1. If $f(x) = 2x + 3$ and $h(x) = 6x + 5$, find a function g such that $g \circ f = h$

2. Simplify the quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x) = -\frac{1}{x}$

3. Consider the vectors $u_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, u_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}$. Suppose that α, β and γ are real numbers such that $v = \alpha u_1 + \beta u_2 + \gamma u_3$. Find $\alpha - 2\beta$.

4. Express the limits as a define integral over $[0, 1]$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \cos^2(2\pi x_i^*)\, \Delta x$$

a. $\int_0^1 \cos^2(2\pi)\, dx$

b. $\int_0^1 \cos^2(\frac{2\pi}{x})\, dx$

c. $\int_{-1}^1 \cos^2(2\pi x)\, dx$

d. $\int_0^1 \cos^2(2\pi x)\, dx$

5. Let A be the point in the line $x = t, y = 2t, z = t$ that is closest to the point $(1, -2, 3)$. Determine the first coordinate of A.

6. Find all numbers a such that the set of vectors $\{(1, -1, 0), (2, 0, 1), (0, 1, a)\}$ is dependent.

7. Let $A = \begin{bmatrix} 1 & 2 \\ -5 & 4 \end{bmatrix}$. Choose the correct statement.

a. $A^2 + 5A - 14I = 0$

b. $A^2 - 5A + 14I = 0$

c. $A^2 - 5A - 14I = 0$

d. $A^2 + 5A + 14I = 0$

8. Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

9. Find all the constant c that makes g discontinuous at $x = 4$:

$$g(x) = \begin{cases} x^2 + c^2, & x < 4 \\ cx + 13, & x \geq 4 \end{cases}$$

10. Find the (3, 1) – cofactor of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 3 & 0 & 6 \end{bmatrix}$

11. A particle moves along a straight line with displacement given by $s(t) = t^2 - 8t + 18$. What is the instantaneous velocity when $t = 4$.

12. Use Newton's method with the initial approximation $x_1 = 1$ to find x_3, the third approximation to the root of the equation $x^5 - 10 = 0$. Round your answer to 4 decimal places.

13. Find $\frac{d^4y}{dx^4}$ for $y = \sqrt[3]{x}$

14. Let A=$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$. Compute $\det(2A^3A^TA^{-1})$

15. Find the average value of $f(x) = x^2 - 1$ on the interval [0, 3]

16. Given $A = \begin{bmatrix} 3 & 5 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 4 & 0 \\ 5 & 2 & 2 \end{bmatrix}$, $u = [-18 \quad 5 \quad 1]^T$ and $v = [1 \quad 1 \quad -7]^T$. Choose the correct statement:

a. v is a common eigenvector of A and B

b. u is a common eigenvector of A and B

c. Both u and v are common eigenvectors of A and B

d. None of the other choices is correct

17. Find all eigenvalues of the matrix $\begin{bmatrix} 4 & 0 \\ 1 & 3 \end{bmatrix}$

18. Find the area of the triangle with the vertices $A(3, 0, 1)$, B(5, 1, 0), C(7, 2, -1)

19. The average value of $f(x) = x^2 - x$ over the interval [0, a] is $-\frac{1}{6}$. Find the number a.

20. Let L be the line passing through (1, -1) with slope -1/2. Which of the following point lies in the L?

A. (-2, ½)

B. (-2. 3/2)

C. (2, 1)

D. (2, 0)

E. None of the other choices is correct

21. Use the right-endpoint rule with $n = 4$ to estimate the value of the integral $\int_1^3 f(x)dx$

x	1	1.5	2	2.5	3
f(x)	0.31	0.54	0.36	1.35	2.04

22. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 3x^2 + 3x + 1$ on [0, 2].

23. Determine where the function is increasing and where it is decreasing $f(x) = x^3 - 5x^4$

24. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(u) = [1 \quad 2]^T$, $T(v) = [-1 \quad 0]^T$ for given $u, v \in \mathbb{R}^2$. Find $T(2u - 3v)$

25. Find the parametric equation of the line passing through the origin, intersecting the line $x = 1 + 2t$, $y = 2 - 3t$, $z = t$ and perpendicular to that line.

26. Which of the following statement(s) is/are true?

i. $\mathbb{R}^2 = span\{(1,0),(0,2)\}$

ii. $\mathbb{R}^3 = span\{(1,0,0),(0,1,0),(1,1,0)\}$

27. Find $\int x^3 e^{x^2} dx$

28. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that $T(1,2) = (-1,1)$, $T(0,3) = (-3,3)$. Find the matrix of T.

i. $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ ii. $\begin{bmatrix} 1 & -4 \\ 7 & 1 \end{bmatrix}$ iii. $\begin{bmatrix} 2 & -1 \\ -1 & 5 \end{bmatrix}$ iv. $\begin{bmatrix} 4 & -1 \\ -1 & 0 \end{bmatrix}$

29. Let $u = [3 \quad 0 \quad 5]^T$, $v = [-1 \quad 2 \quad 2]^T$. Find the vector x such that

$$2u - v = \|v\|(5x + v)$$

30. Find conditions on a, b, c such that the following system has only trivial solution

$$\begin{cases} x + ay = 0 \\ y + bz = 0 \\ z + cx = 0 \end{cases}$$

31. Let $A = \begin{pmatrix} 2 & -1 \\ 3 & 0 \\ 5 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 3 \\ 2 & 5 \\ 7 & 8 \end{pmatrix}$, $A = \begin{pmatrix} 0 & 4 \\ 12 & 5 \\ 7 & 13 \end{pmatrix}$. Find the (2, 1) – entry of the matrix A – 2B + 3C.

32. Find the number k for which the matrix $A = \begin{bmatrix} 1 & 2 & k \\ 3 & -1 & 1 \\ 5 & 3 & -5 \end{bmatrix}$ has no inverse.

33. Determine whether U is a subspace of $\mathbb{R}^3$

i. $U = \{[0 \quad 1 \quad s]^T : s \in \mathbb{R}\}$

ii. $U = \{[0 \quad a \quad b]^T : a, b \in \mathbb{R}\}$

iii. $U = \{[a \quad b \quad a+1]^T : a, b \in \mathbb{R}\}$

34. Find two positive numbers (a, b) whose product is 64 and whose sum is minimum.

35. Let A be a 4 x 7 matrix. Assume that rank A = 1. Find the dimension of the null space of A.

36. Find $\frac{d}{dx}\int_{29}^{x^3} \sin t \, dt$

37. Find the solution of the linear system whose augmented matrix is

$$\begin{bmatrix} 1 & 2 & 4 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

38. Evaluate $\int_0^1 \frac{1}{\sqrt{3-x}} dx$

39. Use the trapezoidal rule with $n = 4$ to estimate the value of the integral $\int_4^6 f(x)dx$

x	4	4.5	5	5.5	6
f(x)	4.19	4.53	4.84	5.13	5.38

40. If $y = x^3 - 3x$ and $\frac{dx}{dt} = 3$, find dy/dt when $x = 5$

41. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 7 \\ 1 & 1 \end{bmatrix}$, and X such that $AX = B$. Find the second row of X.

42. Find dy/dx by implicit differentiation $\cos(xy) + 2y = 3$.

43. Find conditions on a, b, c such that the system has infinitely many solutions

$$\begin{cases} x - y + 2z = a \\ 3x + y - z = b \\ 5x + 3y - 4z = c \end{cases}$$

44. Find the reflection of $v = (0, -1, 3)$ in the plane with equation $2x + y - 3z = 0$

45. Let $h(x) = \sin(f(x))$. Given that $f(0) = \pi$ and $f'(0) = 2$. Find $h'(0)$.

46. Let {u, v, w} be independent. Which of the following sets are independent?

i. {u, v-w, w}

ii. {u, u-v, u+v, w}

47. Evaluate $\int \frac{(\ln x)^3}{x} dx$

i. $\left(\frac{1}{4x}\right)(\ln x)^4 + C$

iii. $\frac{1}{2}(\ln x)^2 + C$

ii. $4(\ln x)^4 + C$

iv. $\frac{1}{4}(\ln x)^4 + C$

48. Let $\lim_{x\to 2} f(x) = 1$ and $\lim_{x\to 2} g(x) = -2$. Find $\lim_{x\to 2} \frac{f(x)-g(x)}{3g(x)+7}$.

49. Find all a, b, c such that the set

$$\begin{bmatrix}1 & 2 & 1 & 1\end{bmatrix}, \begin{bmatrix}2 & 1 & -1 & -3\end{bmatrix}, \begin{bmatrix}a & b & c & 3\end{bmatrix}$$

is orthogonal

50. Find all values of x and y so that the matrix $\begin{bmatrix}y & 1 & x\\ 0 & x & y\end{bmatrix}$ is reduced row-echelon.

A. $x = 0, y = 0$ only

B. $x = y = 0$ or $x = 0, y = 1$

C. None of the other choices is correct

D. $x = y = 1$

E. $x = 1, y = 0$