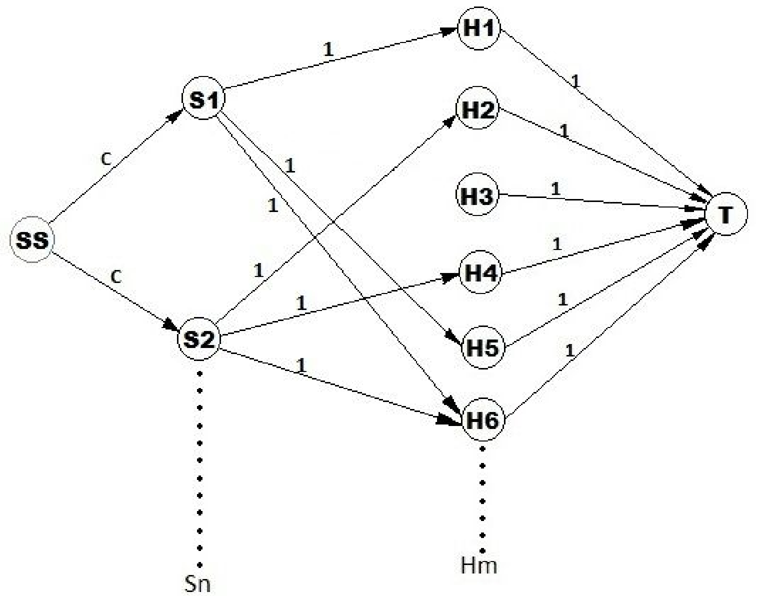
To: Professor Kearns
From: Anh Nguyen
CPE 349 - Section 9
Assignment: Holiday Scheduling

<u>**Flow A: Without constraint #3**</u> (no need to describe a holiday period since 3rd constraint is neglected)

1. Symbols:

SS : the SuperSource of the flow.
{S1, S2, S3, …, Sn} : Each vertex S represent a supervisor.
{H1, H2, H3, …, Hm}: Each vertex H represent a holiday.
T : The Sink of the flow.
C : The maximum number of holidays each supervisor can be assigned.



2. Satisfies the constraints:

Each vertex S represent a supervisor. Each of the edge from a S to a H represent a holiday that supervisor willing to work on. The total number of edge going out of an S is i (i is 3 in my example graph). The set of i edges going out a S represent the list of the holidays that supervisor would be willing to work. There possible is some holiday which no one want to work (H3 in my example graph)⇒ Satisfies the first constraint.

Each S only has one input edge which is the edge from SuperSource, and that edge has capacity C ⇒ The max flow into vertex S is C ⇒ the max number of paths going through S is C ⇒ because each path will assign one holiday to one person, the max number of holiday will be assign to a supervisor is C ⇒ Satisfies the second constraint.

2. How this diagram solve the problem

Since there is only one edge coming out of H, and it has capacity 1 ⇒ there will be maximum 1 person working on that holiday. Only H has edge connecting to Sink, and each of the edge from a H to the sink has capacity 1 ⇒ the total units received at the sink is the total number of holidays have been assigned. Since the goal of network flow problem is to find the max value of the flow (which is max number received at Sink), finding the max value of the flow will generate the schedule with most number of holiday with a person work on ⇒ Solve the problem.

3. How to tell if there is no solution

If the size of the flow is less than the total number of holiday (m) ⇒ There is no solution

Flow B: With constraint #3

1. Symbols:

SS : the SuperSource of the flow.

{S1, S2, S3, …, Sn} : Each vertex S represent a supervisor.

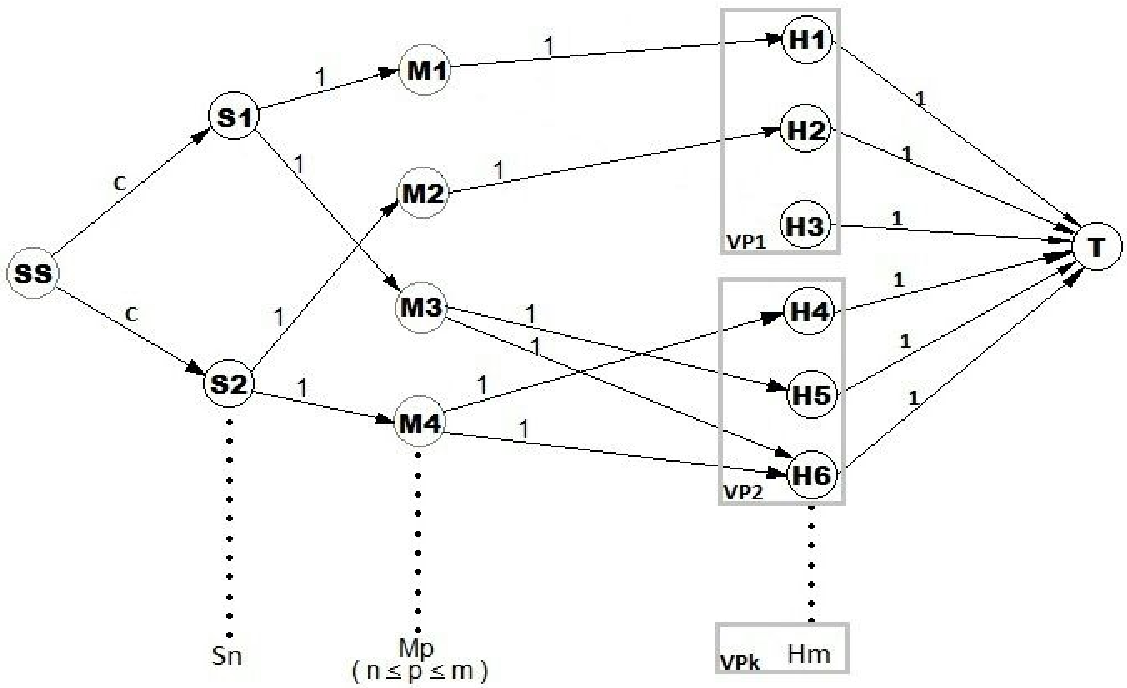
{M1, M2, M3, …, Mp} : Each vertex M represent a holiday period which a supervisor willing to work on some specific days in that period. Each S can connect to multiple M because each Supervisor possible sign up to work on different holiday period.

{H1, H2, H3, …, Hm}: Each vertex H represent a holiday.

{VP1, VP2, VP3, …, VPk} :Each region VP represent a holiday period. Size of region is Dj

T : The Sink of the flow.

C : The maximum number of holidays each supervisor can be assigned.



2. Satisfies the constraints:

Each vertex M represent a holiday period that a supervisor willing to work. Each edge from a M to a H represent a specific holiday that supervisor willing to work. The set of all M connect to a common S represent all periods that supervisor signed up for. The set of i edges going out from that set of M represent the list of all holidays that supervisor would be willing to work ⇒Satisfies the 1st constraint.

Each S only has one input edge which is the edge from SuperSource, and that edge has capacity C ⇒ The max flow into vertex S is C ⇒ the max number of paths going through S is C ⇒ because each path will assign one holiday to one person, the max number of holiday will be assign to a supervisor is C ⇒ Satisfies the second constraint.

From a vertex S to a VP region, the flow must go through one specific M. Because the total input of each M is 1 ⇒ the flow from a S to a VP will have at most 1 unit. ⇒ Which means a supervisor will only work at most 1 day in a holiday period ⇒ This satisfies the third constraint

2. How this diagram solve the problem

Similar with flow A, the total units received at the sink is the total number of holidays have been assigned. Since the goal of network flow problem is to find the max value of the flow (which is max number received at Sink), finding the max value of the flow will generate the schedule with most number of holiday with a person work on ⇒ Solve the problem.

3. How to tell if there is no solution

If the size of the flow is less than the total number of holiday (m) ⇒ There is no solution