Task 1: Systolic Pressure (12 point)

Question 1: Explore and Manipulate the data

In [28]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [29]:
```
# Task 1
from sklearn.linear_model import LinearRegression, LogisticRegression, Ridge
from sklearn.preprocessing import PolynomialFeatures
from sklearn.pipeline import Pipeline
from sklearn.model_selection import cross_val_score # Calculate MSE
# Task 2
from sklearn.decomposition import PCA
from sklearn.model_selection import train_test_split
from sklearn.metrics import confusion_matrix, precision_score, accuracy_score
```

In [30]:
```
# Import data in Pandas form, make sure having the same namefile
pressure = pd.read_csv("bloodpressure.csv")
pressure.head()
```

Out[30]:

	ID-NUMBER	AGE	ED-LEVEL	SMOKING STATUS	EXERCISE	WEIGHT	SERUM-CHOL	SYSTOLIC	IQ	SODIUM	GENDER	MARITAL-STATUS	NAME
0	1	27	2	1	1	120	193	126	118	136	F	M	Braund, Mr. Owen Harris
1	2	18	1	0	1	145	210	120	105	137	M	S	Cumings, Mrs. John Bradley (Florence Briggs Th...
2	3	32	2	0	0	118	196	128	115	135	F	M	Heikkinen, Miss. Laina
3	4	24	2	0	1	162	208	129	108	142	M	M	Futrelle, Mrs. Jacques Heath (Lily May Peel)
4	5	19	1	2	0	106	188	119	106	133	F	S	Allen, Mr. William Henry

In [31]:
```
systolic = pressure['SYSTOLIC']
systolic = systolic.to_numpy().reshape(-1, 1)
```

In [32]:
```
pressure_predictor = pressure.drop(["ID-NUMBER", "SYSTOLIC", "NAME"], axis=1)
pressure_predictor.head()
```

Out[32]:

	AGE	ED-LEVEL	SMOKING STATUS	EXERCISE	WEIGHT	SERUM-CHOL	IQ	SODIUM	GENDER	MARITAL-STATUS
0	27	2	1	1	120	193	118	136	F	M
1	18	1	0	1	145	210	105	137	M	S
2	32	2	0	0	118	196	115	135	F	M
3	24	2	0	1	162	208	108	142	M	M
4	19	1	2	0	106	188	106	133	F	S

In [33]:
```
# Dummies for multiple linear regression part
pressure_predictor = pd.get_dummies(pressure_predictor, drop_first=True)
# To keep all values in the numeric value
# Change categorical value to integer to use the linear regression (categorical value do not work)
pressure_predictor["GENDER_M"] = pressure_predictor["GENDER_M"].astype(np.int8)
pressure_predictor["MARITAL-STATUS_M"] = pressure_predictor["MARITAL-STATUS_M"].astype(np.int8)
pressure_predictor["MARITAL-STATUS_S"] = pressure_predictor["MARITAL-STATUS_S"].astype(np.int8)
pressure_predictor["MARITAL-STATUS_W"] = pressure_predictor["MARITAL-STATUS_W"].astype(np.int8)
pressure_predictor.head()
```

Out[33]:

	AGE	ED-LEVEL	SMOKING STATUS	EXERCISE	WEIGHT	SERUM-CHOL	IQ	SODIUM	GENDER_M	MARITAL-STATUS_M	MARITAL-STATUS_S	MARITAL-STATUS_W
0	27	2	1	1	120	193	118	136	0	1	0	0
1	18	1	0	1	145	210	105	137	1	0	1	0
2	32	2	0	0	118	196	115	135	0	1	0	0
3	24	2	0	1	162	208	108	142	1	1	0	0
4	19	1	2	0	106	188	106	133	0	0	1	0

Question 2: Polynomial Regression

- a. Create polynomial regression models using the whole dataset to predict systolic pressure using the "WEIGHT" feature, for polynomial degrees ranging from 1 to 14.
- b. Perform 10-fold cross-validation
- c. Compute and display the mean RMSEs of the 10-fold cross-validation for each of the 14 polynomial degrees
- d. Produce a cross-validation error plot showing the mean RMSE for polynomial degrees from 1 to 14

In [35]:

```
X = pressure_predictor["WEIGHT"].to_numpy().reshape(-1, 1) # Change into numpy value with 1 column and as many
y = systolic
```

In [36]:

```
poly_List_models = {} # Store polynomial models
poly_positive_mse_10foldcv = {}  # Store MSE
poly_rmse = {} # Store mean of RMSE of each model

for degree in range(1,15):
    # a. Create polynomial regression models to predict Systolic using "Weight", from degree 1 to 14
    poly_features = PolynomialFeatures(degree = degree, include_bias = False)  # Set a standard polynomial featu
    model = Pipeline([
        ("poly_features", poly_features), # X will be transformed into X_poly with corresponding degree
        ("lin_reg", LinearRegression()),  # Then X_poly will be put in the LinearRegression
    ])
    model.fit(X,y) # Transforming then Fitting
    poly_List_models[f'model{degree}'] = model # Storing

    # b. 10-fold cross validation
    positive_mse_scores = -cross_val_score(model, # Fitted from Pipeline
                                           X, y, scoring='neg_mean_squared_error', cv=10)
    poly_positive_mse_10foldcv[f'MSE{degree}'] = positive_mse_scores # Storing

    # c. RMSEs of 10-fold CV for each degree
    rmse_scores = np.sqrt(positive_mse_scores)
    mean_rmse_scores = np.mean(rmse_scores)
    poly_rmse[f'RMSE{degree}'] = mean_rmse_scores # Storing
```

In [37]:

```
# Testing for prediction a random degree
testing_degree = input('Enter a degree from 1-14:')
model_degree = 'model'+testing_degree

testing_model = poly_List_models[model_degree]

X_new = [[120], [100], [150]]

testing_model.predict(X_new)
```

Out[37]:

```
array([[123.19841446],
       [112.62543661],
       [131.90888099]])
```

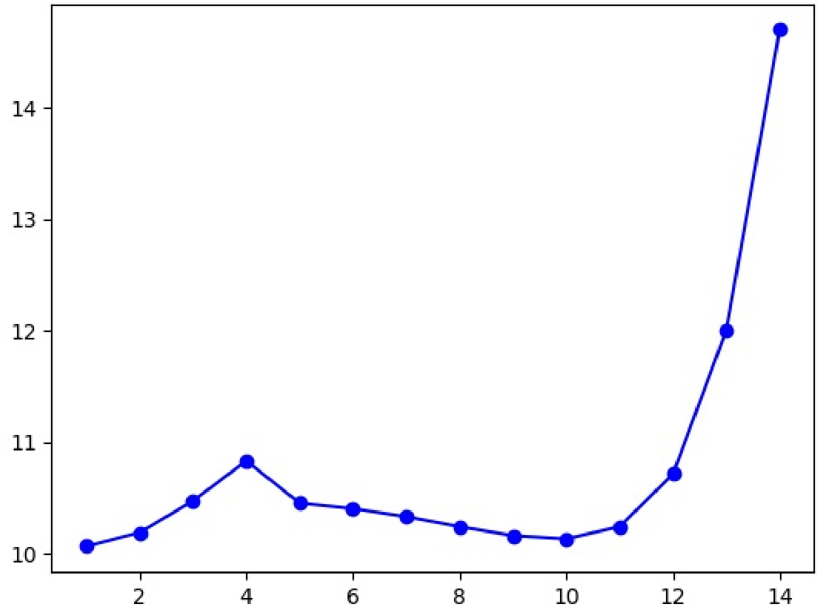
In [38]:

```
# d. Visualisation
print('RMSE of 14 models', poly_rmse.values())
plt.plot(range(1,15), poly_rmse.values(), "bo-")
```

```
RMSE of 14 models dict_values([10.063566348213604, 10.187166706053542, 10.470321681840364, 10.829963377150463, 1
0.45133716702582, 10.4018597256369, 10.329004499108617, 10.240305476454346, 10.159177954821084, 10.1302942634471
53, 10.240212879403142, 10.712133802678924, 12.002013373747856, 14.697035935941656])
```

Out[38]:

```
[<matplotlib.lines.Line2D at 0x14080a3f0>]
```



Question 3: Model selection for polynomial regression

- a. Select the best polynomial degree and briefly explain your choice
- b. Print the intercept and coefficients of the selected model

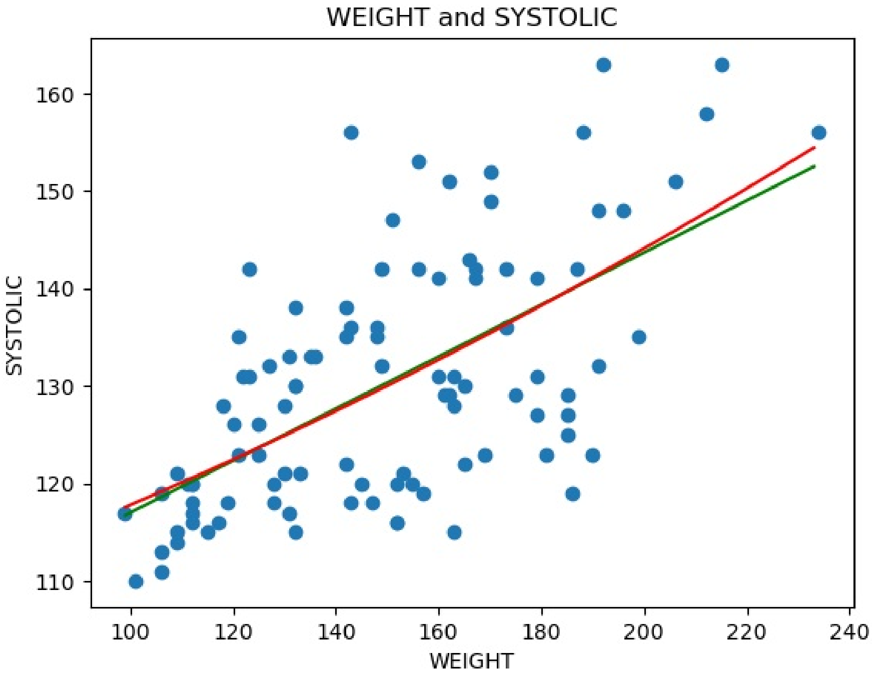
In [40]:
```
# a. Choose best model
list_poly_rmse = np.array(list(poly_rmse.values()))
index_poly_rmse = np.argsort(list_poly_rmse)+1
# Index start at 0, plus 1 to make it easier to read
index_poly_rmse
```

Out[40]:
```
array([ 1, 10,  9,  2, 11,  8,  7,  6,  5,  3, 12,  4, 13, 14])
```

Consider first four degree because they give the smallest RMSE among 14 degrees.

- Degree 10 and 9 would be a bit overfitting so they will be ignored
- Then having a look at the below graph to see how degree 1 and 2 work

In [42]:
```
X_plot = np.array(range( 99, 234, 1)) # min and max of X
## Take intercepts and coefficients in the draft and only show the best one below
y_degree1 = 90.28431413226751 + 0.2669545938540424*X_plot
y_degree2 = 98.911250919365 + 0.15125235*X_plot + 0.00037347 * X_plot**2
## Visualising
plt.scatter(X, y) # orginal data in blue
plt.plot(X_plot, y_degree1, 'g') # degree 1 in green
plt.plot(X_plot, y_degree2, 'r') # degree 2 in red
plt.xlabel('WEIGHT')
plt.ylabel('SYSTOLIC')
plt.title('WEIGHT and SYSTOLIC')
plt.show()
```



2 models produce a relatively similar in the the regression line. Although model of degree 1 indeed offers a smaller amount of RMSE, I would rather go to model of degree 2.

- Model 1, which is basically linear, could be underfitting in some extent because when looking at the graph above, we could see there is no clear pattern of a basic linearity.
- On the other hand, despite having a slightly higher number in RMSE, it could still serve the same regression as model 1 does in the particular range. Besides, when the value X (Weight) reach 200-ish, the red model (degree 2) could identify the difference and start moving up in the line graph.
- Therefore, the model with degree 2 will be chosen.

In [44]:
```
best_degree = 2
print("Degree", best_degree)
```

```
Degree 2
```

In [45]:
```
# b. Print intercept and coefficients of the model
best_model = poly_List_models['model'+str(best_degree)]
# Result would be the Pipeline with 2 steps: Poly transform and Linear regression
lin_reg_best_model = best_model.named_steps['lin_reg']   # Only access the Linear Regression step

intercept = lin_reg_best_model.intercept_[0]
coefficients_1 = lin_reg_best_model.coef_[0,0]
coefficients_2 = lin_reg_best_model.coef_[0,1]
print("Intercept", intercept)
print("Coefficients", coefficients_1, 'and', coefficients_2)
print(f'Best equation: Systolic = {intercept} + {coefficients_1} Weight + {coefficients_2} Weight^2')
```

```
Intercept 98.911250919365
Coefficients 0.15125234579213007 and 0.00037346540241700046
Best equation: Systolic = 98.911250919365 + 0.15125234579213007 Weight + 0.00037346540241700046 Weight^2
```

Question 4: Multiple Linear Regression

- a. Create a multiple linear regression model to predict systolic pressure using all the other relevant useful features in the dataset.
- b. Print the intercept and coefficients of the model.
- c. Perform 10-fold cross-validation.
- d. Compute and display the mean RMSE for the 10-fold cross-validation

In [47]:
```
y_multi = systolic # Different name for different model, but the same value
X_multi = pressure_predictor.to_numpy() # Whole remaining dataset, including dummies
```

In [48]:
```
# a. Create model with all other features
lin_reg_multi = LinearRegression()
lin_reg_multi.fit(X_multi, y_multi)
```

Out[48]:
```
▾ LinearRegression
LinearRegression()
```

In [49]:
```
# b. Intercept and Coefficients
```

```
multi_intercept = lin_reg_multi.intercept_[0]
multi_coefficients = lin_reg_multi.coef_
print("Intercept", multi_intercept)
print("Coefficients", multi_coefficients)
```

```
Intercept 69.14256088196578
Coefficients [[  0.37180203  -0.83445173  -0.08319725  -0.11105294   0.3004112
    0.01360296  -0.05463374   0.08452103 -10.87181364   0.60239764
   -0.11053997  -4.47617349]]
```

In [50]:

```
# c. 10-fold cross-validation
mse_score_multi = -cross_val_score(lin_reg_multi, X_multi, y_multi, cv=10, scoring='neg_mean_squared_error')
print("MSE scores for 10-fold:", mse_score_multi)
# d. mean RMSE
rmse_multi = np.mean(np.sqrt(mse_score_multi))
print("Mean RMSE:", rmse_multi)
```

```
MSE scores for 10-fold: [75.42004641 53.94956458 74.57654424 39.51401931 55.51622802 50.61795498
 32.80881046 74.47990128 67.63895163 54.308156  ]
Mean RMSE: 7.546861673936118
```

Question 5: Ridge Regression

- a. Build a Ridge regression model for the multiple linear regression model created in item 4 with a regularization parameter $\alpha = 0.1$
- b. Print the intercept and coefficients of the model
- c. Perform 10-fold cross-validation
- d. Compute and display the mean RMSE for the 10-fold cross-validation.

In [52]:

```
y_ridge = systolic # Different name for different model, but the same value
X_ridge = pressure_predictor.to_numpy() # Whole remaining dataset with dummies
alpha = 0.1
```

In [53]:

```
# a. Ridge model with alpha = 0.1
ridge_reg = Ridge(alpha = alpha, solver = "cholesky") # Update new alpha
ridge_reg.fit(X_ridge, y_ridge)

# b. Intercept and Coefficients
ridge_intercept = ridge_reg.intercept_[0]
ridge_coefficients = ridge_reg.coef_
print("Intercept", ridge_intercept)
print("Coefficients", ridge_coefficients)
```

```
Intercept 69.13966661213357
Coefficients [[  0.3722703   -0.83170596  -0.07951758  -0.12025351   0.29872371
    0.0134391   -0.05442173   0.08578891 -10.76454194   0.62400999
   -0.09656377  -4.38823966]]
```

In [54]:

```
# c. 10-fold cross-validation
mse_score_ridge = -cross_val_score(ridge_reg, X_ridge, y_ridge, cv=10, scoring='neg_mean_squared_error')
print("MSE scores for 10-fold:", mse_score_ridge)
# d. mean RMSE
rmse_ridge = np.mean(np.sqrt(mse_score_ridge))
print("Mean RMSE:", rmse_ridge)
```

```
MSE scores for 10-fold: [75.70264387 54.27396138 74.3962837  39.42073141 55.04450843 50.61497325
 32.37145079 74.43886849 67.6710702  54.38217405]
Mean RMSE: 7.5423410378856115
```

Question 6: Model Comparison

Best RMSE of 3 models to predict Systolic are:

- 10.187166706053542 of **Polynomial Regression Model** with **degree 2** for Weight
- 7.546861673936118 of **Multiple Linear Regression Model** with all features considered
- 7.5423410378856115 of **Ridge Regression Model** with all features considered

I choose Ridge Regression because of some of following reasons:

- Ridge Regression has the lowest score in terms of Mean RMSE compared to other methods
- Polynomial Regression only uses 1 variable to predict while Systolic pressure could be derived from multiple factors
- Some features in Multiple Linear Regression may have multicollinearity, for example: EXERCISE and WEIGHT, WEIGHT and SERUM-CHOL, AGE and IQ. Hence using Ridge Regression could handle multicollinearity by adjusting coefficients of those correlated features for better prediction.

Task 2: MNIST Digit Classification (8 point)

1. Load the renowned MNIST ('mnist 784') dataset, which consists of a large collection of handwritten digit images. Your task is to reduce the number of features first, and then build a binary classification model to distinguish between the digit "7" and all other digits (not "7").

In [58]:
```
from sklearn.datasets import fetch_openml

mnist = fetch_openml('mnist_784', version=1, as_frame=False)
print(list(mnist.keys()))
```

```
['data', 'target', 'frame', 'categories', 'feature_names', 'target_names', 'DESCR', 'details', 'url']
```

In [59]:
```
print(mnist["data"])
print(mnist["target"]) # type: object
```

```
[[0 0 0 ... 0 0 0]
 [0 0 0 ... 0 0 0]
 [0 0 0 ... 0 0 0]
 ...
 [0 0 0 ... 0 0 0]
 [0 0 0 ... 0 0 0]
 [0 0 0 ... 0 0 0]]
['5' '0' '4' ... '4' '5' '6']
```

In [60]:
```
X = mnist["data"]     # predictor variables
y = mnist["target"]   # target variable
y = (y == '7')        # '7' is object varible instead of number
# Change value into True if it is '7'. Otherwise, it will be False
y = y.astype(np.int8) # 1 when True. 0 when False
print(y)
```

```
[0 0 0 ... 0 0 0]
```

2. Perform Principal Component Analysis (PCA) on the feature data to reduce its dimensionality while retaining 90% of the overall explained variance.

In [62]:
```
pca = PCA(n_components = 0.9)
X_reduced = pca.fit_transform(X)
```

3. Split the data into training and testing sets, using a common split ratio of 80% for training and 20% for testing.

In [64]:
```
X_train, X_test, y_train, y_test = train_test_split(X_reduced, y, test_size = 0.2)
```

In [65]:
```
### Check size and number of observation having value 7
print(y.shape)    # Size 70000 x (1)
print(np.sum(y)) # In 70000 values, there are 7293 values 1 (representing for '7')

print(X.shape)         # Size 70000 x 784
print(X_reduced.shape) # Size 70000 x 87
                       # Decrease from 784 features to 87 features with 90% remaining
```

```
(70000,)
7293
(70000, 784)
(70000, 87)
```

4. Create a Logistic Regression model using the reduced feature dataset.

In [67]:
```
log_reg = LogisticRegression(max_iter = 100000)
# max_iter = 70000 make sure it take all into consideration
log_reg.fit(X_train, y_train)
```

Out[67]:
```
LogisticRegression
LogisticRegression(max_iter=100000)
```

5. Use this model to predict the labels for both the training and testing dataset.

In [69]:
```
train_prediction = log_reg.predict(X_train)
test_prediction = log_reg.predict(X_test)
```

6. Print the number of principal components preserved. Print the prediction accuracy (proportion of correct predictions) of your model on the training set. Also, print the prediction accuracy, the confusion matrix, and the misclassified digits (i.e. wrong predicitons) of your model on the testing set.

In [71]:
```
print("PCs preserved:", X_reduced.shape[1], "\n")

print("Training set Prediction \n", confusion_matrix(train_prediction, y_train))
print("Training Precision of classifier:", round(precision_score(y_train, train_prediction),4))
print("Training Accuracy rate:", round(accuracy_score(y_train, train_prediction),4))
```

```python
print("\nTesting set Prediction \n", confusion_matrix(test_prediction, y_test))
print("Testing Precision of classifier:", round(precision_score(y_test, test_prediction),4))
print("Testing Accuracy rate:", round(accuracy_score(y_test, test_prediction),4))

index_misclassed_digits = np.where(test_prediction != y_test)
index_string = ', '.join(map(str, index_misclassed_digits[0]))
print("\nThe model classifies wrong", len(index_misclassed_digits[0]), "places for testing dataset at indexes:"
```

```
PCs preserved: 87

Training set Prediction
 [[49863   543]
 [  313  5281]]
Training Precision of classifier: 0.944
Training Accuracy rate: 0.9847

Testing set Prediction
 [[12443   127]
 [   88  1342]]
Testing Precision of classifier: 0.9385
Testing Accuracy rate: 0.9846

The model classifies wrong 215 places for testing dataset at indexes: 59, 99, 104, 112, 159, 170, 203, 226, 232,
305, 329, 603, 646, 746, 757, 888, 907, 1051, 1057, 1127, 1262, 1321, 1402, 1459, 1586, 1776, 1779, 1898, 1985,
1992, 2019, 2099, 2112, 2284, 2325, 2326, 2341, 2479, 2494, 2497, 2500, 2573, 2634, 2681, 2751, 2755, 2805, 2857
, 2923, 2955, 2976, 3059, 3119, 3137, 3145, 3227, 3252, 3268, 3669, 3936, 3998, 4043, 4157, 4184, 4223, 4283, 43
35, 4368, 4482, 4518, 4554, 4592, 4633, 4682, 4694, 4742, 4853, 4904, 4991, 5043, 5141, 5146, 5197, 5222, 5348,
5388, 5440, 5477, 5521, 5551, 5569, 5668, 5704, 5718, 5748, 5826, 6000, 6015, 6103, 6105, 6206, 6229, 6265, 6358
, 6376, 6523, 6541, 6729, 6765, 6771, 6811, 6827, 6984, 7013, 7015, 7116, 7154, 7169, 7309, 7330, 7387, 7391, 74
60, 7520, 7641, 7653, 7671, 7675, 7943, 7959, 7966, 8323, 8398, 8445, 8466, 8494, 8572, 8651, 8786, 9016, 9041,
9093, 9101, 9157, 9167, 9231, 9252, 9277, 9442, 9772, 9783, 9825, 9827, 9829, 9862, 9980, 10006, 10014, 10034, 1
0049, 10071, 10175, 10313, 10350, 10418, 10578, 10617, 10661, 10724, 10813, 10874, 10997, 11008, 11091, 11100, 1
1155, 11202, 11309, 11363, 11668, 11707, 11788, 11789, 11812, 11816, 11927, 12062, 12133, 12142, 12172, 12173, 1
2298, 12365, 12372, 12391, 12642, 12695, 12732, 12738, 12841, 12856, 12881, 12961, 12998, 13058, 13062, 13086, 1
3330, 13428, 13526, 13568, 13653, 13673, 13734, 13925
```

7. Evaluate the model: What do you think of the model generated (good, underfitting, overfitting)? Briefly explain your reasoning.

In my opinion, this model works well due to the high accuracy rate in both training and validating data. Although I only used PCA with 90% of explained variance (reducing dimension to 87 remaining from 784 original features), it still achieves a high level of predicting accuracy (around 98%), which is really impressive. On top of that, when reducing dimensions, it could keep the model from being overfitting, and 90% is a sufficient number of features to keep the model good, instead of being underfitting.