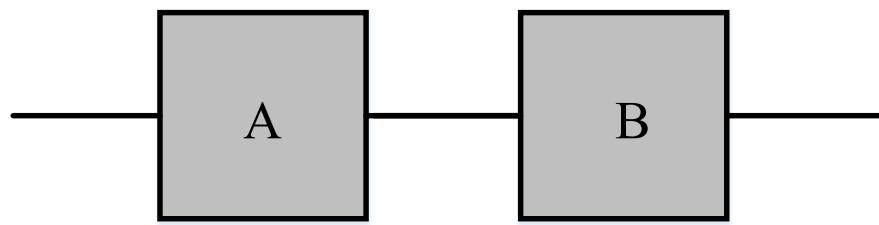
NCU Gate



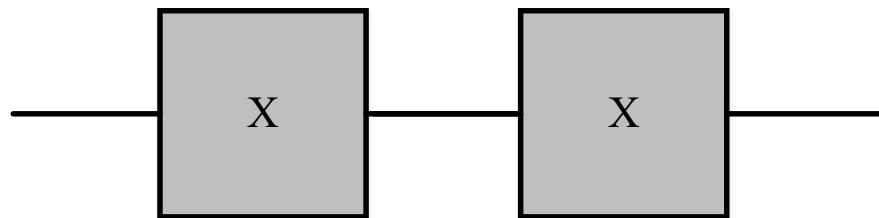
Quantum Circuit



Quantum Circuit Series Connection

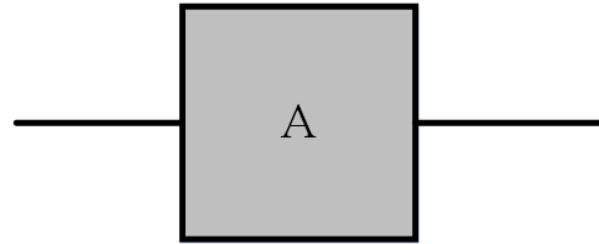


$$B \cdot A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

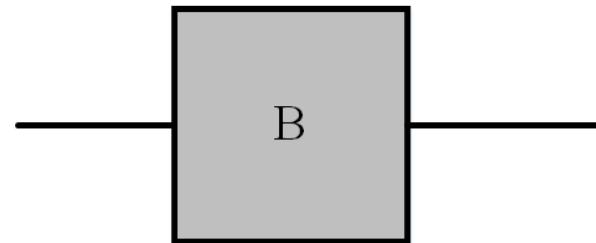


$$x \cdot x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Quantum Circuit Parallel Connection



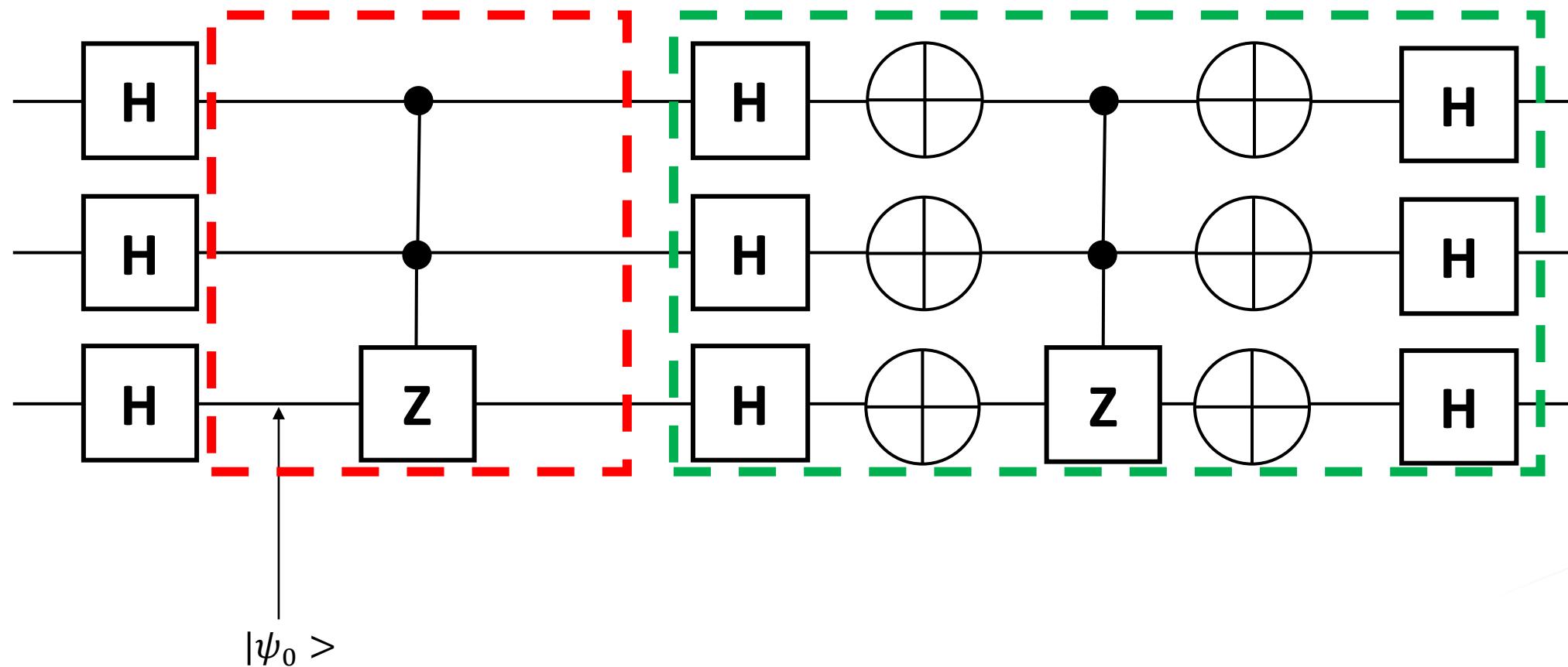
$$C = B \otimes A$$



$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}, \quad B = \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix},$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \otimes \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} = \begin{bmatrix} a_{1,1} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{1,2} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \\ a_{2,1} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} & a_{2,2} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} a_{1,1}b_{1,1} & a_{1,1}b_{1,2} & a_{1,2}b_{1,1} & a_{1,2}b_{1,2} \\ a_{1,1}b_{2,1} & a_{1,1}b_{2,2} & a_{1,2}b_{2,1} & a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} & a_{2,1}b_{1,2} & a_{2,2}b_{1,1} & a_{2,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,1}b_{2,2} & a_{2,2}b_{2,1} & a_{2,2}b_{2,2} \end{bmatrix}.$$

StateVector Simulation



StateVector Simulation

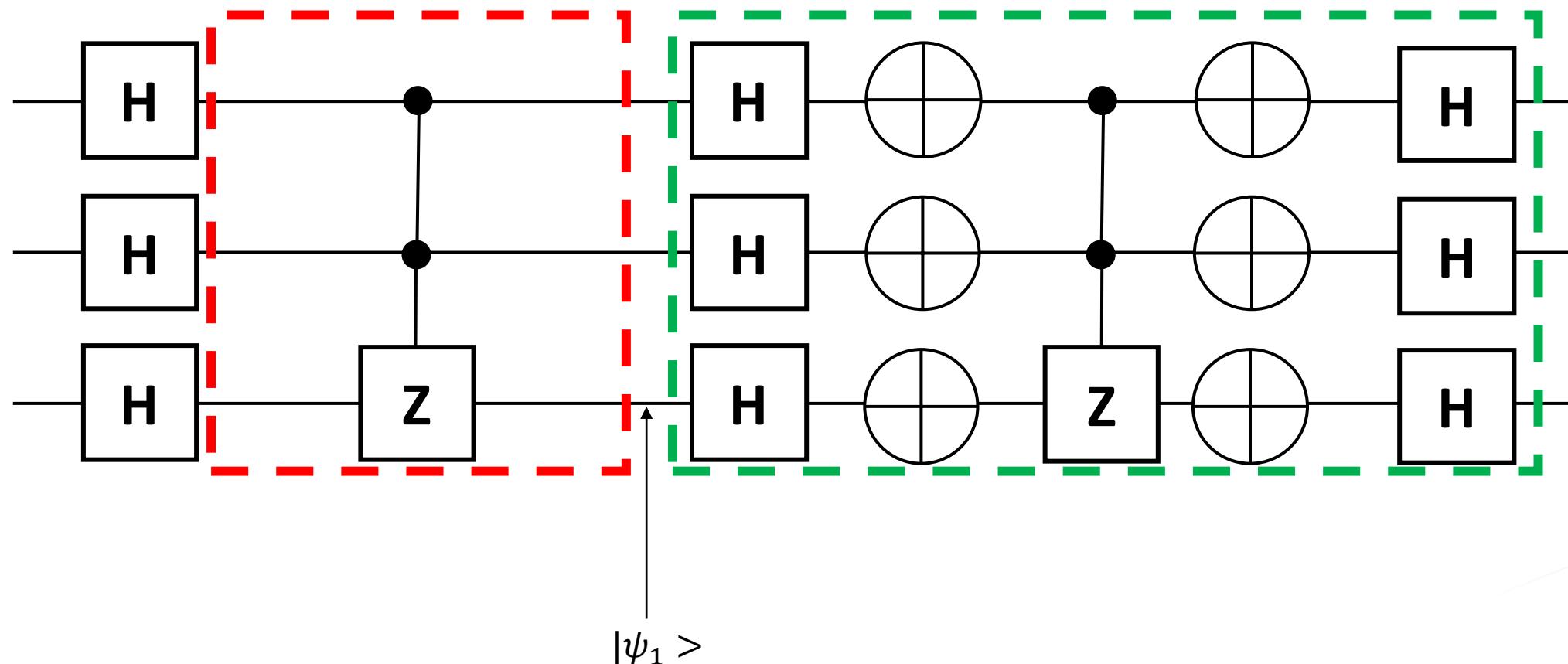
$$H^{\otimes 3} = H \otimes H \otimes H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$
$$= \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \end{bmatrix}$$

$$|000\rangle = |0\rangle \otimes |0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

StateVector Simulation

$$\begin{aligned} |\psi_0\rangle &= H^{\otimes 3} |000\rangle = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \\ &= \frac{1}{\sqrt{8}} \left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right) \\ &= \frac{1}{\sqrt{8}} (|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle) \\ &= \boxed{\frac{1}{\sqrt{8}} (|0\rangle + |1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + |6\rangle + |7\rangle)} \end{aligned}$$

StateVector Simulation



StateVector Simulation

$$|\psi_1\rangle = U_f |\psi_0\rangle$$

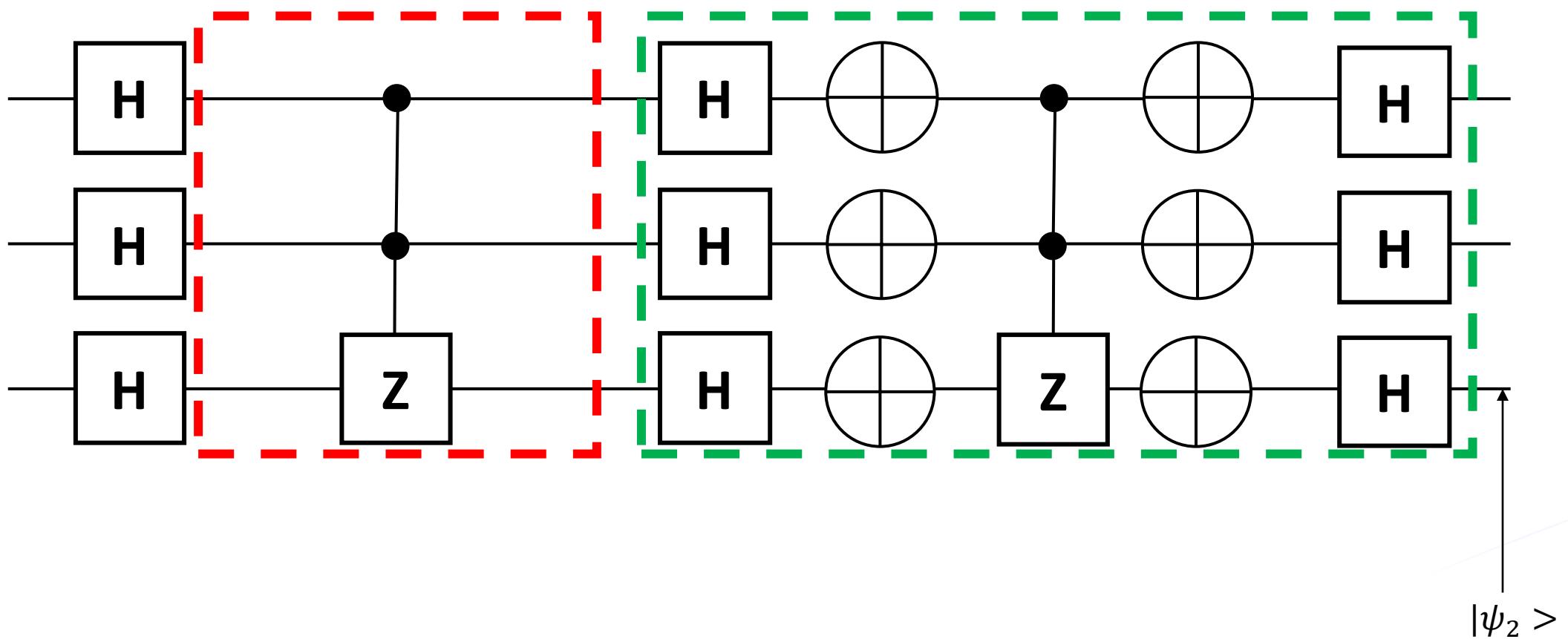
$$= U_f H^{\otimes 3} |000\rangle$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{8}} (|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle - |111\rangle)$$

$$= \frac{1}{\sqrt{8}} (|0\rangle + |1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + |6\rangle - |7\rangle)$$

StateVector Simulation



StateVector Simulation

$$\begin{aligned} |\psi_2\rangle &= (2|\psi\rangle\langle\psi| - \mathbb{I})|\psi_1\rangle \\ &= (2|\psi\rangle\langle\psi| - \mathbb{I})U_fH^{\otimes 3}|000\rangle \end{aligned}$$

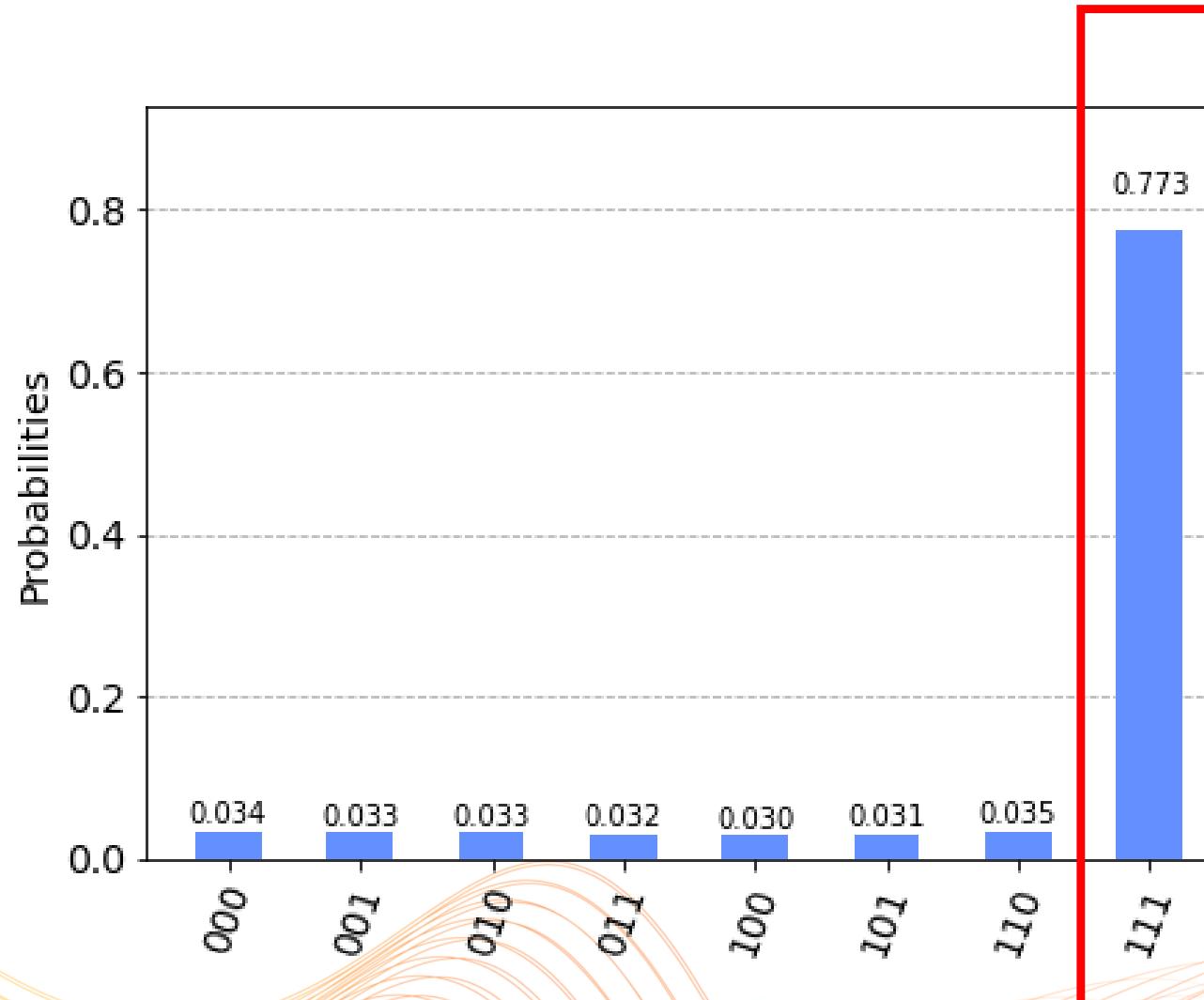
$$= \frac{1}{4} \begin{bmatrix} -3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -3 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -3 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -3 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -3 \end{bmatrix} \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 10 \end{bmatrix} = \frac{1}{4\sqrt{8}} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 10 \end{bmatrix} = \frac{1}{2\sqrt{8}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 5 \end{bmatrix}$$

$$= \frac{1}{2\sqrt{8}} (|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + 5|111\rangle)$$

$$= \frac{1}{2\sqrt{8}} (|0\rangle + |1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + |6\rangle + 5|7\rangle)$$

狀態 $|7\rangle$ 的機率和其他的比起來最大，所以我們找到 $|7\rangle$

StateVector Simulation

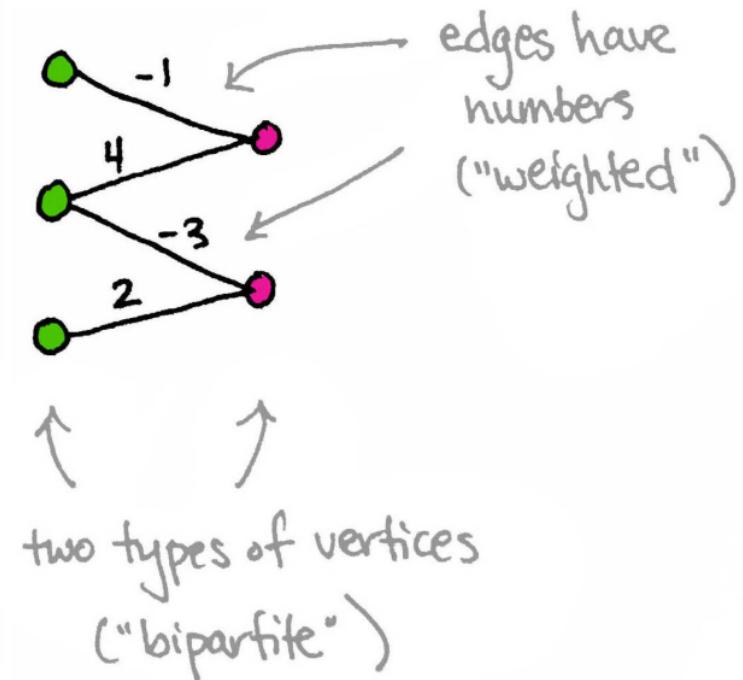


Tensor Network Representation

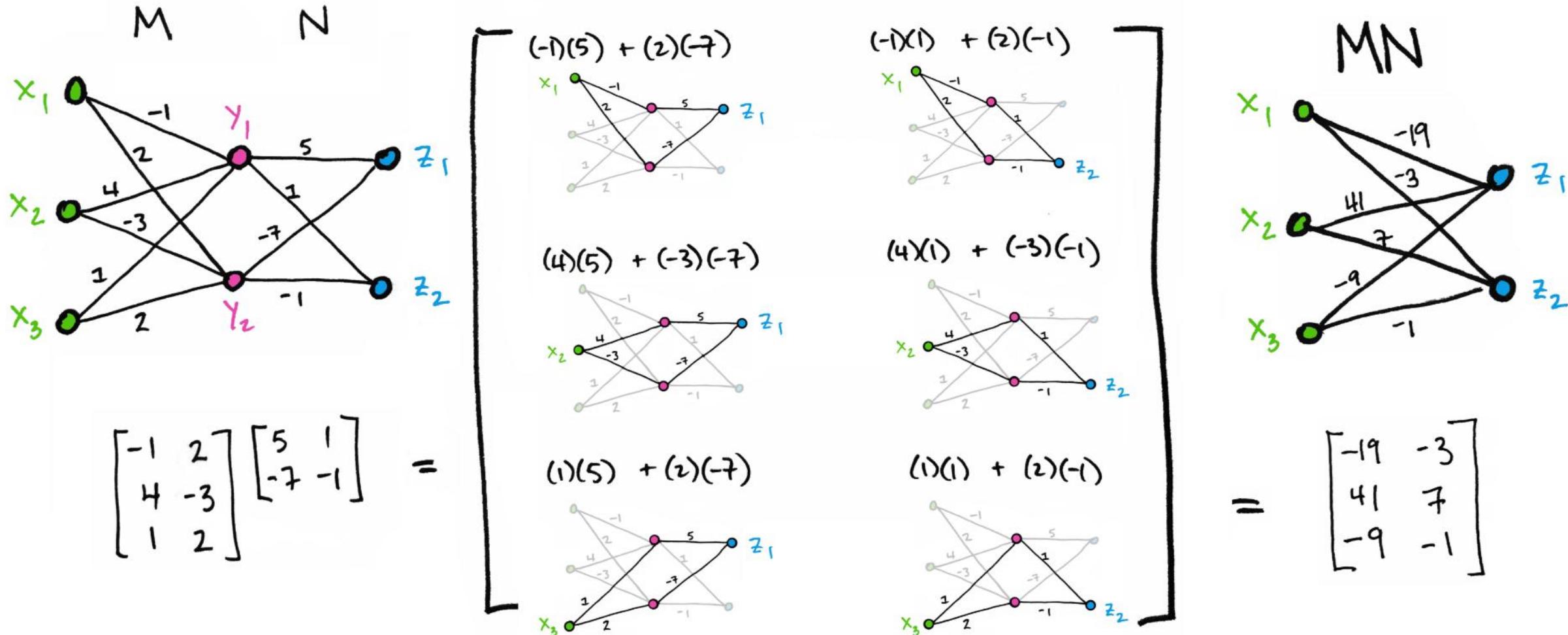
Tensor Network

M has three rows & two columns and corresponds to this weighted bipartite graph:

$$M = \begin{bmatrix} -1 & 0 \\ 4 & -3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 4 & -3 \\ 0 & 2 \end{bmatrix}$$

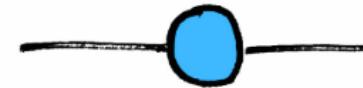


Tensor Network

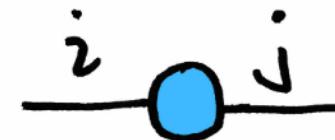


Tensor Network

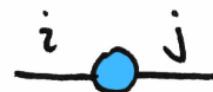
a matrix is a node



the matrix M_{ij} is



M



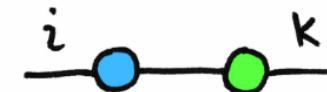
M_{ij}

N



N_{jk}

MN



$$(MN)_{ik} = \sum_j M_{ij} N_{jk}$$

Tensor Network

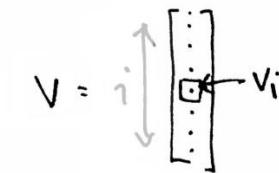
a number



a vector



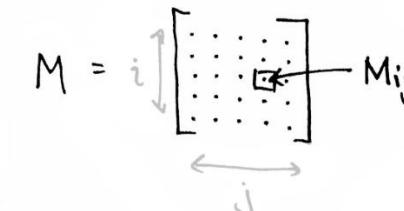
v_i



a matrix



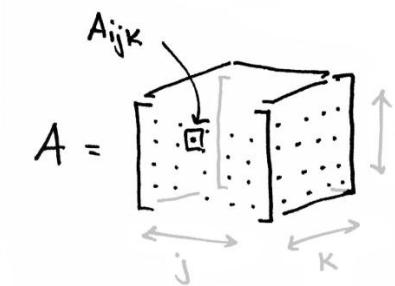
M_{ij}



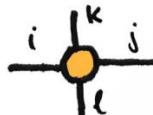
a 3-tensor



A_{ijk}



a 4-tensor



B_{ijkl}

$B = \left(\begin{array}{c} 4\text{-dimensional} \\ \text{cube} \dots \end{array} \right)$

Tensor Network

—○—○— can be thought of as



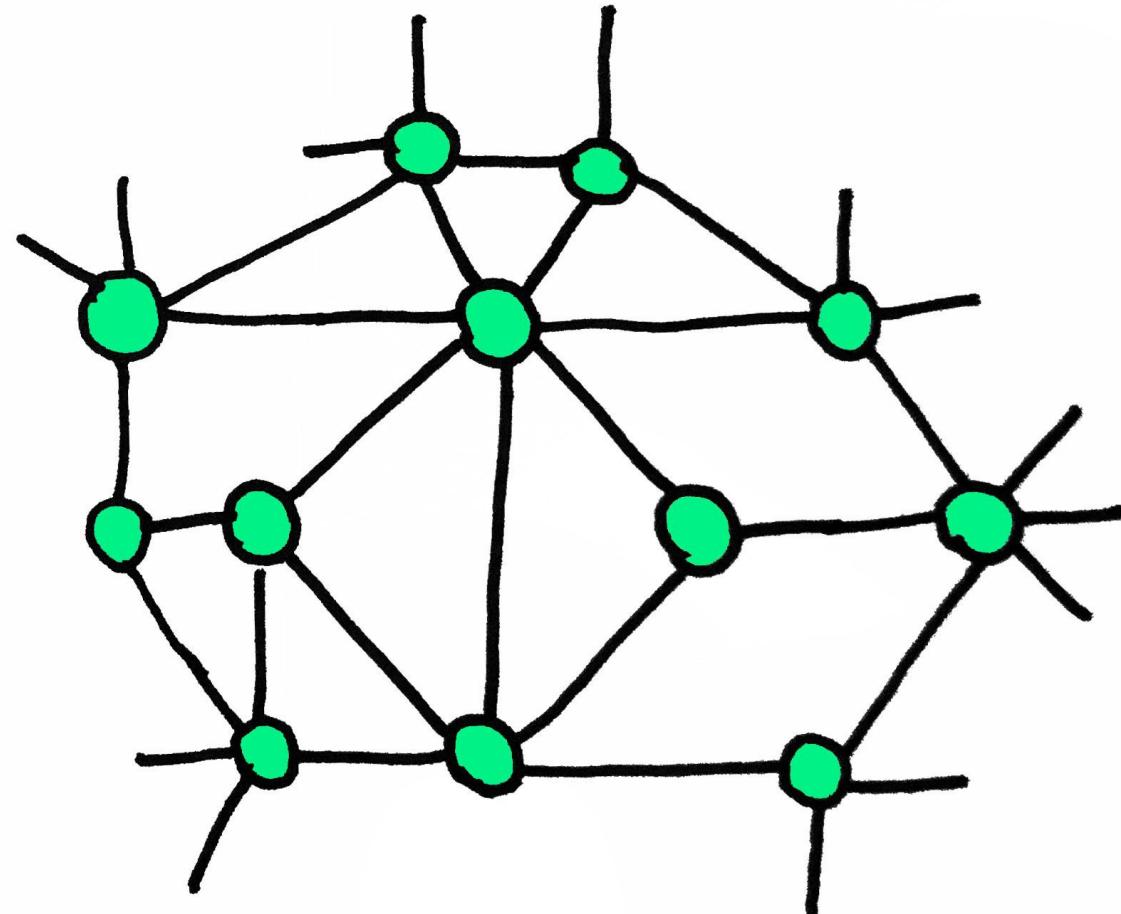
A diagram illustrating the contraction of two tensors. Two circles, one blue and one green, are connected by a horizontal line. A dashed oval encloses them with arrows indicating they are being "smooshed together". To the right of the equals sign is a single teal circle connected by a horizontal line.

which is a matrix ✓

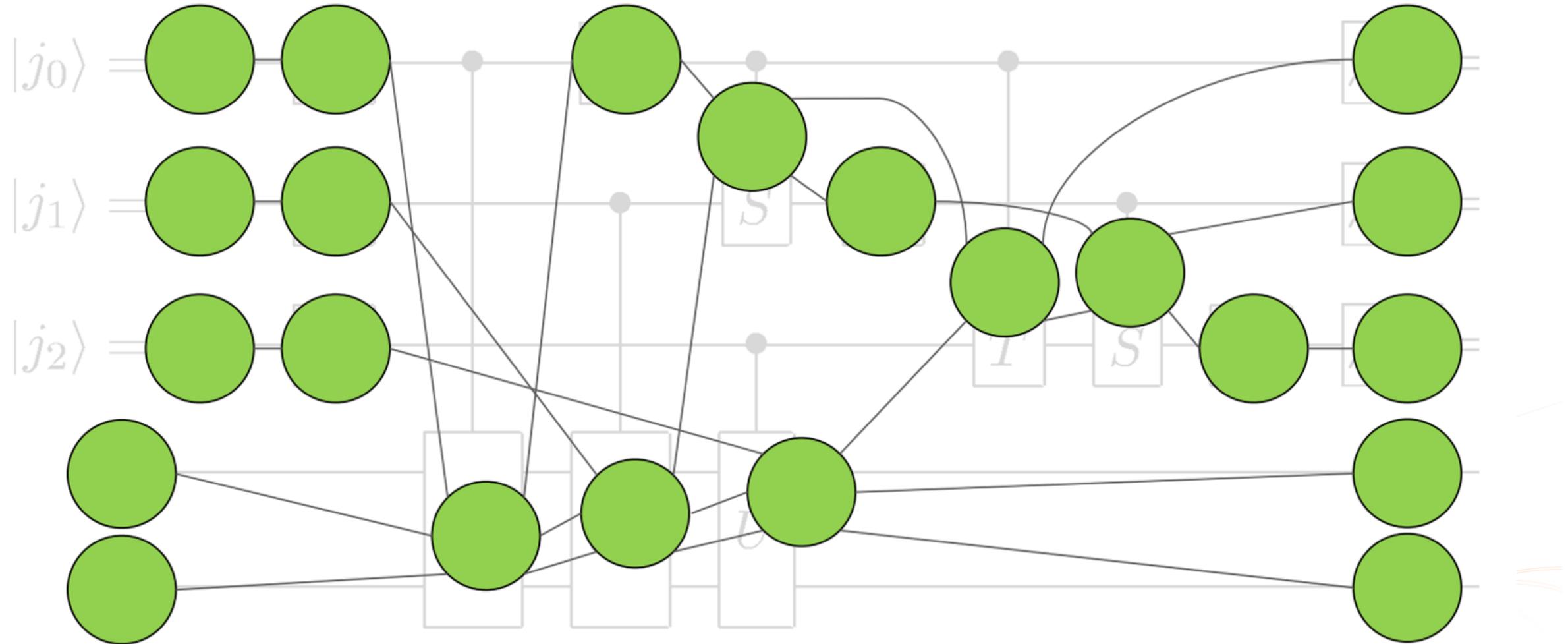
Mv is —○—○— = —○—

M $= UDV^+$

—○— = —→—○—○—←—

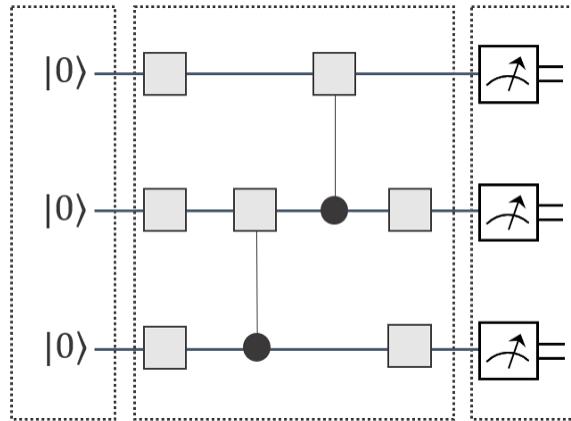


Tensor Network Representation



Quantum Circuit Simulation

Quantum Circuit
(Gate-Based Quantum Computing)



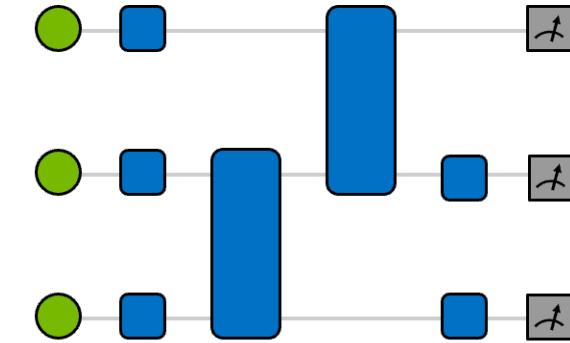
Put **dataset** (Classical or Quantum)
into Quantum Algorithm

Noisy Intermediate-Scale QC
(QPU)



Limited **qubits** and **gates**
with **insufficient accuracy**

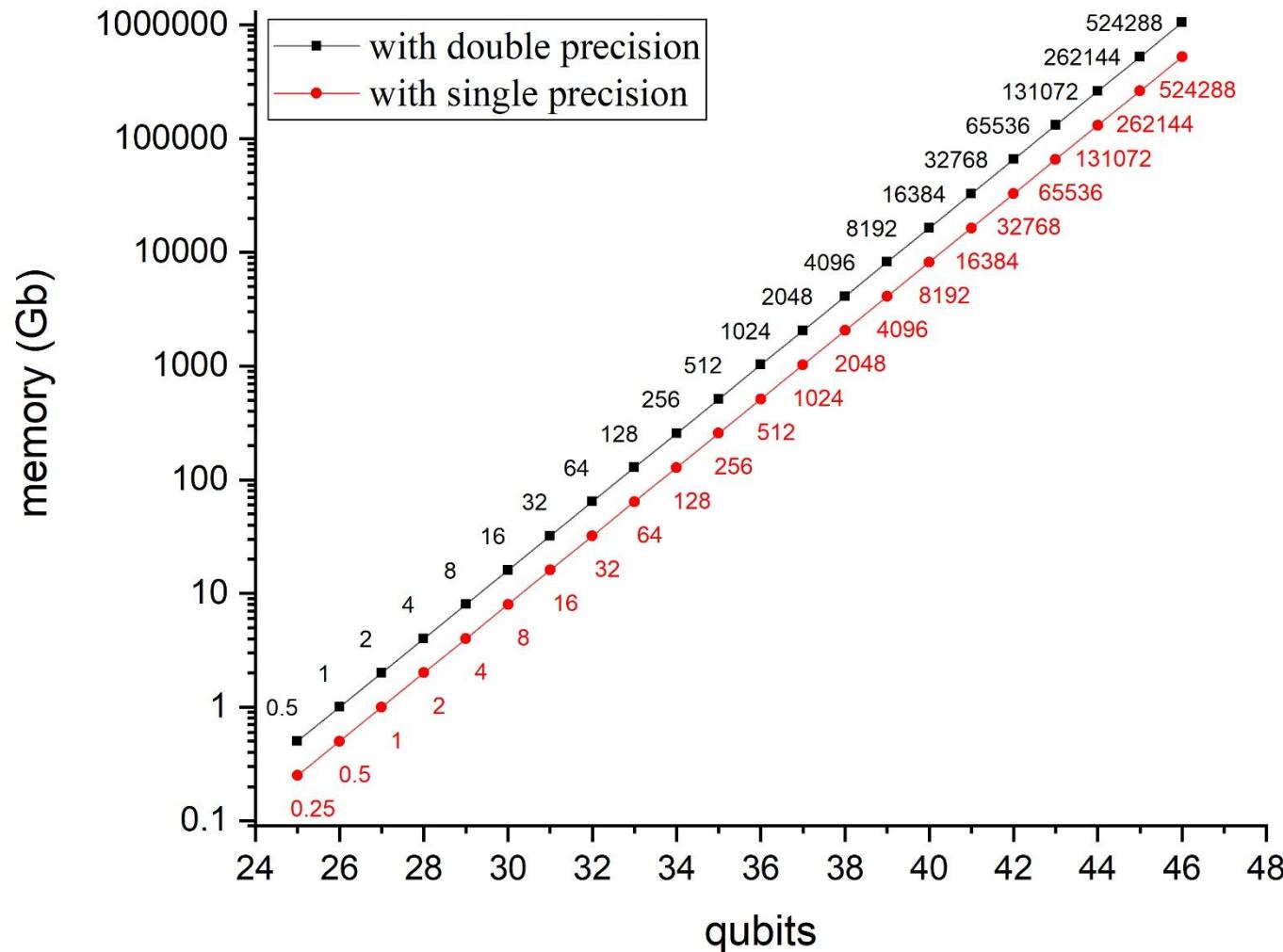
Quantum Circuit Simulation
(HPC platform)



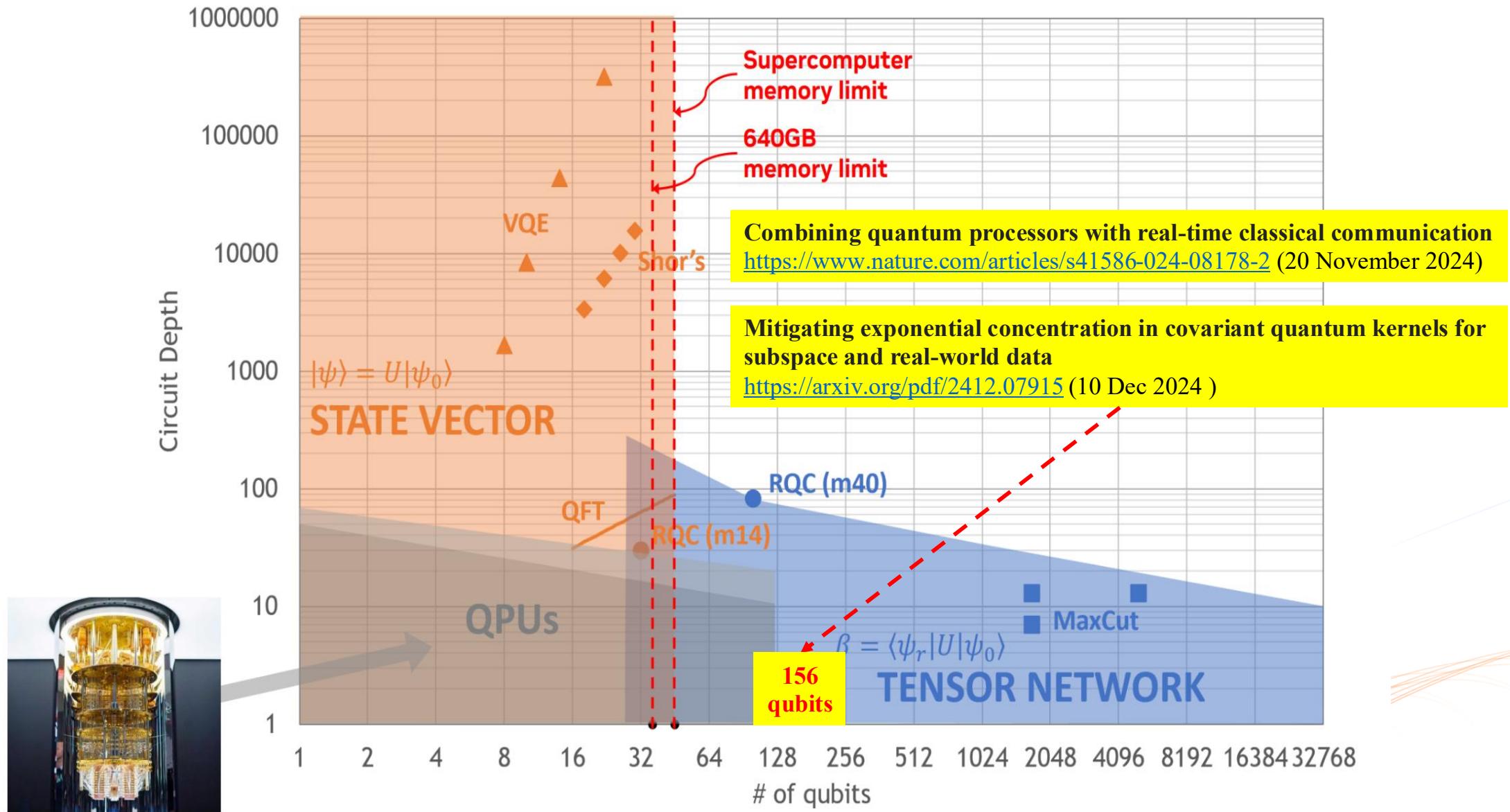
A series of matrix operations on
CPUs and GPUs
Scalable with accurate results

Quantum Circuit Simulation

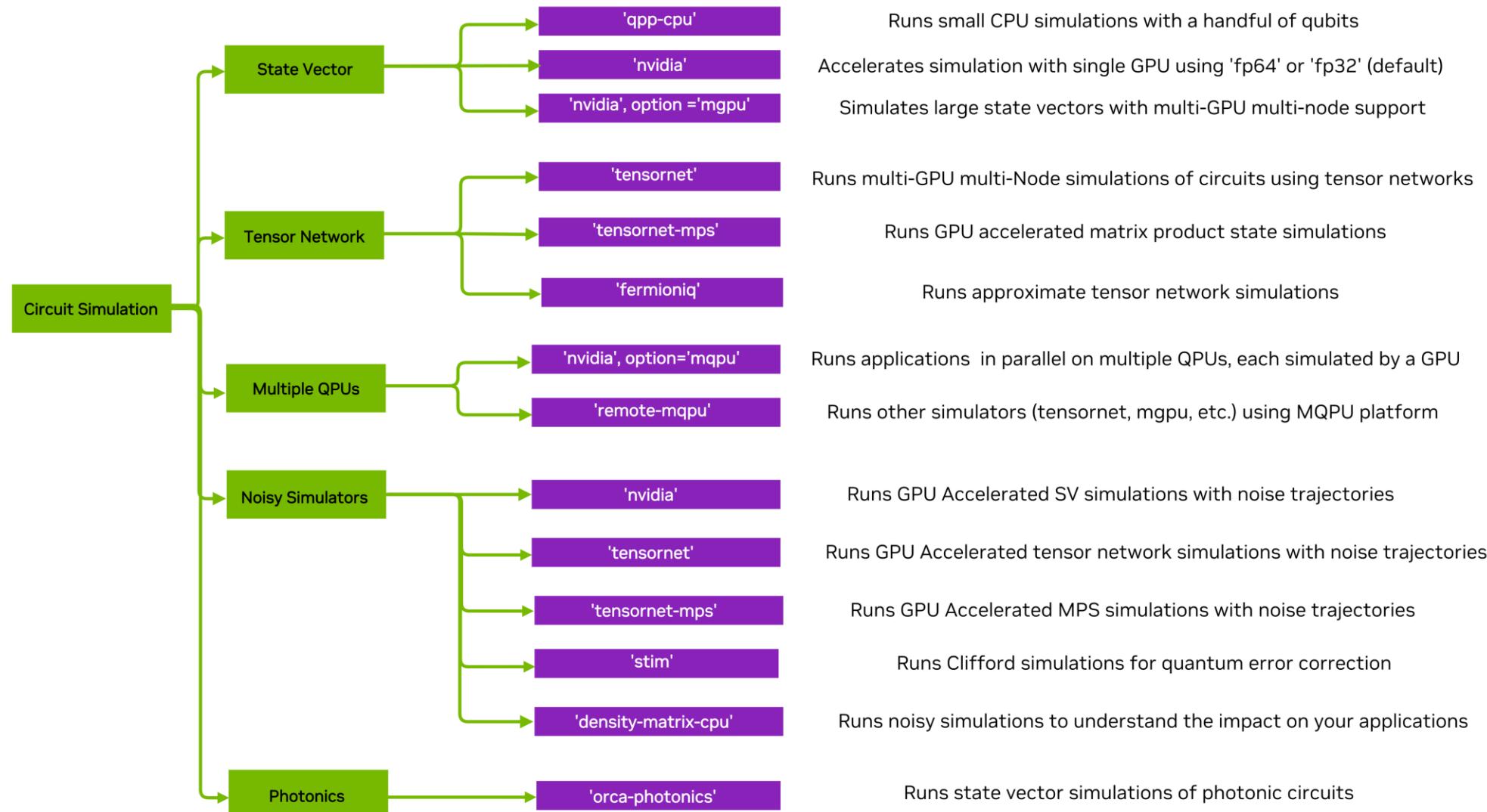
$2^{(qubits)} \times 16(\text{complex values[Bytes]})$



Quantum Circuit Simulation



Why CUDA-Q ?



Why CUDA-Q ?

Physical QPUs

Ion Trap



QUANTINUUM

Neutral Atom



Pasqal

IQuEra>
Computing Inc.

Superconducting



IQM

OQC

Photonic



Aggregators

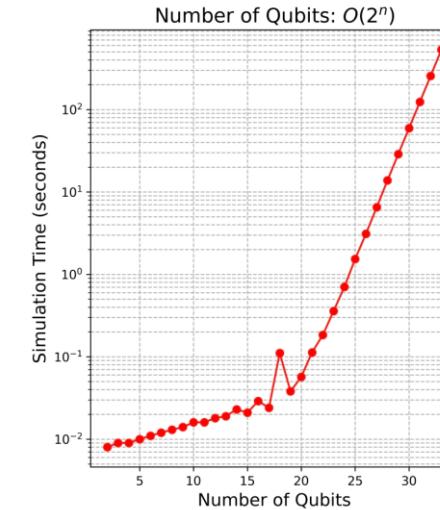
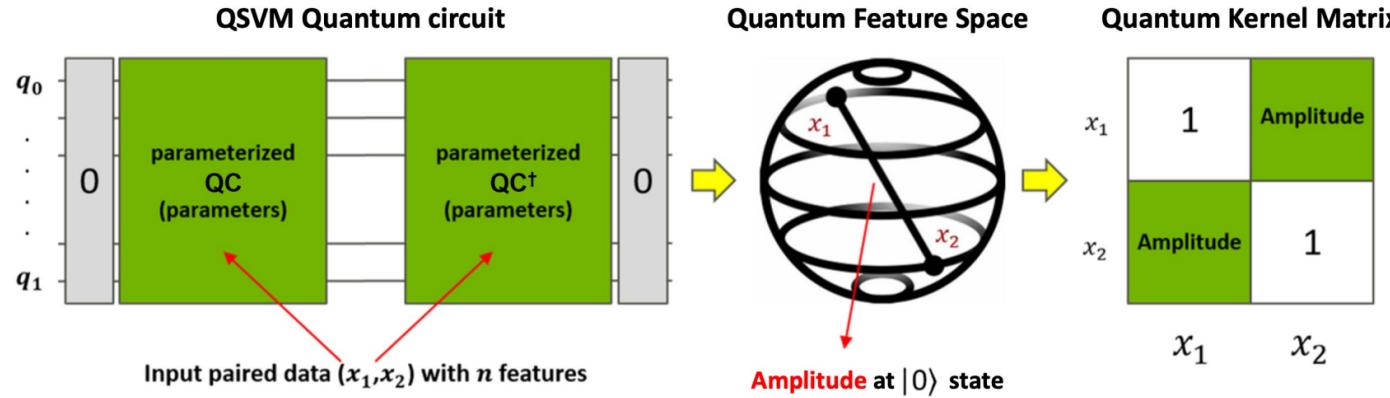


Research of Quantum Machine Learning in NCHC

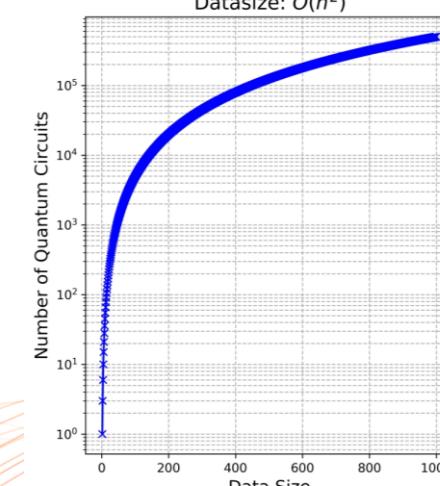
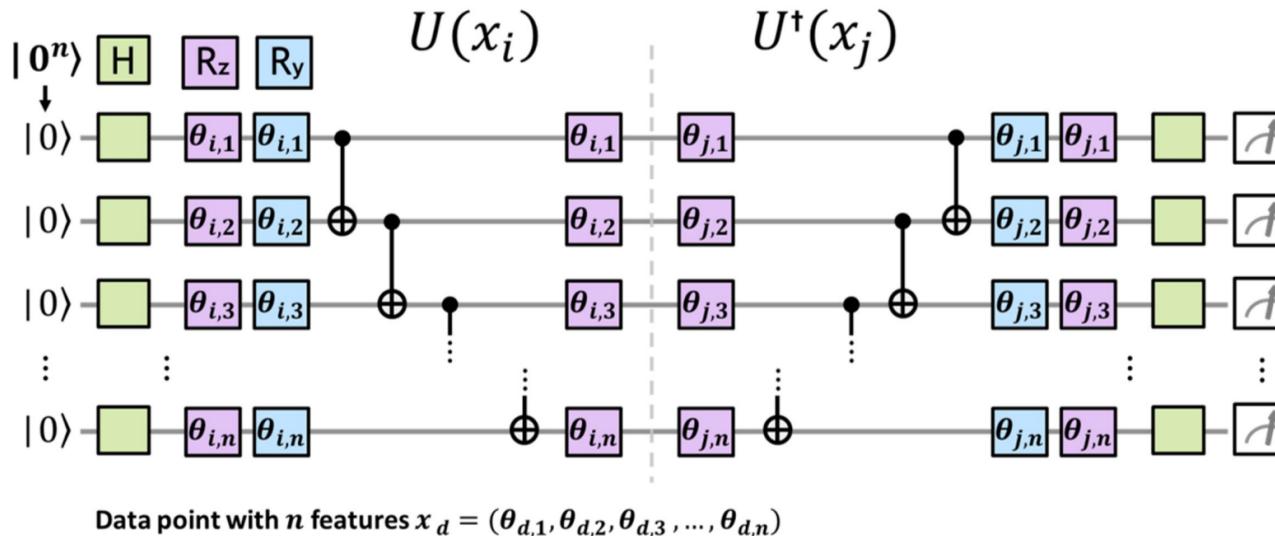
Scalable Quantum Support Vector Machine Simulation

QSVM Workflow and Computational Complexity on HPC Platform

(a)



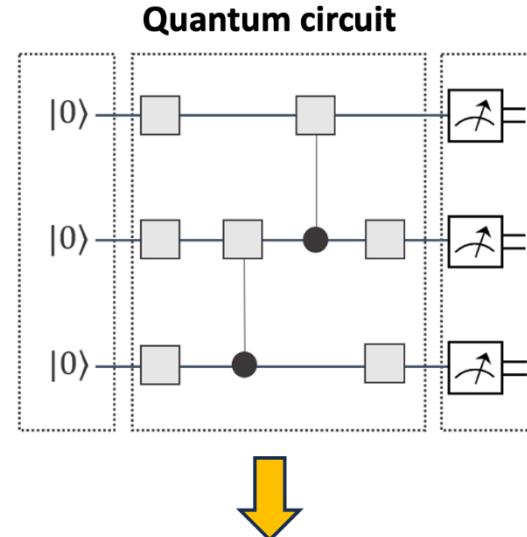
(b)



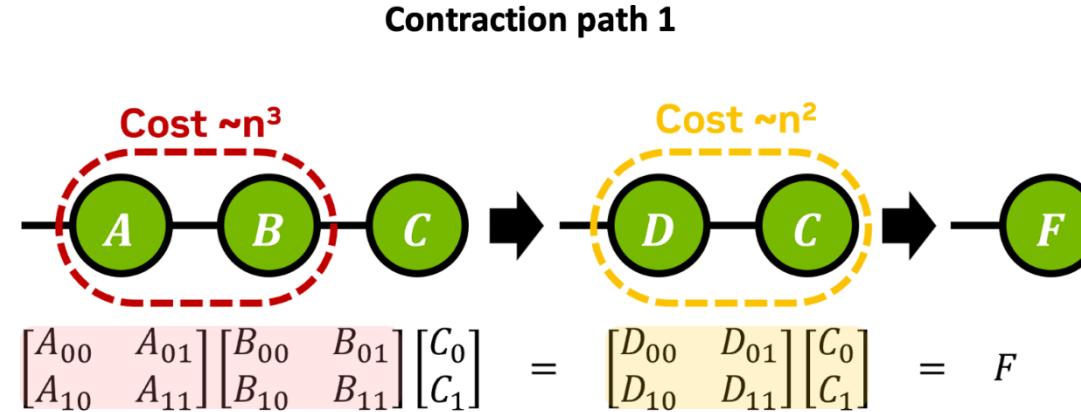
Scalable Quantum Support Vector Machine Simulation

Tensor Network Simulation using cuQuantum

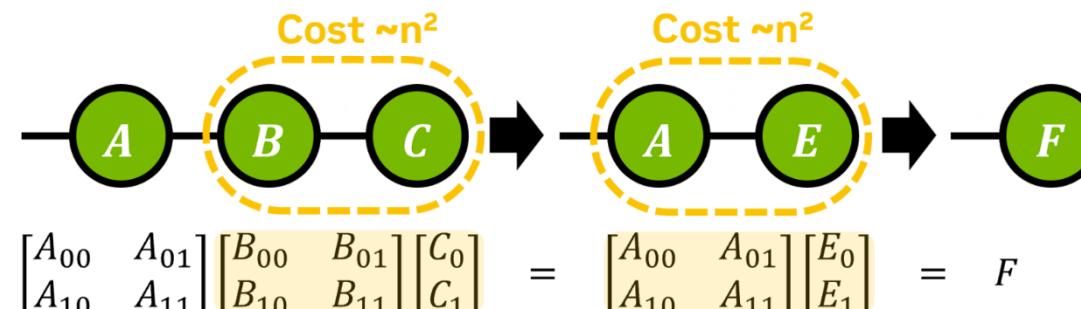
(a)



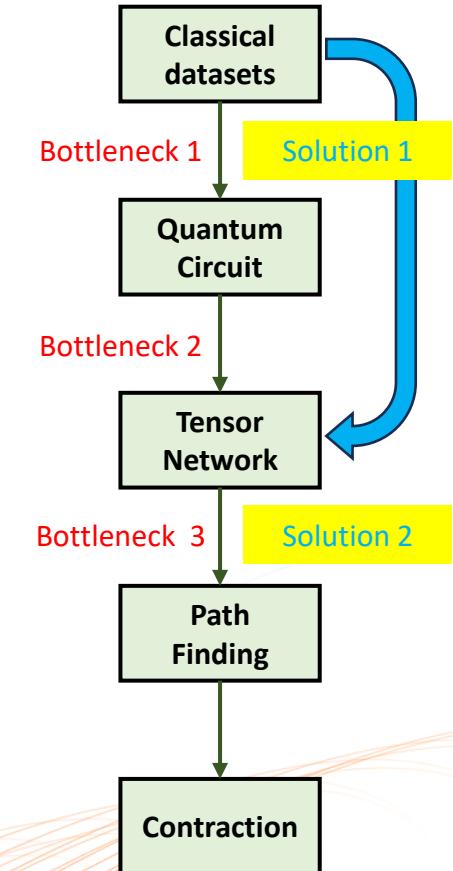
(b)



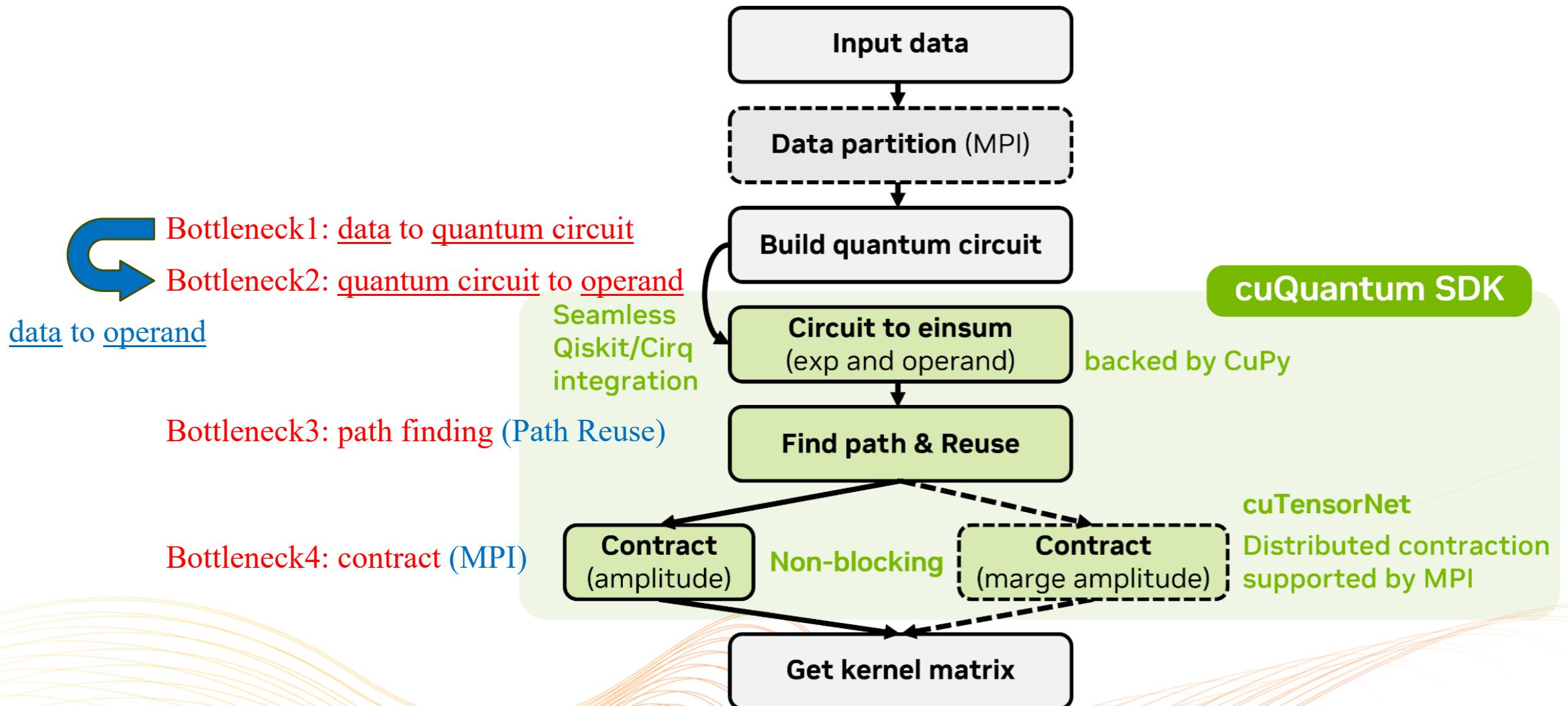
Contraction path 2



Optimize Workflow

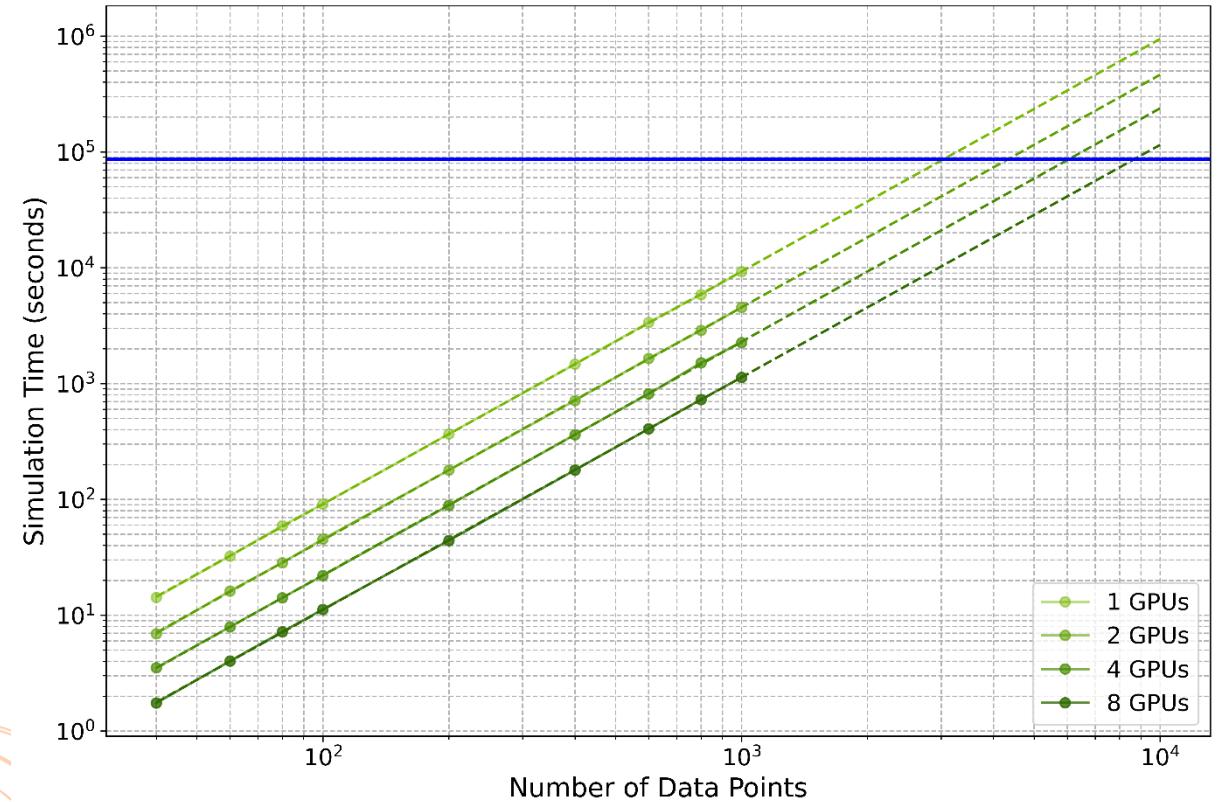
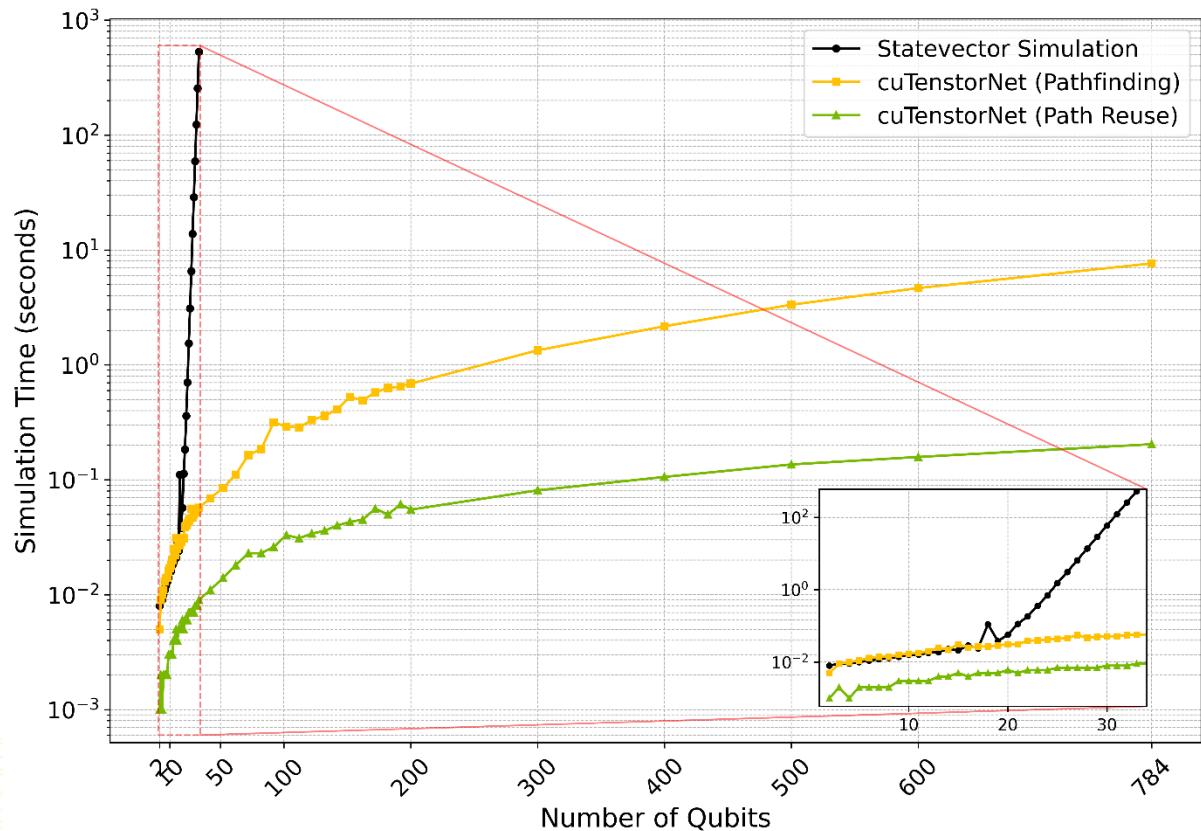


Scalable Quantum Support Vector Machine Simulation



Scalable Quantum Support Vector Machine Simulation

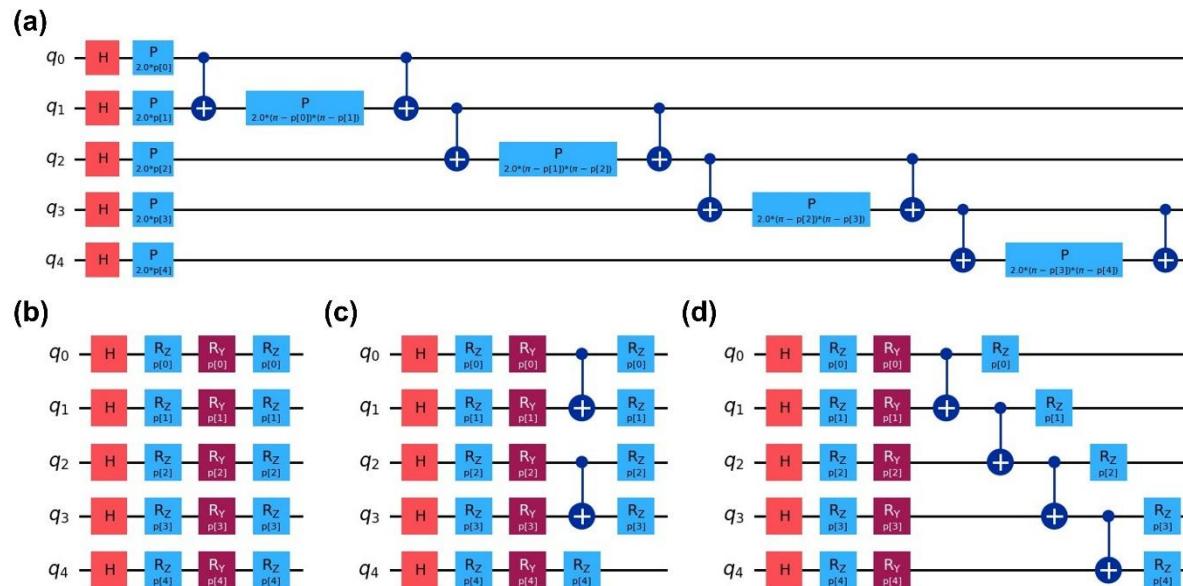
Single-GPU and Multi-GPUs Benchmark



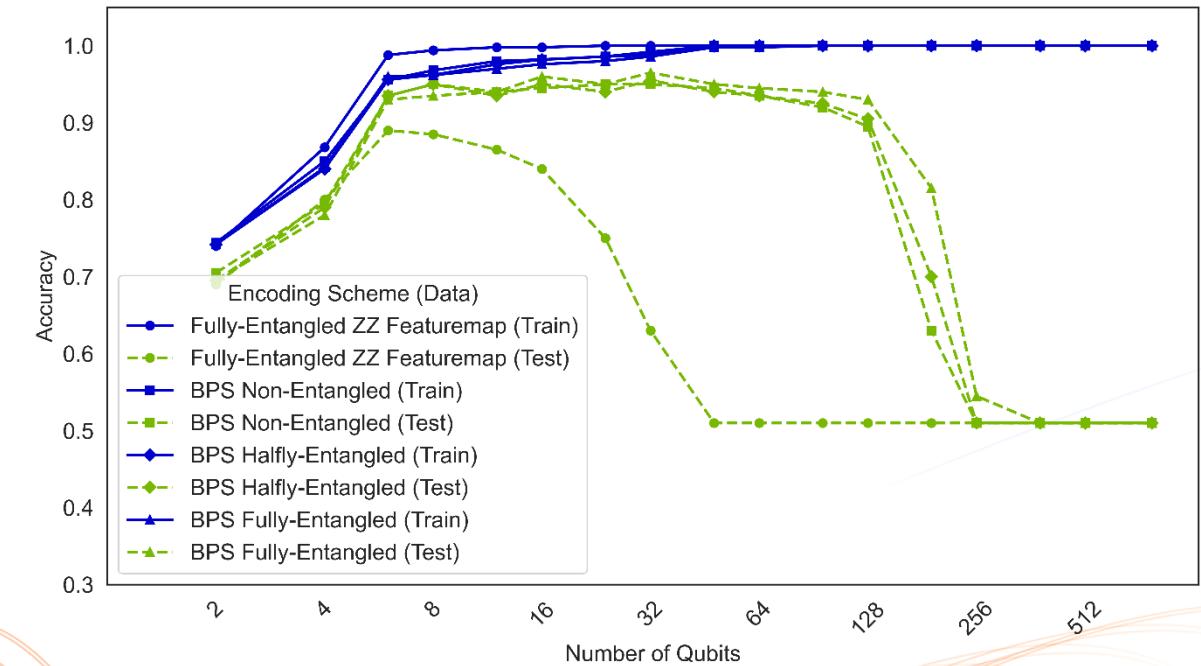
Scalable Quantum Support Vector Machine Simulation

Performance of Quantum Circuit Architectures and Entanglement Levels

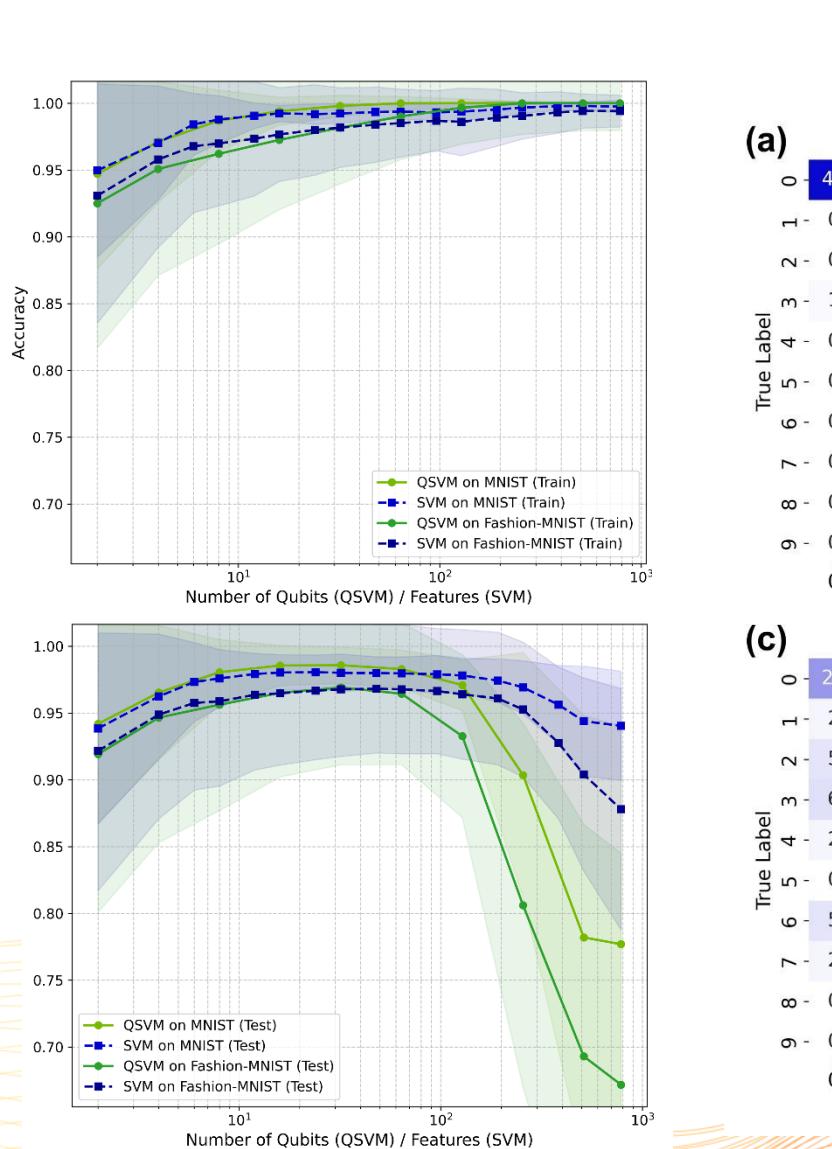
Different Quantum Circuit Architectures and Entanglement Levels



Performance



Scalable Quantum Support Vector Machine Simulation



Multi-class Classification

QSVM

(a)

		Predicted Label									
		0	1	2	3	4	5	6	7	8	9
True Label	0	47	0	0	0	0	1	0	1	1	1
	1	0	42	1	1	1	0	0	3	1	1
2	0	0	44	1	1	0	0	1	2	1	1
	3	1	1	1	42	2	0	0	1	1	1
4	0	1	0	0	0	43	1	2	0	2	1
	5	0	0	0	0	0	46	0	1	0	0
6	0	0	0	2	1	0	46	0	1	0	0
	7	0	0	0	0	0	0	41	2	1	1
8	0	0	0	1	0	2	0	0	49	1	0
	9	0	0	0	2	1	0	1	0	44	2
		0	1	2	3	4	5	6	7	8	9

SVM

(b)

		Predicted Label									
		0	1	2	3	4	5	6	7	8	9
True Label	0	47	0	0	0	0	0	1	0	1	1
	1	0	42	1	1	1	0	0	0	3	1
2	0	0	42	1	1	0	0	1	2	1	1
	3	1	1	1	40	3	0	0	0	1	1
4	0	1	0	0	0	45	0	2	0	1	1
	5	0	0	0	0	0	47	0	0	1	0
6	0	0	0	2	0	0	42	2	1	1	1
	7	0	0	0	0	0	0	49	1	0	0
8	0	0	0	1	0	1	0	0	45	2	0
	9	0	0	0	2	1	0	1	0	44	2
		0	1	2	3	4	5	6	7	8	9

MNIST

(c)

		Predicted Label									
		0	1	2	3	4	5	6	7	8	9
True Label	0	20	0	3	5	0	8	8	0	5	1
	1	2	40	1	2	1	0	0	0	1	3
2	5	0	24	1	3	5	6	1	2	3	1
	3	6	0	2	23	3	4	6	0	3	3
4	2	0	8	3	20	4	6	0	6	1	1
	5	0	0	0	0	0	40	0	3	6	1
6	5	0	2	0	2	3	26	0	12	0	0
	7	2	0	2	4	1	1	1	29	6	4
8	0	0	0	1	0	0	0	0	49	0	0
	9	0	0	0	0	0	5	1	2	6	36
		0	1	2	3	4	5	6	7	8	9

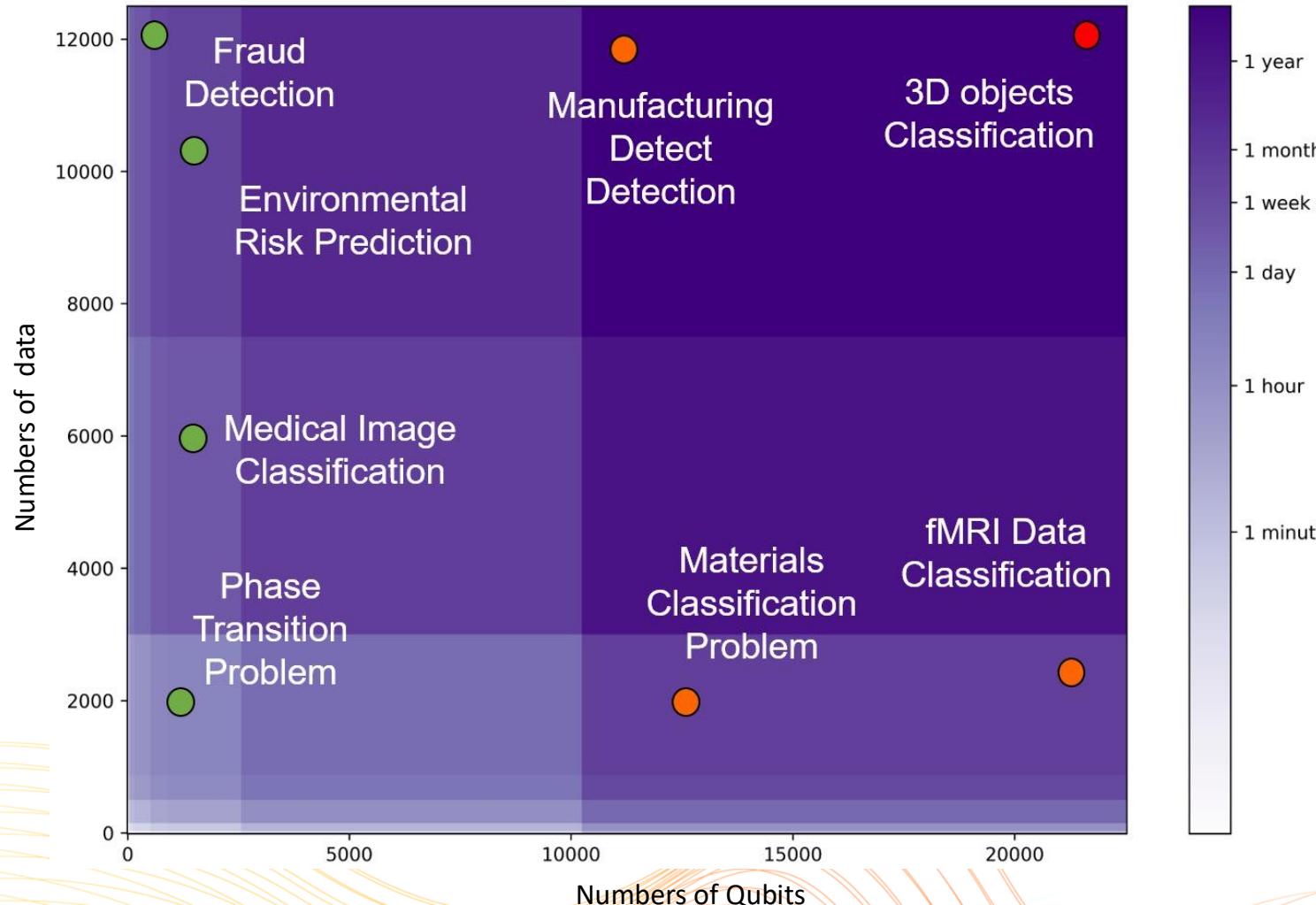
(d)

		Predicted Label									
		0	1	2	3	4	5	6	7	8	9
True Label	0	26	0	4	6	0	5	6	0	3	0
	1	1	42	0	2	1	0	1	0	1	2
2	5	0	26	1	4	2	6	1	3	2	2
	3	7	0	3	2	22	4	7	0	3	2
4	3	0	7	2	2	4	7	0	4	3	1
	5	0	0	0	0	0	0	42	0	4	3
6	7	0	6	0	2	3	24	0	8	0	0
	7	2	0	1	2	1	1	1	35	3	4
8	0	0	0	1	0	0	0	0	49	0	0
	9	0	0	0	0	2	1	2	3	42	0
		0	1	2	3	4	5	6	7	8	9

Fashion-MNIST

Scalable Quantum Support Vector Machine Simulation

Call for Various Datasets



(we are ready!)



Scalable Quantum Support Vector Machine Simulation

Better Algorithms with Large-Scale Simulations and AI

IonQ/AWS/AstraZeneca
AF-QMC simulation for
electronic structure modeling

HPE Labs
Distributed QC with
Adaptive Circuit Knitting

NCHC
784 qubit simulation to
validate QML approaches

Qubits

Better QPUs

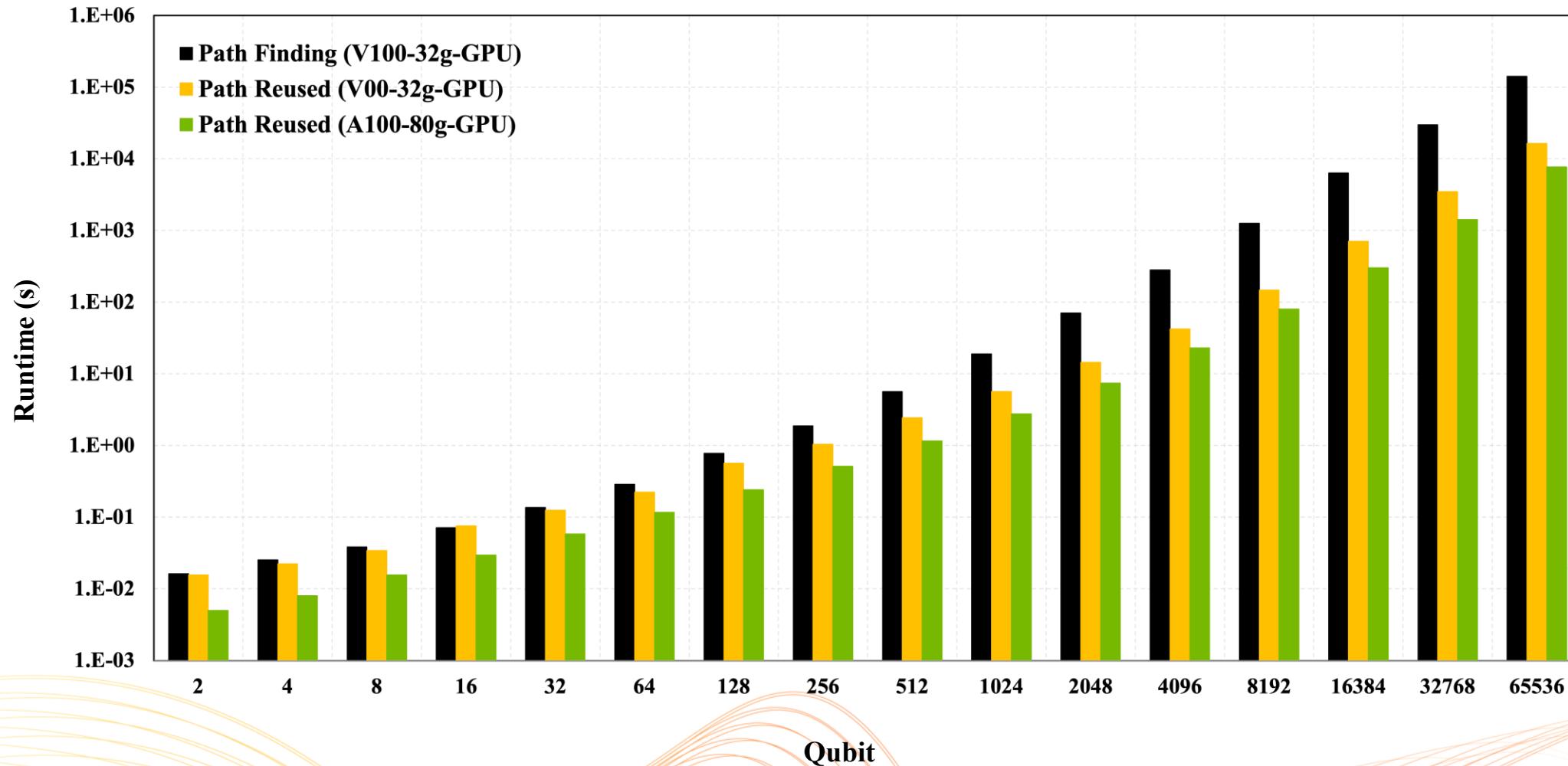
Better Algorithms

Scaling QEC

Accelerated Quantum Supercomputers

Chen et. al. cuTN-QSVM: cuTensorNet-accelerated Quantum Support Vector Machine with cuQuantum SDK.
<https://arxiv.org/html/2405.02630v2> (2024)

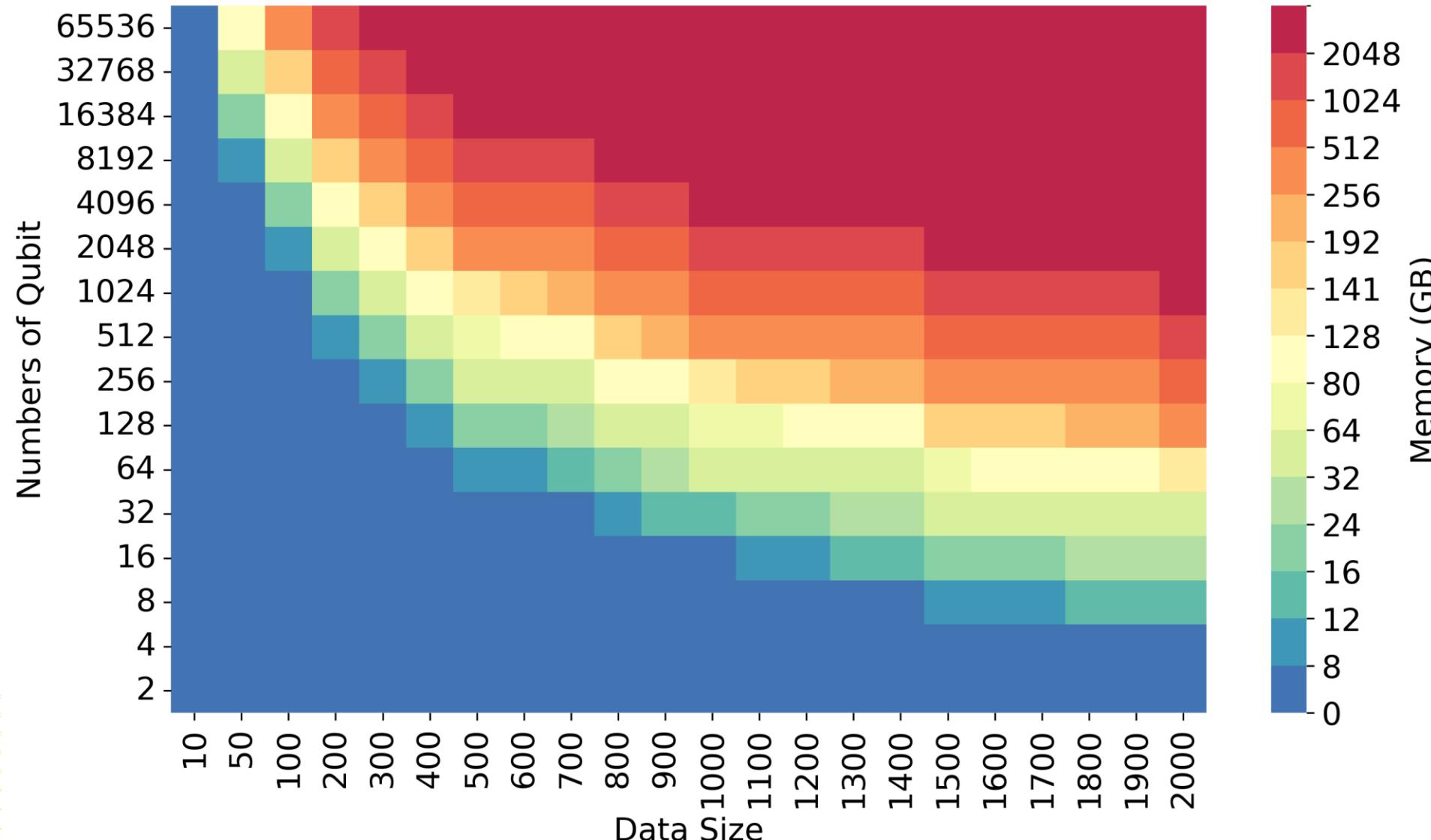
Scalable Quantum Support Vector Machine Simulation



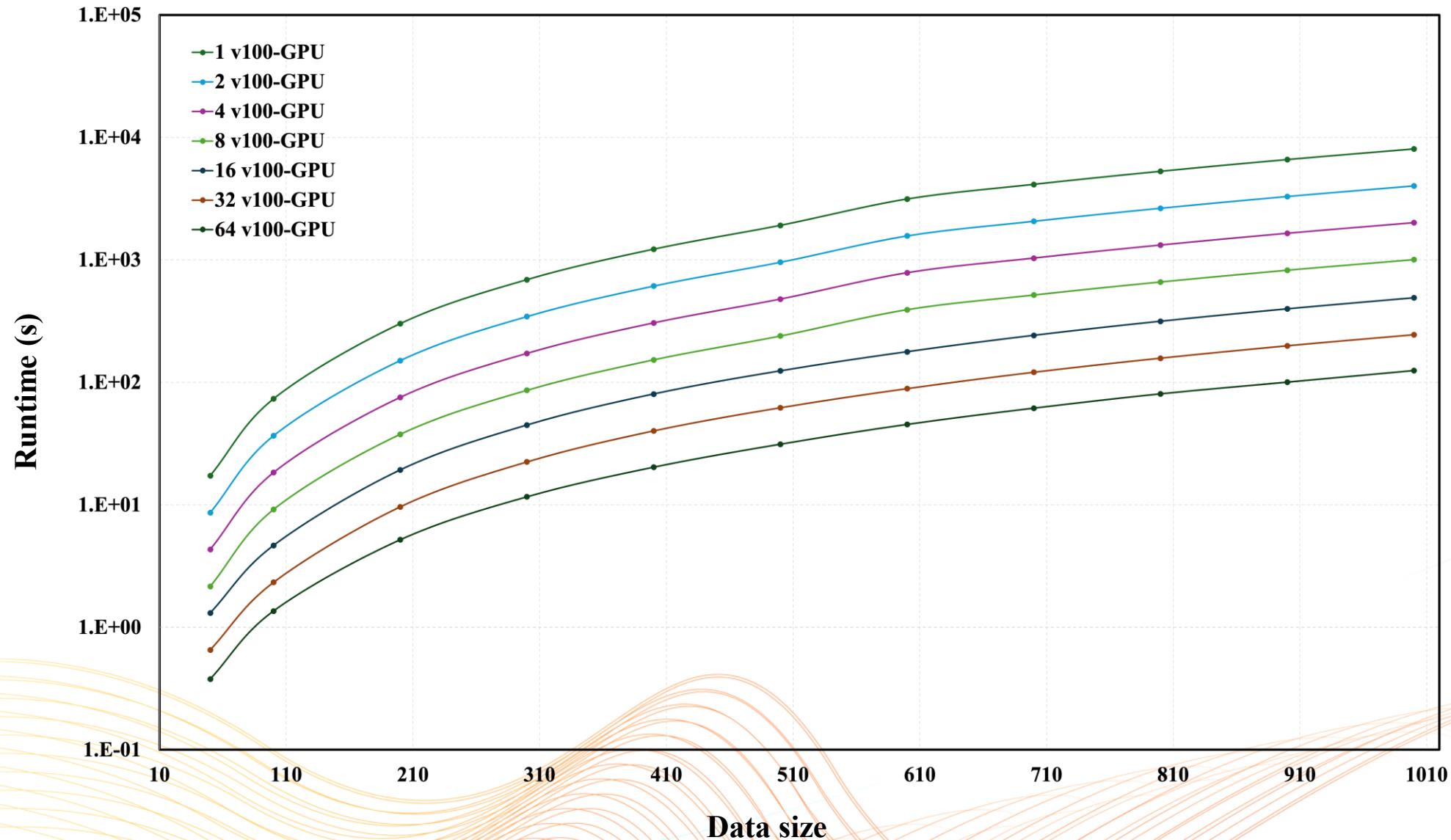
Scalable Quantum Support Vector Machine Simulation

Qubit	Build QC and TN (s)	Renew TN (s)	Build Environment (s)	Get Path (s)	Contraction (s)	Total Time (s)
2	0.01	0.00	0.00	0.00	0.00	0.02
4	0.00	0.00	0.00	0.01	0.00	0.02
8	0.01	0.01	0.00	0.01	0.00	0.03
16	0.01	0.01	0.04	0.03	0.00	0.10
32	0.03	0.02	0.03	0.05	0.01	0.14
64	0.05	0.04	0.02	0.11	0.01	0.23
128	0.26	0.07	0.01	0.22	0.02	0.58
256	0.19	0.29	0.03	0.46	0.05	1.03
512	0.61	0.58	0.07	1.06	0.10	2.41
1024	1.41	1.17	0.17	2.56	0.19	5.50
2048	3.24	2.21	0.31	7.03	0.38	13.17
4096	6.65	4.50	1.07	22.03	1.01	35.26
8192	13.39	9.06	3.50	79.15	1.47	106.57
16384	28.29	18.25	11.57	299.87	2.82	360.80
32768	56.86	39.26	36.61	1420.52	5.76	1559.01
65536	123.92	87.08	159.42	7724.41	11.32	8106.16

Scalable Quantum Support Vector Machine Simulation



Scalable Quantum Support Vector Machine Simulation

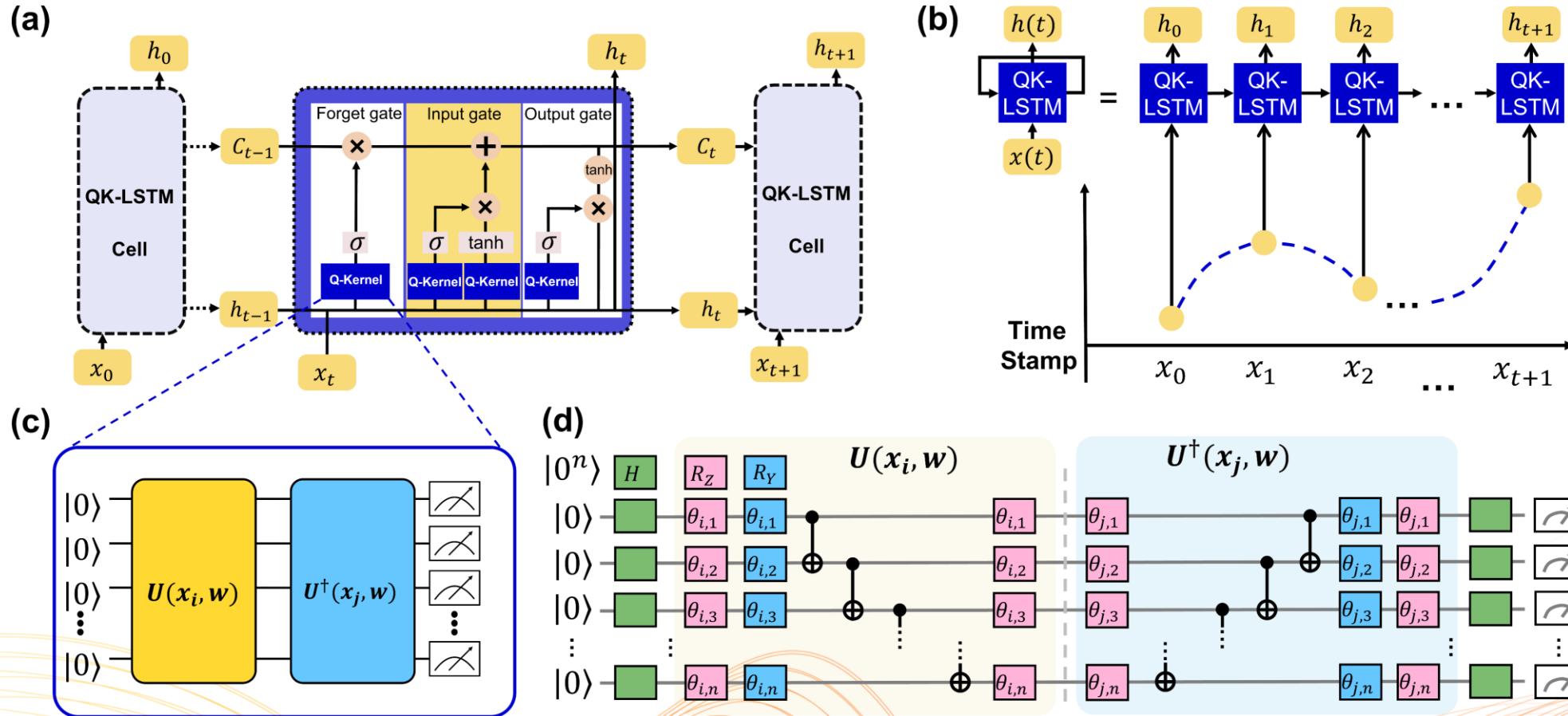


Scalable Quantum Support Vector Machine Simulation

Qubit	Data Size	Quantum Circuit	Quantum Circuit/GPU	GPUs	Build QC and TN (s)	Renew TN (s)	Build Environment (s)	Get Path (s)	Contraction (s)	Total Time (s)
2048	50	1225	20	64	1.374	3.098	0.37	8.637	4.845	18.324
2048	100	4950	78	64	1.348	5.716	0.382	8.673	18.647	34.766
2048	200	19900	311	64	1.36	16.201	0.373	8.698	83.4	110.032
2048	300	44850	701	64	1.38	41.769	0.406	8.789	183.441	235.785
2048	400	79800	1247	64	1.375	73.437	0.394	8.962	326.574	410.742
2048	500	124750	1950	64	1.382	110.838	0.396	8.923	513.117	634.656
2048	600	179700	2808	64	1.392	156.375	0.418	9.28	739.758	907.223
2048	700	244650	3823	64	1.397	212.694	0.405	9.241	1003.388	1227.125
2048	800	319600	4994	64	1.386	288.6	0.419	9.728	1317.922	1618.055
2048	900	404550	6322	64	1.404	350.633	0.424	9.746	1671.988	2034.195
2048	1000	499500	7805	64	1.389	447.505	0.432	10.242	2072.545	2532.113

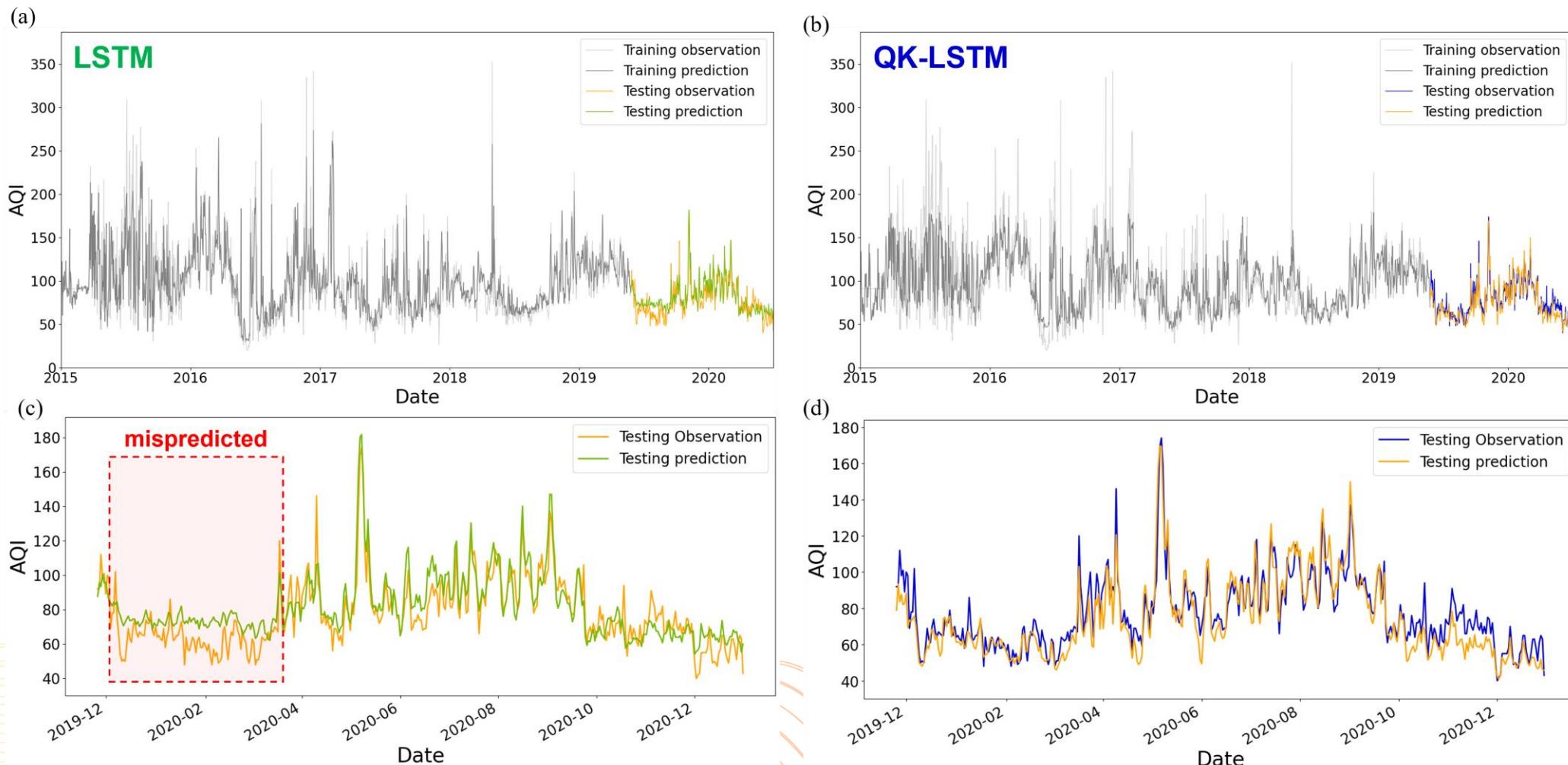
Quantum Kernel-based Long Short-Term Memory

Overview of the QK-LSTM architecture



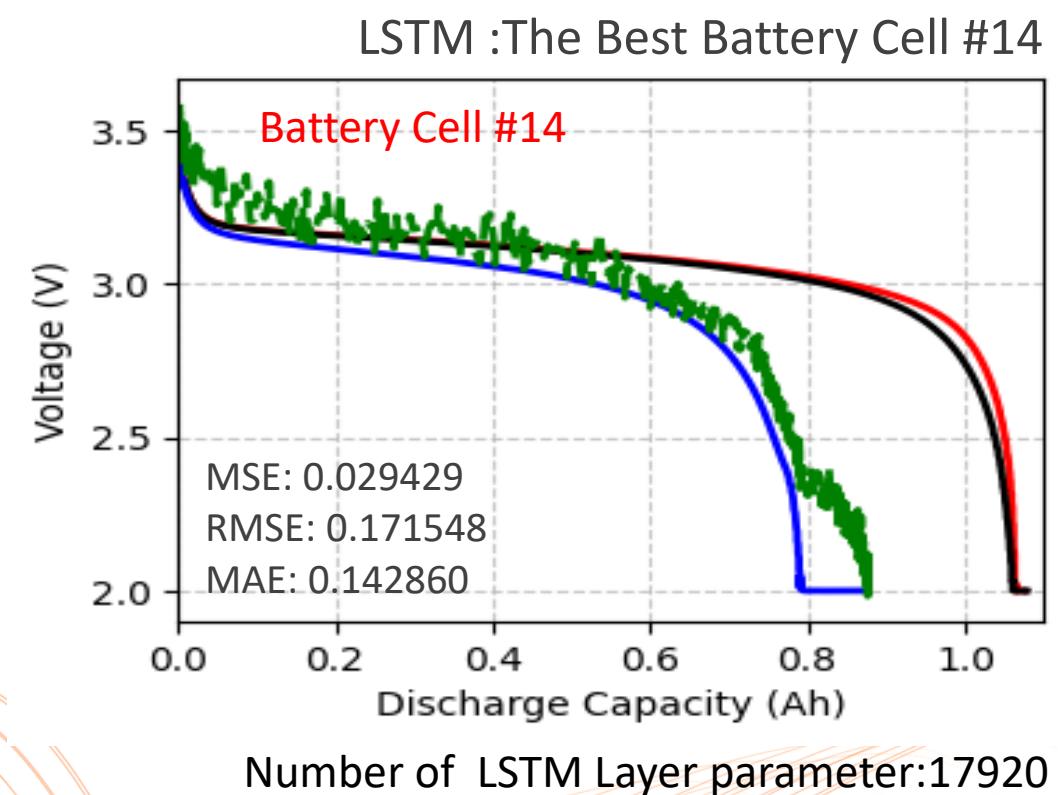
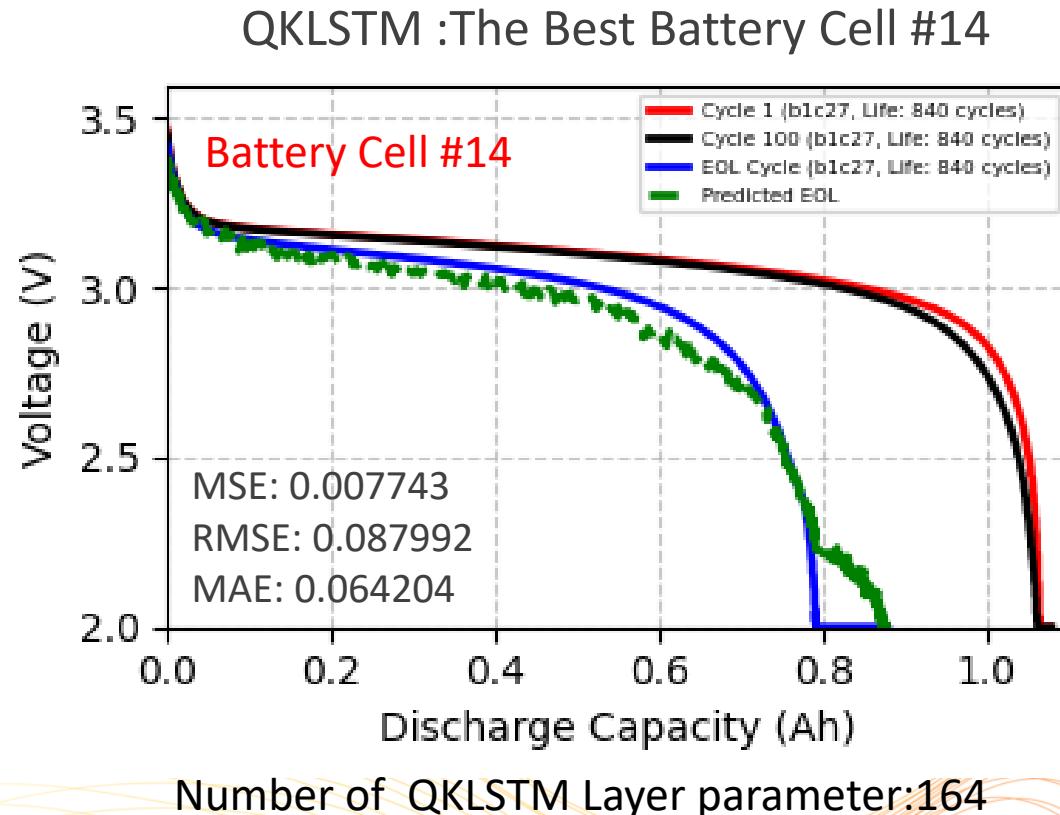
Quantum Kernel-based Long Short-Term Memory

Comparative analysis of air quality prediction performance using classical LSTM and QK-LSTM models



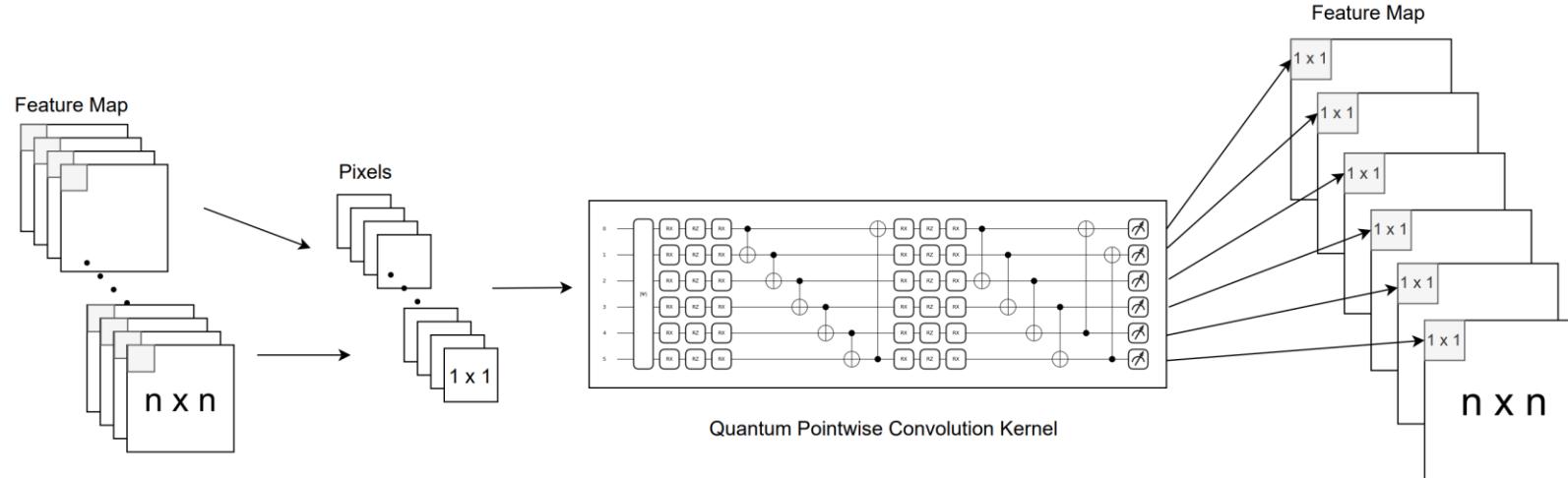
Quantum Kernel-based Long Short-Term Memory

- Input: The first 100 cycles of each battery [capacitance, voltage, current, temperature]
 - Target: $V(Q)$ of each batter's EOL (last discharge curve)
- Training dataset: 91 batteries
 - Testing dataset: 23 batteries

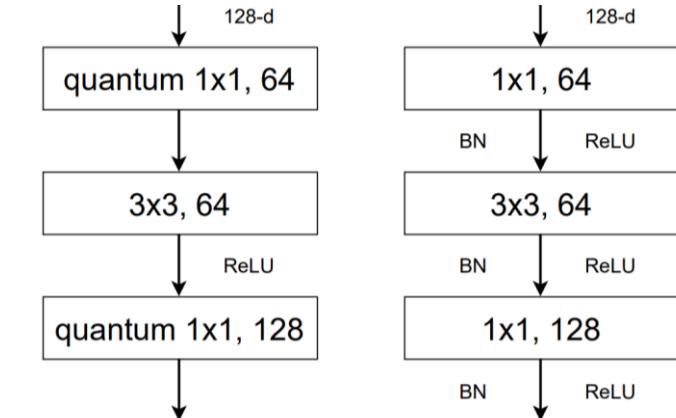


Quantum Pointwise Convolution Networks

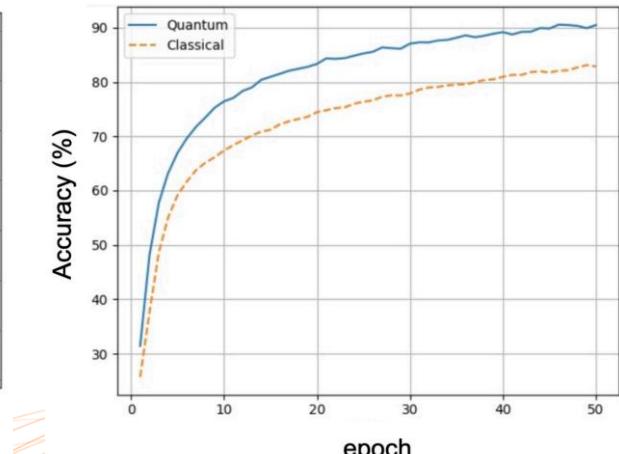
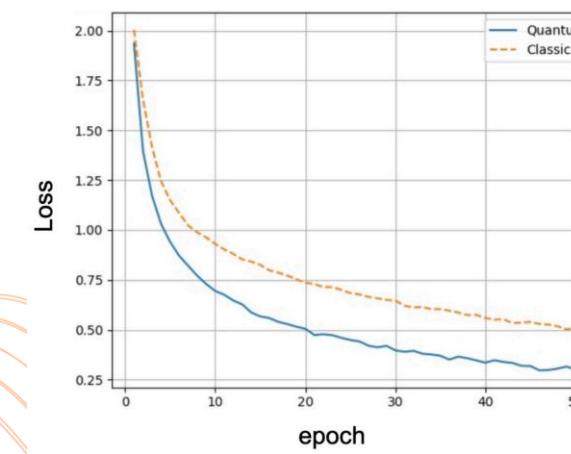
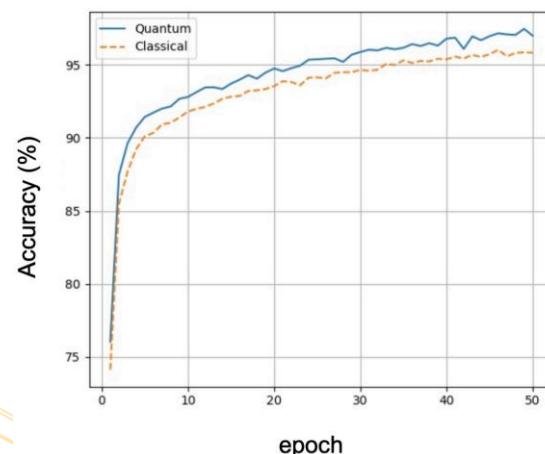
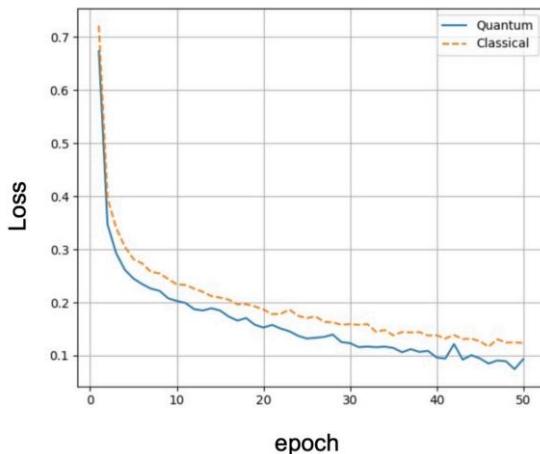
(a) Overview of the Quantum Pointwise Convolution Networks (QPWCNN)



(b) With and without QPWCNN layer

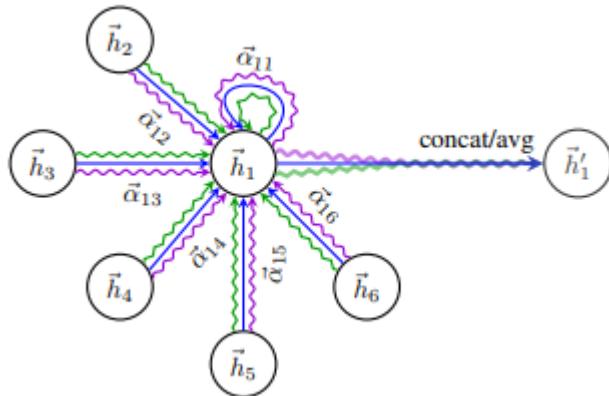


(c) Fashion mnist Result

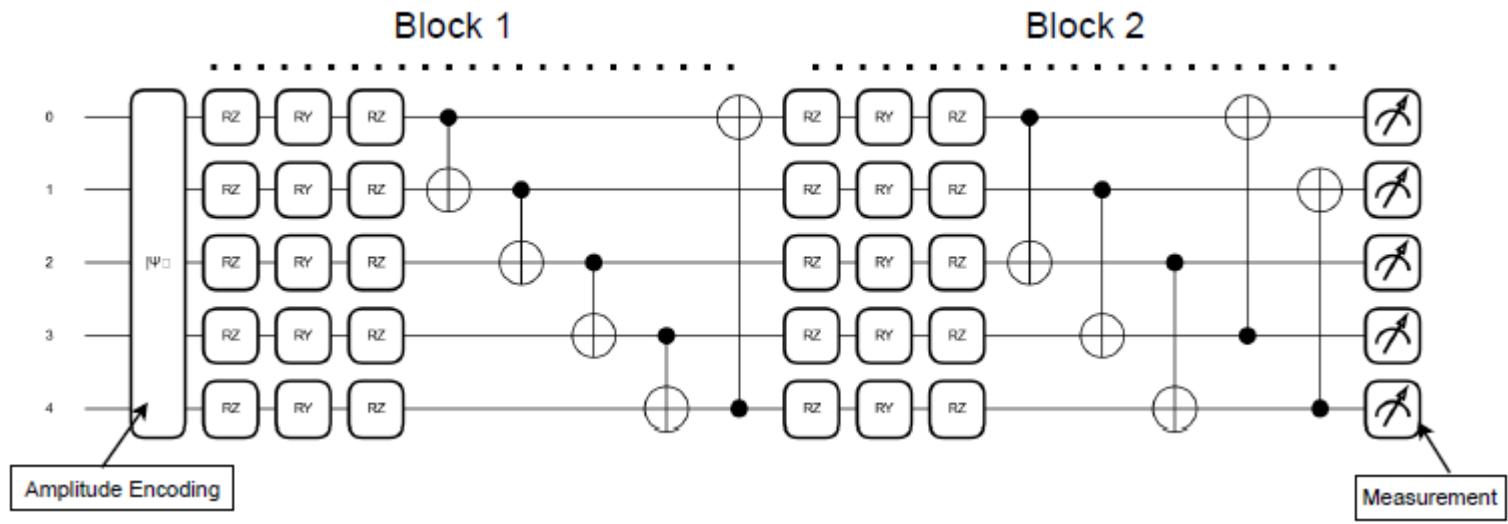


Quantum Graph Attention Network

A Novel Quantum Multi-Head Attention Mechanism for Graph Learning



An illustration of multi-head attention (with $K = 3$ heads) by node 1 on its neighborhood. Different arrow styles and colors denote independent attention computations. The aggregated features from each head are concatenated or averaged to obtain \vec{h}'_1 .
(<https://arxiv.org/abs/1710.10903>)



Example of a quantum circuit used in QGAT

Quantum Graph Attention Network

Benchmark results

Table 1: Accuracy (%) on transductive node classification benchmarks. All results are averaged over 5 runs; standard deviations are reported.

Model	Pubmed	ogbn-arxiv	ogbn-products
GAT [11]	78.1 ± 0.59	71.54 ± 0.3	79.04 ± 1.54
GATv2 [11]	78.5 ± 0.38	71.87 ± 0.25	80.63 ± 0.7
QGAT	79.2 ± 0.62	73.62 ± 0.42	82.10 ± 2.31

Table 2: Inductive node classification performance on PPI and ogbn-proteins. Metric: Micro-F1 (PPI) and ROC-AUC (ogbn-proteins). All results are averaged over 5 runs; standard deviations are reported.

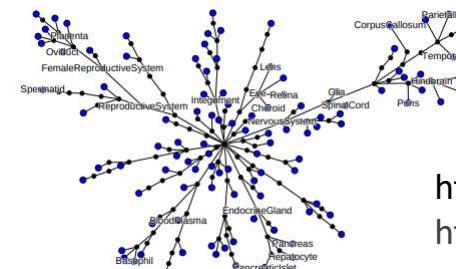
Model	PPI (Micro-F1)	ogbn-proteins (ROC-AUC)
GAT [6]	97.3 ± 0.20	78.63 ± 1.62
GATv2	98.2 ± 0.25	79.52 ± 0.55 [11]
QGAT	98.9 ± 0.12	79.41 ± 0.21

Table 3: Link prediction performance on OGB benchmarks. Metric: Hits@50 (%) for *ogbl-collab*, and Mean Reciprocal Rank (MRR) for *ogbl-citation2*. All results are averaged over 5 runs; standard deviations are reported.

Model	ogbl-collab	ogbl-citation2
GAT [11]	46.63 ± 2.80	75.95 ± 1.31
GATv2 [11]	49.7 ± 3.08	80.14 ± 0.71
QGAT	51.2 ± 1.92	82.2 ± 1.27

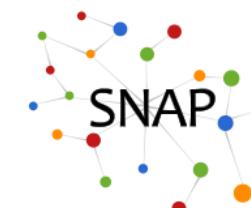


<https://arxiv.org/abs/2005.00687>



<https://arxiv.org/abs/1707.04638>

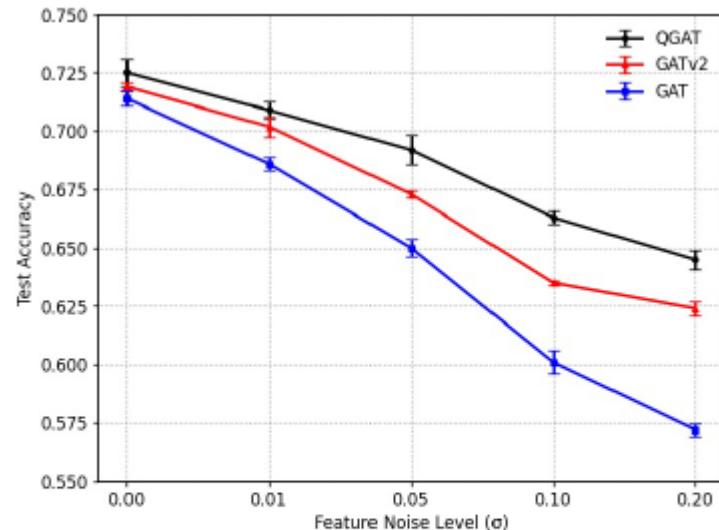
<https://arxiv.org/abs/1706.02216>



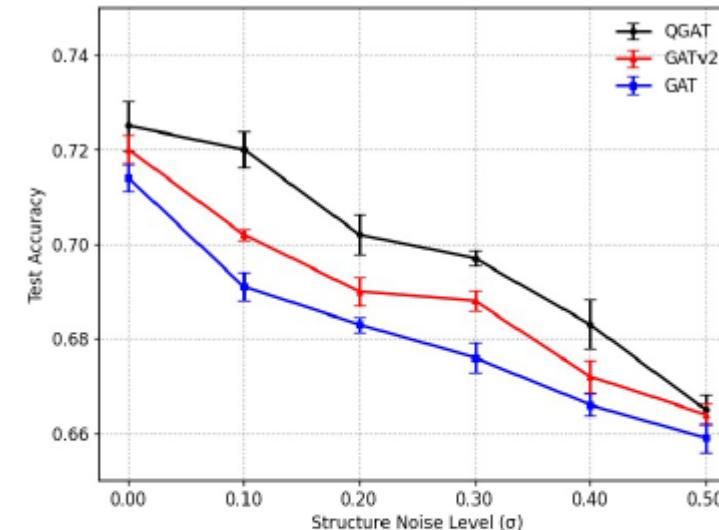
<https://snap.stanford.edu/>

Quantum Graph Attention Network

Robustness to Noise



(a) Feature noise. Test accuracy under Gaussian perturbation with σ .



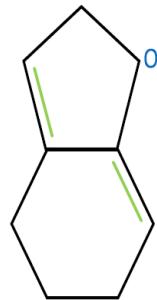
(b) Structural noise. Test accuracy under random edge insertion with σ .

Figure 2: Robustness comparison of QGAT, GATv2, and GAT under feature and structural noise. Each point is averaged over 5 runs with standard deviation shown as error bars.

Quantum Circuit for Molecular Generation

Overview of the quantum-based molecular generator (QMG)

QM9 datasets: Only C,N,O atoms and 9 heavy atoms at most (ignore F atom).

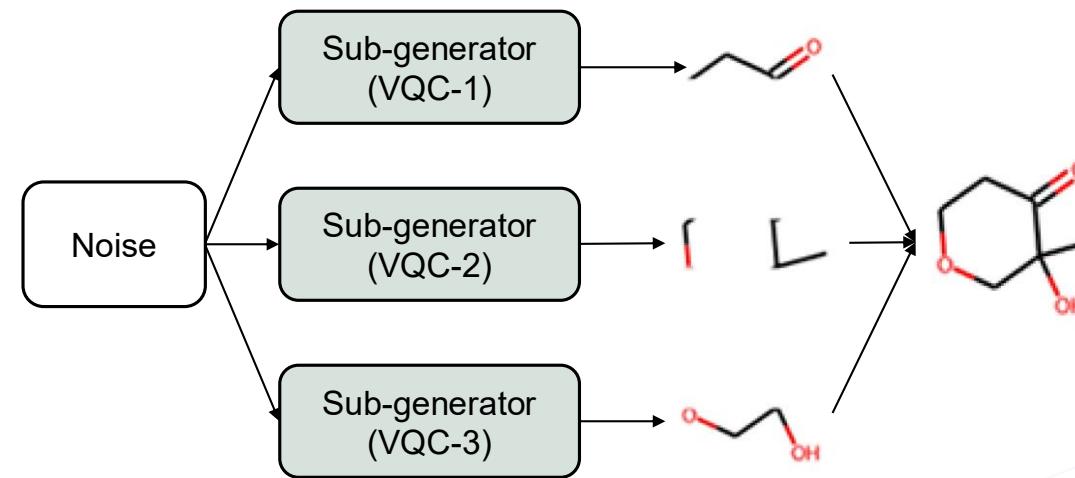


$$B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$A = [1 \quad 3 \quad 1 \quad 2 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$$

$$a_i \in \{\text{None, C, N, O}\}^9$$

$$b_{ij} \in \{\text{None, single, double, triple}\}^{36}$$

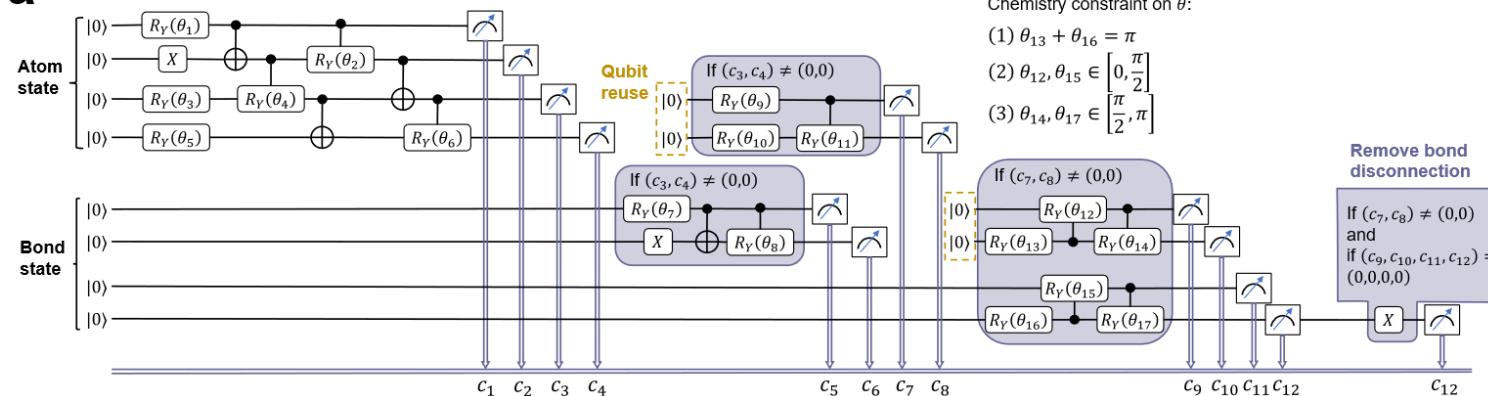
$$\log_2 4^9 + \log_2 4^{36} = 90 \text{ qubits}$$



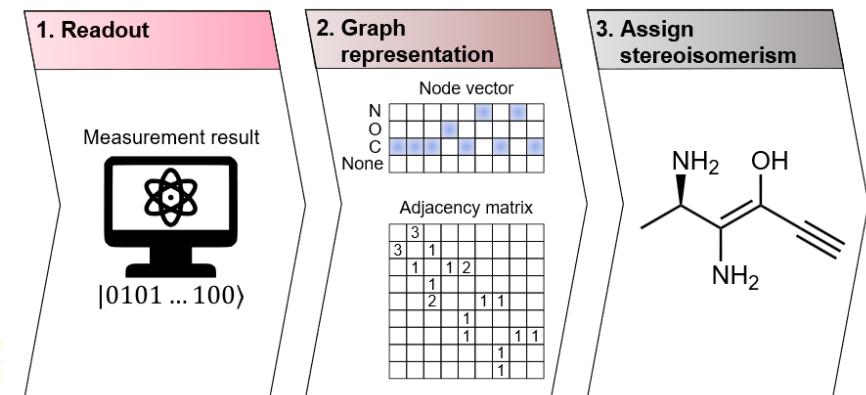
Quantum Circuit for Molecular Generation

Overview of the quantum-based molecular generator (QMG)

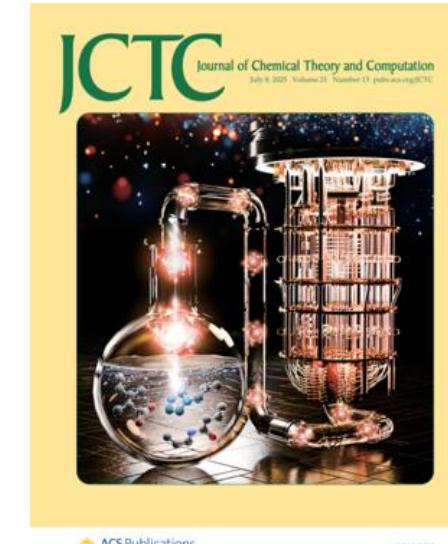
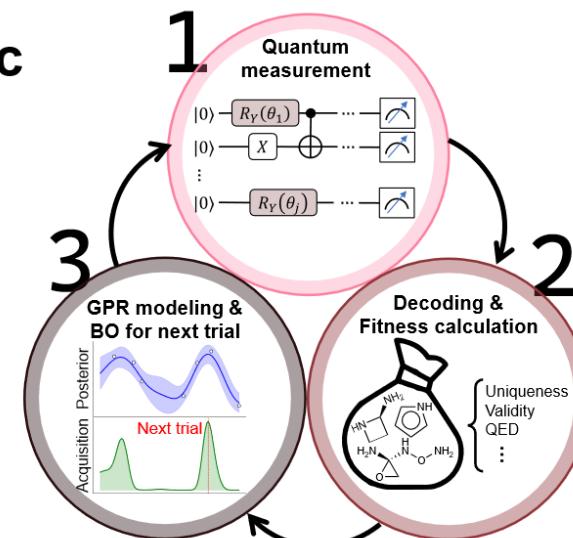
a



b



c



Overview of the molecular structure generation and using dynamic quantum circuit. (a) An illustration figure of 3-heavy atom quantum-based molecule generator (QMG). (b) The measurement outcome (binary value) would be post-processed to molecular structures on classical computers. (c) The parameters in quantum generator take the Bayesian optimization (BO) routine for enhancing properties and validity.

Quantum Circuit for Molecular Generation

Overview of the quantum-based molecular generator (QMG)

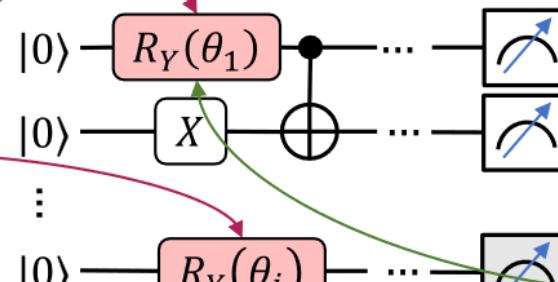
Classical computing

2.2. Scaffold-retained generation
for molecular design
(User specifies substructure)

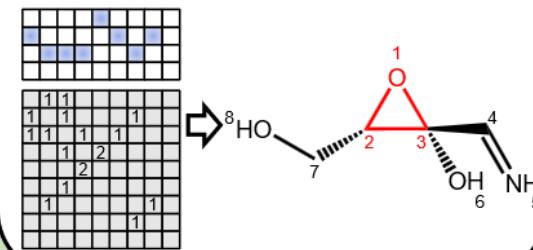


Quantum computing

2.1. Ansatz of quantum dynamic circuits
(User specifies chemical space)



2.3. Post-processing of 2D
molecular graphs with
stereochemical details

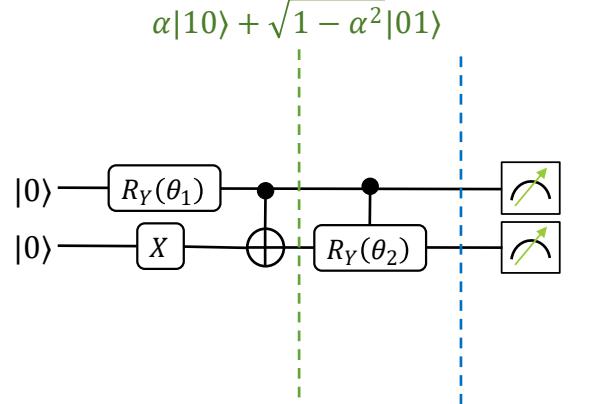


2.4 Bayesian optimization (BO) of
QMG parameters

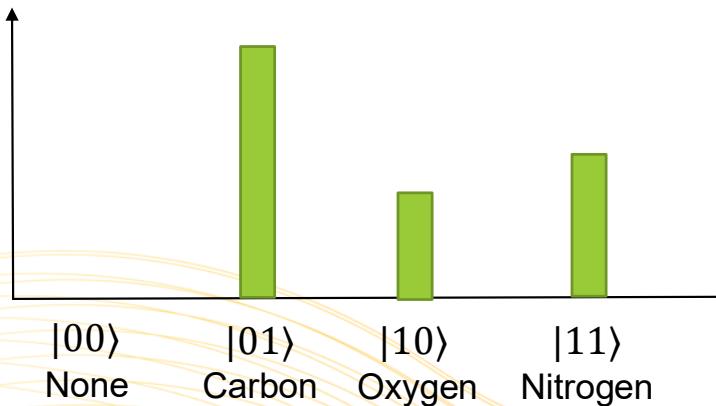
**Overview of the
quantum-based
molecular generator
(QMG) construction
process.**

Quantum Circuit for Molecular Generation

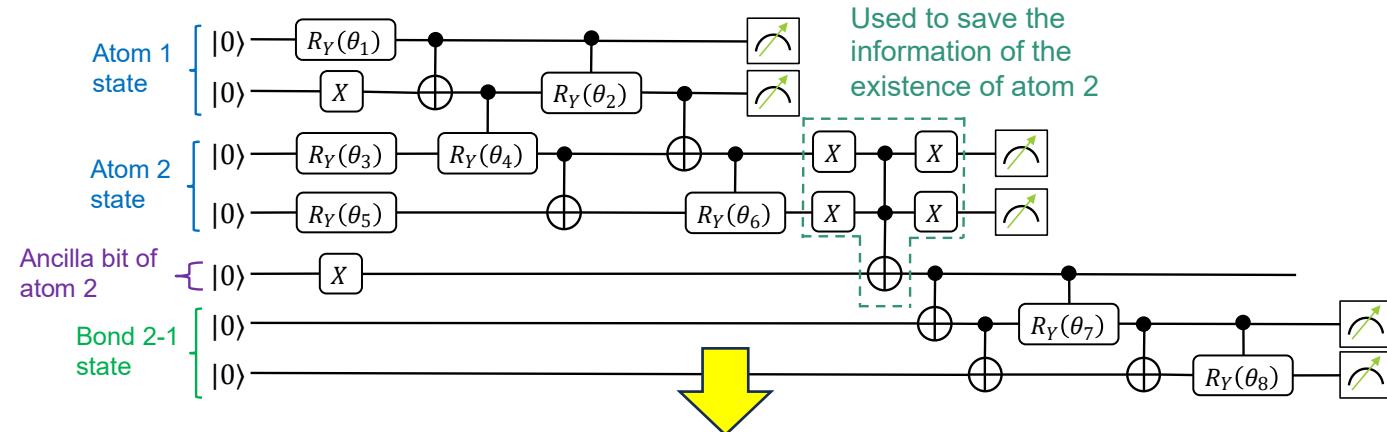
(a) One-heavy atom system



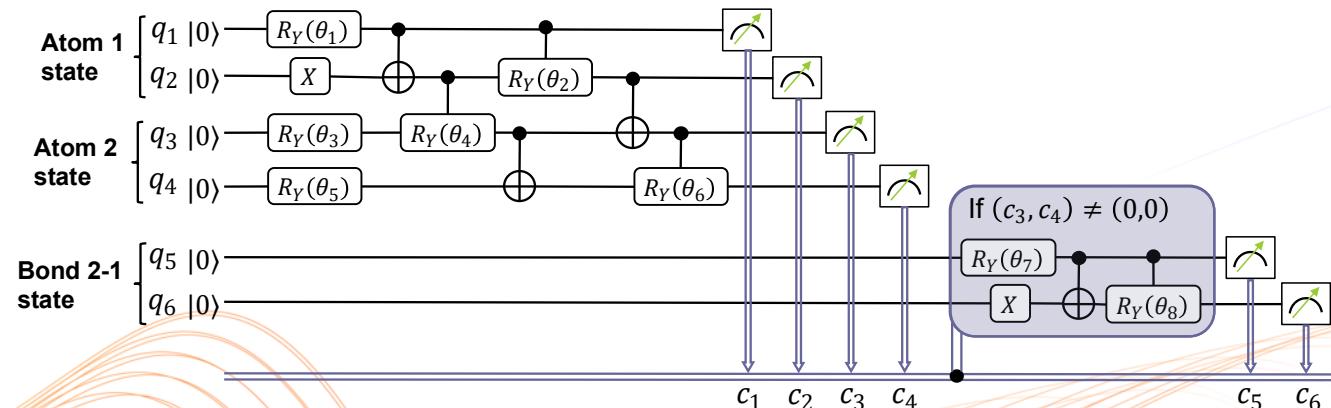
$$\alpha\beta|10\rangle + \alpha\sqrt{1 - \beta^2}|11\rangle + \sqrt{1 - \alpha^2}|01\rangle$$



(b) Two-heavy atom system : Static Circuit



(c) Two-heavy atom system : Dynamic Circuit



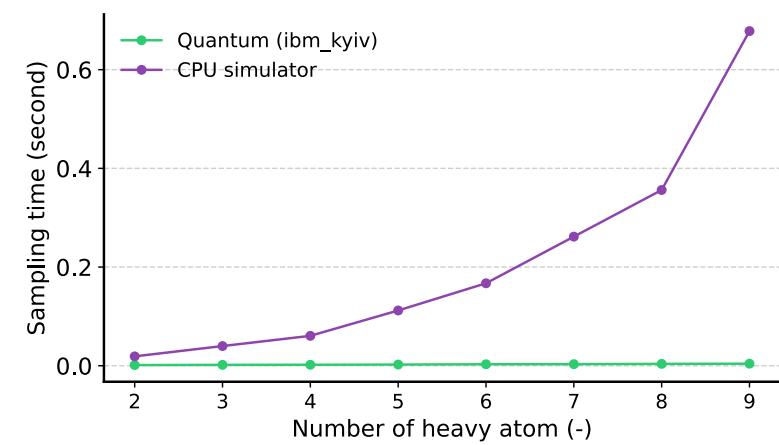
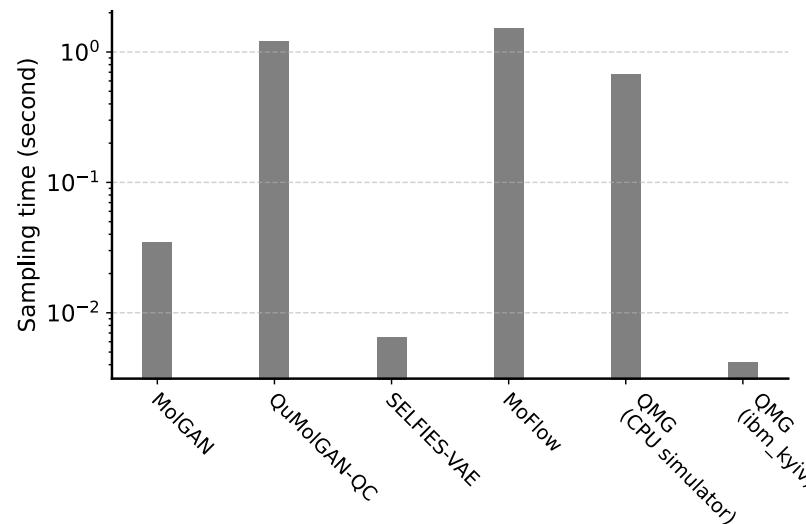
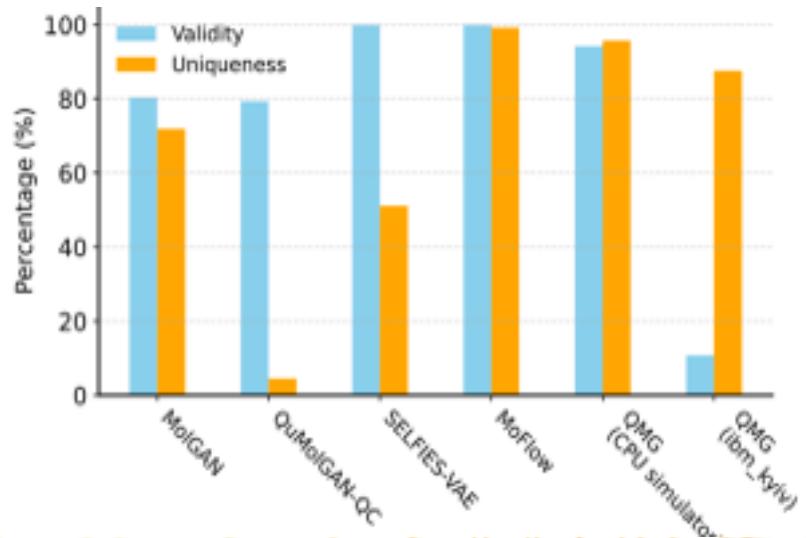
Quantum Circuit for Molecular Generation

No. of heavy atoms	No. of qubits required for static circuit	No. of qubits required for dynamic circuit	No. of parameters in dynamic circuit	Valid proportion of quantum state (%)
2	6	6	8	45.32
3	12	8	17	10.67
4	20	10	29	1.10
5	30	12	44	0.04
6	42	14	62	-
7	56	16	83	-
8	72	18	107	-
9	90	20	134	-
N	$N(N + 1)$	$2(N + 1)$	$8 + 3(N - 2)(N + 3)/2$	-

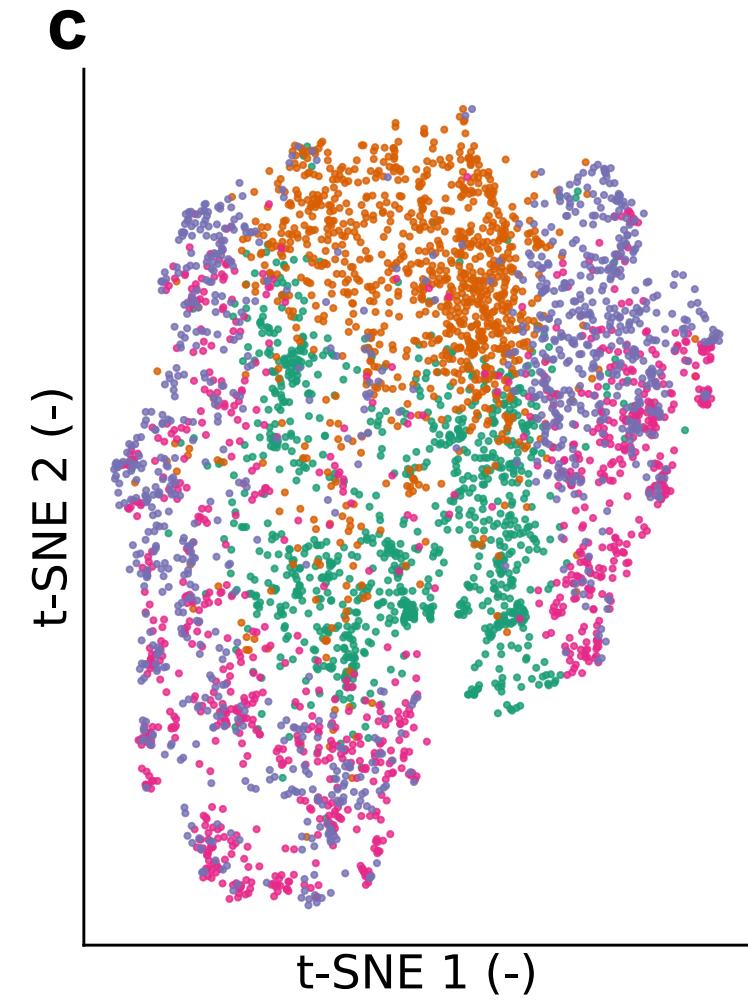
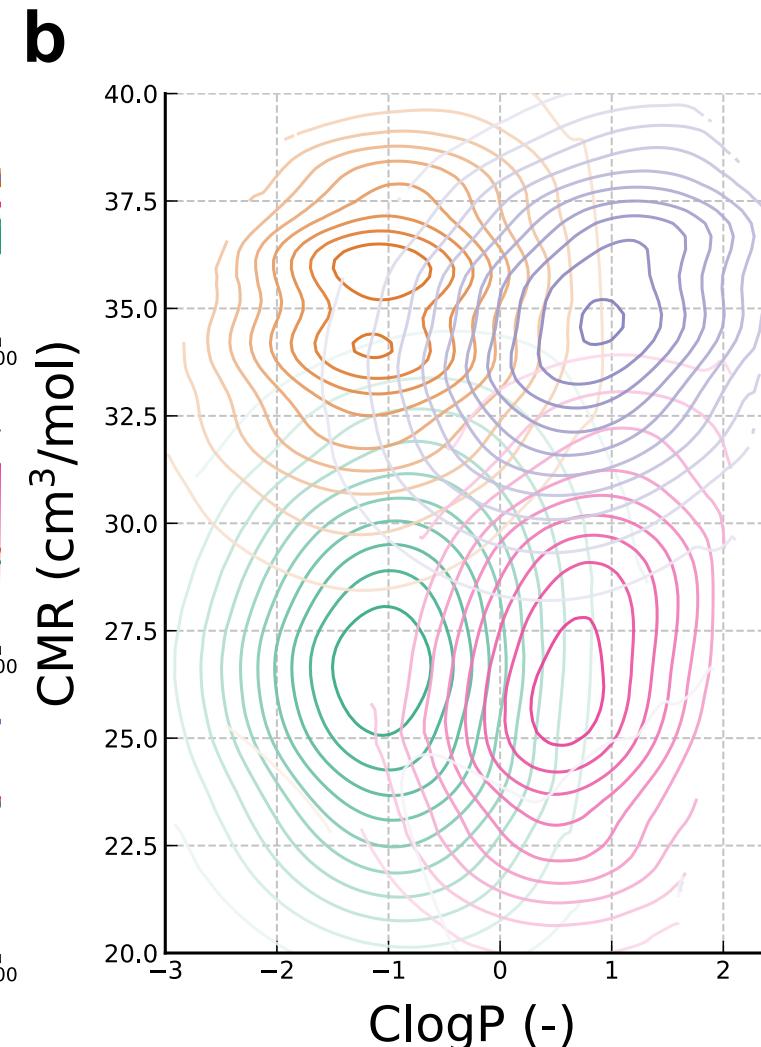
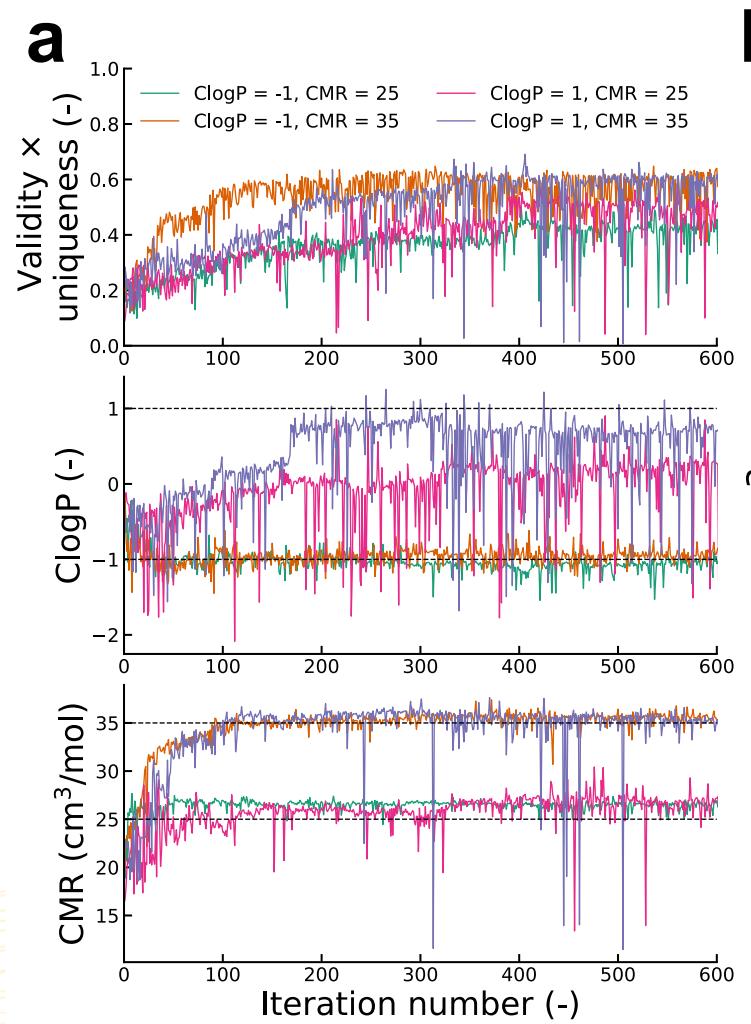
Quantum Circuit for Molecular Generation

Benchmark Result

Model	Validity (%) (\uparrow)	Uniqueness (%) (\uparrow)	No. of parameters (\downarrow)
MolGAN [1]	80.40	71.89	\sim 576,000
QuMoIGAN-QC [2]	79.39	4.49	\sim 90,000
SELFIES-VAE [3]	100	51.08	\sim 77,000
MoFlow [4]	100	99.26	\sim 2,712,000
QMG (Ours)	93.9	94.6	134

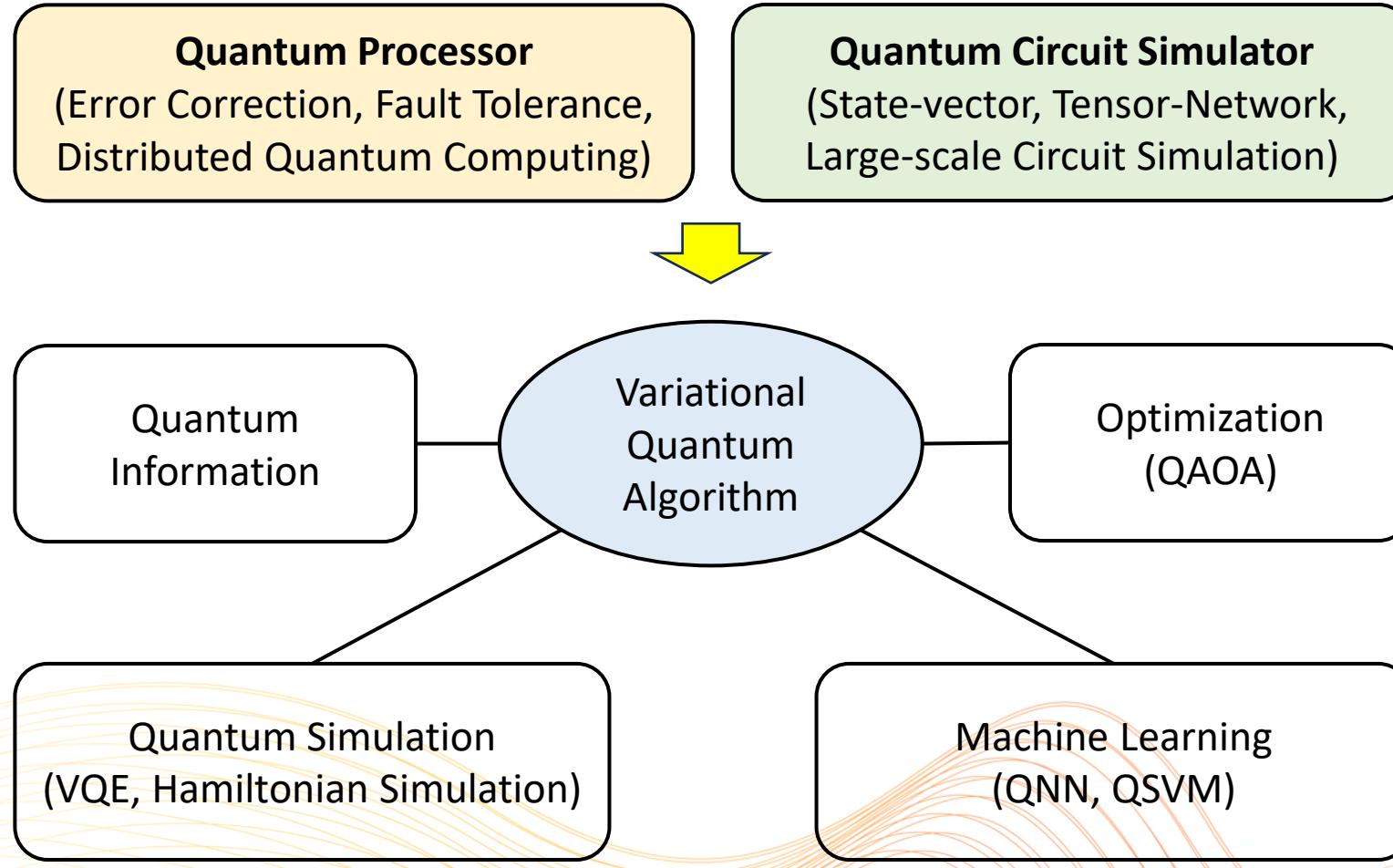


Quantum Circuit for Molecular Generation

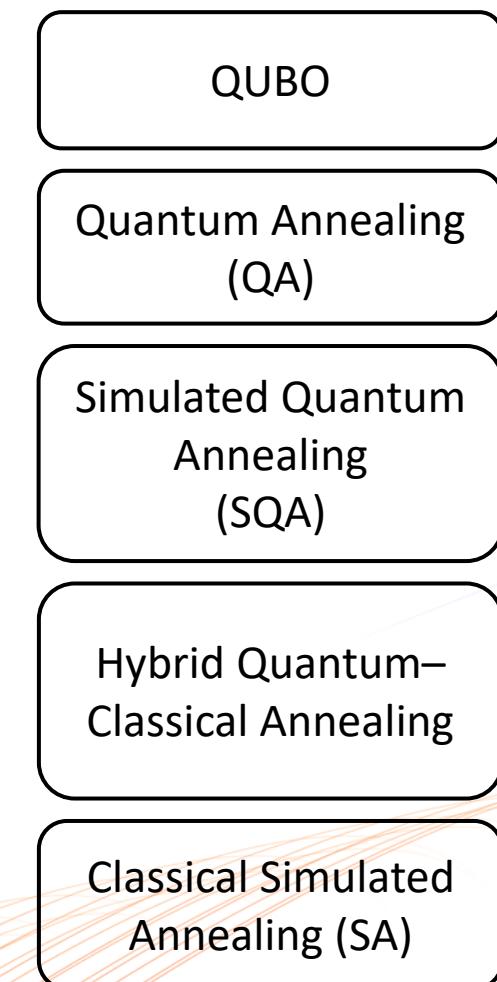


Future Works

Gate-base system



Annealing-base system



Collaborators

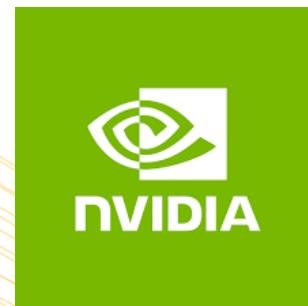


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國家高速網路與計算中心
National Center for High-performance Computing

陳南佑/國網中心
李泰岳/國網中心
陳冠朋/國網中心
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林俊鈺/國網中心



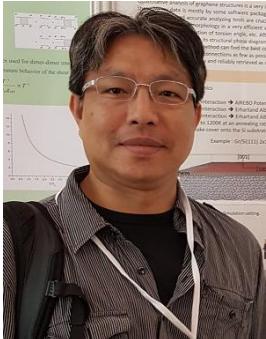
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甯安/KAIST
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董甄茵/台大電機系



王允遠/NVIDIA
陳冠丞/QuEST

關於我們

科學計算小組



陳南佑 博士

李玟頽 博士

楊安正 博士

陳冠朋

李泰岳

清華大學物理博士

中山大學機械博士

成功大學機械博士

中央大學應數碩士

東華大學物理博士

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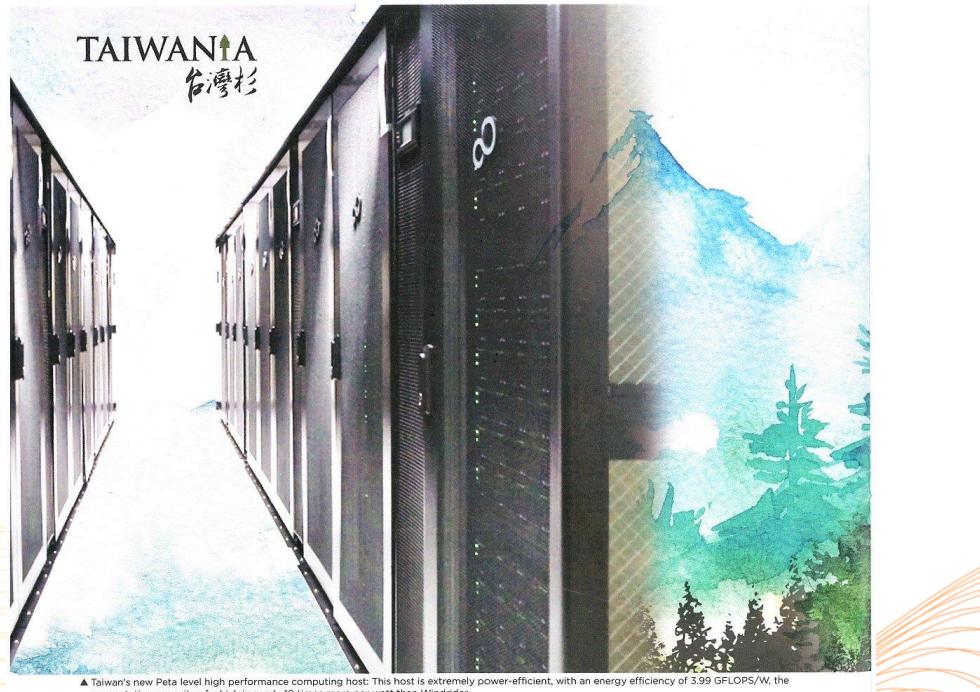
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