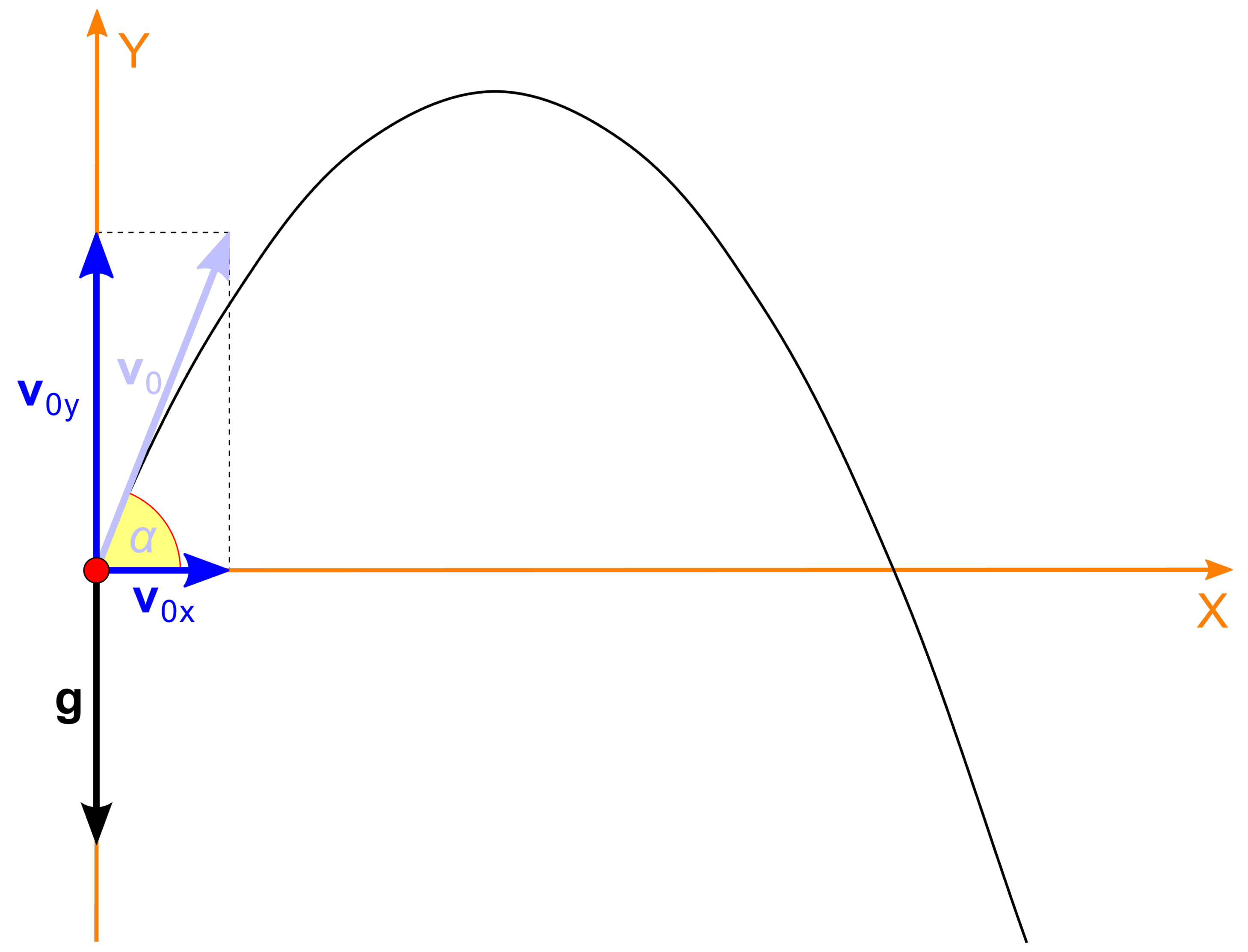
[Lab1] Physics-Informed approach to an AI for Science application

CK, May 25, 2023

Hands-on with Modulus

Projectile motion

Problem description



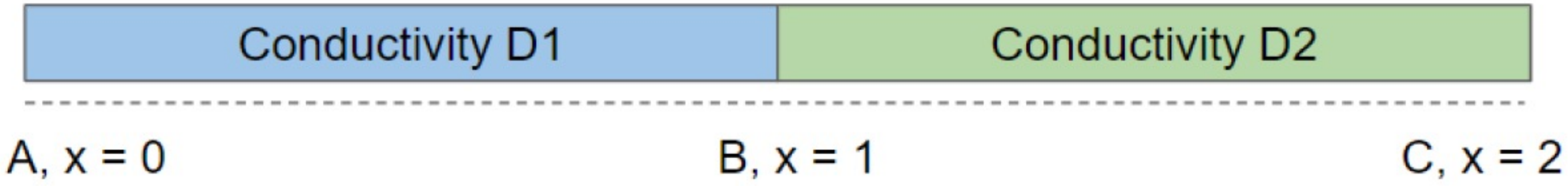
- Solving projectile motion in the case of absence of air-resistance on the surface on Earth.
- Equations: Equations of motions are as follows.

$$\frac{d^2}{dt^2}S_x = 0 \qquad\qquad \frac{d^2}{dt^2}S_y = -g$$

- Let us solve these two equations with the ball starting at origin (0,0)

1D diffusion

Problem description



- Composite bar with material of conductivity $D_1 = 10$ for $x \in (0,1)$ and $D_2 = 0.1$ for $x \in (1,2)$. Point A and C are maintained at temperatures of 0 and 100 respectively
- Equations: Diffusion equation in $1D$

$$\frac{d}{dx}\left(D_1 \frac{dU_1}{dx}\right) = 0 \qquad \text{When } 0 < x < 1$$

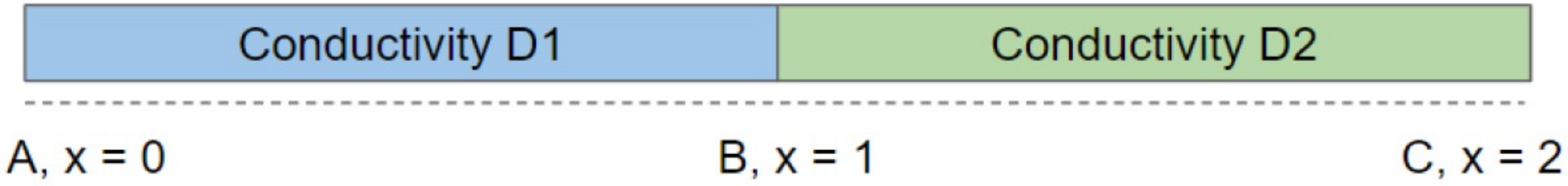
$$\frac{d}{dx}\left(D_2 \frac{dU_2}{dx}\right) = 0 \qquad \text{When } 1 < x < 2$$

- Flux and field continuity at interface (x=1)

$$\left(D_1 \frac{dU_1}{dx}\right) = \left(D_2 \frac{dU_2}{dx}\right)$$
$$U_1 = U_2$$

Parameterized 1D diffusion

Problem description



- Composite bar with material of conductivity D_1 for $x \in (0,1)$ and $D_2 = 0.1$ for $x \in (1,2)$.
- Solve the problem for multiple values of D_1 in the range $(5, 25)$ in a single training
- Same boundary and interface conditions as before
- Equations: Diffusion equation in $1D$

$$\frac{d}{dx}\left(D_1 \frac{dU_1}{dx}\right) = 0 \qquad \text{When } 0 < x < 1$$

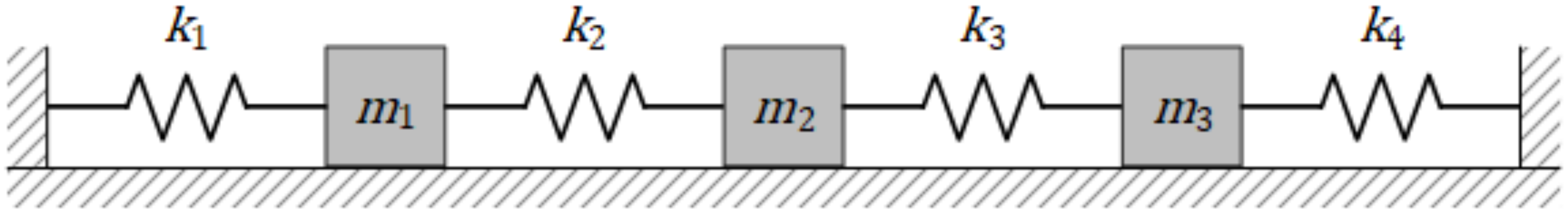
$$\frac{d}{dx}\left(D_2 \frac{dU_2}{dx}\right) = 0 \qquad \text{When } 1 < x < 2$$

- Flux and field continuity at interface (x=1)

$$\left(D_1 \frac{dU_1}{dx}\right) = \left(D_2 \frac{dU_2}{dx}\right)$$
$$U_1 = U_2$$

Optional - Coupled Spring Mass System

Problem description



- Three masses connected by four springs
- System's equations (ordinary differential equations):

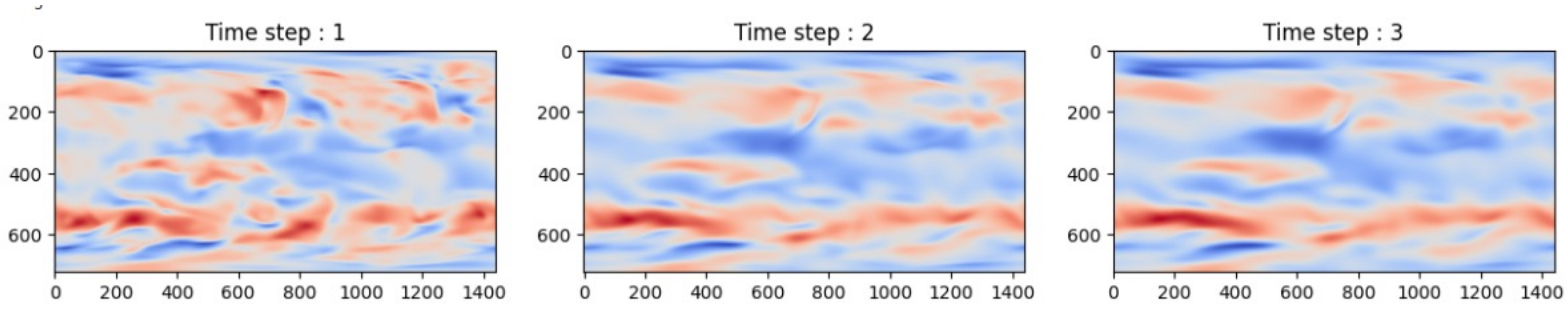
$$
\begin{aligned}
m_1 x_1''(t) &= -k_1 x_1(t) + k_2\big(x_2(t) - x_1(t)\big) \\
m_2 x_2''(t) &= -k_2\big(x_2(t) - x_1(t)\big) + k_3\big(x_3(t) - x_2(t)\big) \\
m_3 x_3''(t) &= -k_3\big(x_3(t) - x_2(t)\big) - k_4 x_3(t)
\end{aligned}
$$

- For given values masses, spring constants and boundary conditions

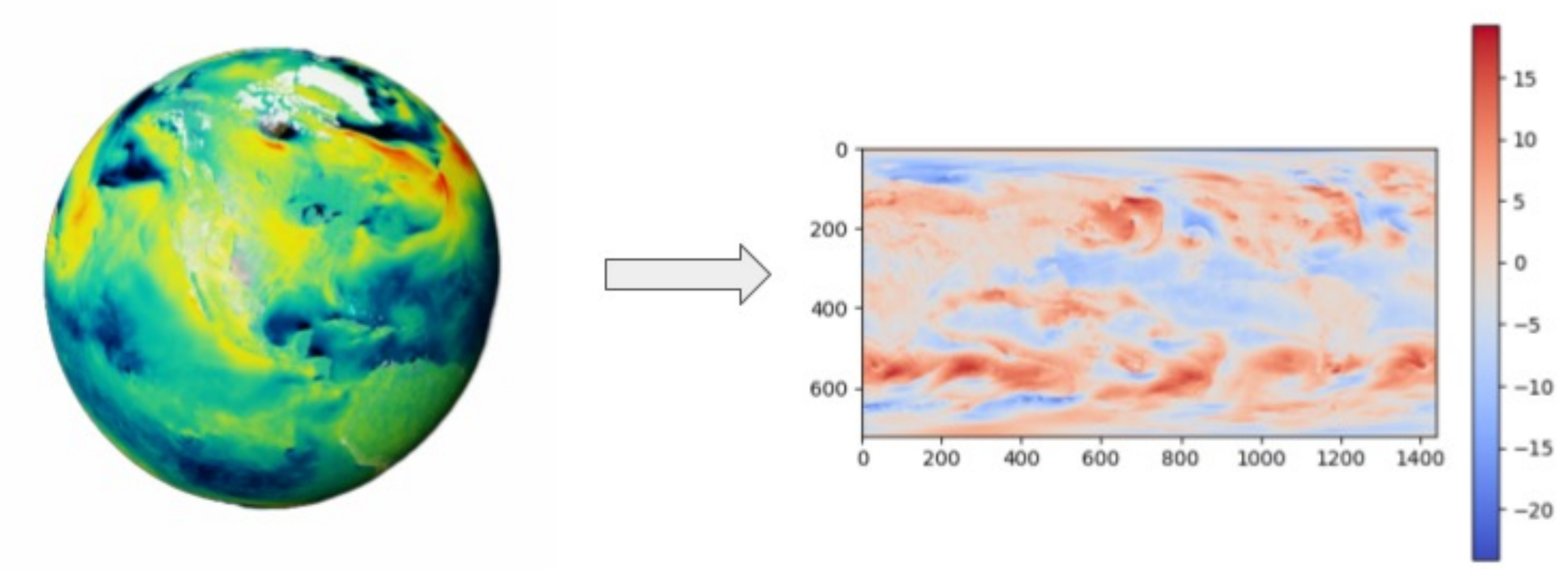
$$
\begin{aligned}
[m_1, m_2, m_3] &= [1, 1, 1] \\
[k_1, k_2, k_3, k_4] &= [2, 1, 1, 2] \\
[x_1(0), x_2(0), x_3(0)] &= [1, 0, 0] \\
[x_1'(0), x_2'(0), x_3'(0)] &= [0, 0, 0]
\end{aligned}
$$

Weather prediction using Navier-Stokes

Problem description

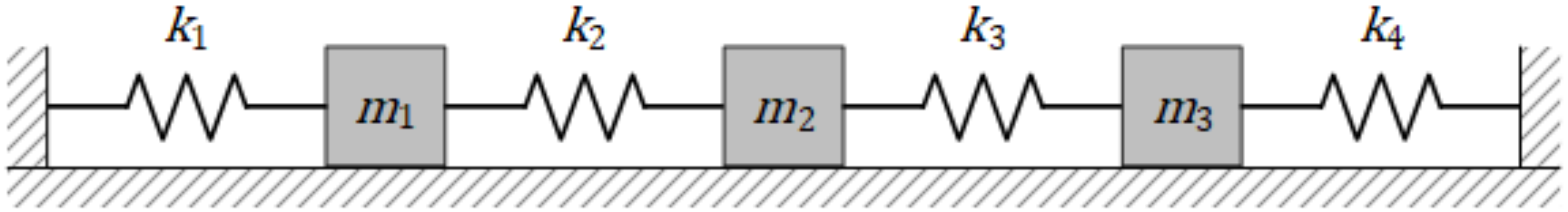


- We aim to predict the velocities for 6-hour timesteps using the Navier-Stokes equation.
- The Navier–Stokes equations mathematically express the conservation of momentum and conservation of mass for Newtonian fluids.
- We will take a 2d projected input from the ERA5 Reanalysis dataset to be used as initial conditions for our input. The process of taking the 3D sphere and projecting it onto a 2D mesh is shown in the diagram below, the 2D mesh is of the size (1440,720)



Optional - Inverse Problem – Coupled Spring Mass System

Problem description



- For the same system, assume we know the analytical solution which is given by:

$$x_1(t) = \frac{1}{6}\cos(t) + \frac{1}{2}\cos(\sqrt{3}t) + \frac{1}{3}\cos(2t);$$
$$x_2(t) = \frac{2}{6}\cos(t) - \frac{1}{3}\cos(2t) \ ;$$
$$x_3(t) = \frac{1}{6}\cos(t) - \frac{1}{2}\cos(\sqrt{3}t) + \frac{1}{3}\cos(2t)$$

- With the above data and the values for m_2, m_3, k_1, k_2, k_3 same as before, use the neural network to find the values of m_1 and k_4

NVIDIA