Uncertainty Quantification for Transport in Porous Media Using Parameterized Physics Informed Neural Networks

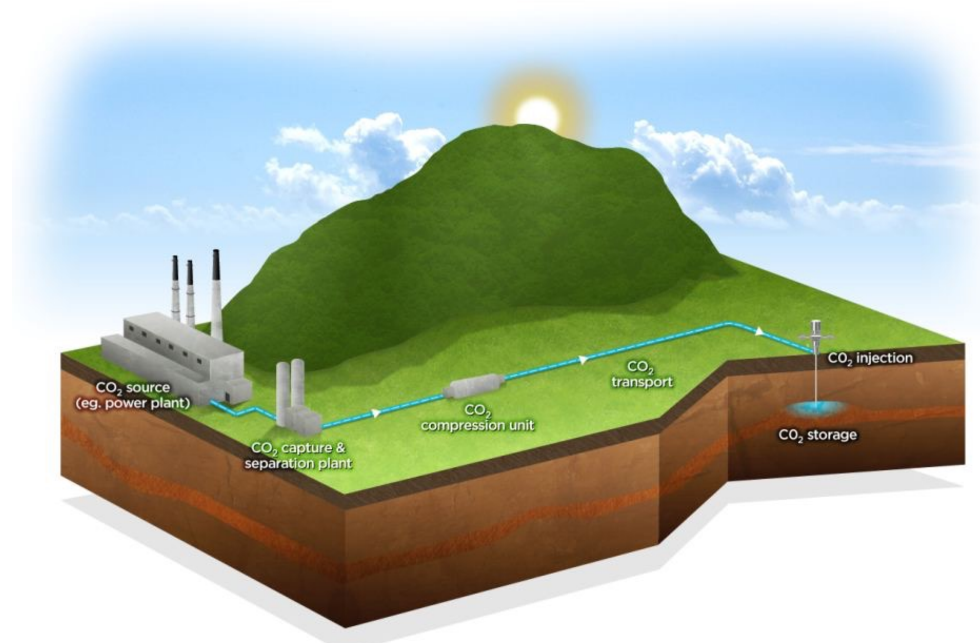
Stanford | SCHOOL OF EARTH, ENERGY & ENVIRONMENTAL SCIENCES

CEDRIC FRACES GASMI
HAMDI TCHELEPI

Motivation

Uncertainty Quantification for Earth Systems

- Uncertainty Quantification for transport in Porous Media
  - Geologic CO_2 Sequestration
  - Contaminant transport
  - Hydrocarbon production
- Fast & Accurate feed forward models
- Sampling method for the resolution of Stochastic Partial Differential Equations (SPDE)



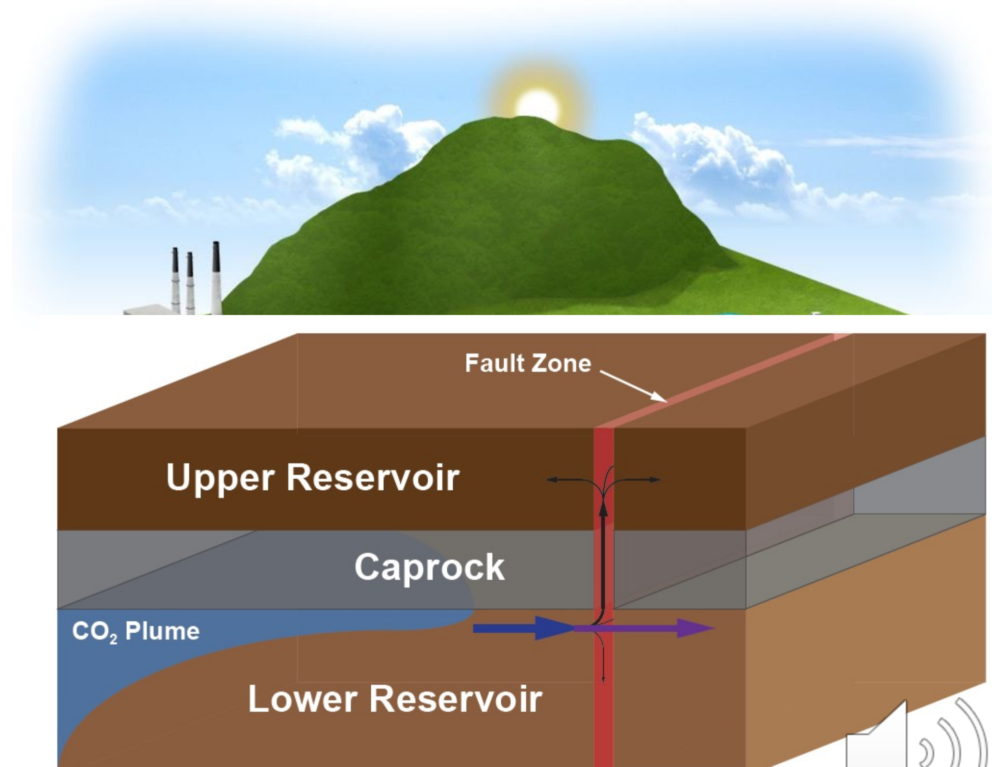
Source: www.greencitytimes.com

CO_2 plume migration

Transport problem

MAJOR UNCERTAINTIES:

- DISTANCE TRAVELED BY THE CO_2 PLUME
- AMOUNT OF CO2 TRAPPED NEAR THE WELLBORE
- PORE SPACE REQUIRED FOR STORAGE
- SURFACE AREA LIABILITY



Related work

Uncertainty Quantification in porous media

- Numerical methods (Monte Carlo sampling, Perturbations, Polynomial Chaos, Method of Moments, Mixture Density Networks,…)
- Limitations:
  - Small perturbations
  - Curse of dimensionality
  - Steady state
  - Closure problem
- Dongxiao Zhang, Hamdi Tchelepi. Stochastic Analysis of Immiscible Two-Phase Flow in Heterogeneous Media, SPE 59250, 1999

Problem Statement

Mesh free simulation of transport in porous media

FIND A FUNCTIONAL FORM

$$S_\theta(x,t) = \sigma\left[\boldsymbol{W}_0 \times \sigma\left(\boldsymbol{W}_1 \dots \sigma\left(\boldsymbol{W}_n \begin{bmatrix} x \\ t \end{bmatrix} + \boldsymbol{b}_n\right) \dots + \boldsymbol{b}_1\right) + \boldsymbol{b}_0\right]$$

Input: Sampled

Model: Deep Neural Network

Parameters:
Boundary conditions (wells BHP and water cut)
Physical laws (Darcy, conservation,…)
Rock properties (permeability, porosity, ...)

Reminder: Deterministic Riemann problem

Mixed wave (Rarefaction/Shock) solution

BUCKLEY LEVERETT FLOW

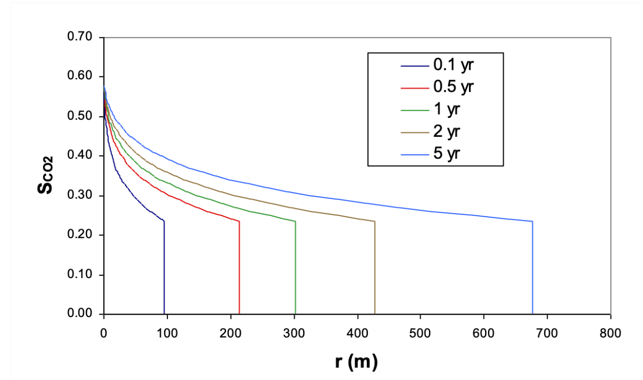
COREY-BROOKS RELATIVE PERMEABILITIES

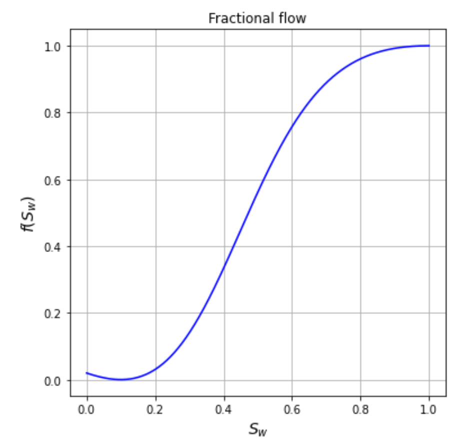
$$\frac{\partial S(x,t)}{\partial t} + \frac{df(S(x,t))}{dx} = 0$$

$$s.t$$

$$\begin{cases} S(x, t=0) = S_{wc} \\ S(x=0, t) = 1 \end{cases}$$

$$f(S) = \frac{(S - S_{wc})^2}{(S - S_{wc})^2 + \frac{(1 - S - S_{or})^2}{M}}$$



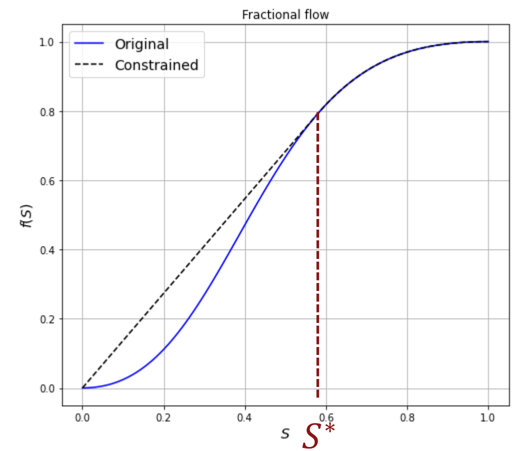


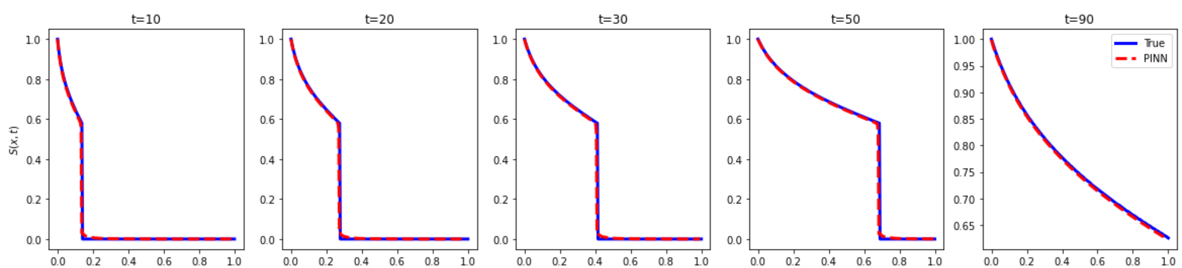
Reminder: Deterministic problem with PINNs

Resolution using PINN

$$\mathcal{L}_{res} = \sum_{i=1}^{N} \left(\frac{\partial S_\theta(x_i, t_i)}{\partial t} + \frac{d\tilde{f}\left(S_\theta(x_i, t_i)\right)}{dx} \right)^2$$

$$\tilde{f}(S) = \begin{cases} 0 & if\ S \le S_{wc} \\ \dfrac{S - S_{wc}}{f(S^*)} & if\ S_{wc} < S \le S^* \\ \dfrac{(S - S_{wc})^2}{(S - S_{wc})^2 + \dfrac{(1 - S - S_{or})^2}{M}} & if\ S^* < S \end{cases}$$



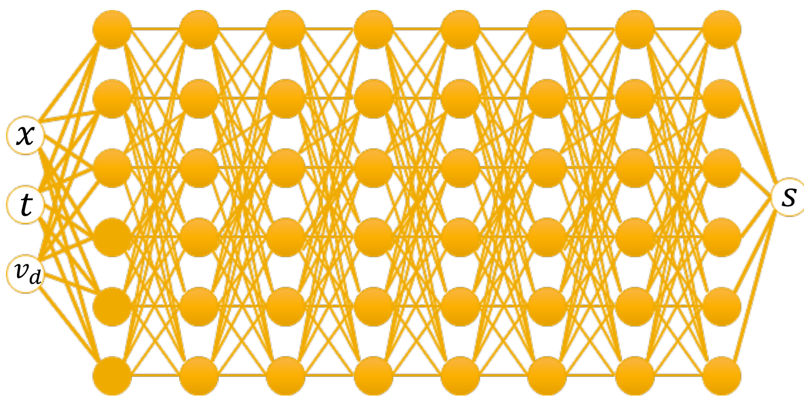


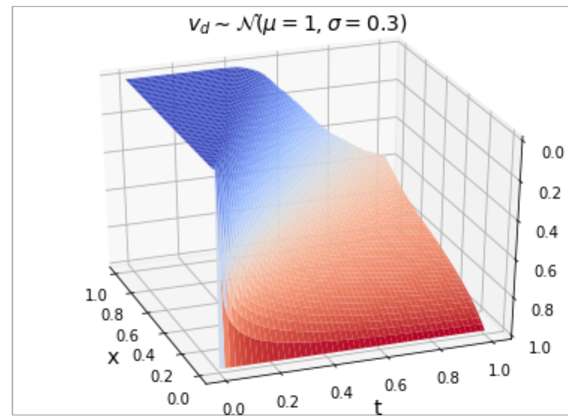
Problem Implementation

Higher dimension, lower gradient

Added input variable v_d

Smoother interpolation





Homogeneous case

Draw a random uniform velocity field

UNCERTAIN DARCY VELOCITY:

$$\frac{\partial S(x,t)}{\partial t} + \boldsymbol{v_d} \frac{df(S(x,t))}{dx} = 0$$

$$\boldsymbol{v_d} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\sigma})$$

PARAMETERIZED PINN (P-PINN) APPROACH:

- Solve PDE parameterized by $\boldsymbol{v_d}$
- Sample $\boldsymbol{v_d}$ randomly during training (along with $\boldsymbol{x}, \boldsymbol{t}, \boldsymbol{S}, \boldsymbol{\partial S}$)
- Added dimension: Smoother solution space

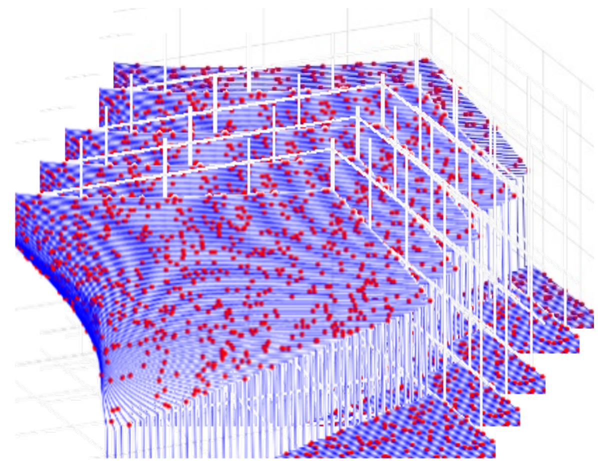
Implementation

Parametrized PINN

- Uniform sampling during training
  - x sampled as bound
  - (t, v_d) sampled as param ranges
- Entropy constrained fractional flow
- Loss function

$$\mathcal{L}_{res} = \sum_{i=1}^{N} \left(\frac{\partial S_\theta(x_i, t_i)}{\partial t} + v_i \frac{d\tilde{f}\big(S_\theta(x_i, t_i)\big)}{dx} \right)^2$$

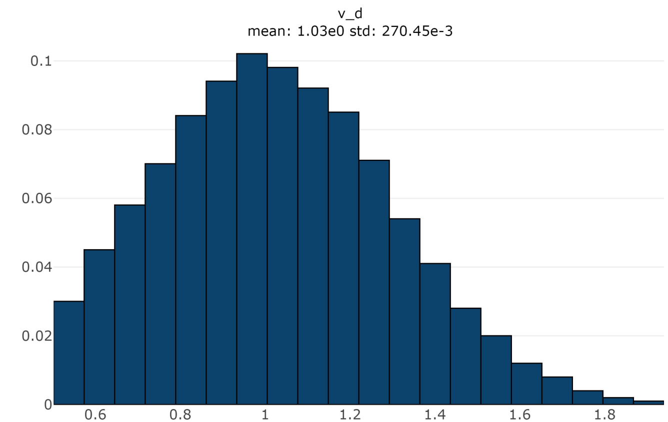
- Non uniform sampling (Normal, binomial,…) during inference

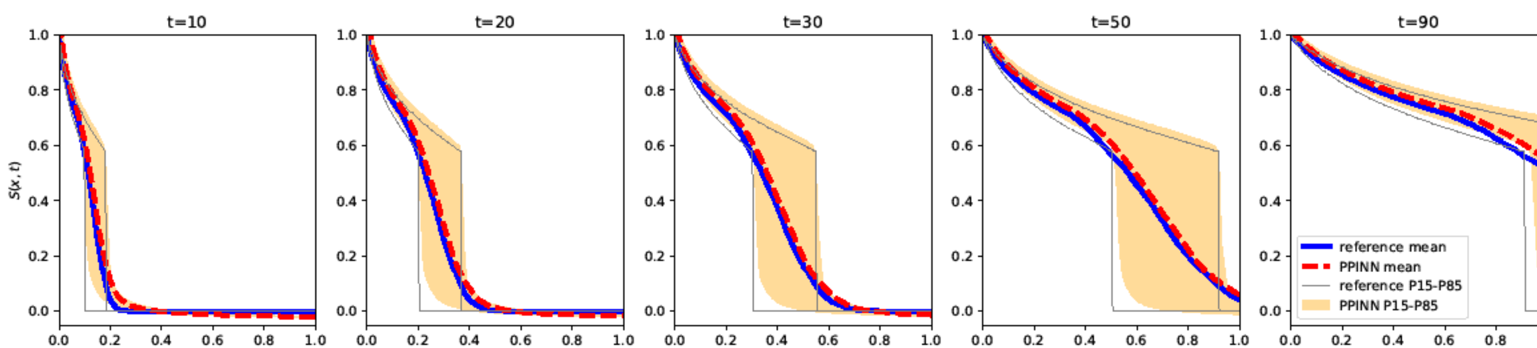


Results

Truncated Normal distribution

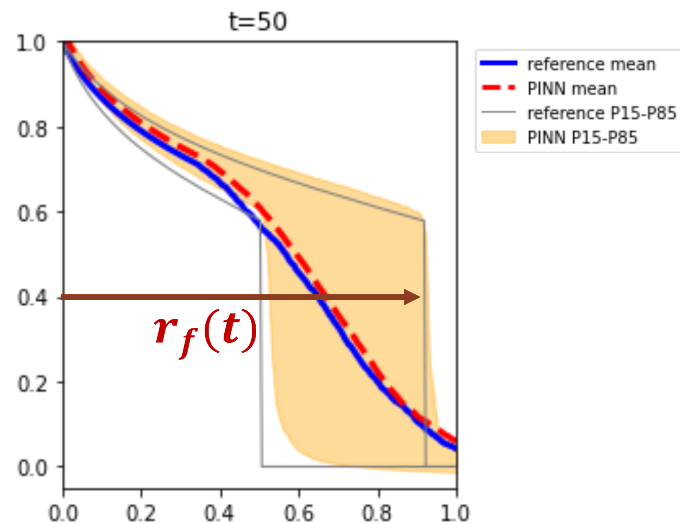
$$v_d \sim \mathcal{N}(\mu = 1, \sigma = 0.3, l = 0.5, u = 2)$$

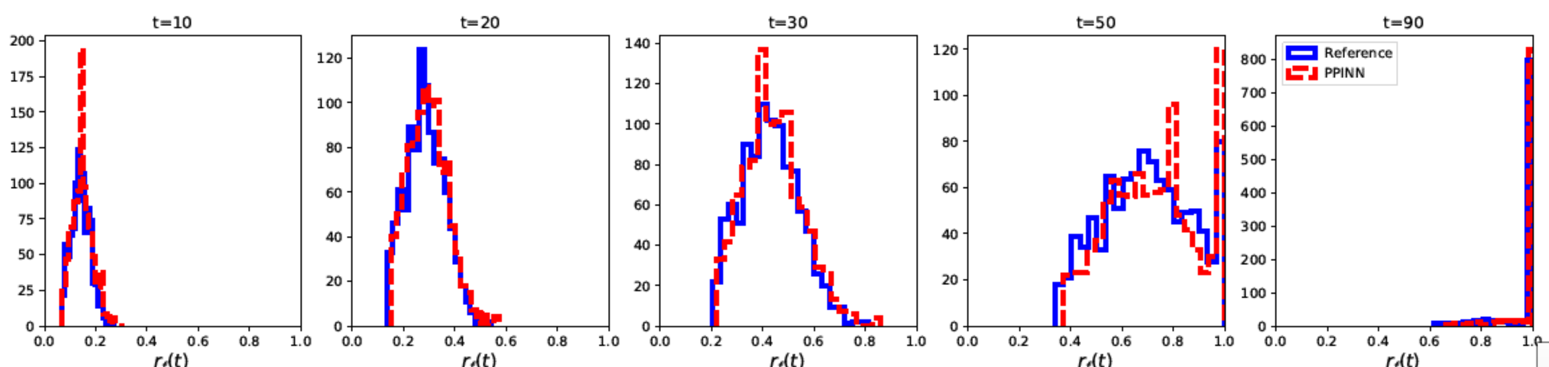




Results

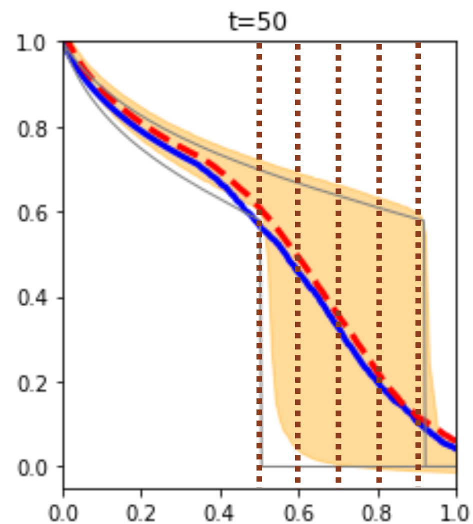
CO_2 Front Radius

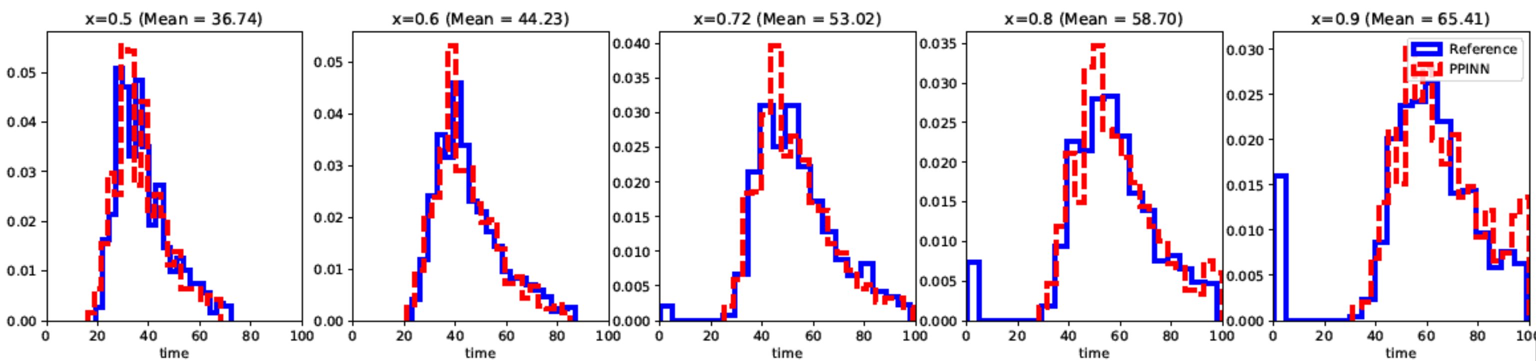




Results

CO_2 BREAKTHROUGH TIME

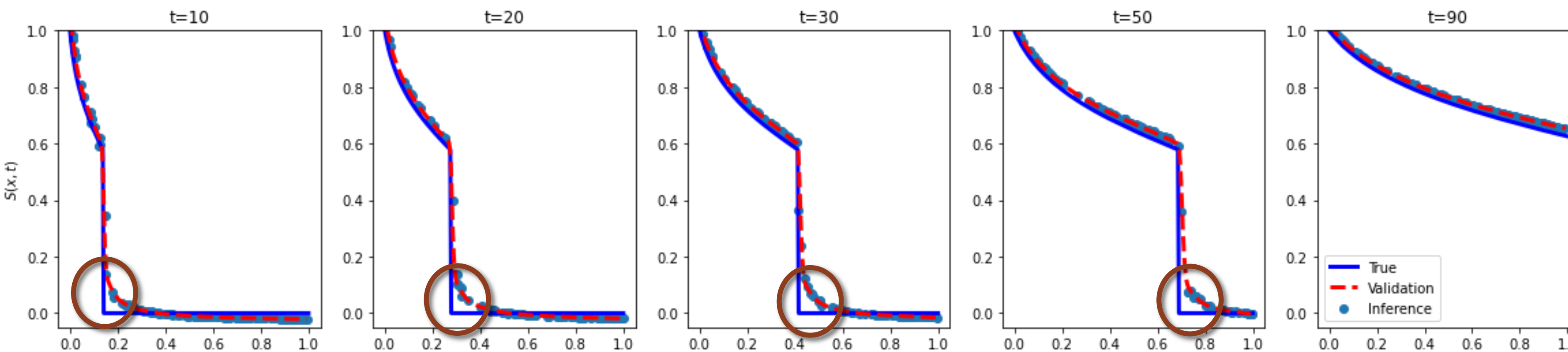




Individual Realizations

Smoother solution

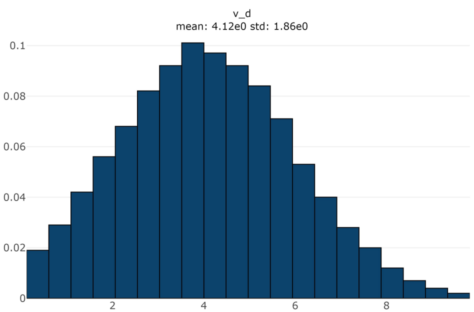
- Loss of accuracy due to added dimension
- Sufficient for distribution for QOI:
- Breakthrough time
- Front Radius

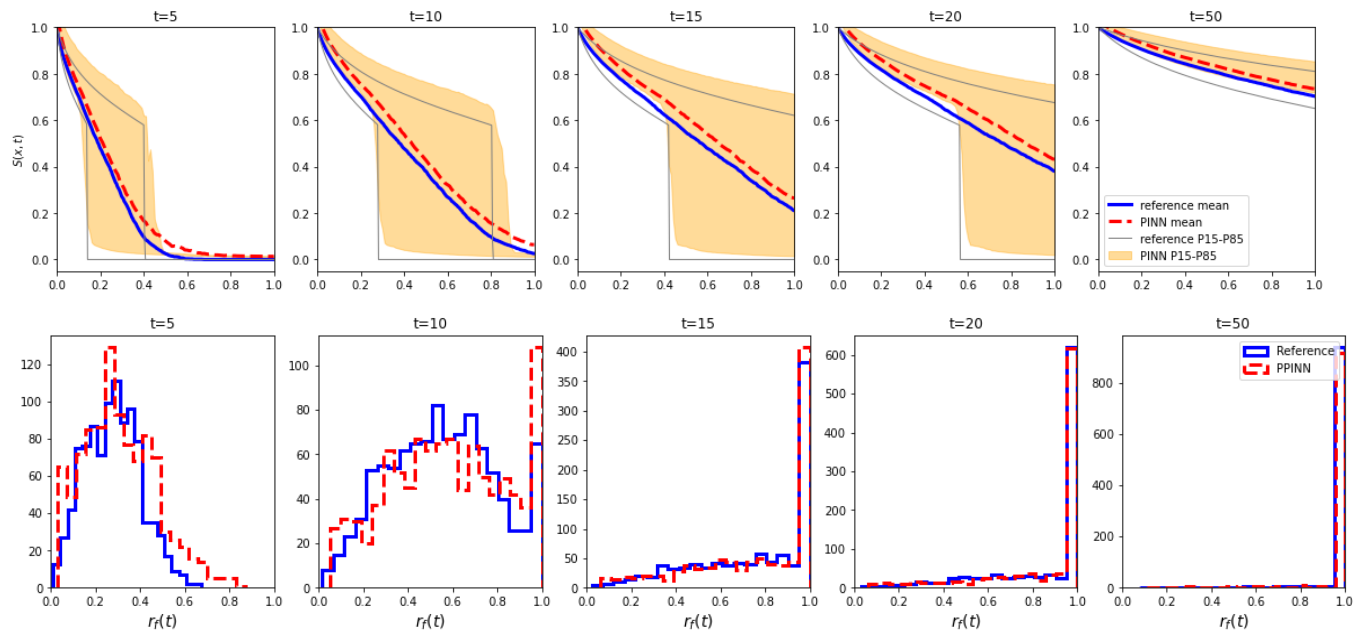


Wider support

Two orders of magnitude between min and max

$$v_d \sim \mathcal{N}(\mu = 4, \sigma = 1, l = 0.1, u = 10)$$

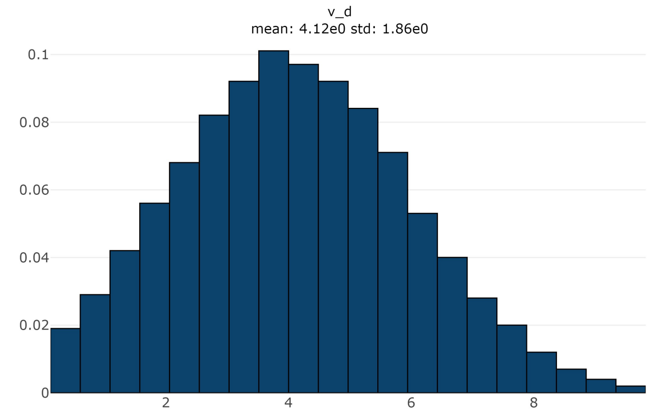


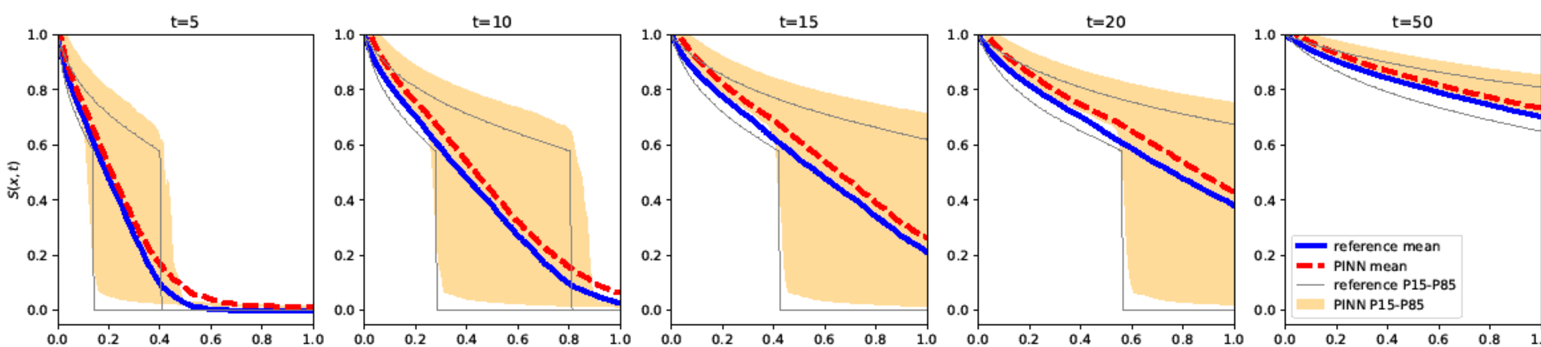


Wider support

Two orders of magnitude between min and max

$$v_d \sim \mathcal{N}(\mu = 4, \sigma = 1, l = 0.1, u = 10)$$

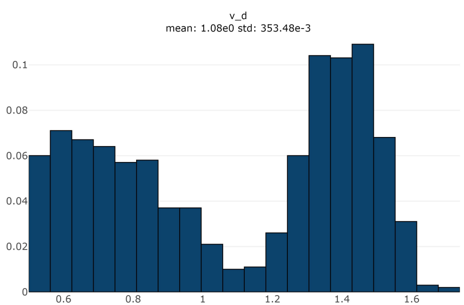


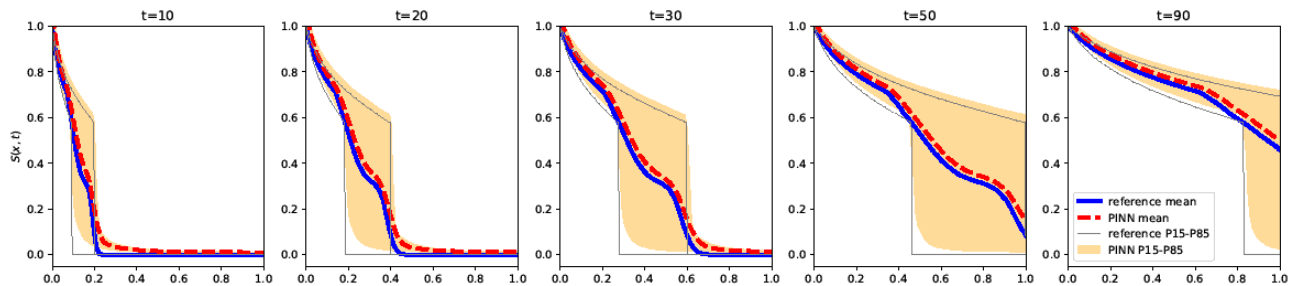


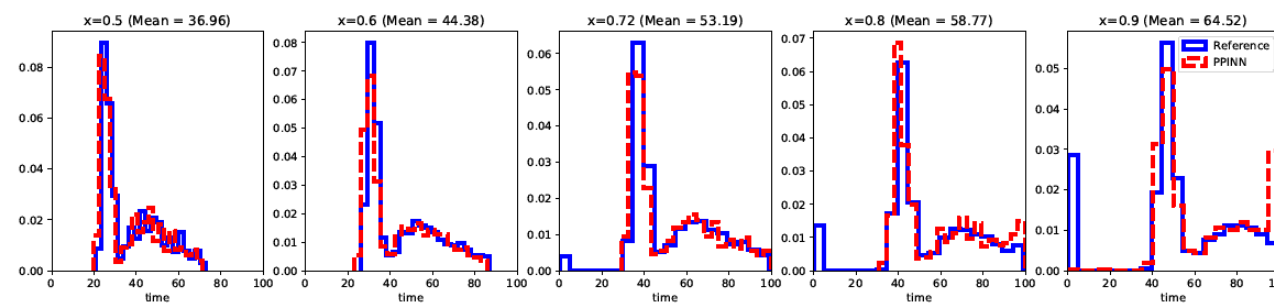
Bimodal

Non Normal distribution

$$v_d \sim Bimodal$$





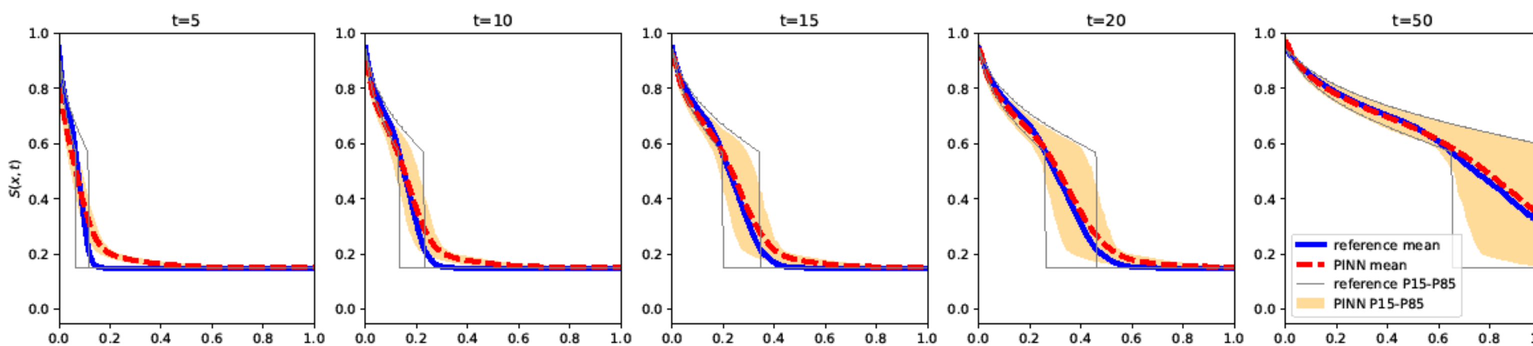


Residual saturations

Trapping, immobile fluids

CONNATE AND RESIDUAL SATURATION NON-NULL

$$f(S) = \frac{(S - S_{wc})^2}{(S - S_{wc})^2 + \frac{\left(1 - S - S_{gr}\right)^2}{M}}$$



Heterogeneous velocity field

Velocity field with Infinite dimensions

UNCERTAIN DARCY VELOCITY:

$$\frac{\partial S(x,t)}{\partial t} + \boldsymbol{v_d}(x)\frac{df(S(x,t))}{dx} = 0$$

$$\boldsymbol{v_d}(\boldsymbol{x}) \sim \langle \boldsymbol{v_d}(\boldsymbol{x}) \rangle + \mathcal{N}(\boldsymbol{0}, \boldsymbol{\sigma}(\boldsymbol{x}))$$

PARAMETERIZED PINN (P-PINN) APPROACH:

- Solve PDE parameterized by $\boldsymbol{v_d}$
- Sample $\boldsymbol{v_d}$ randomly during training (along with $\boldsymbol{x}, \boldsymbol{t}, \boldsymbol{S}, \boldsymbol{\partial S}$)
- Added dimension: Smoother solution space

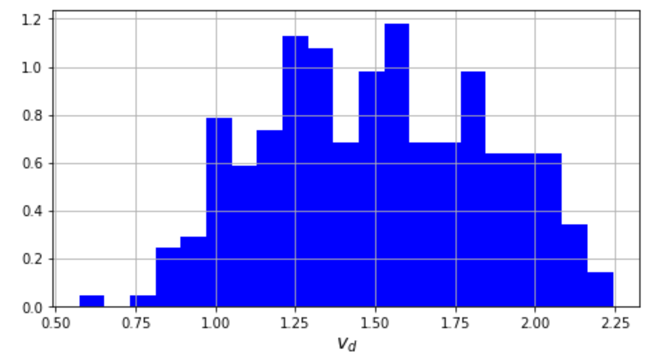
Implementation

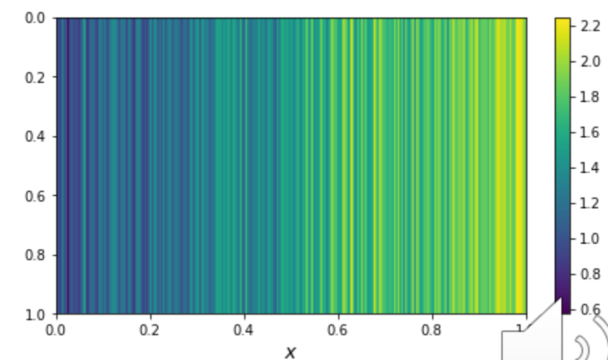
Decoupling stochastic from space domain

- Non-uniform sampling during training
  - x sampled as bound
  - (t, r_1, r_2) sampled as param ranges
  - Normal v_i obtained using a transform (Box-Muller, inverse):

$$v_i = \sqrt{-2log(r_{1,i})}\cos(2\pi r_{2,i})$$

$$\mathcal{L}_{res} = \sum_{i=1}^{N} \left(\frac{\partial S_\theta(x_i, t_i)}{\partial t} + v_i \frac{d\tilde{f}\left(S_\theta(x_i, t_i)\right)}{dx} \right)^2$$



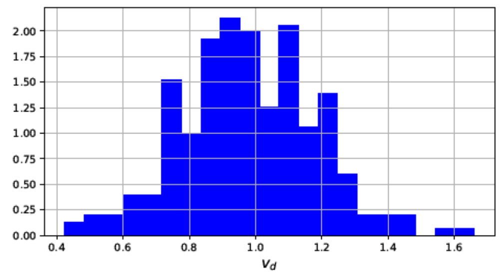


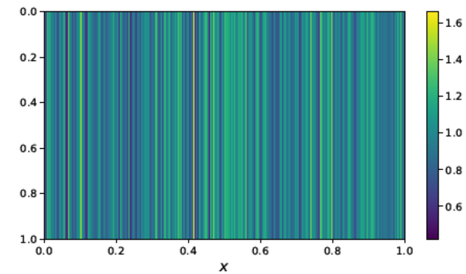
Results

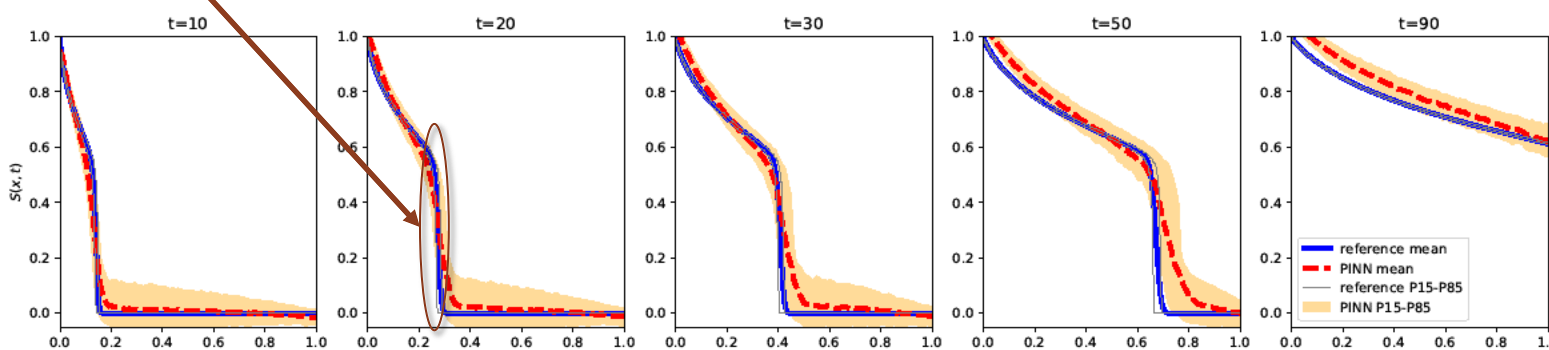
Limitations of the approach

$$v_d \sim \mathcal{N}(\mu = 1, \sigma = 0.3, l = 0.5, u = 2)$$

Narrower uncertainty space





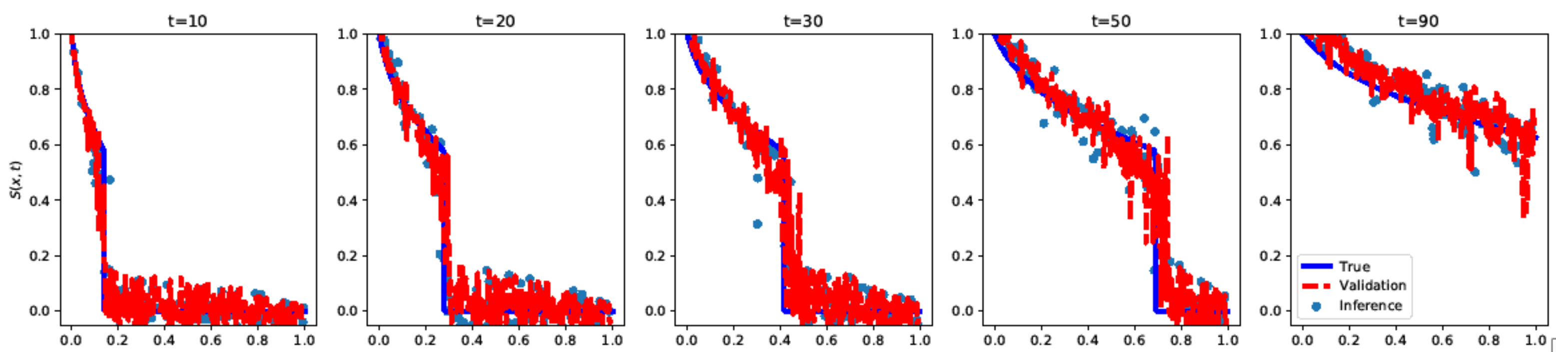


Realizations

Non-physical solutions

NOISY RESULTS USING FOLLOWING IMPLEMENTATIONS:

- ENTROPY CONSTRAINED FRACTIONAL FLOW
- ADDED DIFFUSION TERM
- WEIGHTED LOSS
- NEURAL NET TRAINED TO MIMIC VELOCITY FIELD
- INTEGRAL CONTINUITY

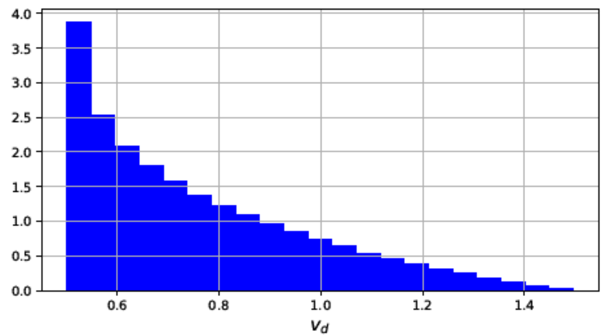


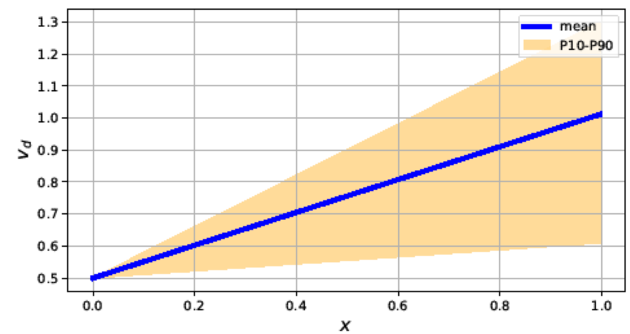
Stochastic affine velocity function

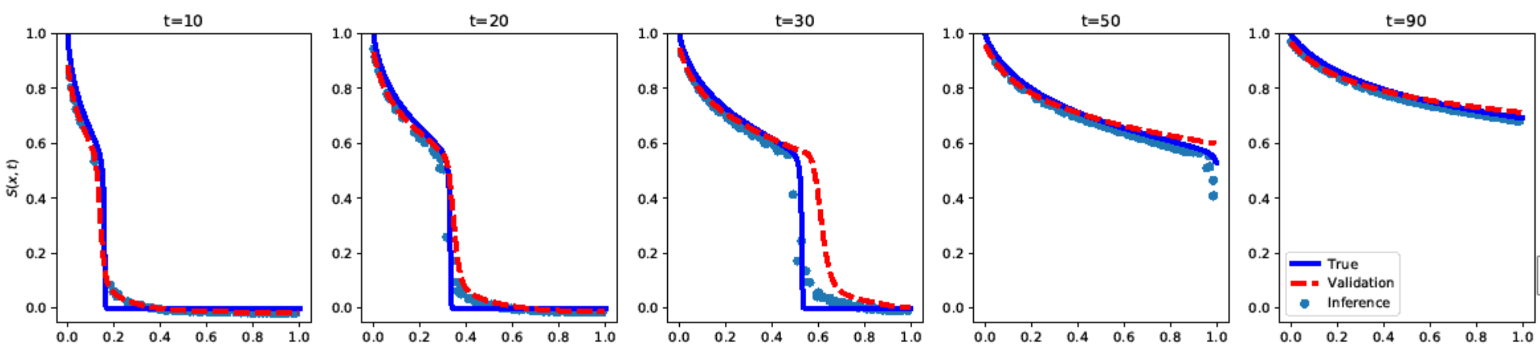
Parametrization of stochastic form

$$v_d(x) = \theta x + b$$

$$\theta \sim \mathcal{N}(\boldsymbol{\mu} = \mathbf{1}, \boldsymbol{\sigma} = \mathbf{0.3})$$

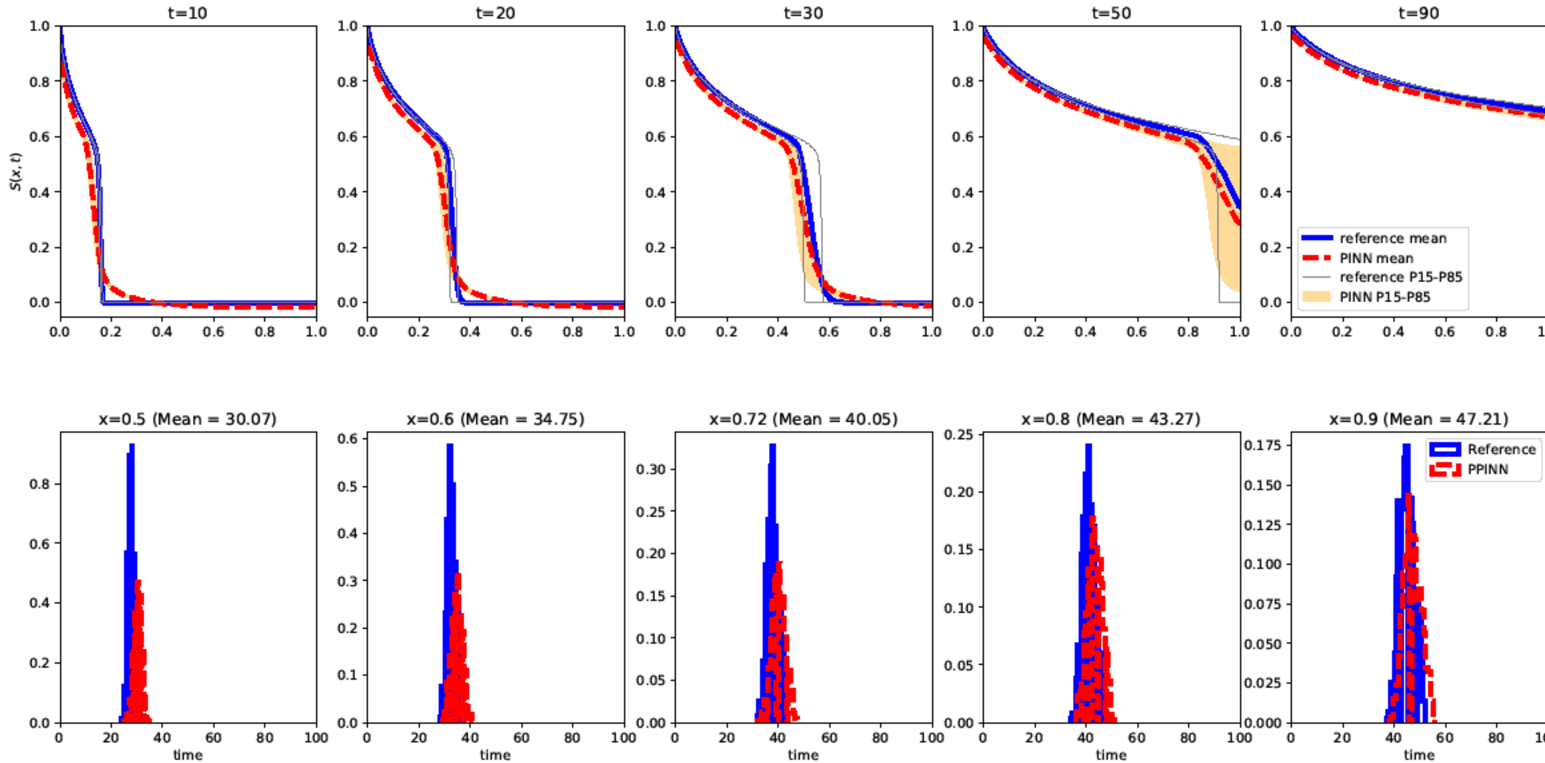






Stochastic affine velocity function

Stochastic and heterogeneous

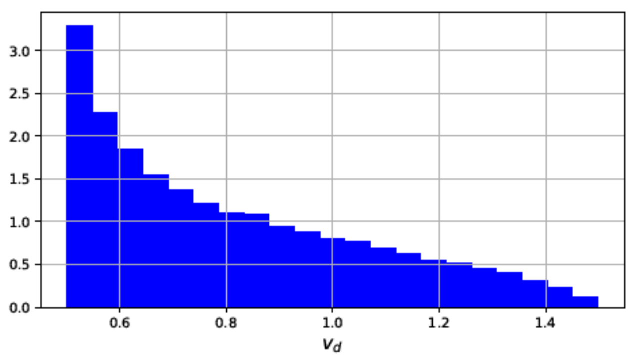


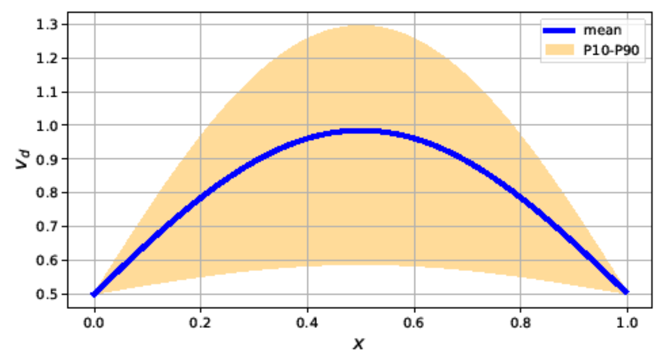
Stochastic periodic velocity function

Non-monotonic velocity

$$v_d(x) = \theta \sin(x) + b$$

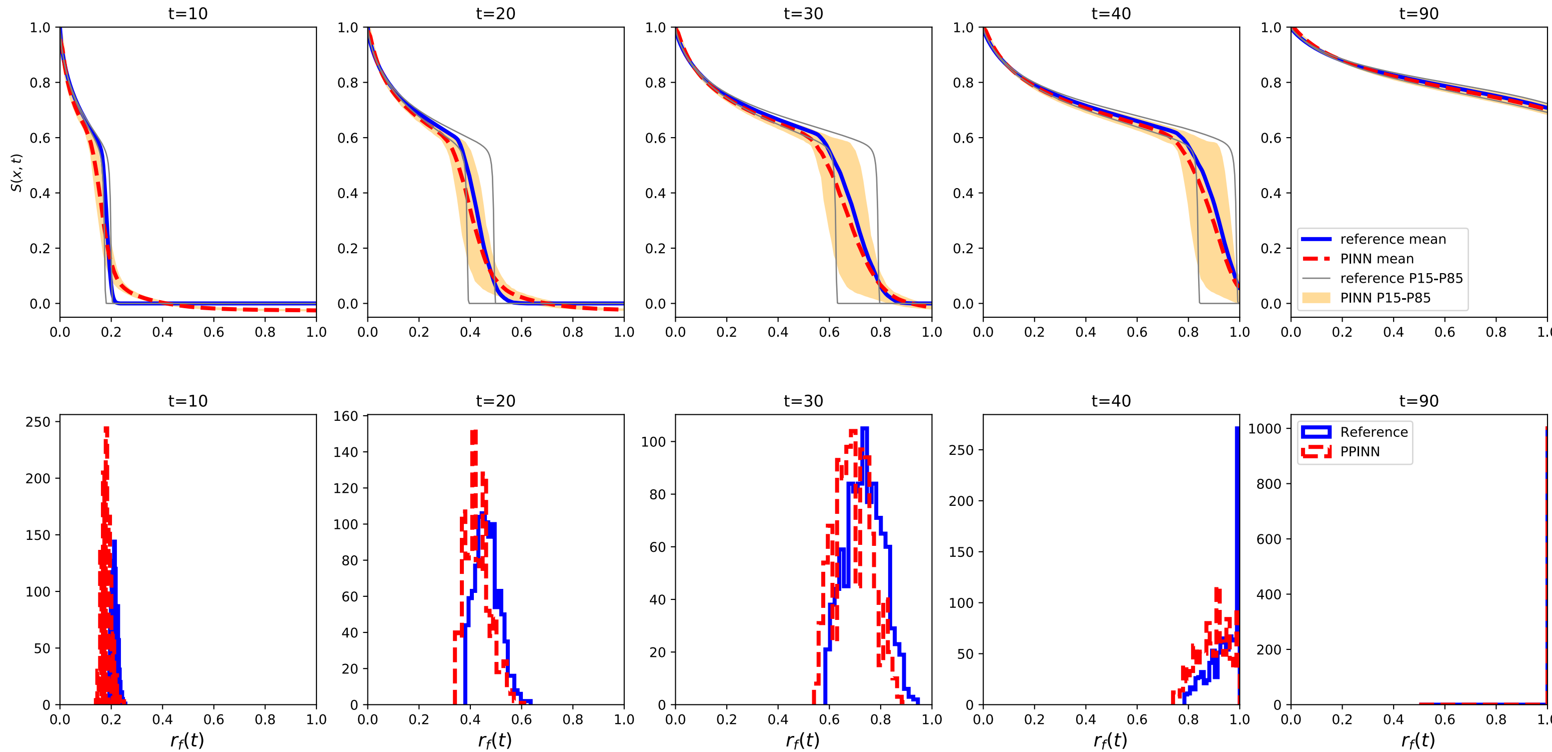
$$\theta \sim \mathcal{N}(\boldsymbol{\mu = 1, \sigma = 0.3})$$





Stochastic periodic velocity function

Uncertainty properly characterized

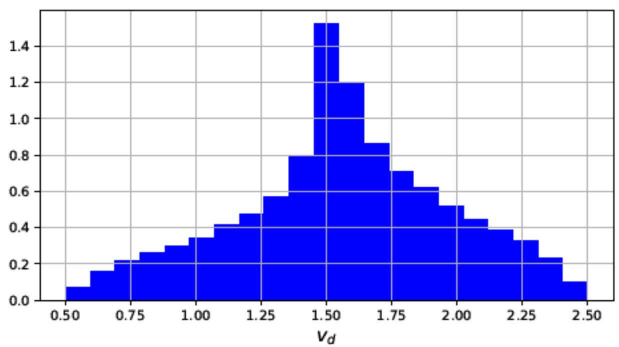


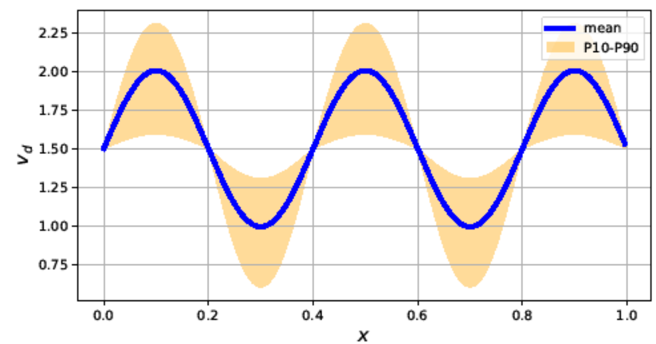
Stochastic periodic velocity function

Increasing the frequency of the velocity

$$v_d(x) = \theta \sin(5x) + b$$

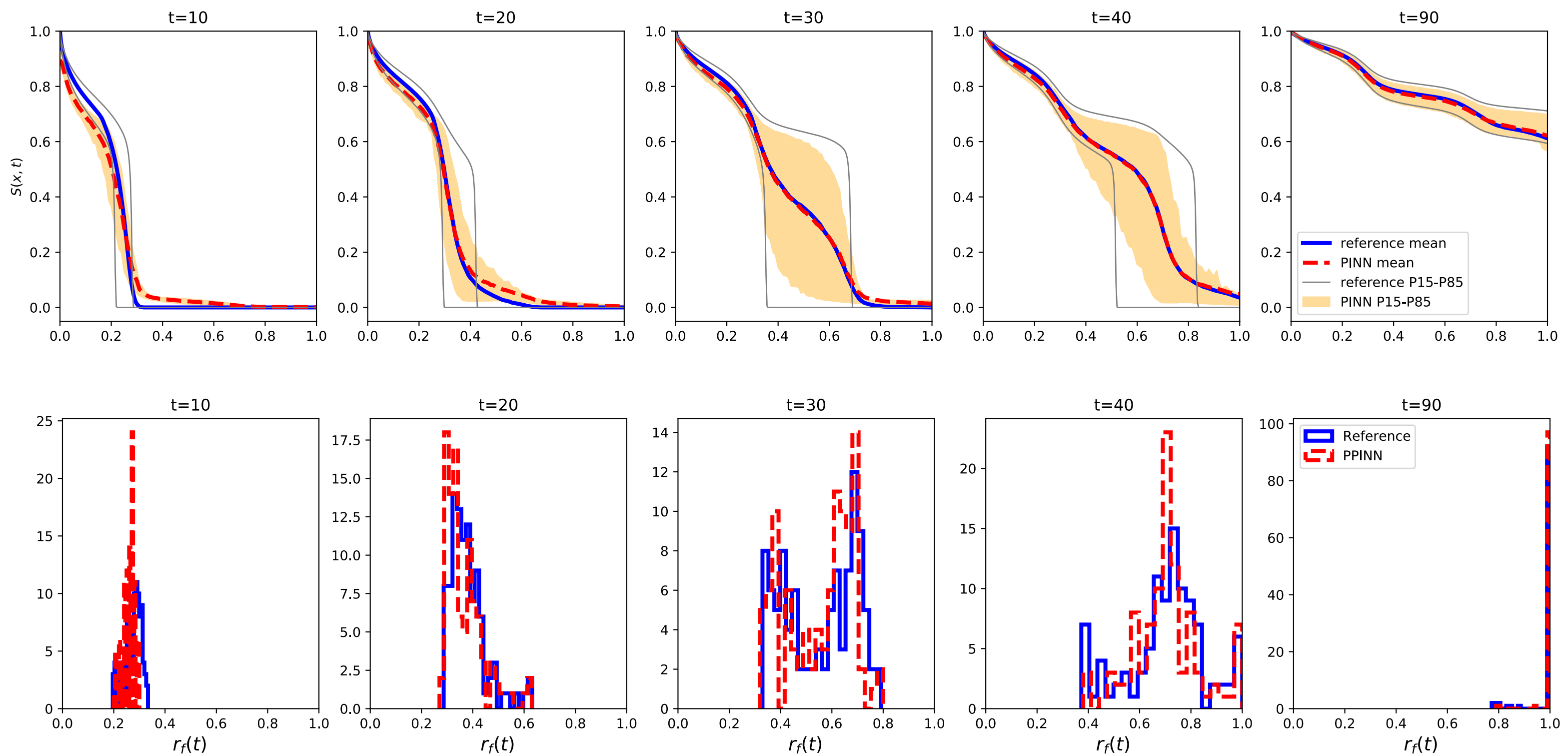
$$\theta \sim \mathcal{N}(\boldsymbol{\mu} = \mathbf{1}, \boldsymbol{\sigma} = \mathbf{0.3})$$





Stochastic periodic velocity function

Uncertainty properly characterized

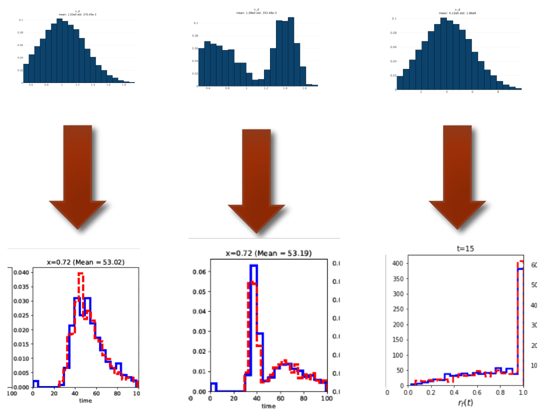


Traditional vs P-PINN

Two orders of magnitude between min and max

Traditional (MCS)

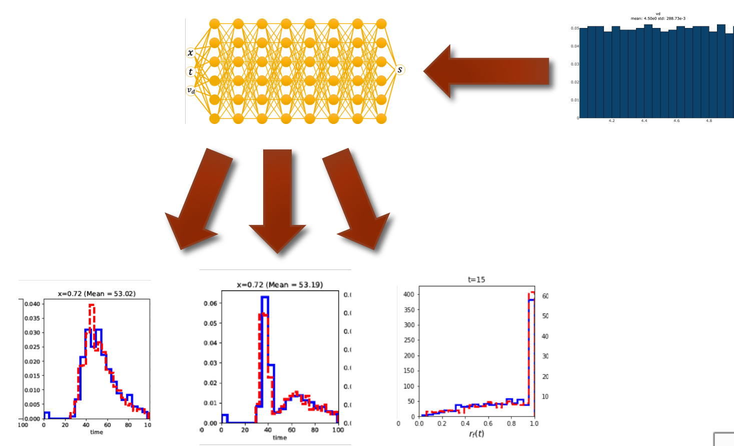
Solve deterministic PDE on prescribed distribution for each distribution of input



Parametrized PINN

Train once (uniform distribution)

Infer on new distributions



Performance

Challenging the State of the Art

- Comparison P-PINN with standard Monte Carlo approach (1000 realizations)
- Problem resolution (256 grid blocks, 100 time-steps)
  - MOC for homogeneous problem
  - Finite Volume with conservative scheme for heterogeneous

	Monte Carlo	P-PINN
Hardware	Intel-i7 3.2GHz (6 core)	Tesla V-100 (16GB)
Training	-	20 min
Inference (Homogeneous)	3 min	1 min
Inference (Heterogeneous)	86 min	1 min

Conclusion, Future Work

- Resolution of Stochastic Riemann problem using P-PINN
- Parameterization of one neural network model used to solve for a range of velocities
- Uncertainty Quantification: Cast problem in high dimensions space with low gradient
- Homogeneous with arbitrary distribution
- Heterogeneous with parametrized models (affine, periodic)
- Limitations for heterogeneous with high local randomness (nugget)
- Next
  - Moment resolution for hyperbolic problem
  - Data assimilation

Acknowledgements

- Stanford School of Earth, Energy and Environmental Science
  - SUPRI-B
- Simnet 2.0
  - Oliver Hennigh
  - Mohammad Nabian
  - Sanjay Choudhry

Thank you

Stanford | SCHOOL OF EARTH, ENERGY & ENVIRONMENTAL SCIENCES