PHYSICS INFORMED NEURAL NETWORKS FOR SOLVING WAVE EQUATION

Harpreet Sethi, 6th November 2021

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ACKNOWLEDGEMENTS

NVIDIA Energy team

- Ken Hester, Doris Pan, Pavel Dimitrov, Gunter Roth

NVIDIA SimNet team

- Sanjay Choudhary, Oliver Hennigh, Kaustabh Sanghali, Zhiwei

AGENDA

- Motivation
- Theory
- Experimental setup
- Numerical examples
- Conclusion and next steps

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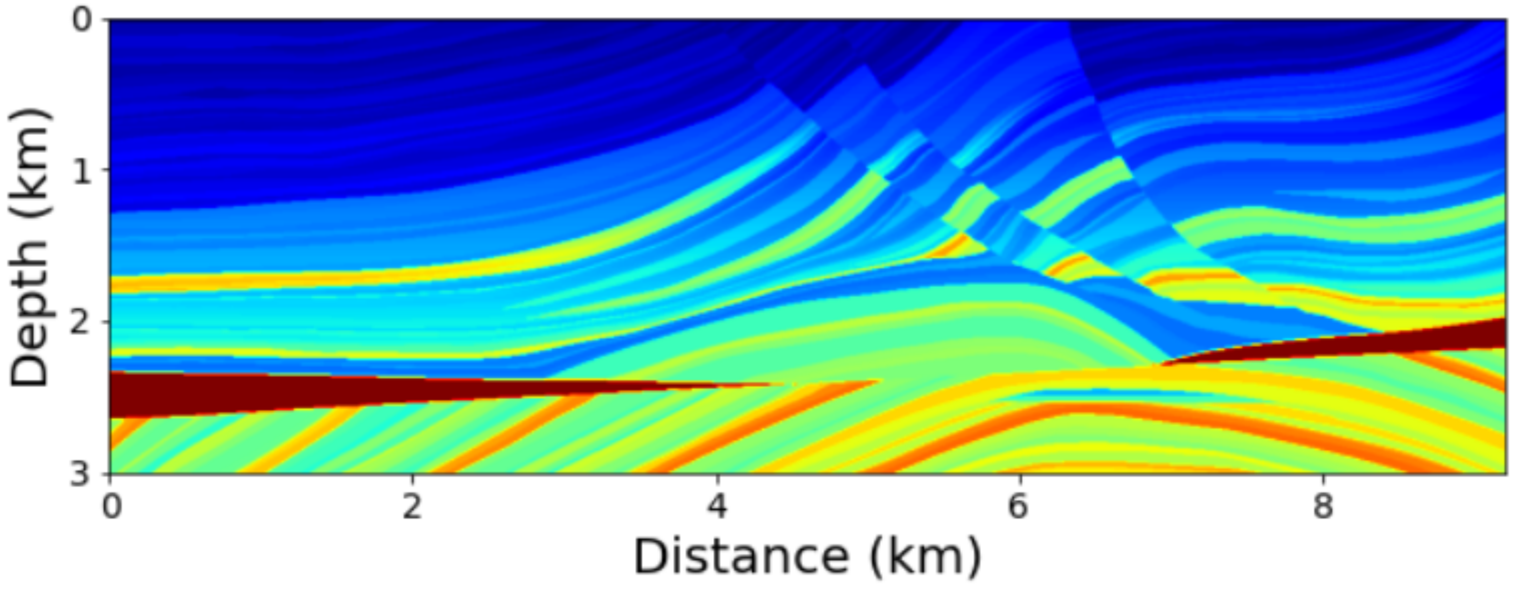
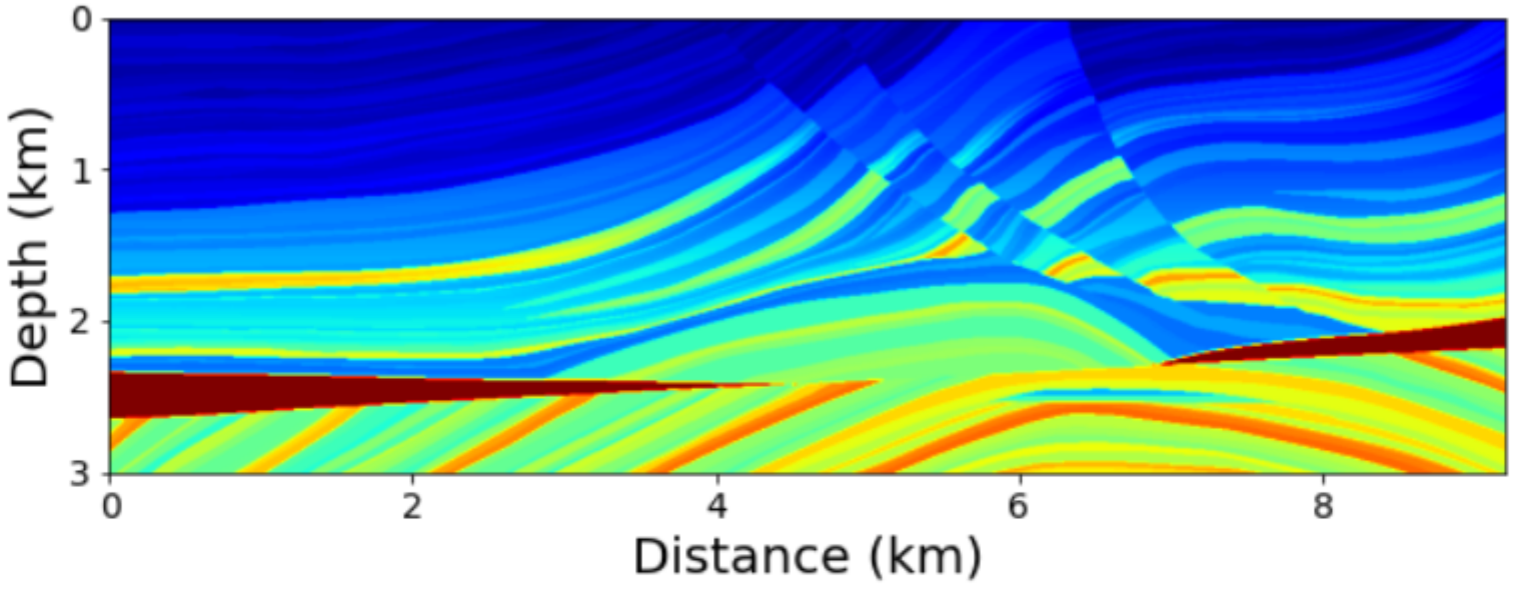
NUMERICAL SOLVERS



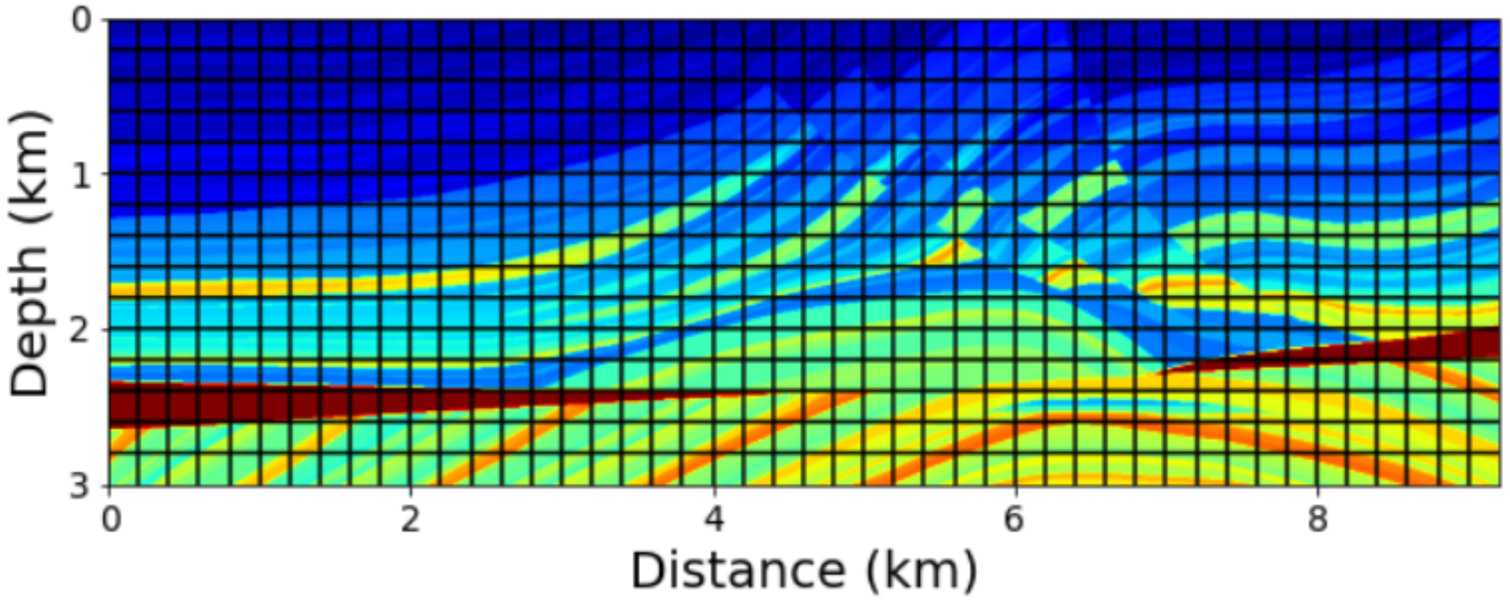
Model domain

Finite-difference (FD), Finite-element (FE) , Finite-volume, Pseudo-spectral methods etc.

NUMERICAL SOLVERS



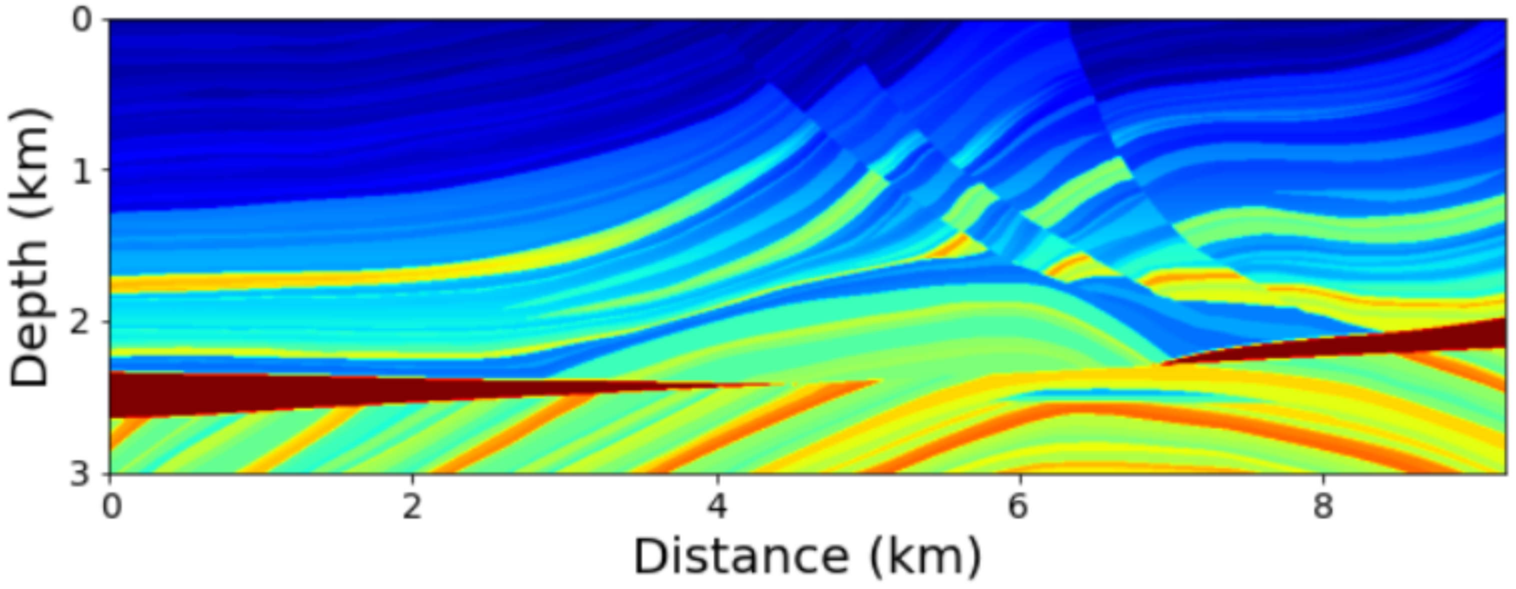
Model domain



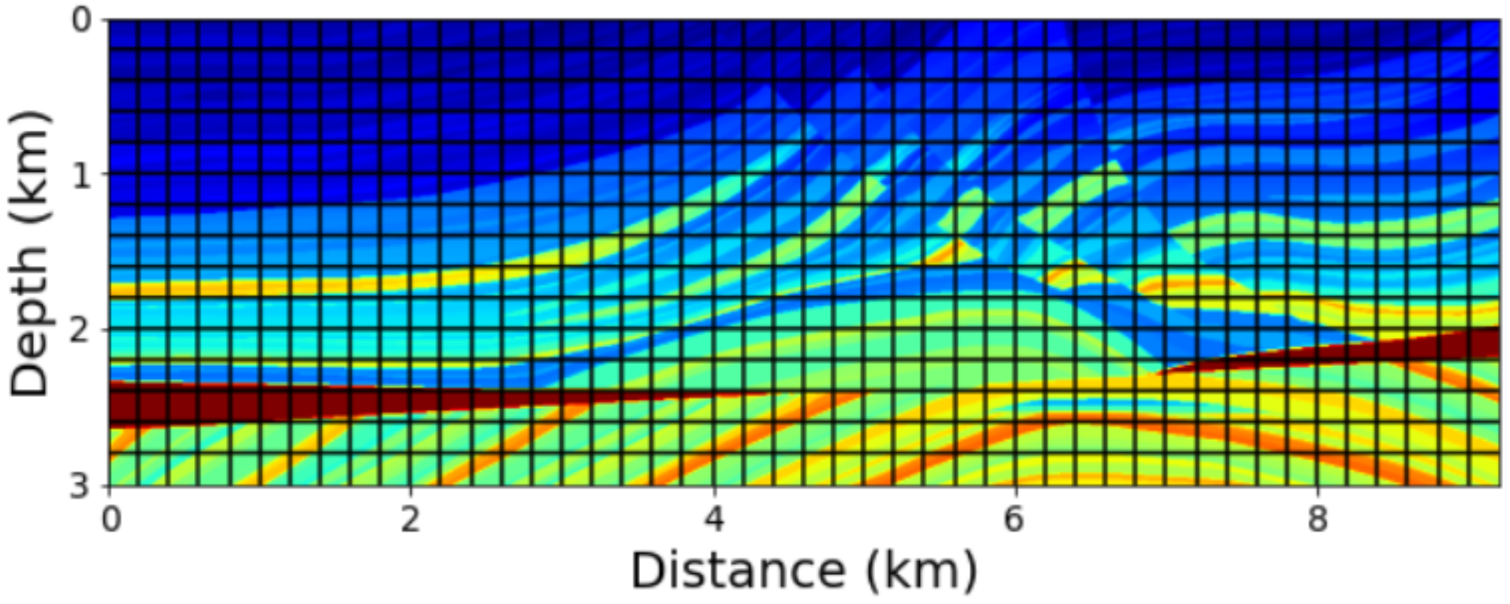
Discretize model/generate a mesh

Finite-difference (FD), Finite-element (FE) , Finite-volume, Pseudo-spectral methods etc.

NUMERICAL SOLVERS



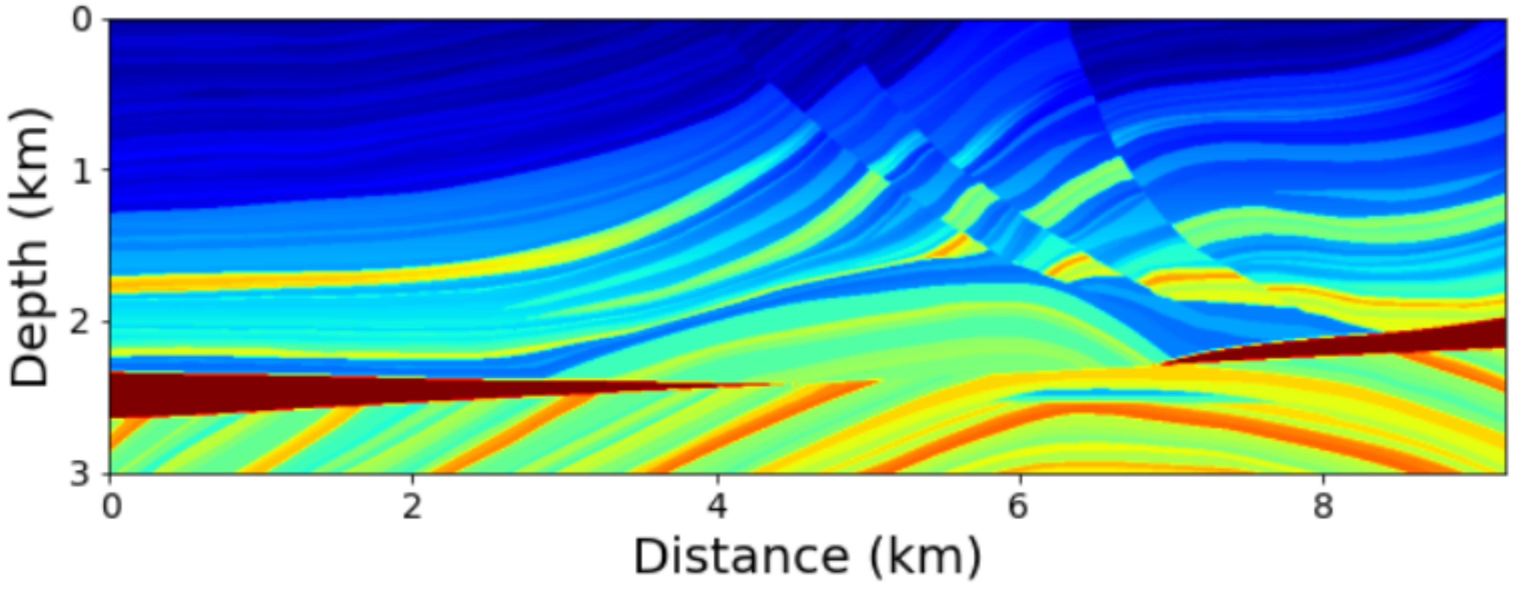
Model domain



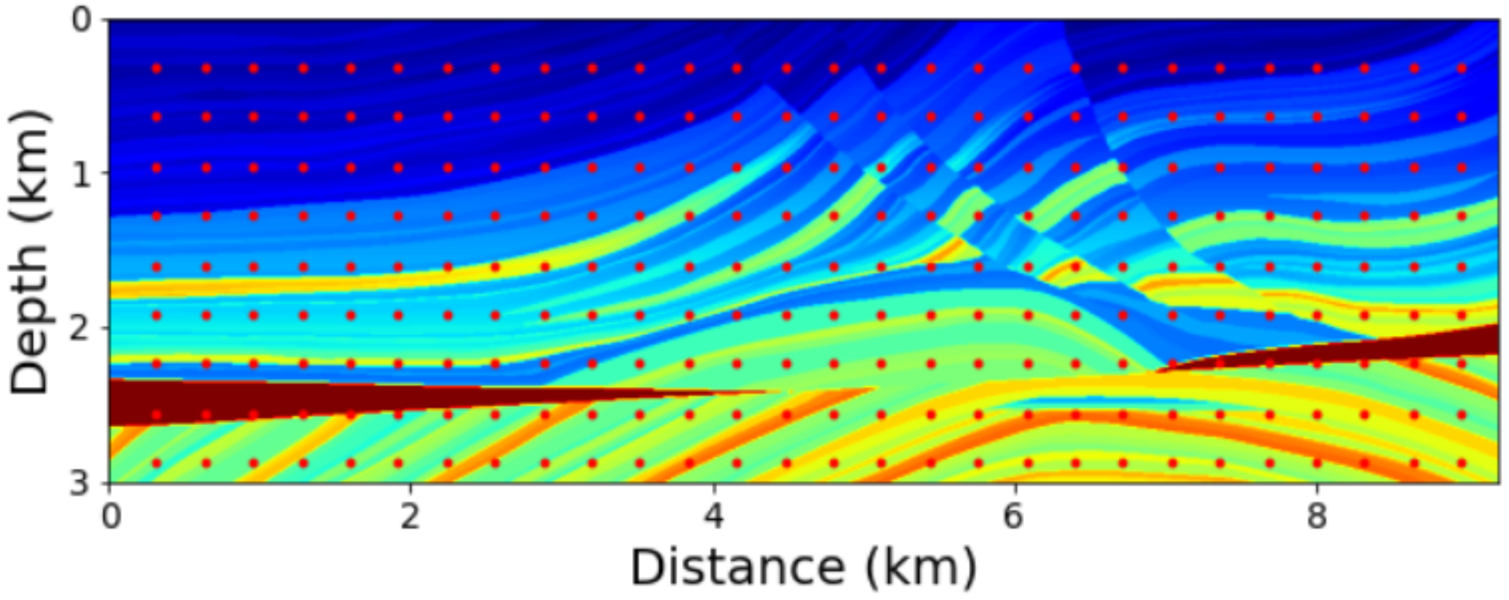
Discretize model/generate a mesh

- Numerical errors such as dispersion, grid anisotropy, truncation etc.

PHYSICS INFORMED NEURAL NETWORK (PINN) SOLVER



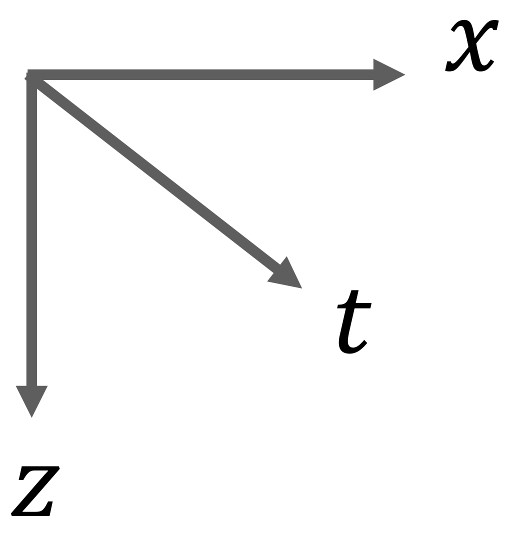
Model domain

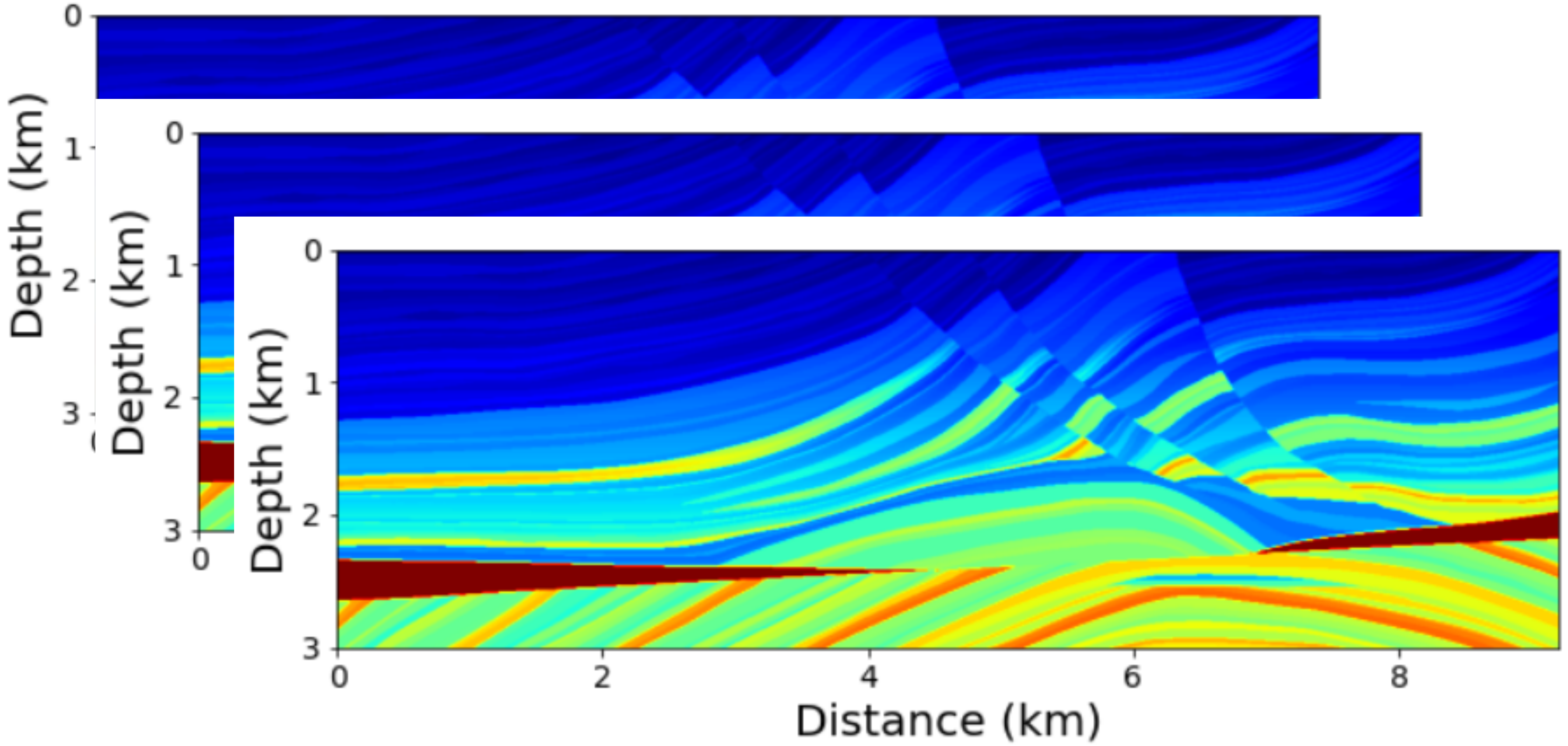


Point cloud representation

- No mesh generation
- Less prone to numerical errors

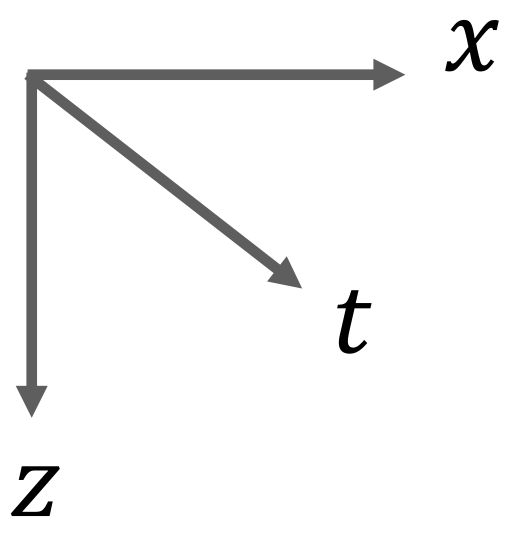
NUMERICAL SOLVERS

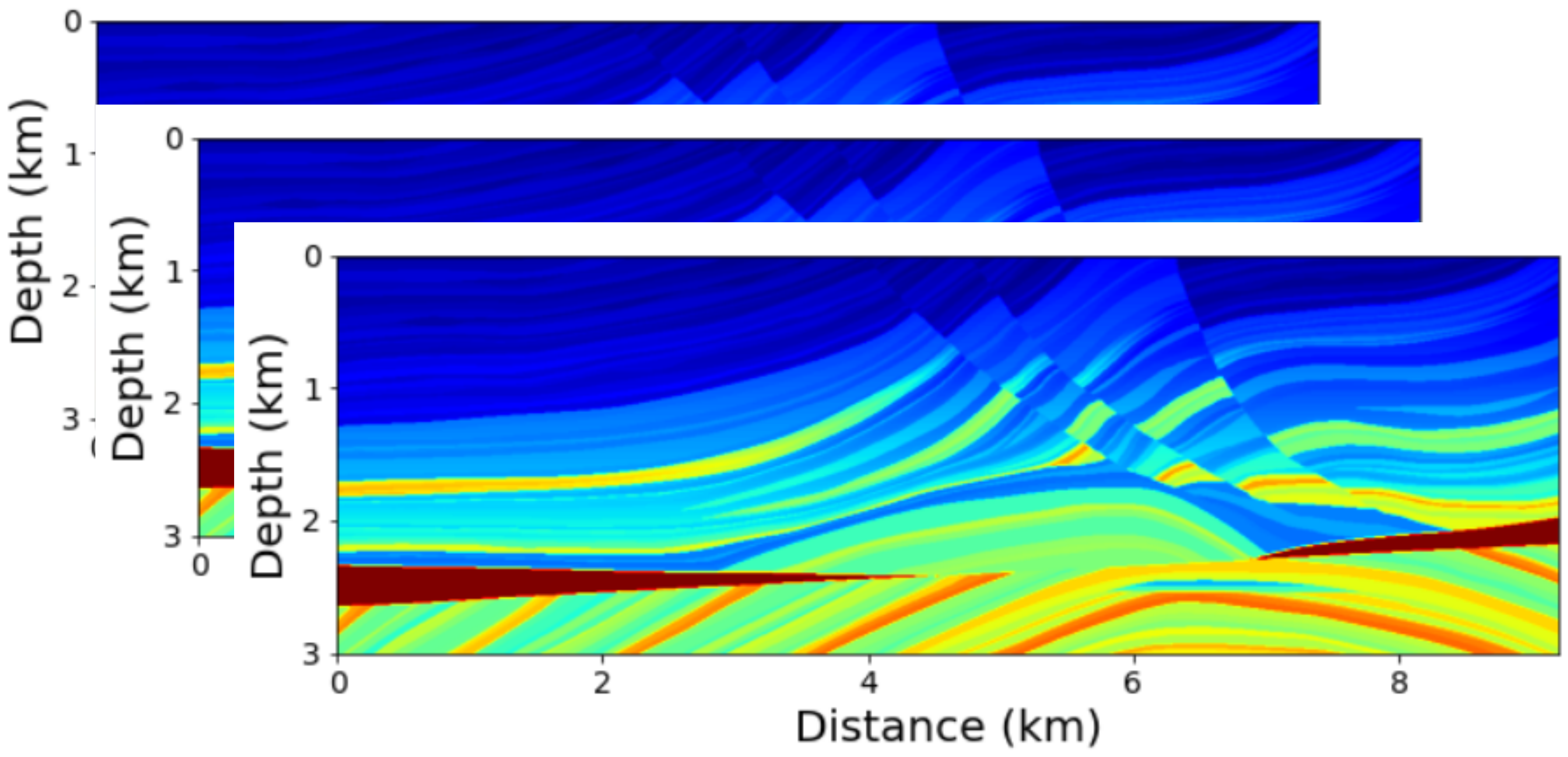




Need to solve for every time step

PINN SOLVER





Resolve complete spatial-temporal domain in single step

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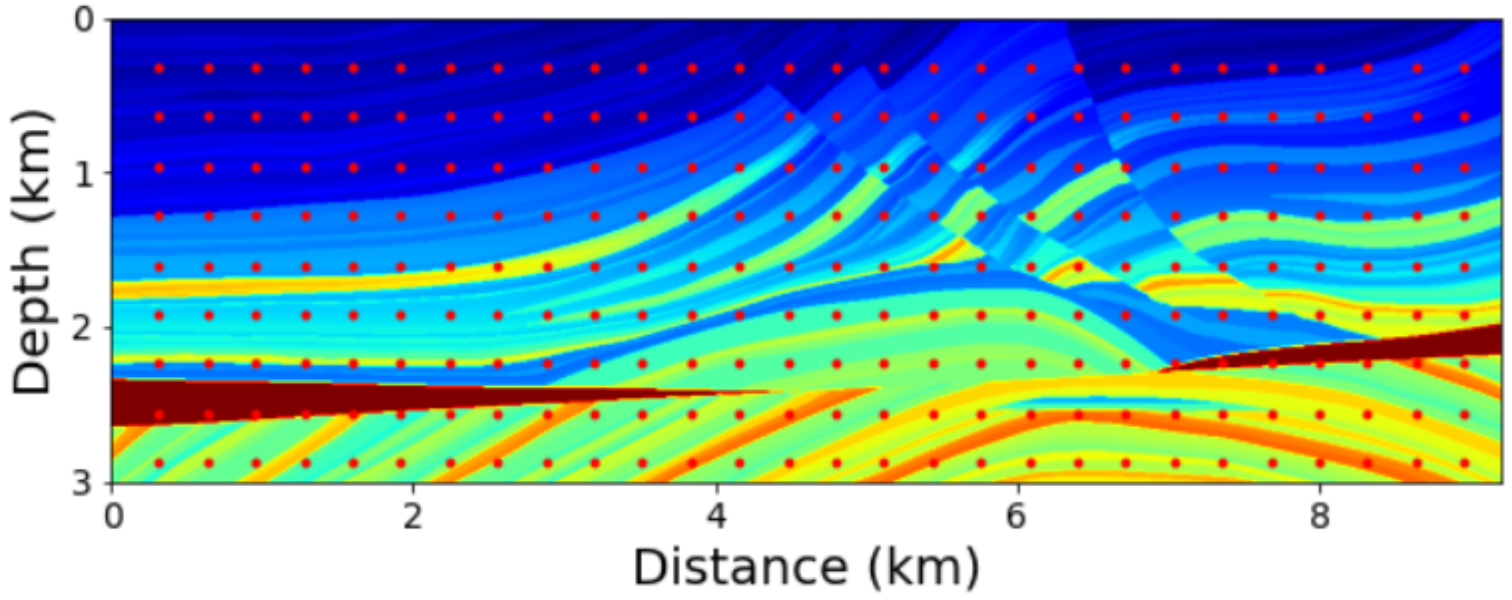
PINN FRAMEWORK



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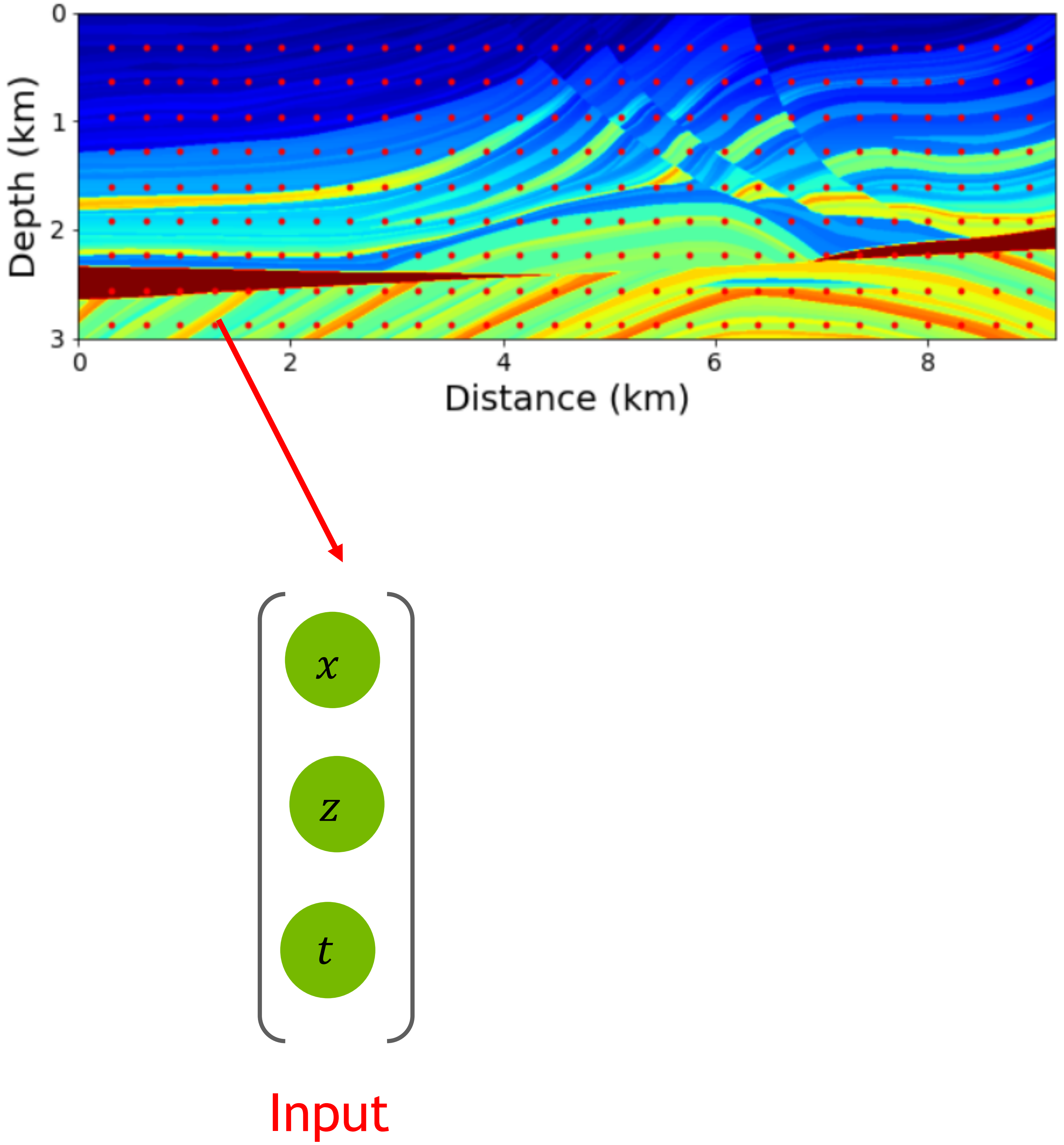

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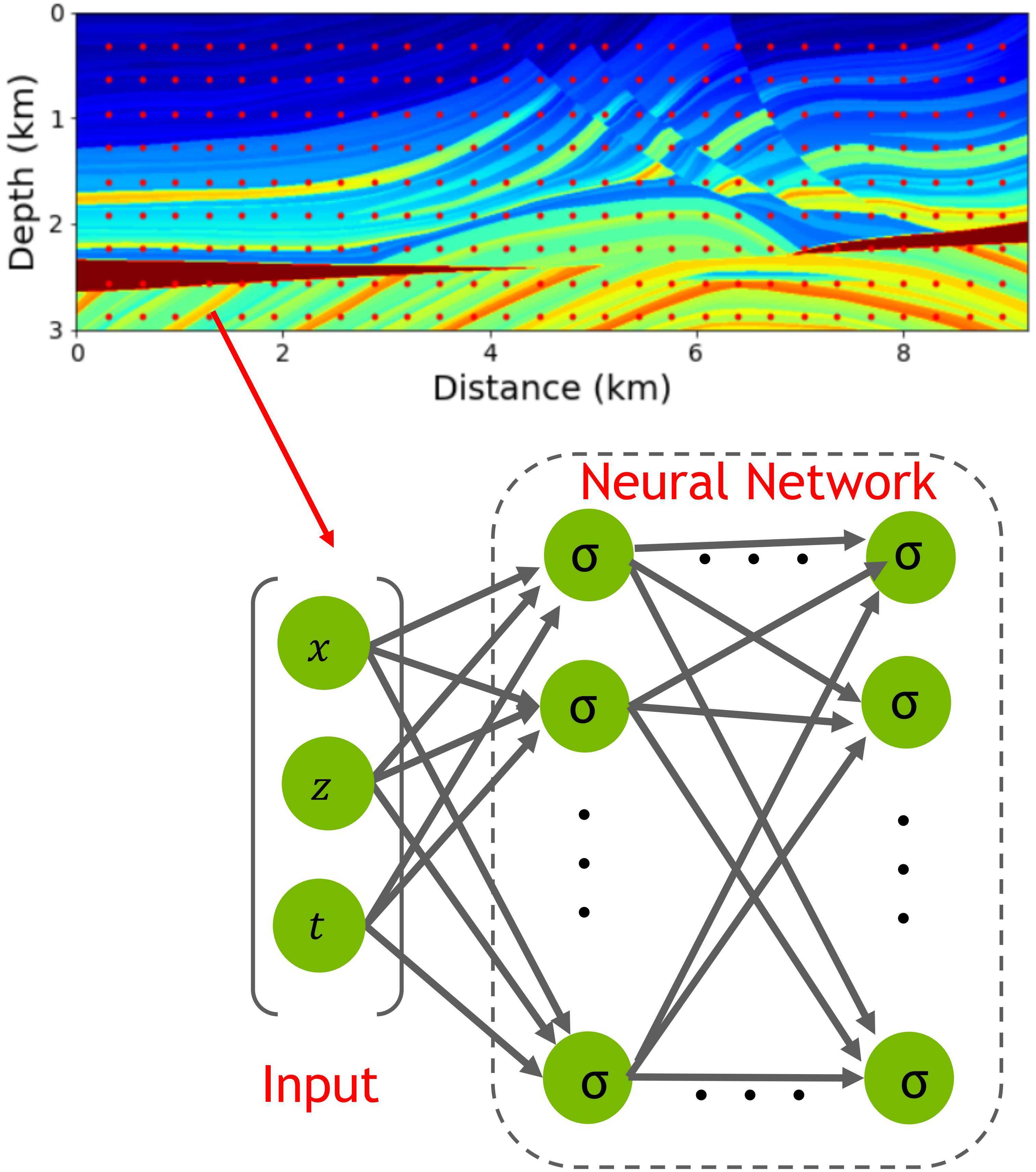

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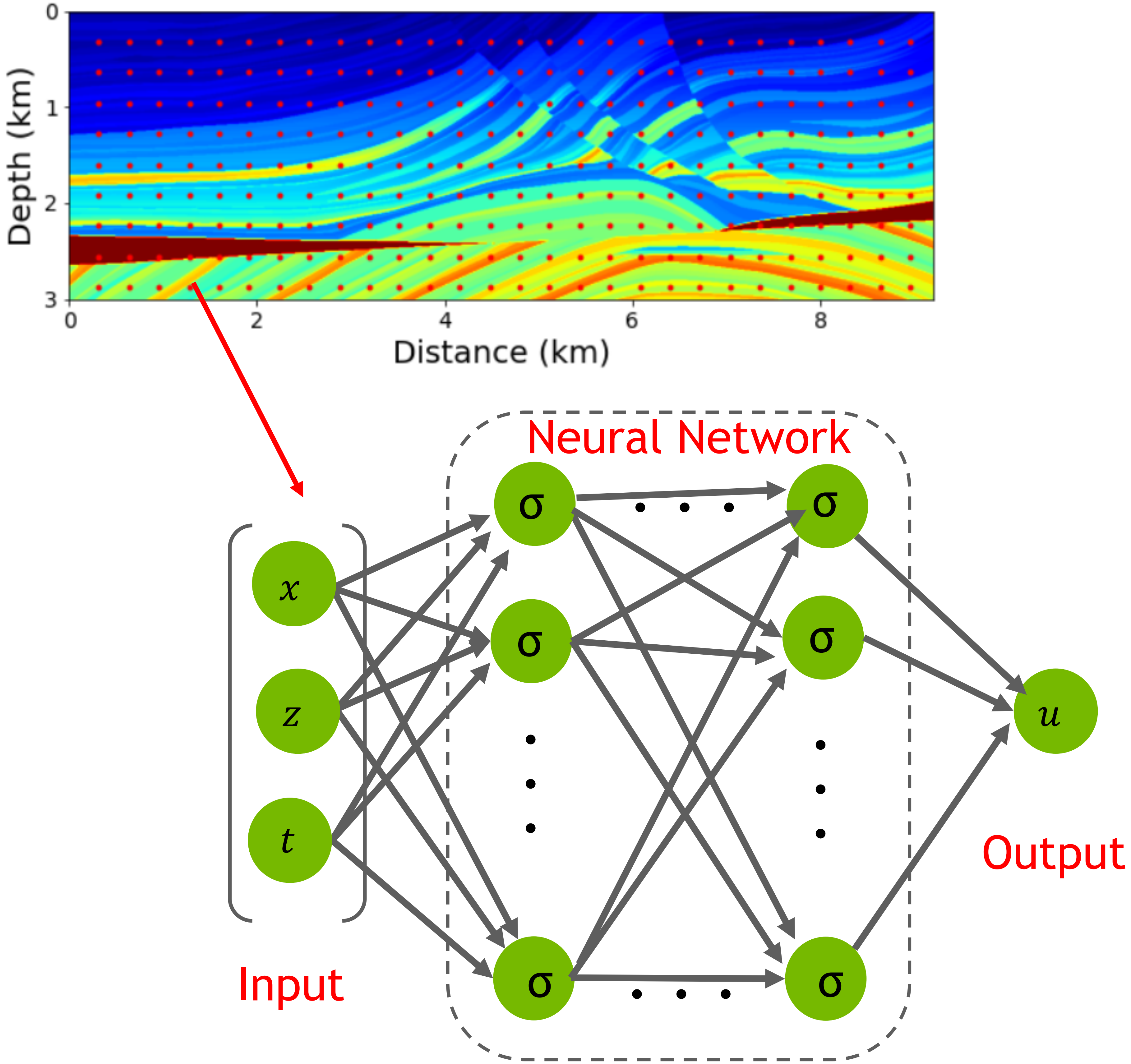

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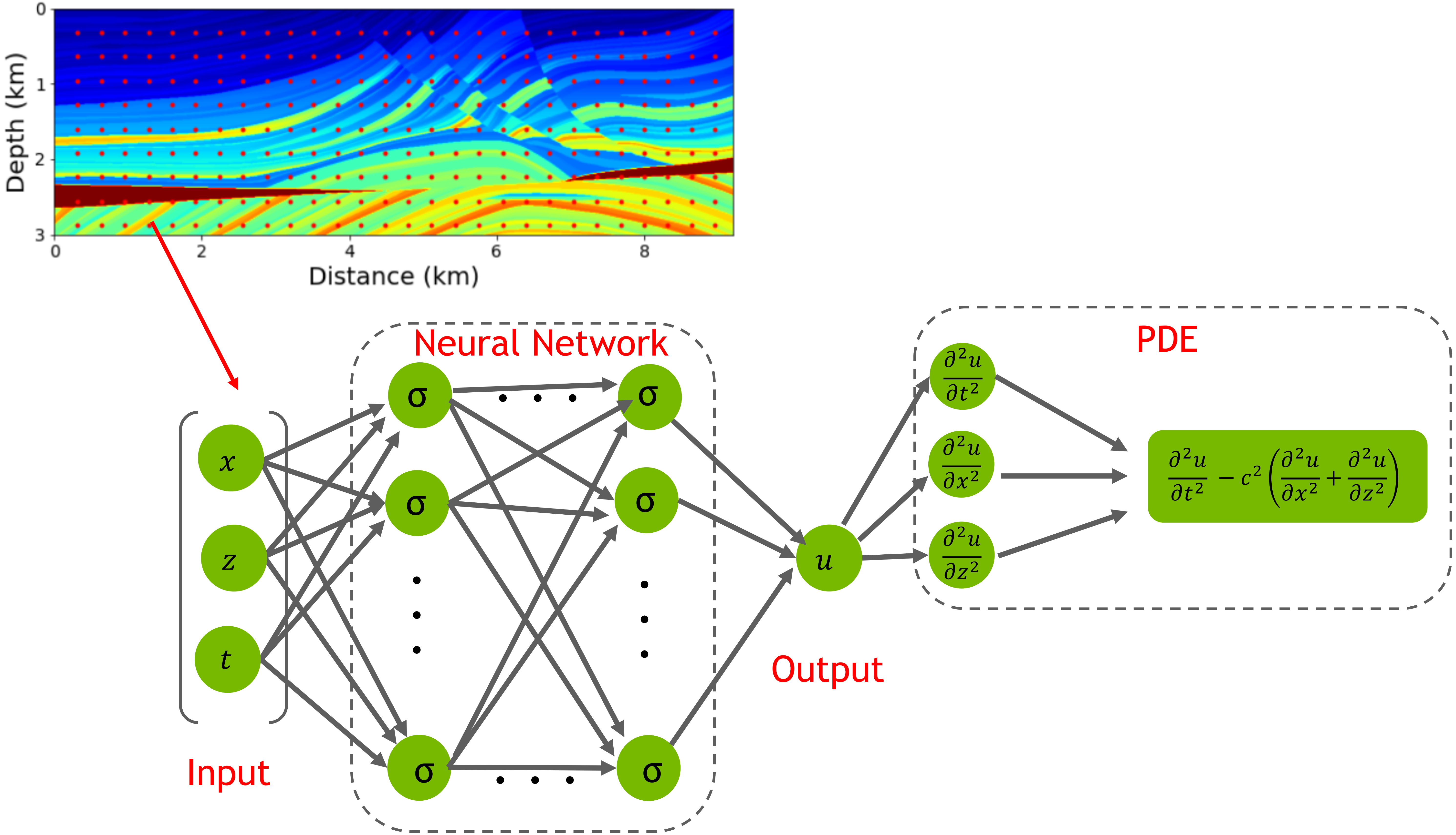

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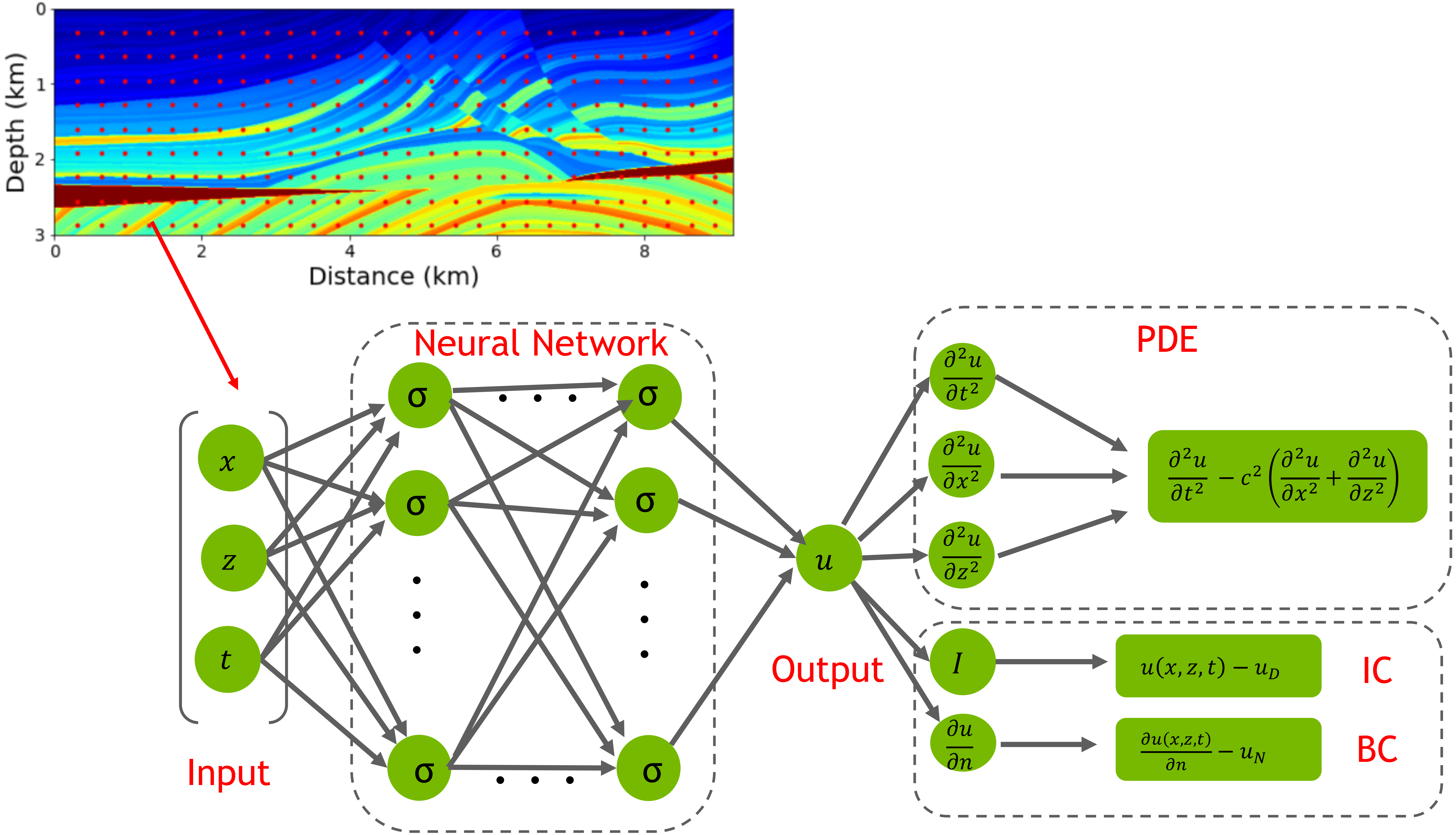

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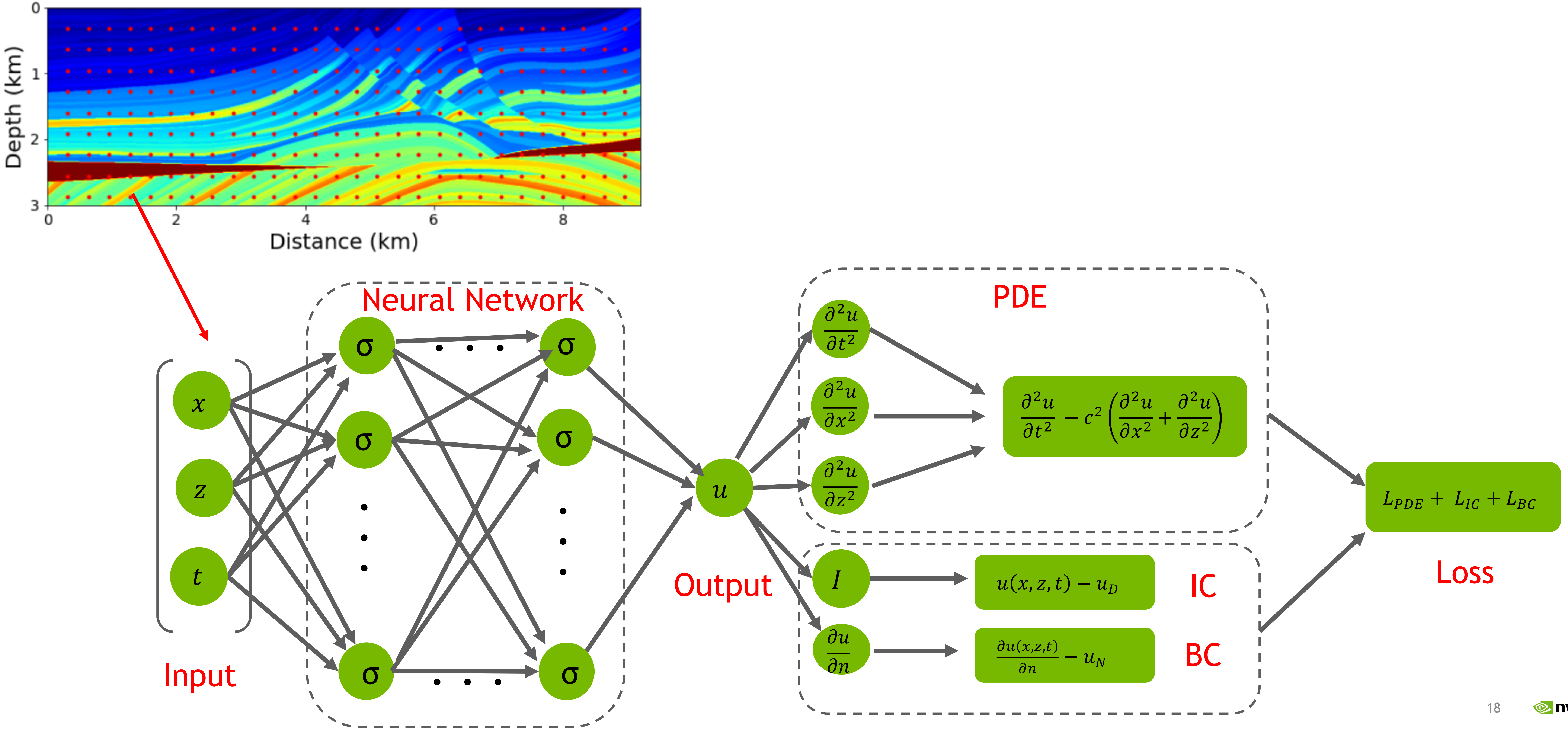
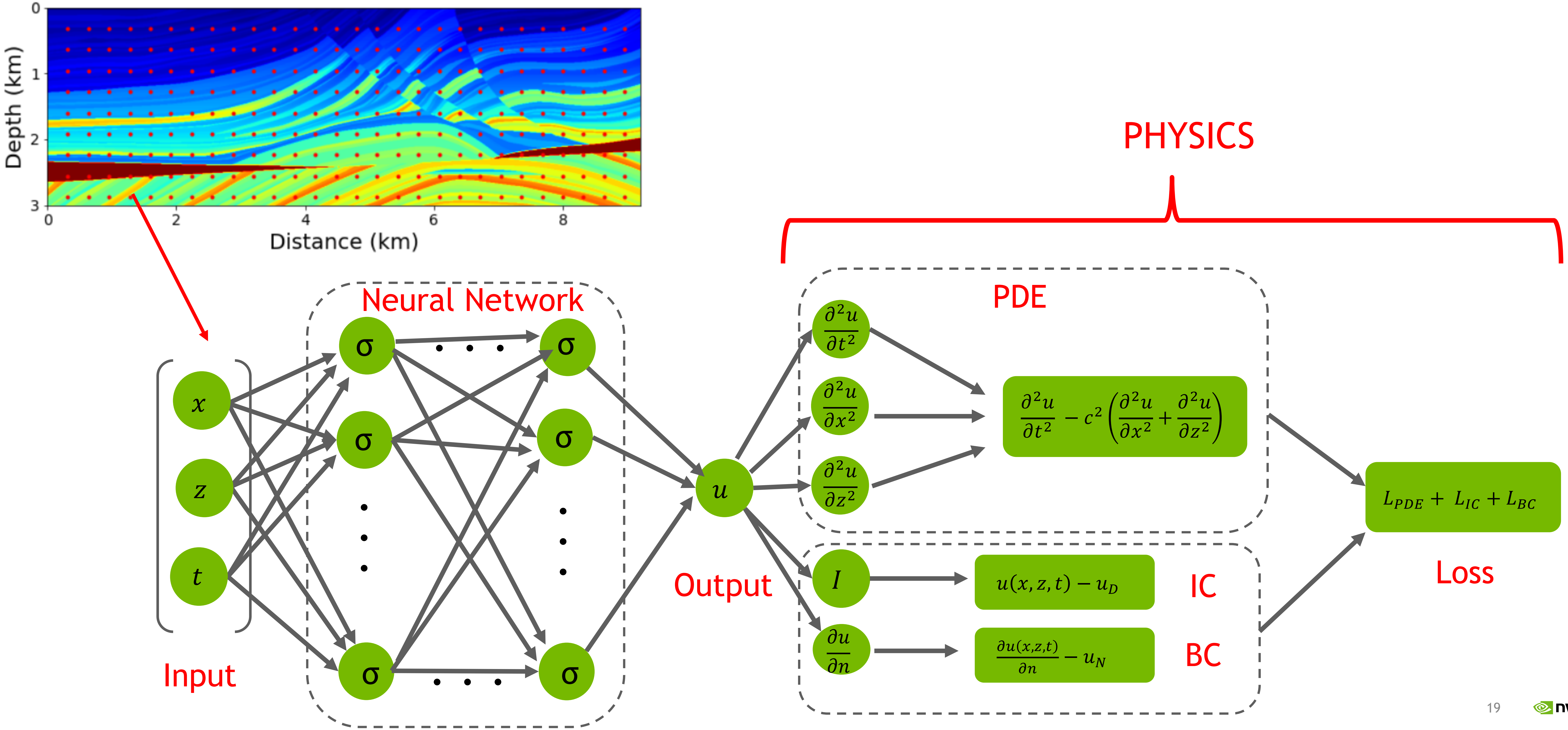
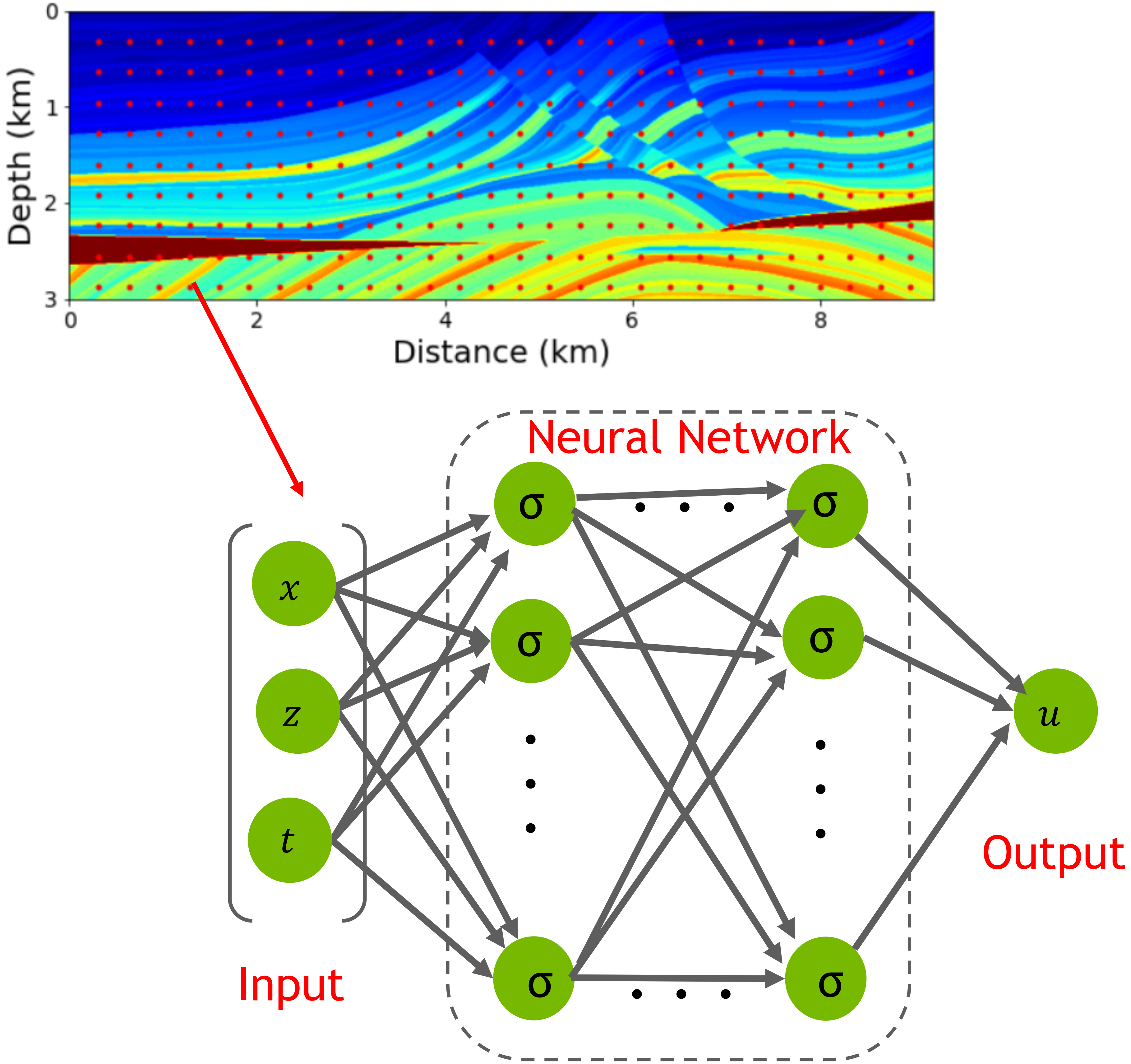


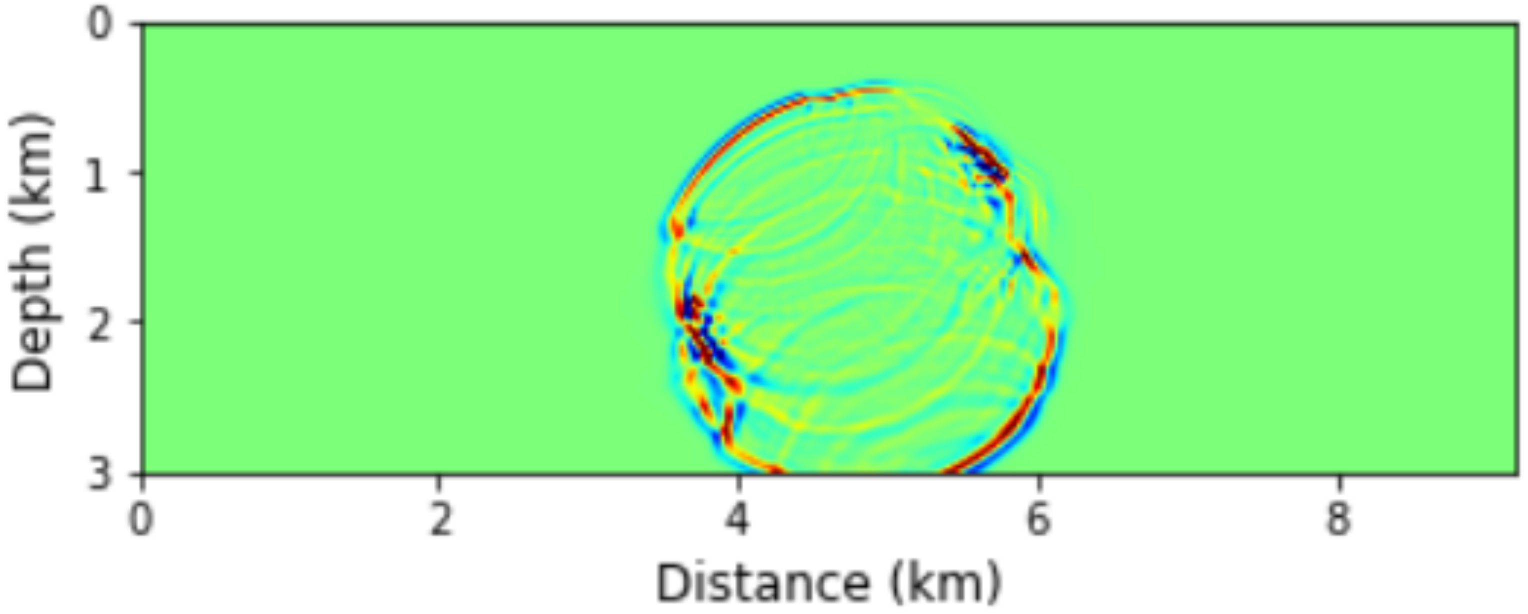
NVIDIA

PINN FRAMEWORK



PINN FRAMEWORK





Goal: To predict wavefield solution

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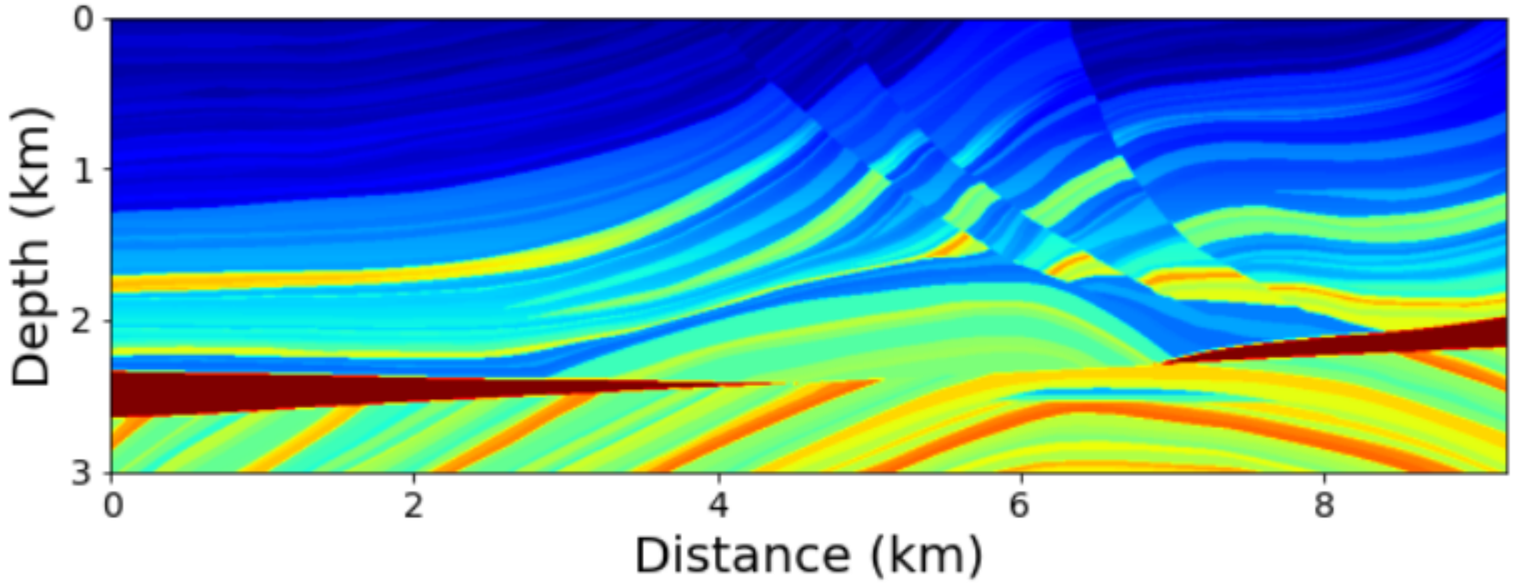
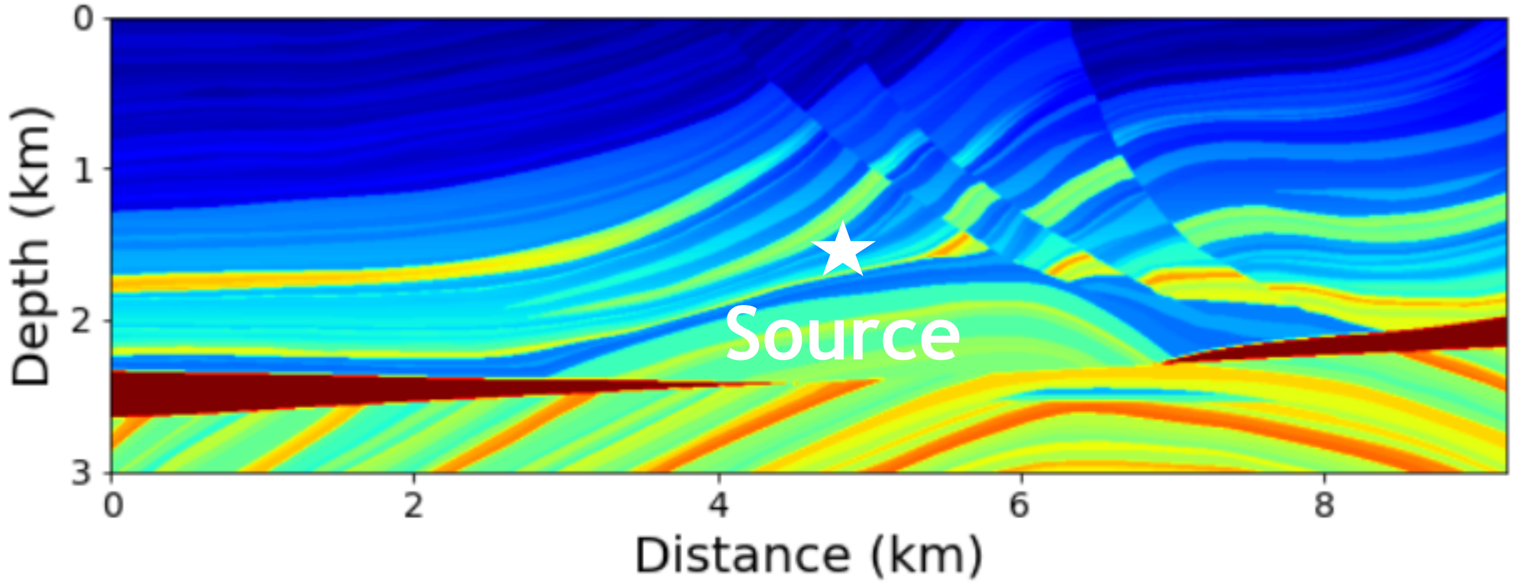
EXPERIMENT SETUP



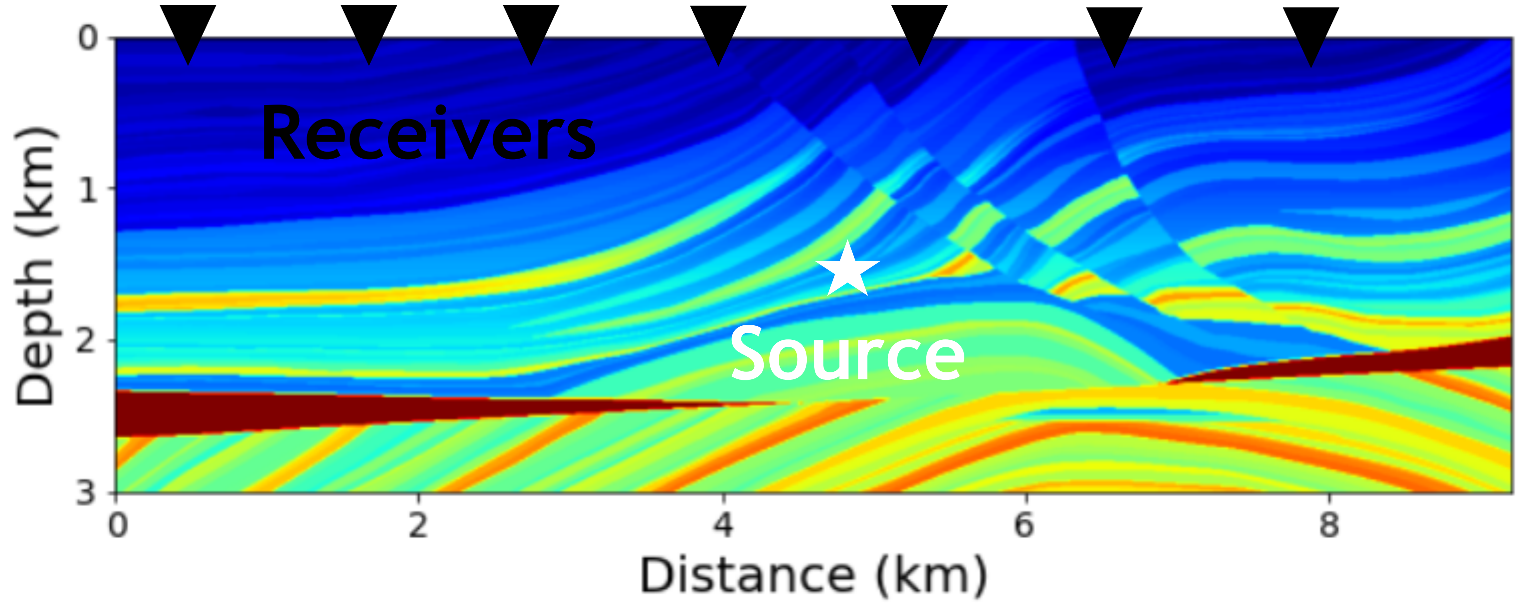
Input velocity model

EXPERIMENT SETUP



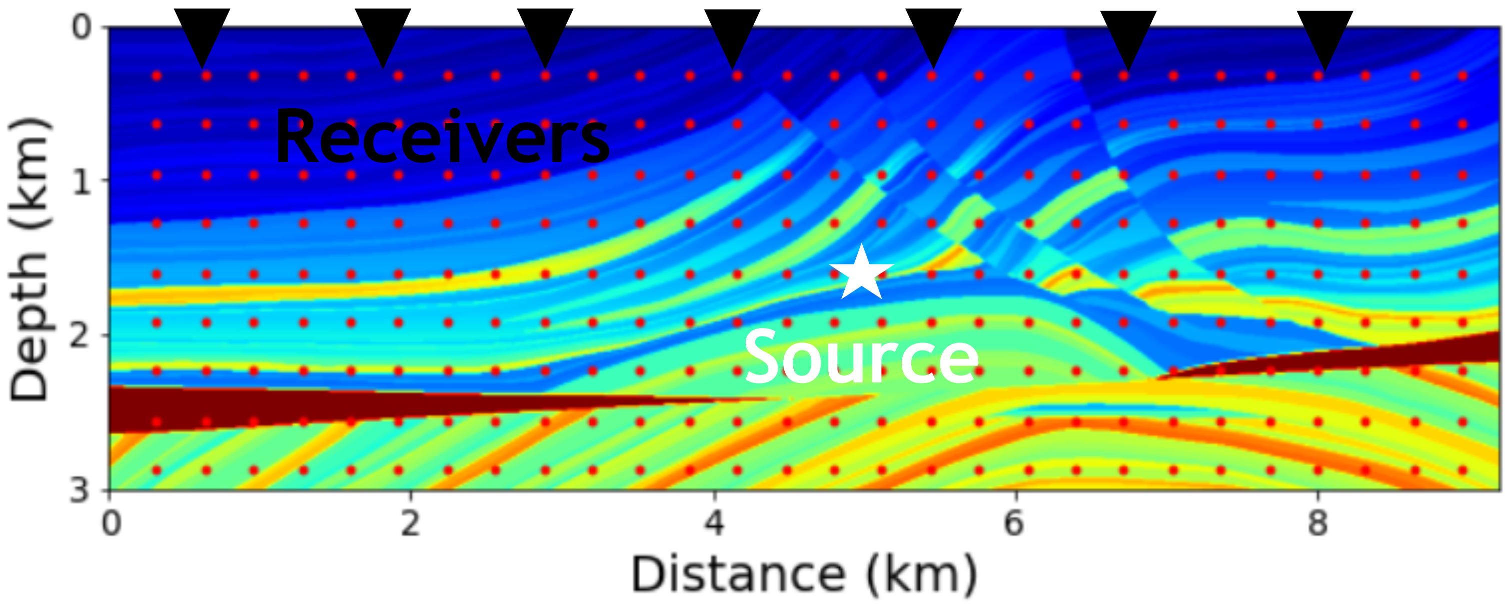
Input velocity model

EXPERIMENT SETUP

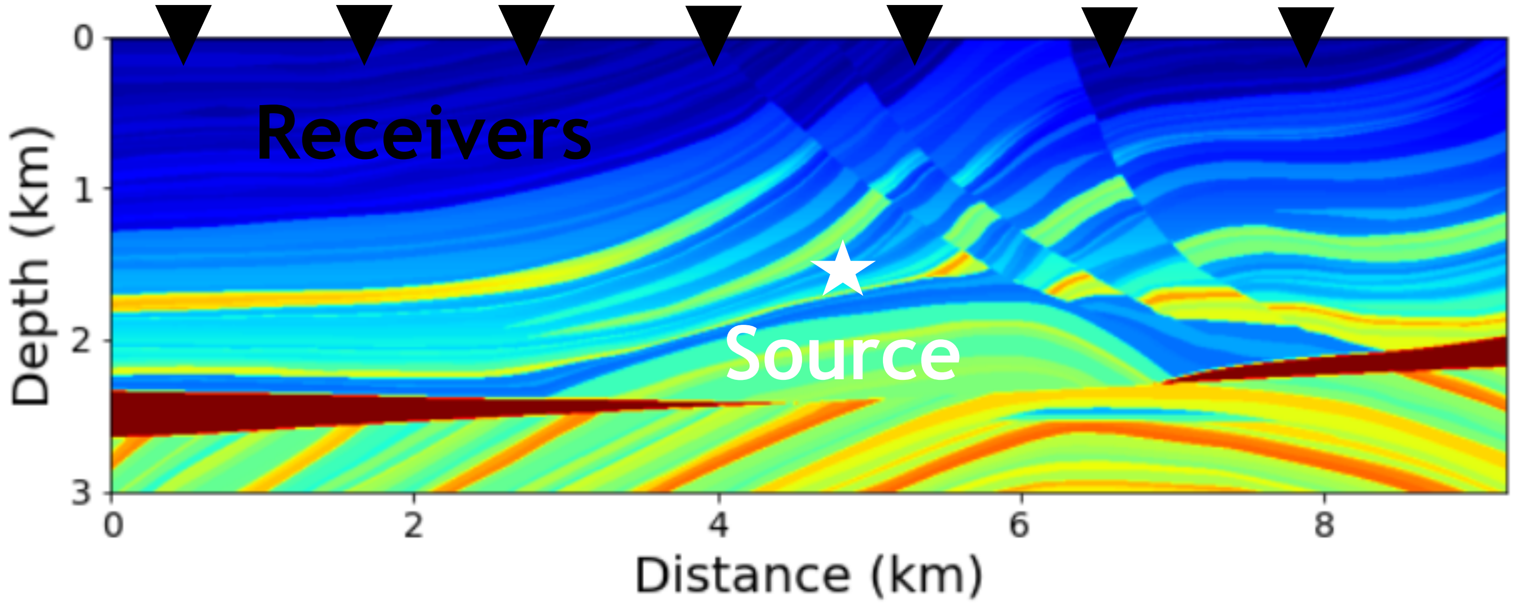


Input velocity model

EXPERIMENT SETUP



Point cloud representation



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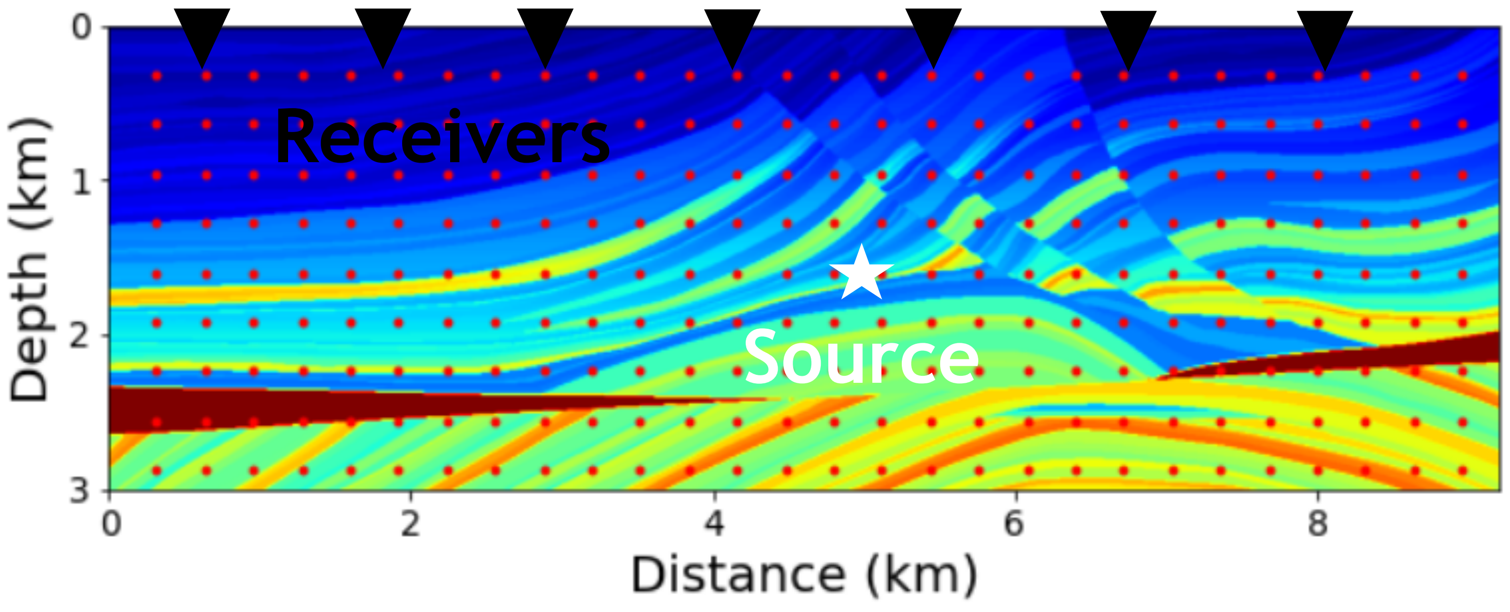
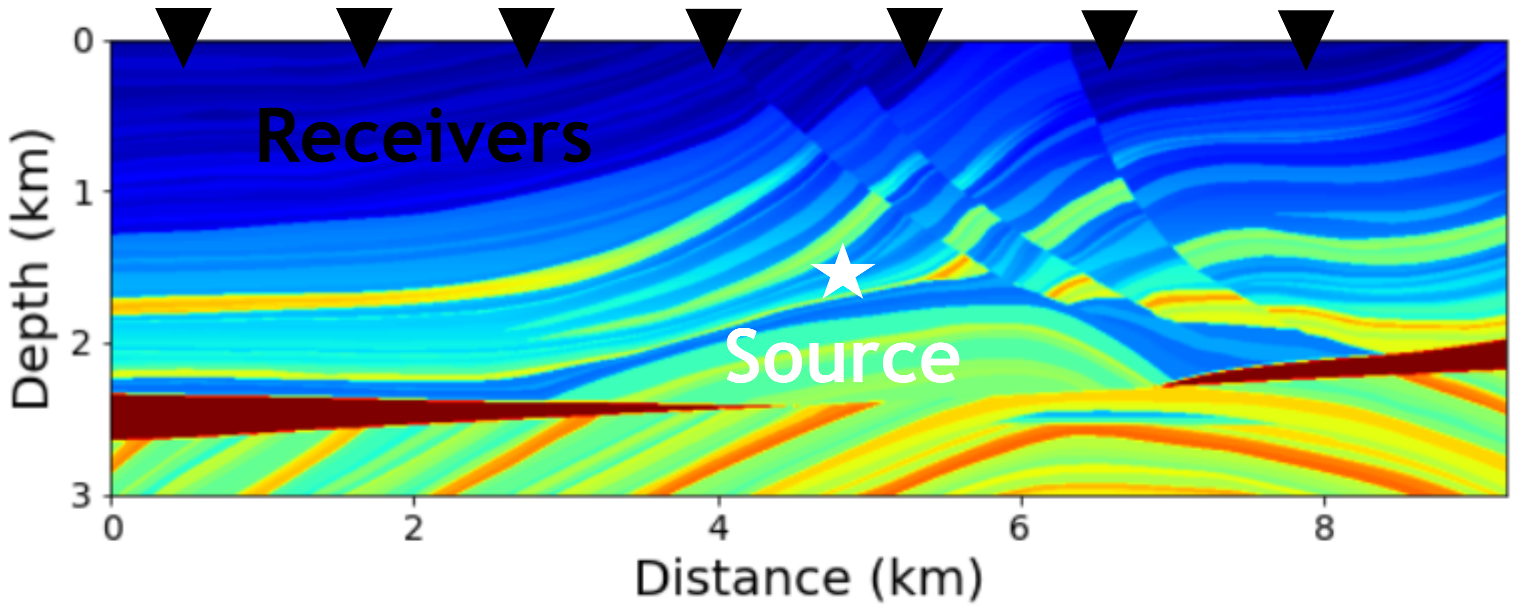
EXPERIMENT SETUP



Point cloud representation



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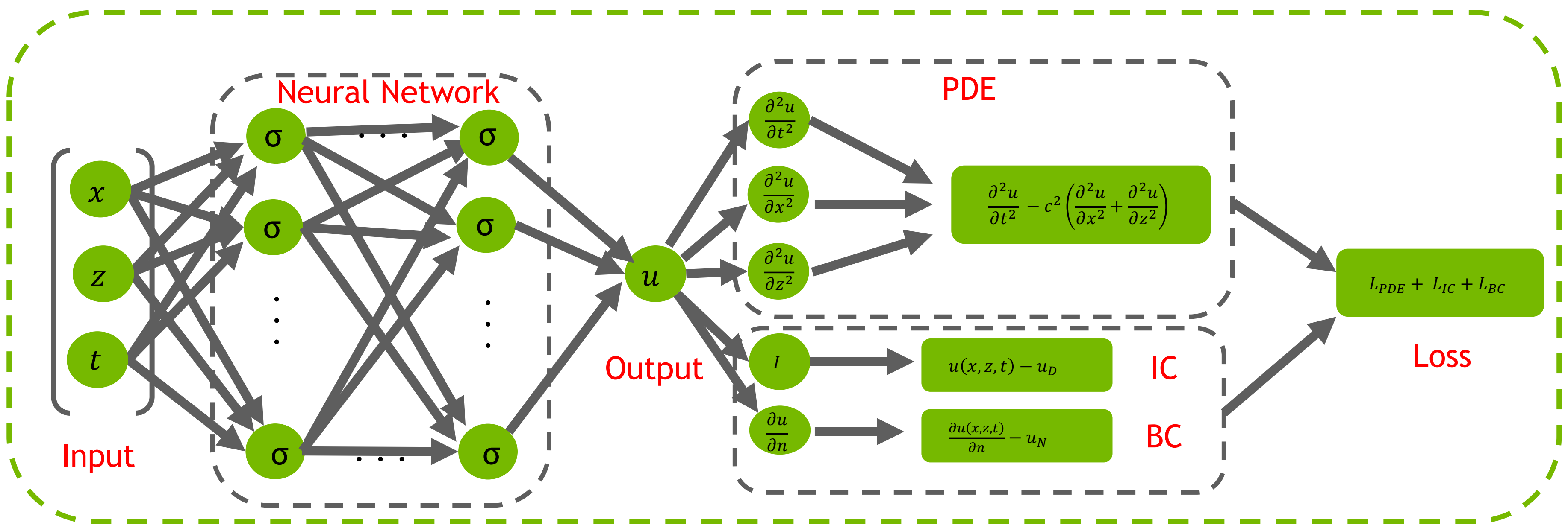
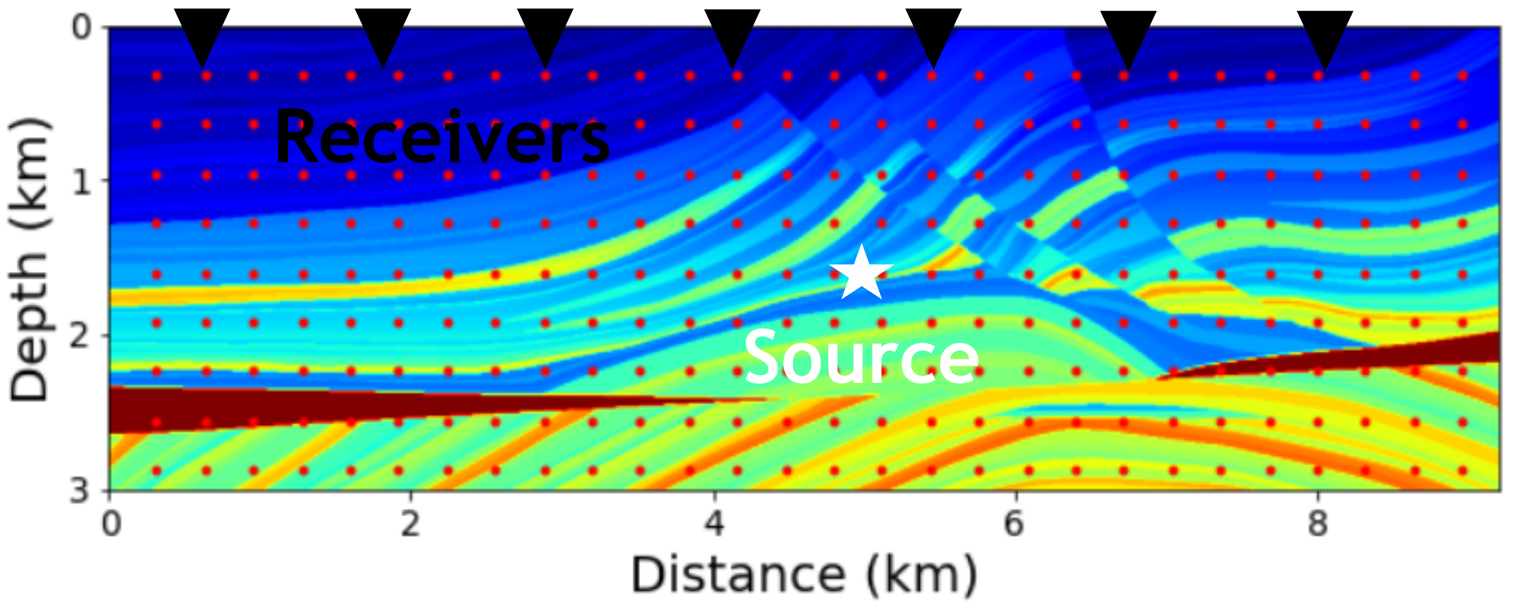


PINN framework

EXPERIMENT SETUP



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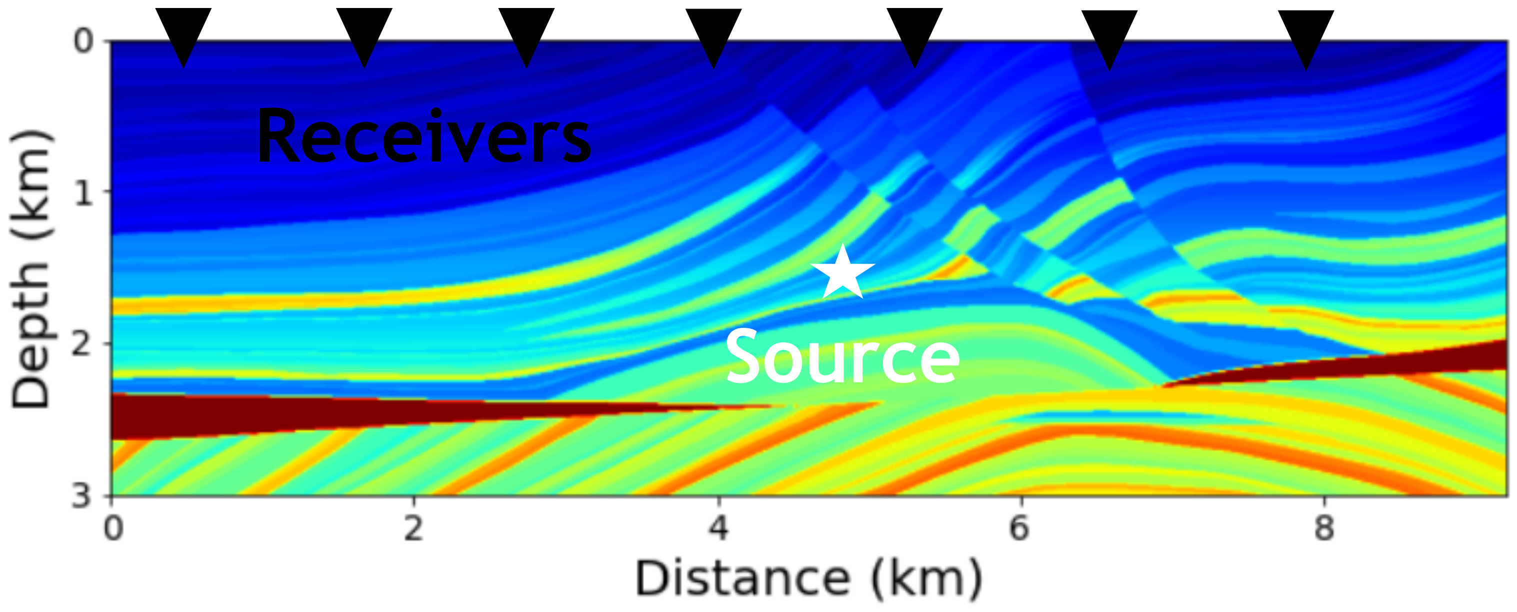

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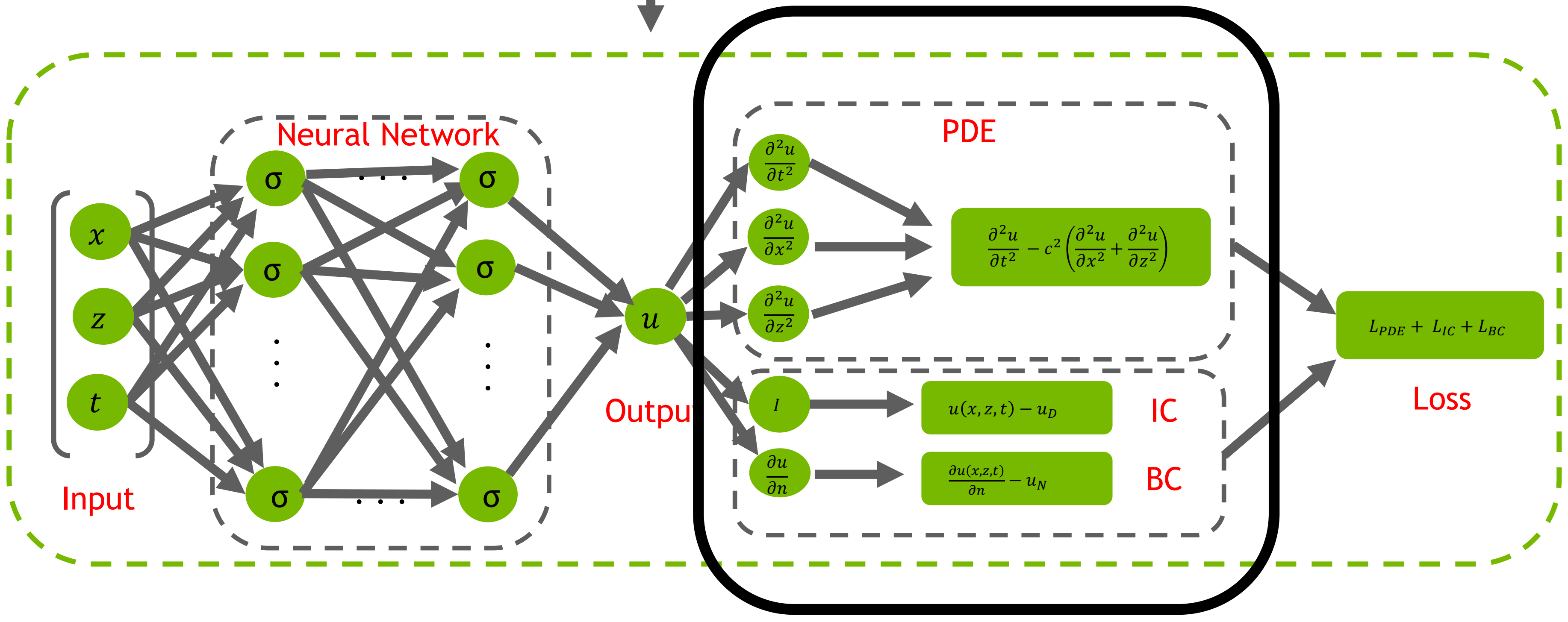
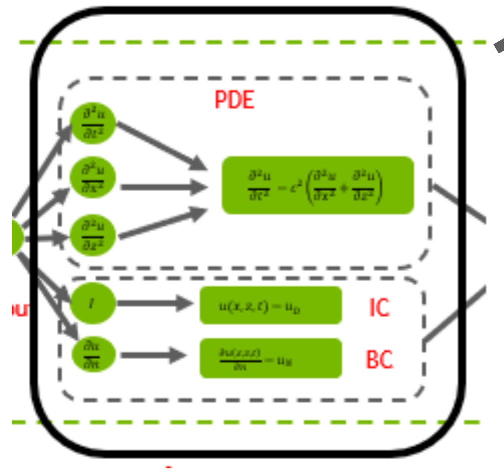


PINN framework

EXPERIMENT SETUP



PDE : $$\frac{\partial^2 u}{\partial t^2} - c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) = f(x,z,t)$$

u : wavefield solution

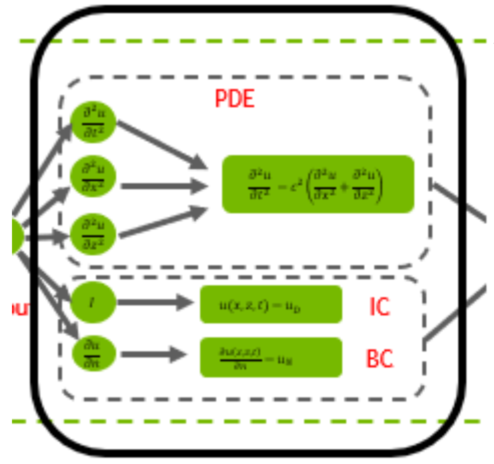
c : velocity

$f(x,z,t)$: source

IC : $$u(x,z,t=0) = 0$$
$$\frac{\partial u(x,z,t=0)}{\partial t} = 0$$

BC : Absorbing boundary conditions (ABC)

EXPERIMENT SETUP



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u : wavefield solution

c : velocity

$$f(x,z,t) = \boldsymbol{r(t)} * s(x,z)$$

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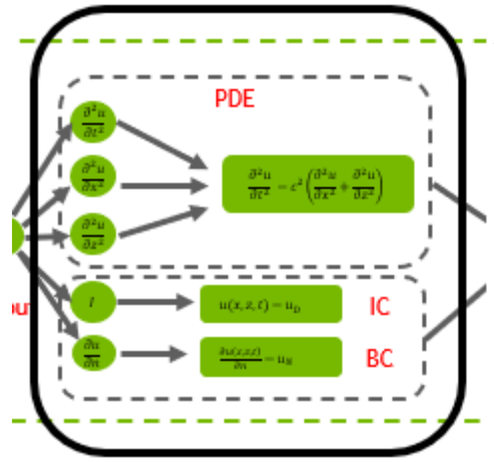
NVIDIA

EXPERIMENT SETUP



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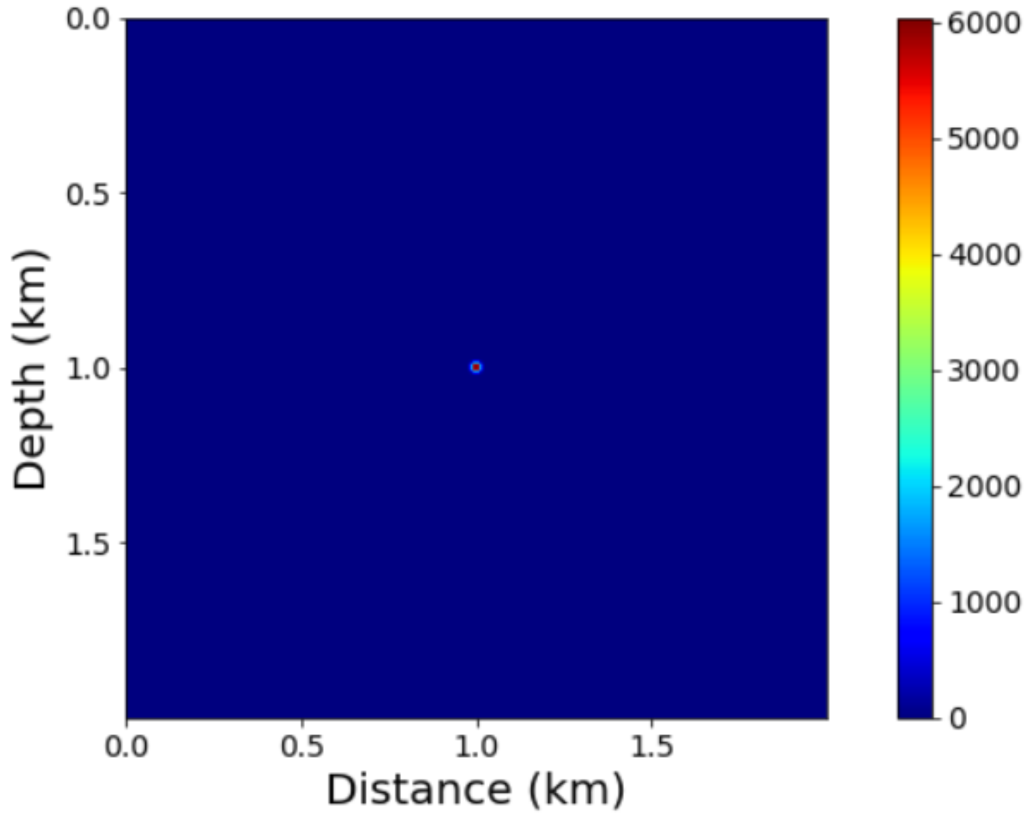
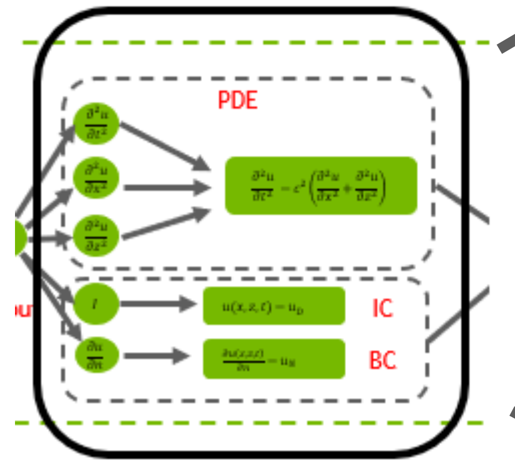
$\boldsymbol{s(x,z)}$:



Gaussian spatial function

EXPERIMENT SETUP



PDE : $$\frac{\partial^2 u}{\partial t^2} - c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) = f(x,z,t)$$

u : wavefield solution

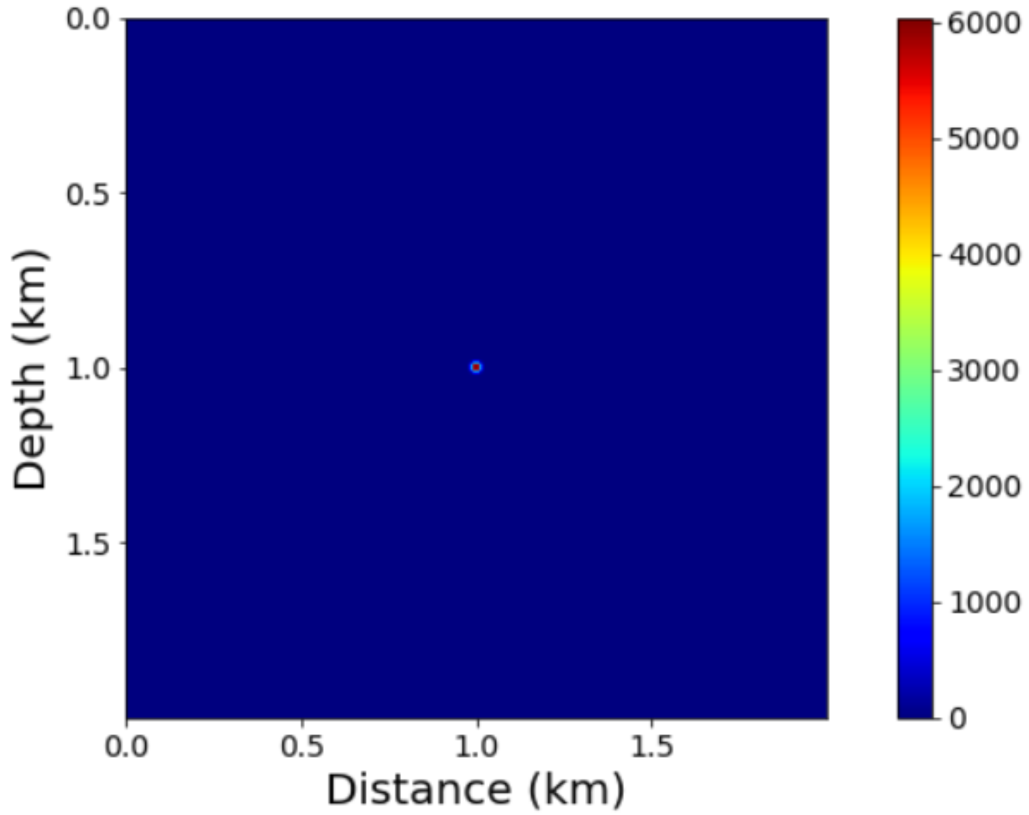
c : velocity

IC : $$u(x,z,t=0) = 0$$
$$\frac{\partial u(x,z,t=0)}{\partial t} = 0$$

BC : Absorbing boundary conditions (ABC)

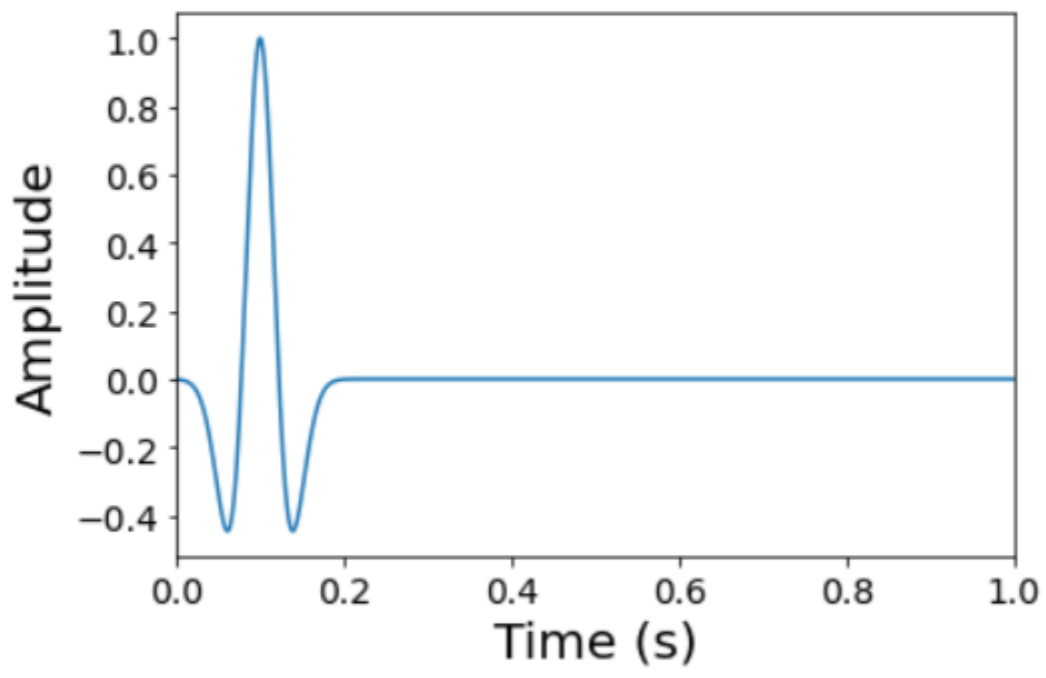
$$f(x,z,t) = \boldsymbol{r(t)} * s(x,z)$$

$s(x,z)$:



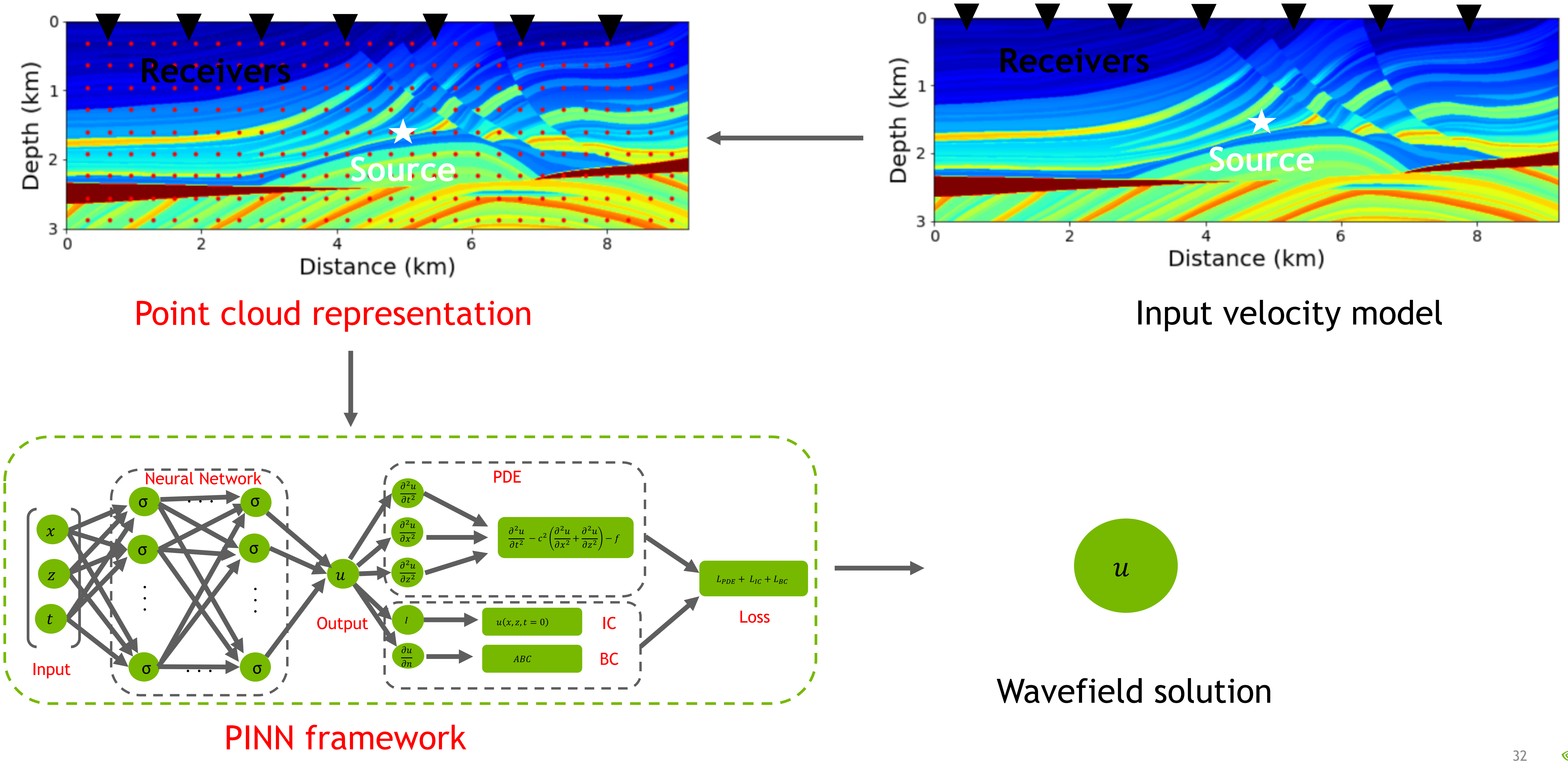
Gaussian spatial function

$\boldsymbol{r(t)}$:

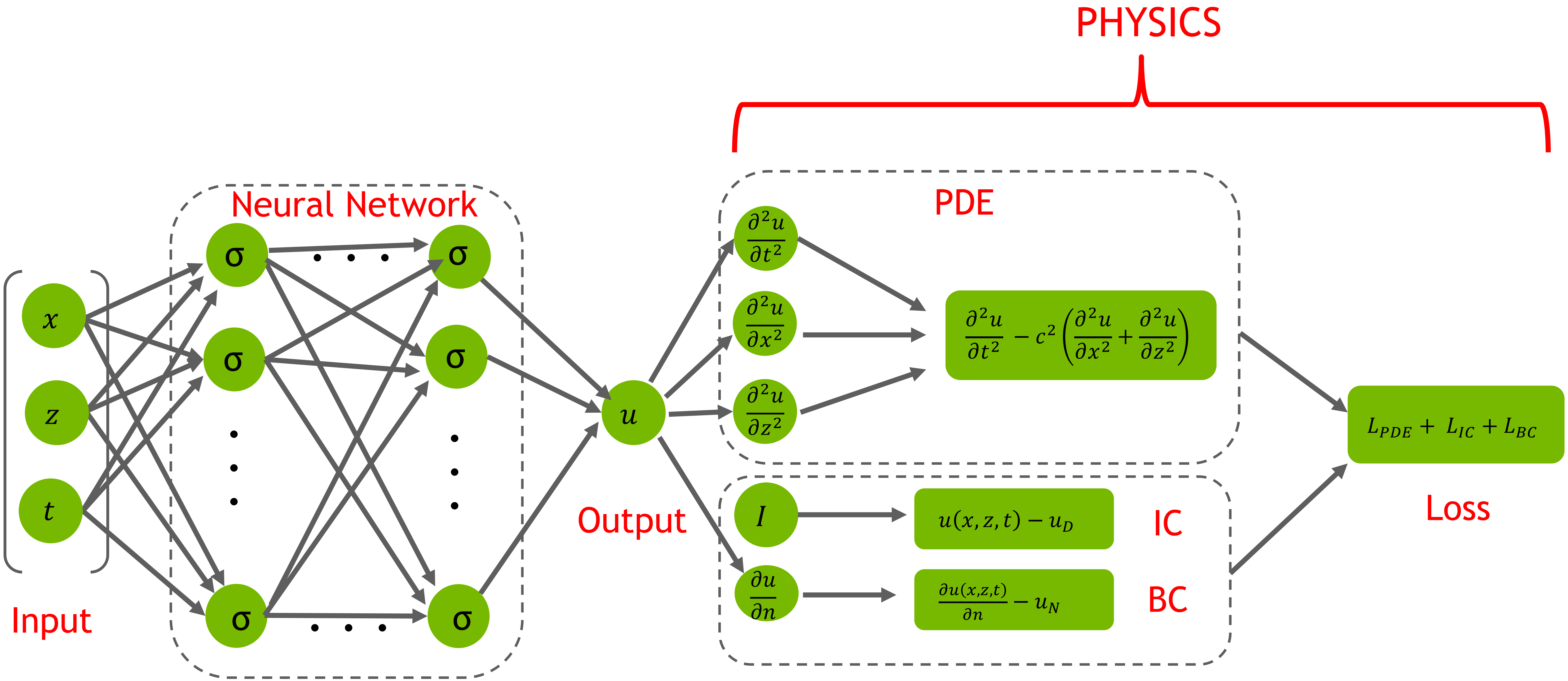


Ricker wavelet

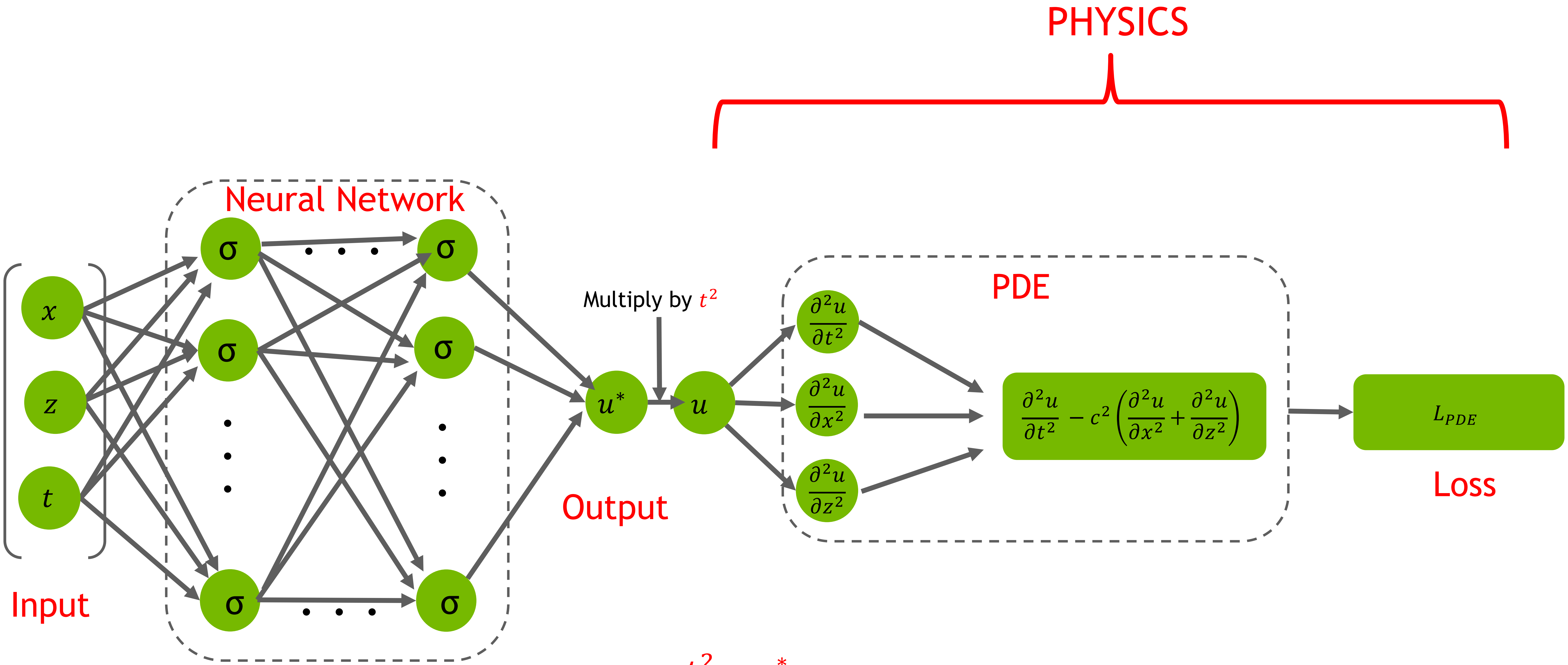
EXPERIMENT SETUP



ENFORCEMENT OF INITIAL CONDITIONS (IC)



ENFORCEMENT OF INITIAL CONDITIONS (IC)



$$u = t^2 * u^*$$

Multiply by t^2 to the Neural Network output u^*

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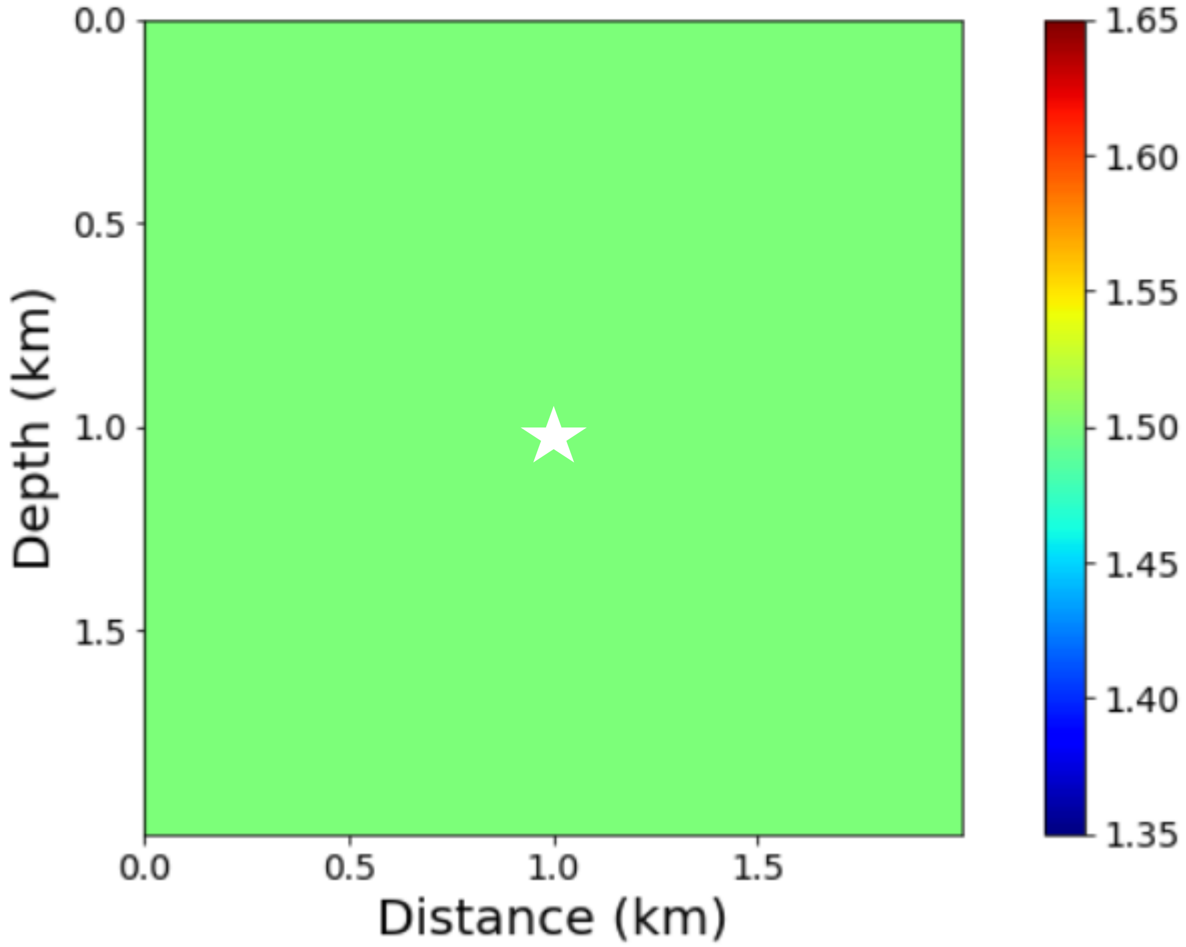
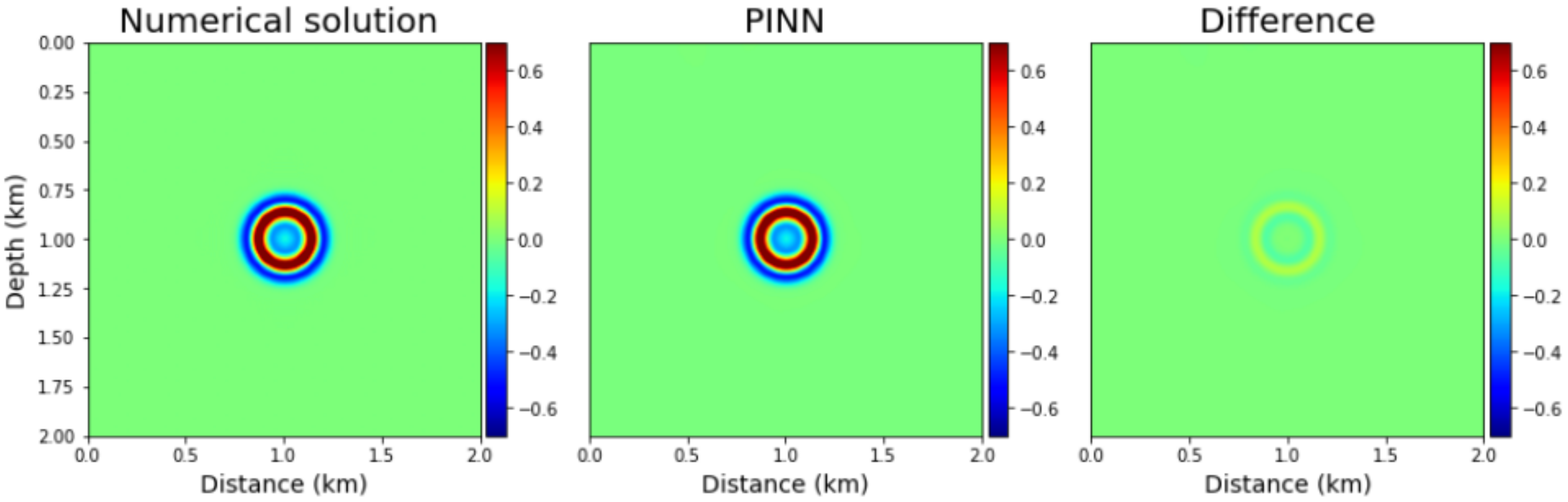
EXPERIMENT-1: HOMOGENEOUS MODEL



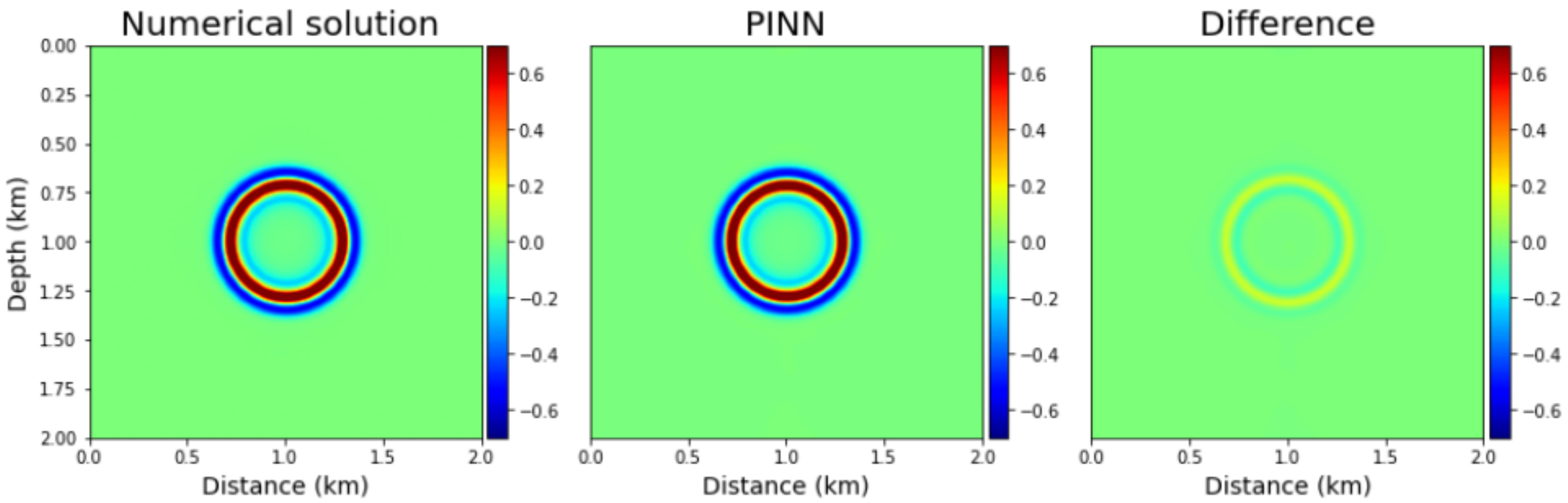
- Ricker wavelet with peak frequency 20 Hz
- Fourier neural network with 20 frequencies

EXPERIMENT-1: HOMOGENEOUS MODEL



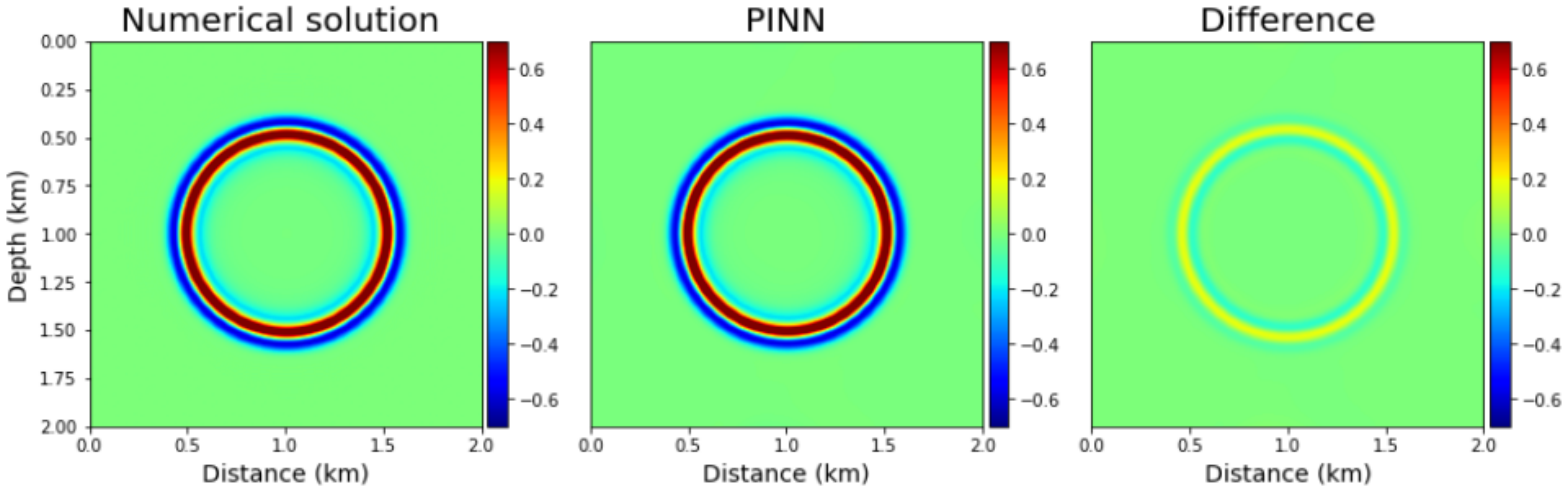
$t = 0.1\ s$

EXPERIMENT-1: HOMOGENEOUS MODEL



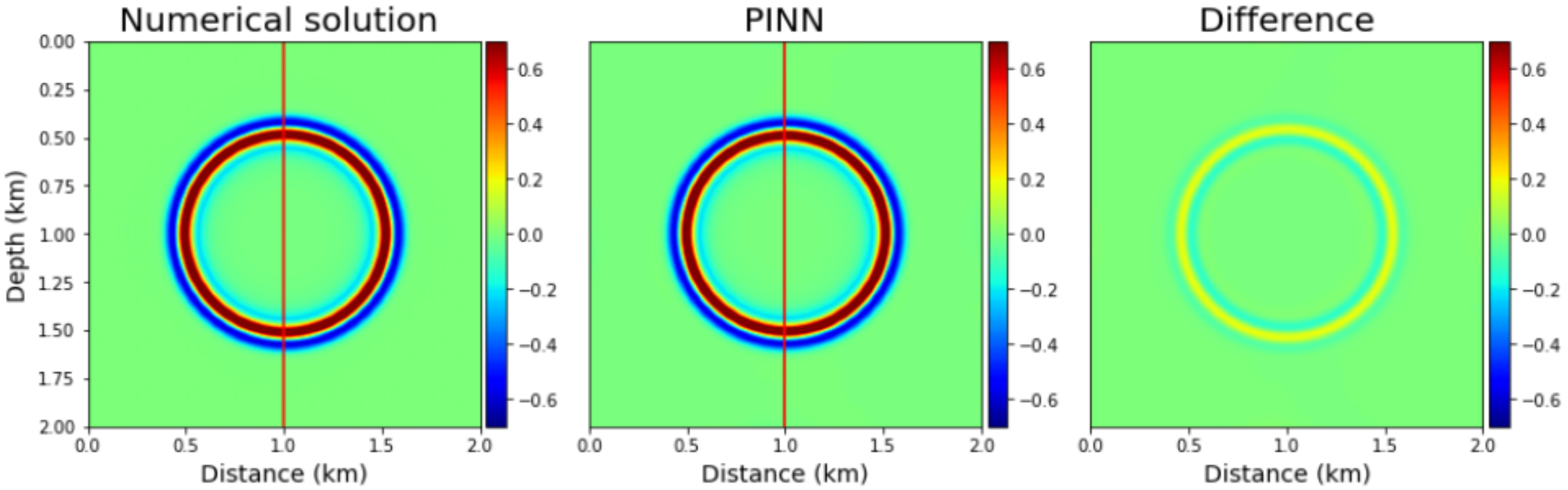
$t = 0.2\ s$

EXPERIMENT-1: HOMOGENEOUS MODEL



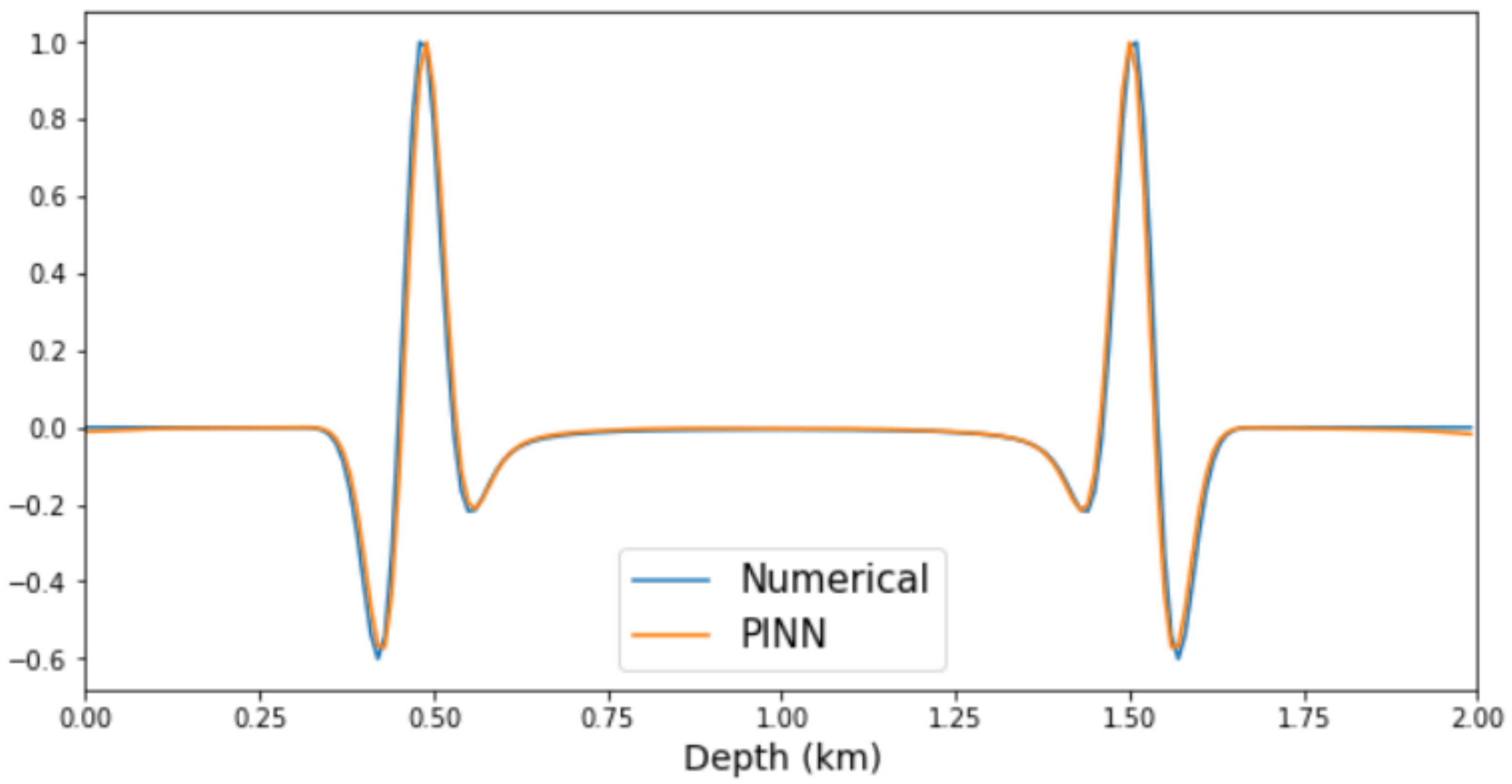
$$t = 0.4\ s$$

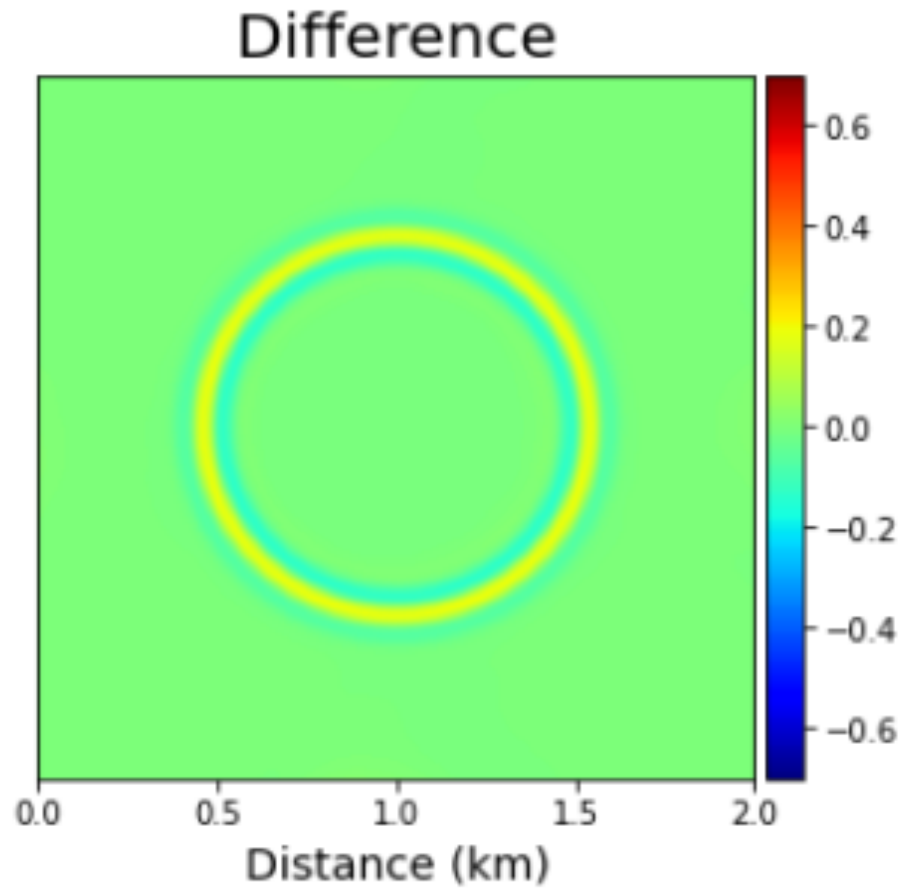
EXPERIMENT-1: HOMOGENEOUS MODEL



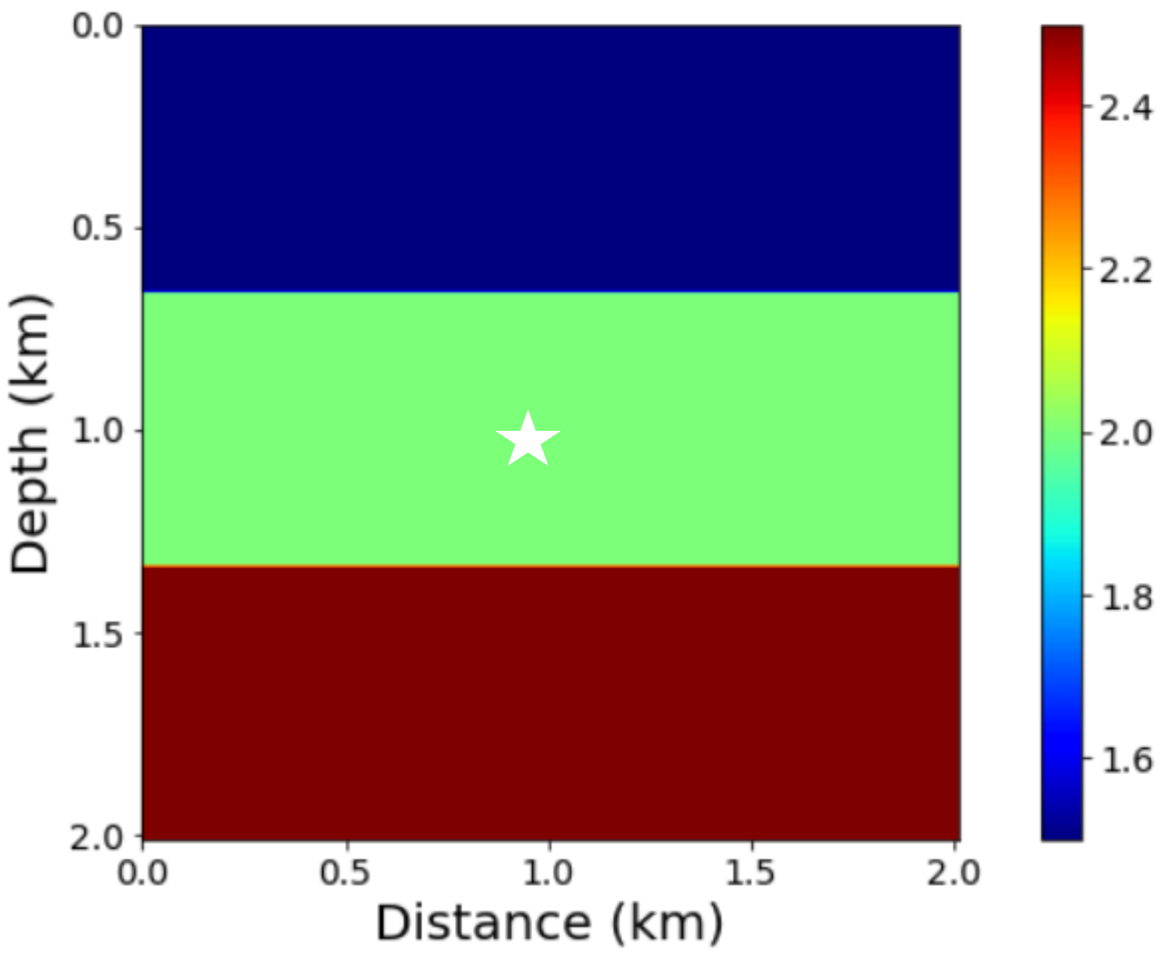
$t = 0.4\ s$

EXPERIMENT-1: HOMOGENEOUS MODEL



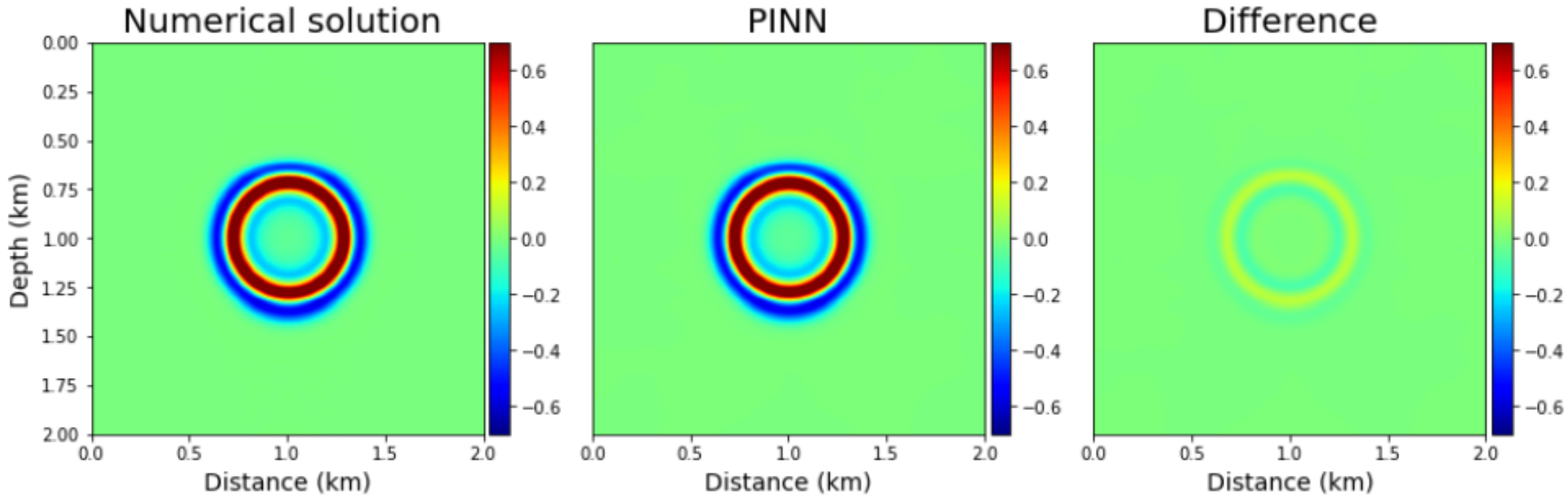


EXPERIMENT-2: LAYERED MODEL



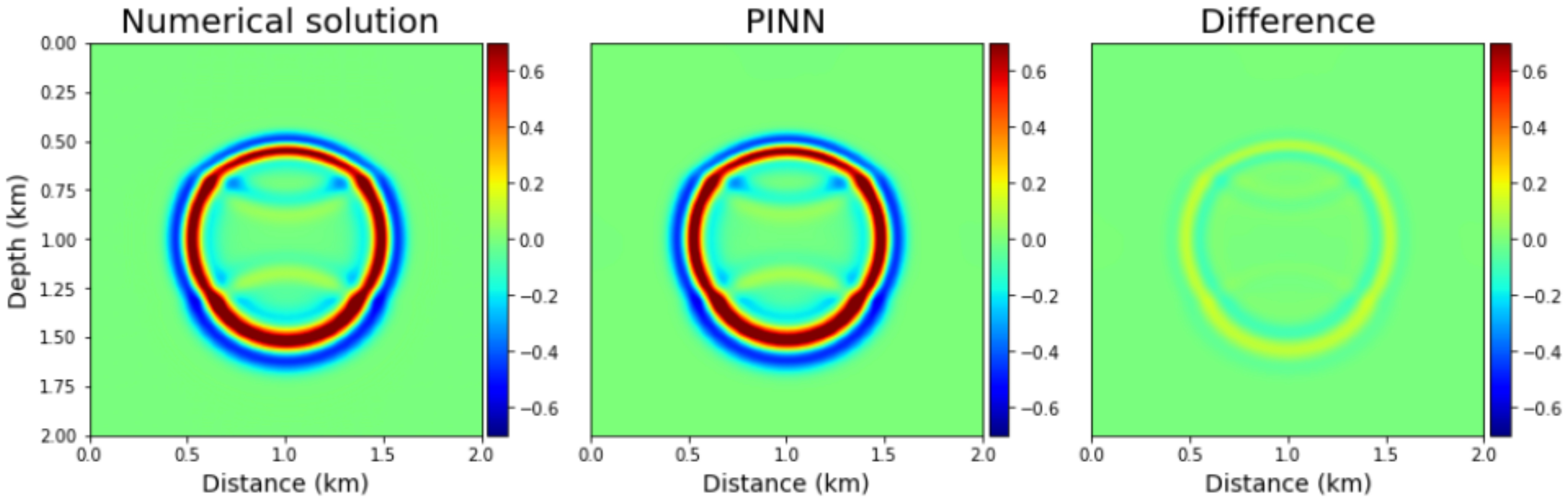
- Ricker wavelet with peak frequency 20 Hz
- Fourier neural network with 20 frequencies

EXPERIMENT-2: LAYERED MODEL



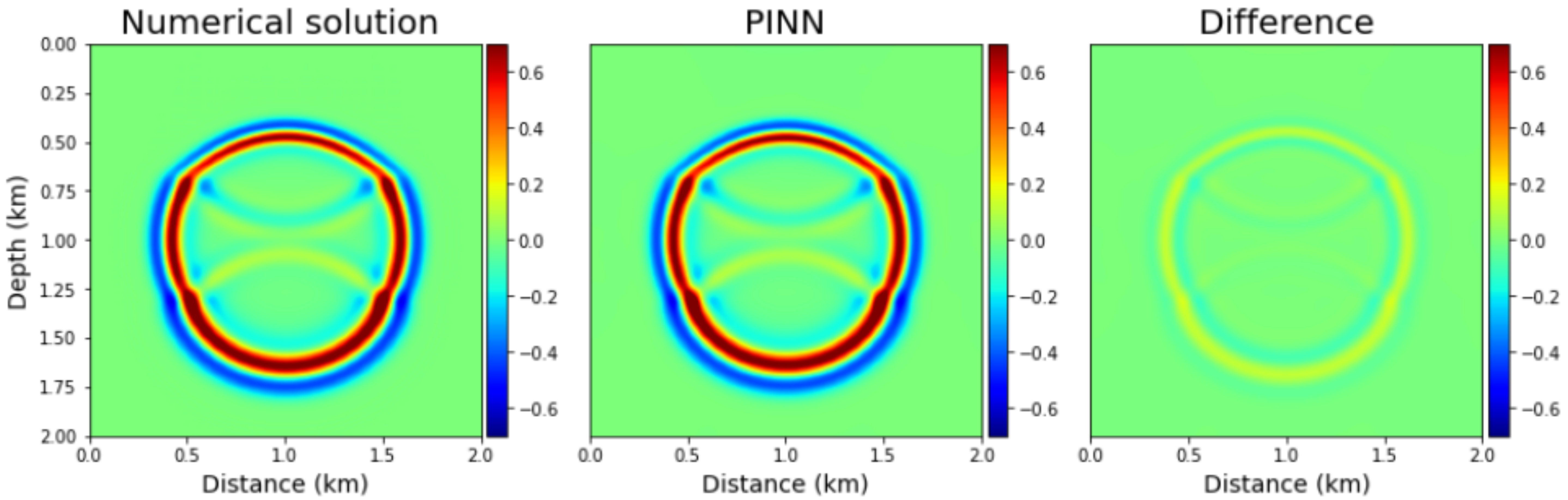
$t = 0.1\ s$

EXPERIMENT-2: LAYERED MODEL



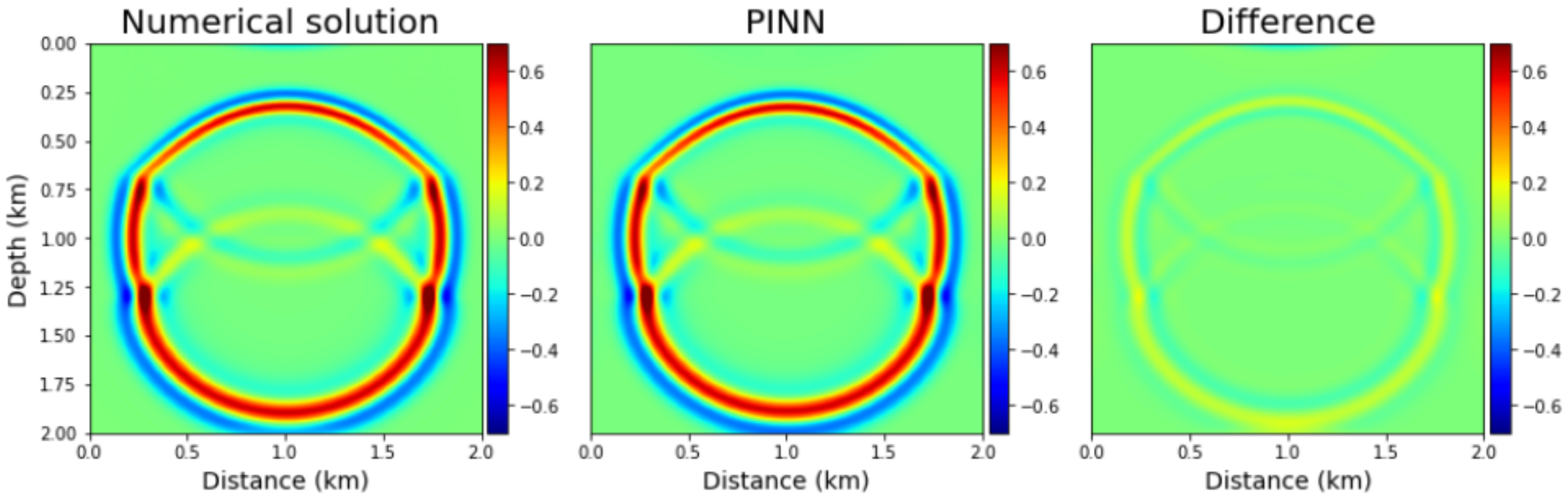
$t = 0.2\ s$

EXPERIMENT-2: LAYERED MODEL



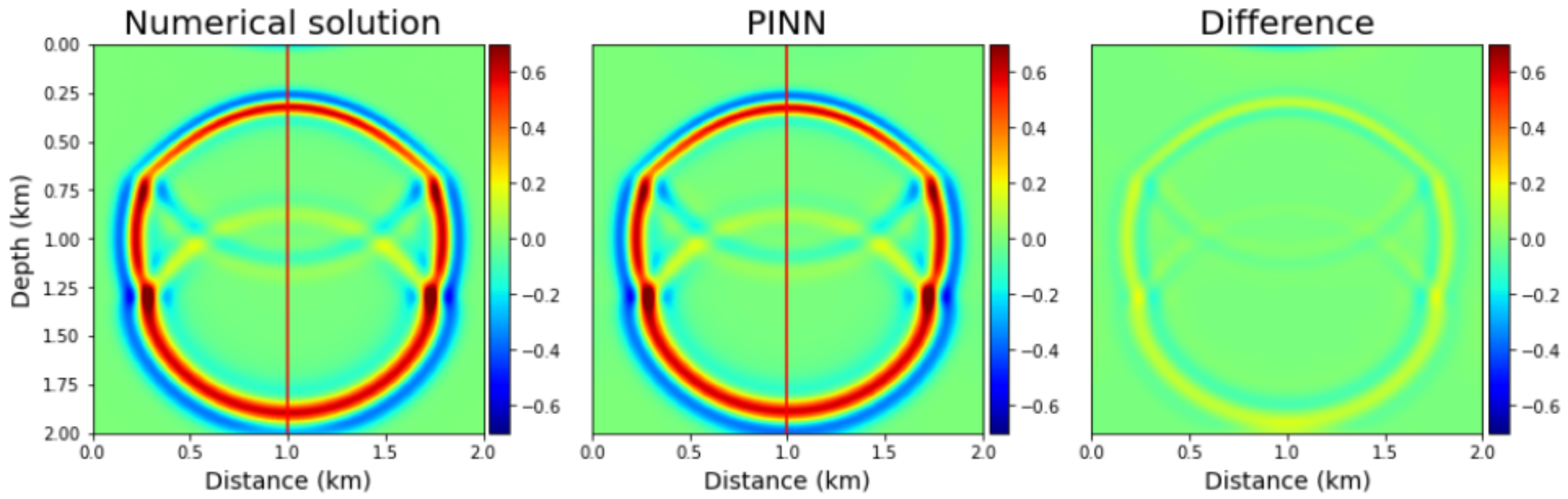
$t = 0.3\ s$

EXPERIMENT-2: LAYERED MODEL



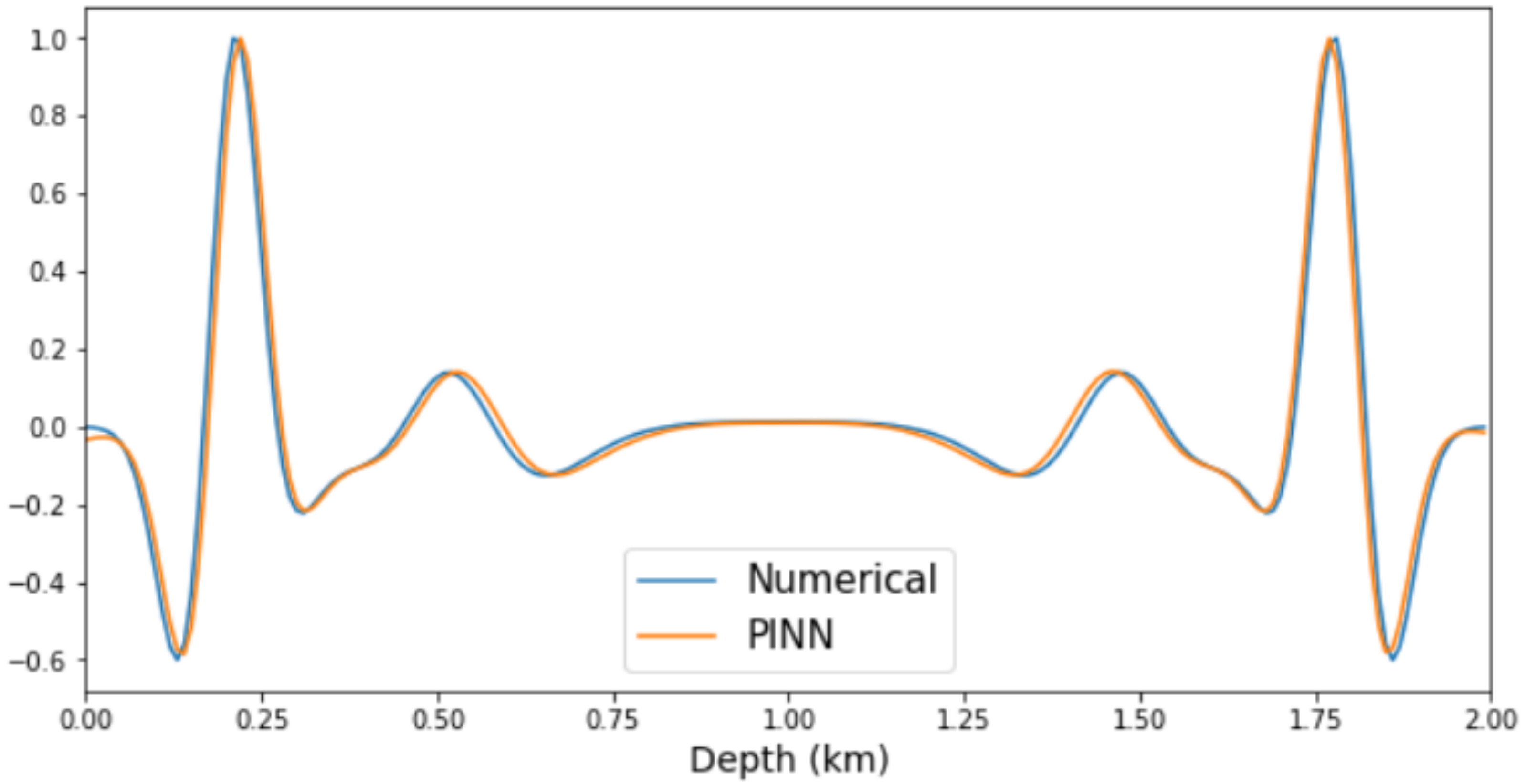
$t = 0.5\ s$

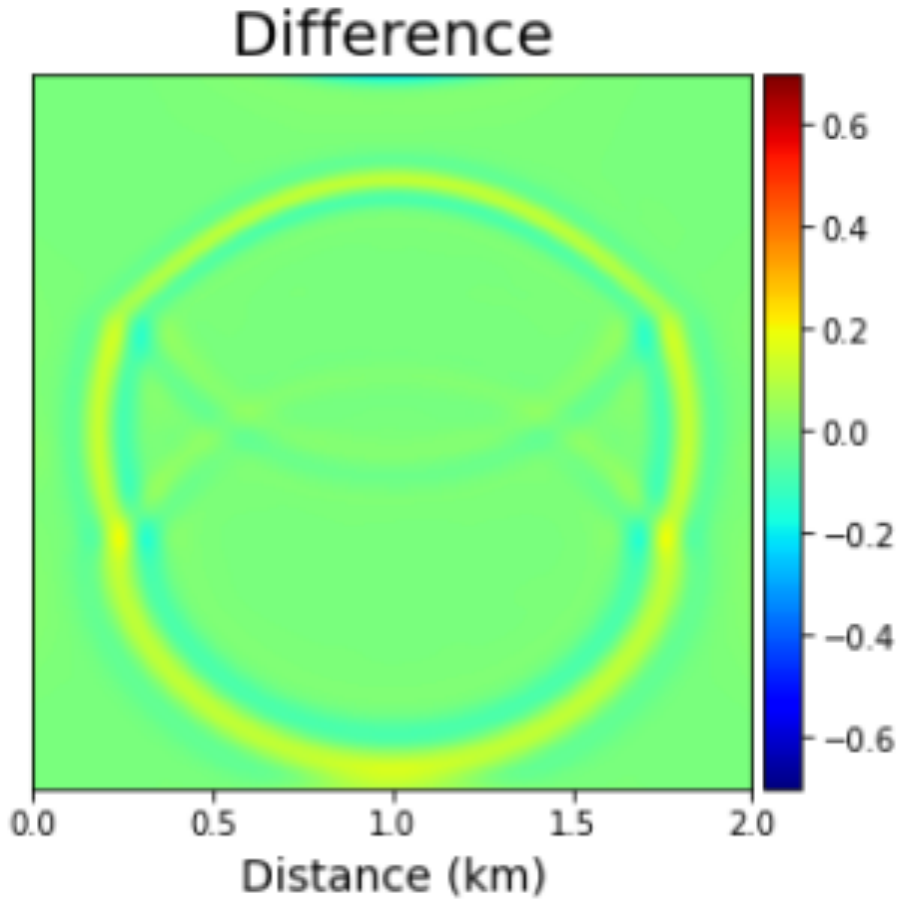
EXPERIMENT-2: LAYERED MODEL



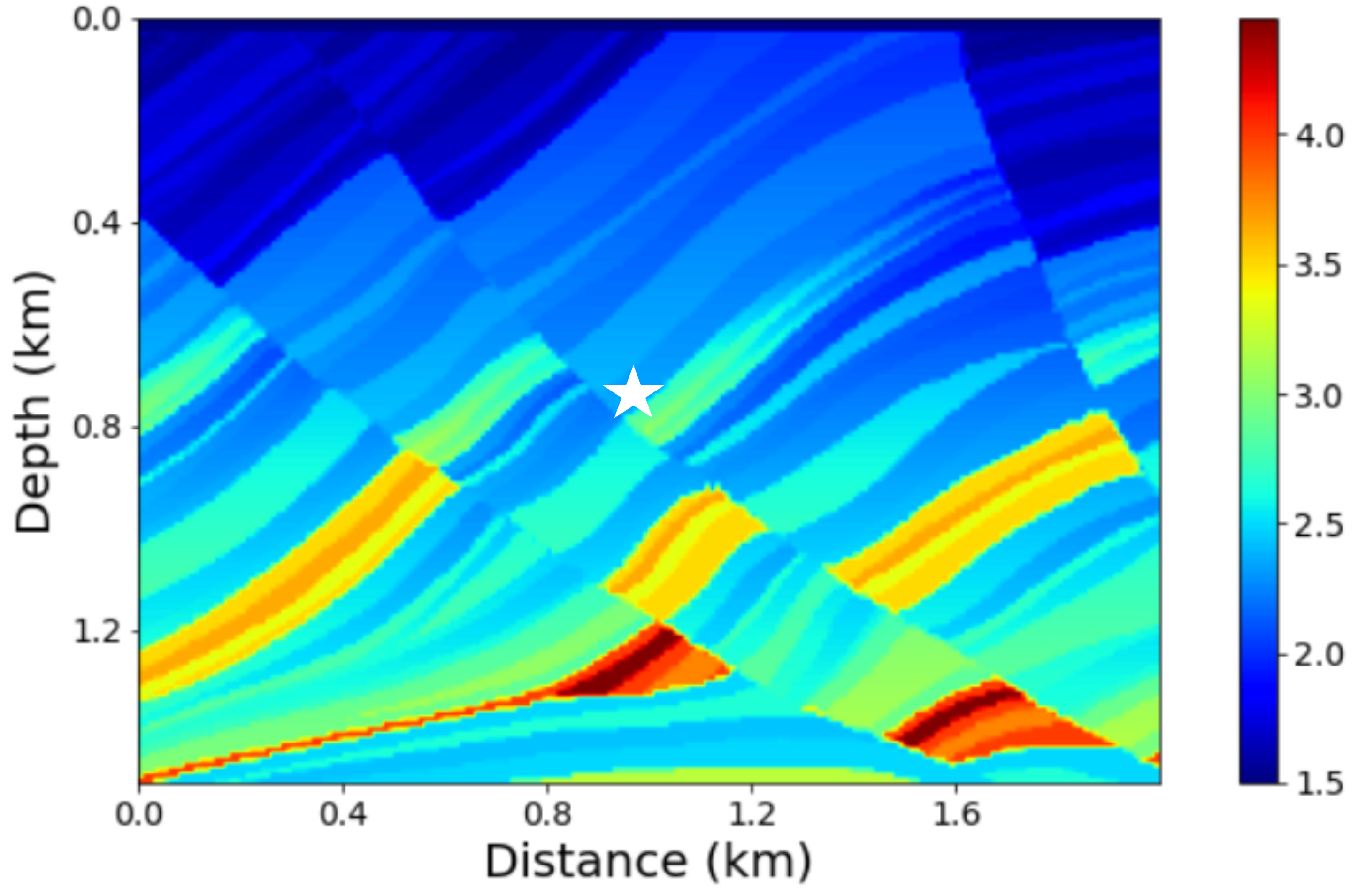
$t = 0.5\ s$

EXPERIMENT-2: LAYERED MODEL



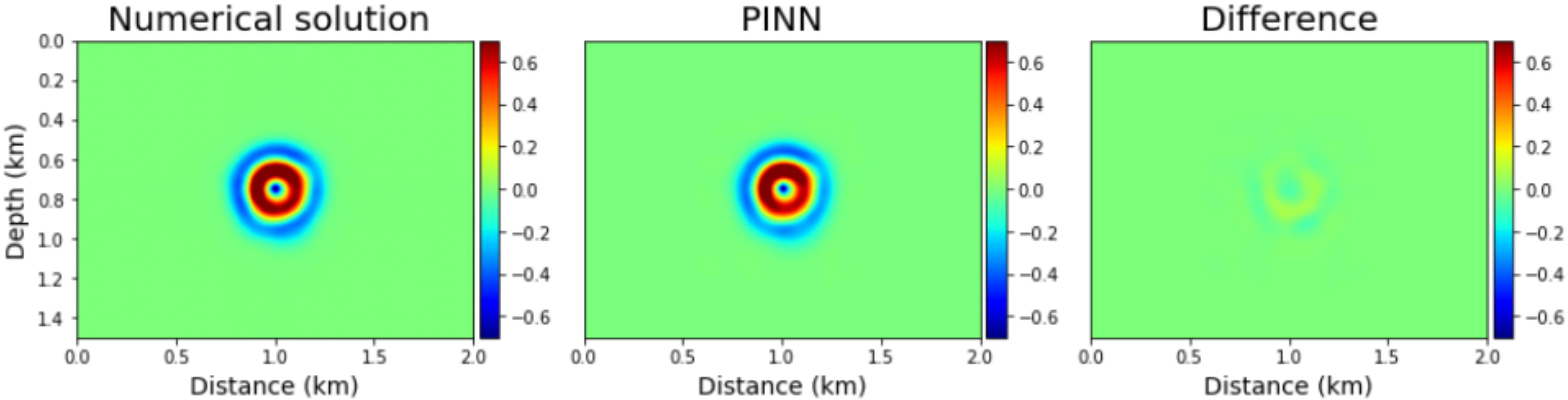


EXPERIMENT-3: MARMOUSI MODEL (MODIFIED)



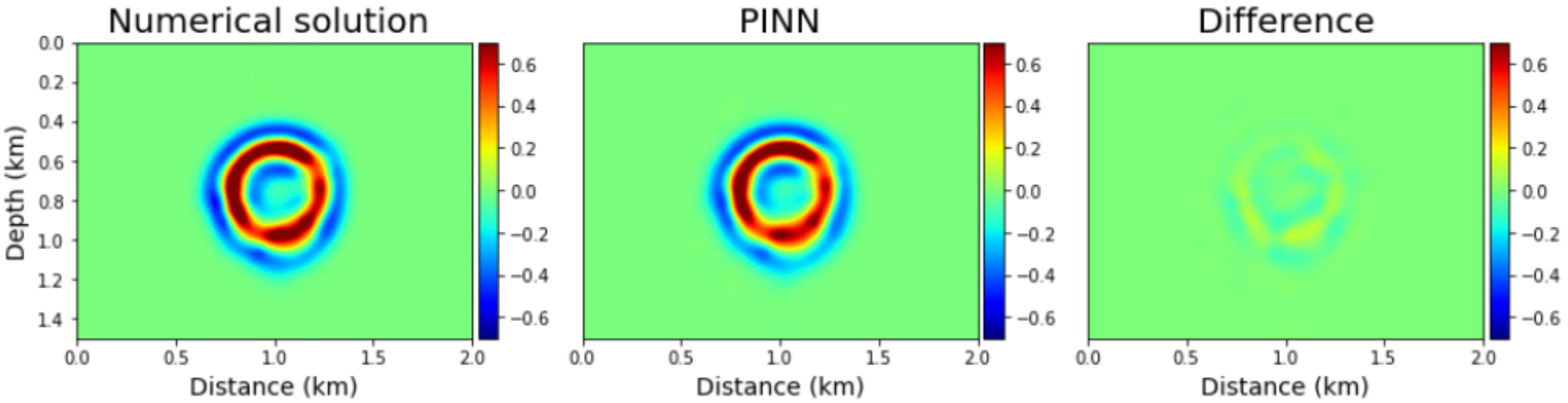
- Ricker wavelet with peak frequency 20 Hz
- Fourier neural network with 50 frequencies

EXPERIMENT-3: MARMOUSI MODEL (MODIFIED)



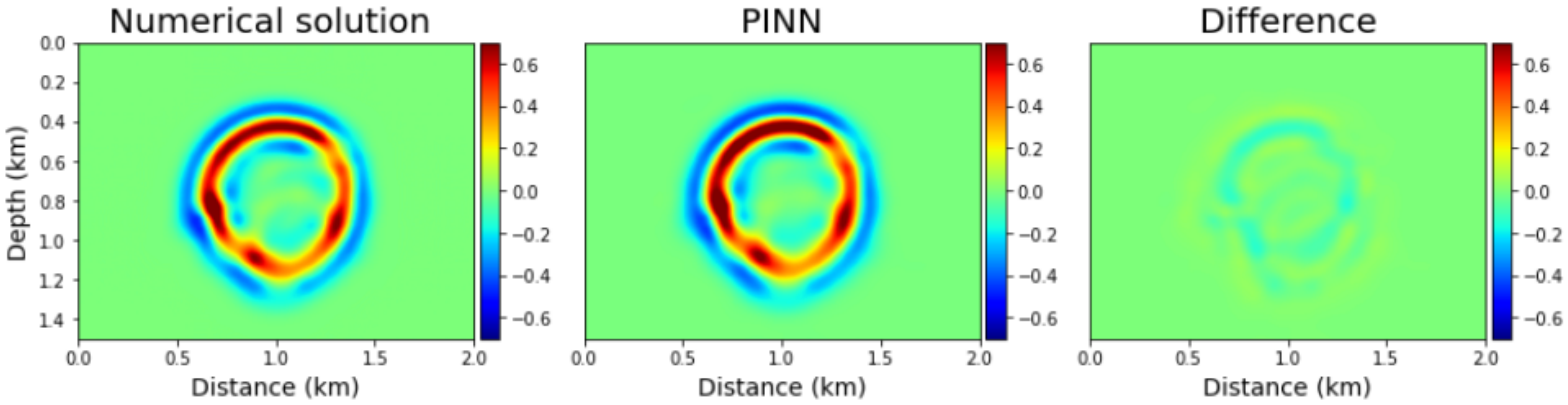
$t = 0.1\ s$

EXPERIMENT-3: MARMOUSI MODEL (MODIFIED)



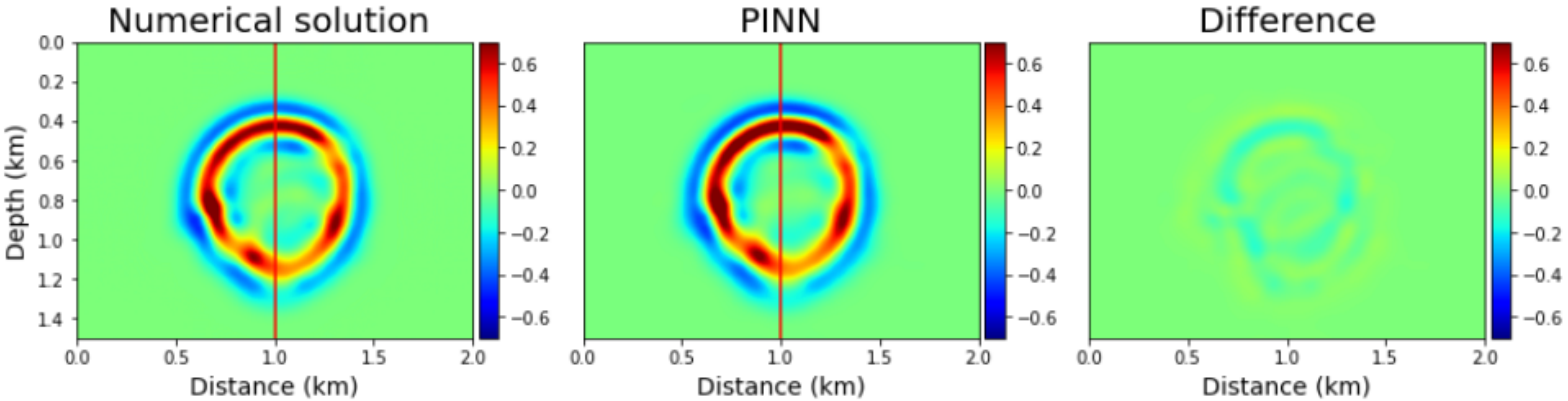
$t = 0.2\ s$

EXPERIMENT-3: MARMOUSI MODEL (MODIFIED)



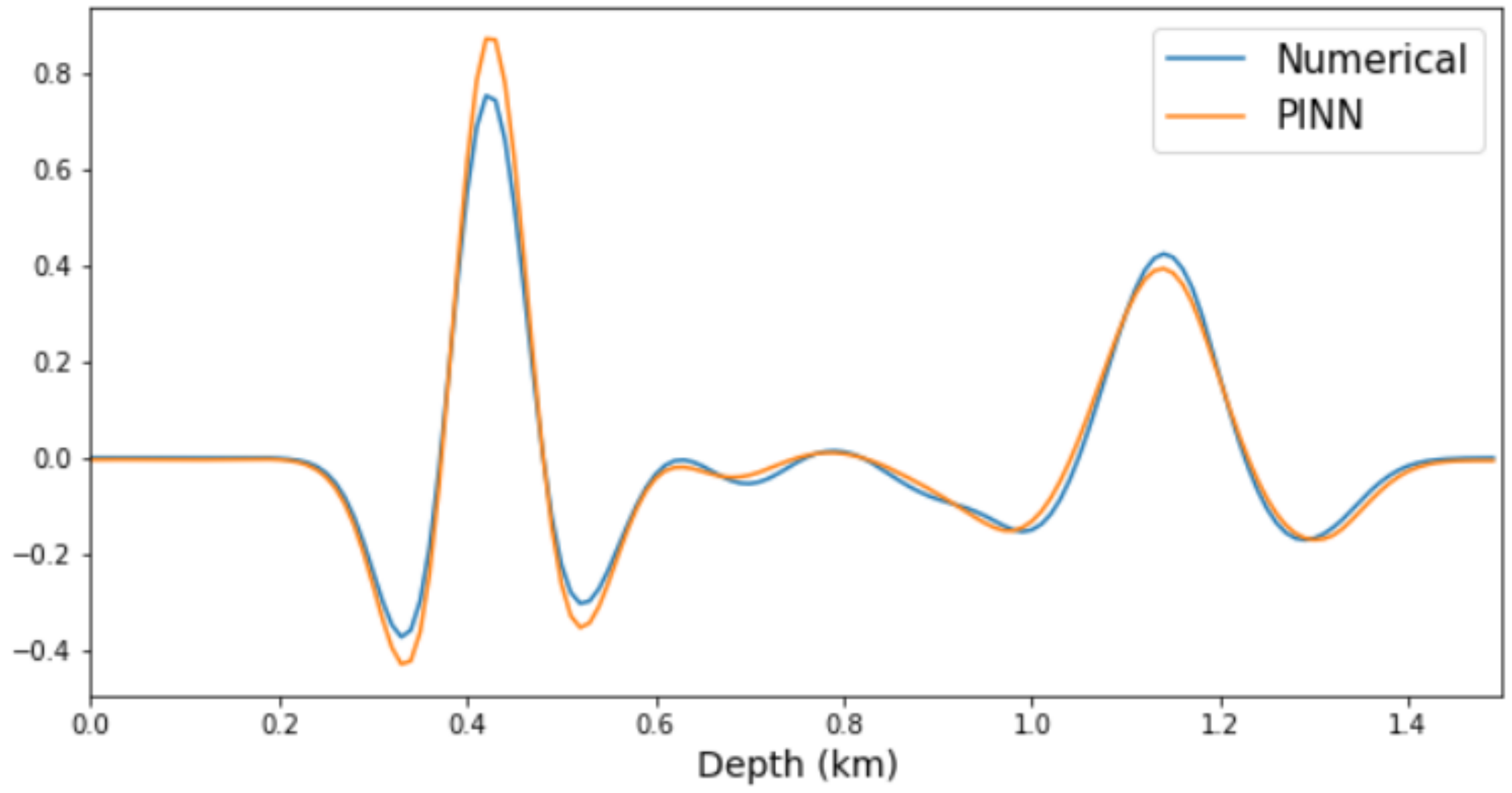
$t = 0.35\ s$

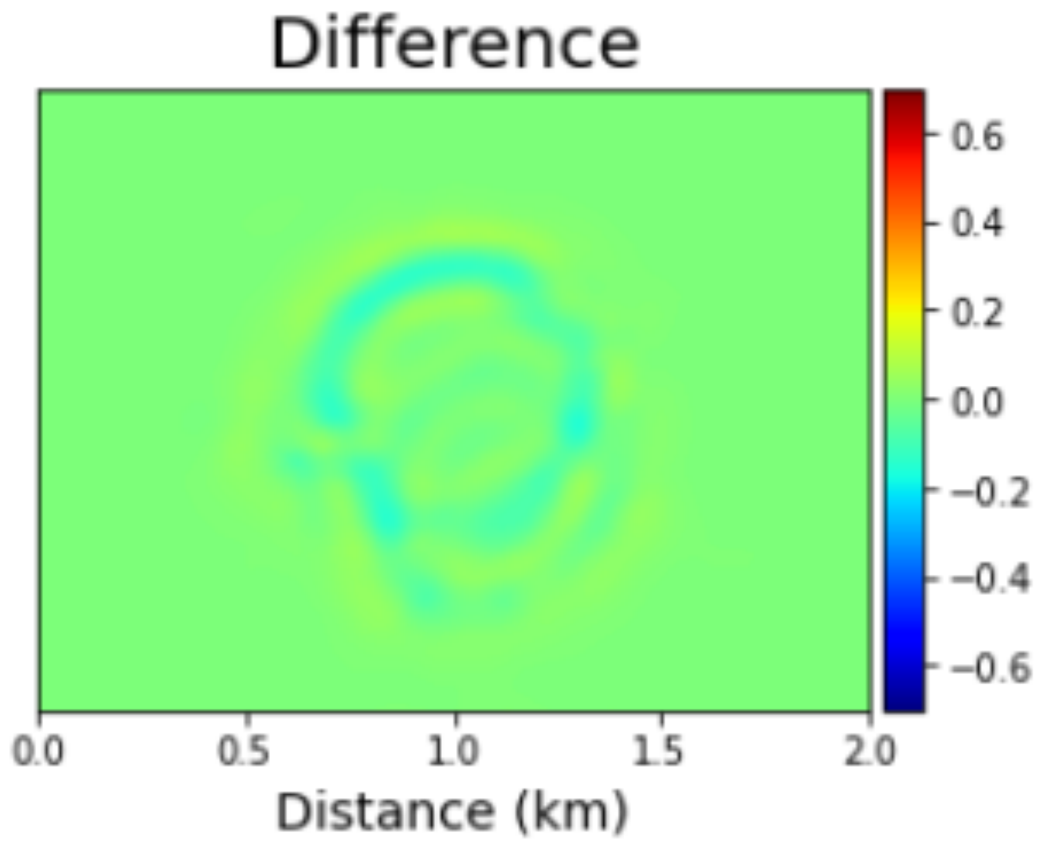
EXPERIMENT-3: MARMOUSI MODEL (MODIFIED)



$t = 0.35\ s$

EXPERIMENT-3: MARMOUSI MODEL (MODIFIED)



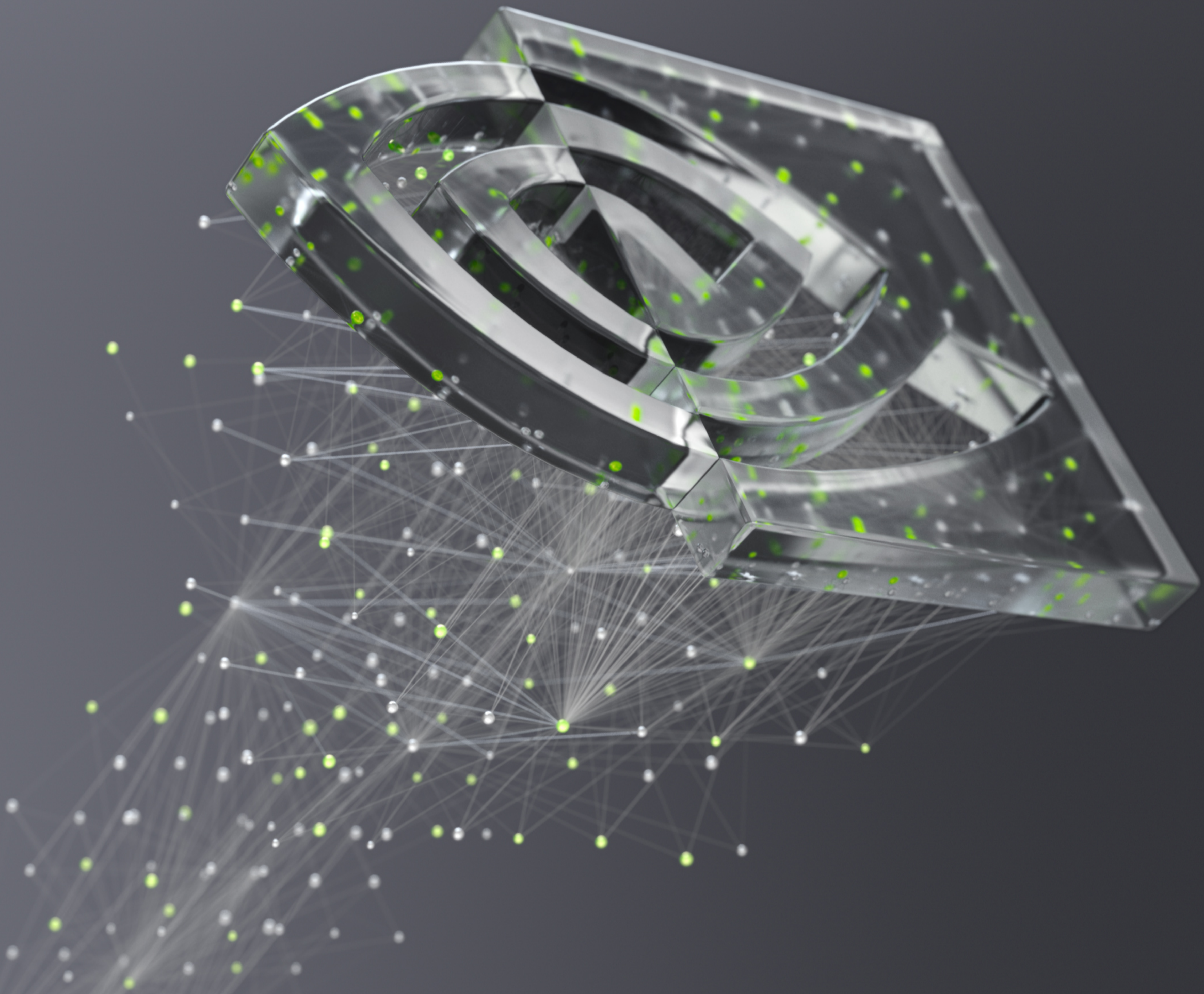


AGENDA

- Motivation
- Theory
- Experimental setup
- Numerical examples
- Conclusion and next steps

NEXT STEPS

- Numerical solutions with reasonable accuracy even for heterogeneous models.
- Hard enforcement of the initial conditions was the key factor to train the NN without using data.
- Extension to waveform inversion applications.



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