

# Batch normalisation backprop

train  $\rightarrow$  batch  
 $(x)$   $(y)$

$$\mu = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \mu)^2$$

$$\hat{x}_i = \frac{x_i - \mu}{\sqrt{\sigma^2 + \epsilon}}$$

$$y_i = \gamma \hat{x}_i + \beta$$



$$\frac{\partial L}{\partial \hat{x}_i} = \frac{\partial L}{\partial y_i} \frac{\partial y_i}{\partial \hat{x}_i} = \frac{\partial L}{\partial y_i} \quad \checkmark$$

$$\begin{aligned} \frac{\partial L}{\partial \mu} = & \frac{\partial L}{\partial \hat{x}_i} \frac{\partial \hat{x}_i}{\partial \mu} + \frac{\partial L}{\partial \mu} \frac{\partial \mu}{\partial x_i} \\ & + \frac{\partial L}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial x_i} \end{aligned}$$

$$\frac{\partial L}{\partial \mu} = \frac{\partial L}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial \mu} + \frac{\partial L}{\partial \hat{x}_i} \frac{\partial \hat{x}_i}{\partial \mu}$$

$$\frac{\partial L}{\partial \sigma^2} = \frac{\partial L}{\partial \hat{x}_i} \frac{\partial \hat{x}_i}{\partial \sigma^2}$$