FUNCTIONS AND GRAPHS

1.2

BASIC CLASSES OF FUNCTIONS

ALGEBRAIC FUNCTIONS

LINEAR MODELS

When we say that y is a **linear function** of x, we mean that the graph of the function is a line.

- So, we can use the slope-intercept form of the equation of a line to write a formula for the function as

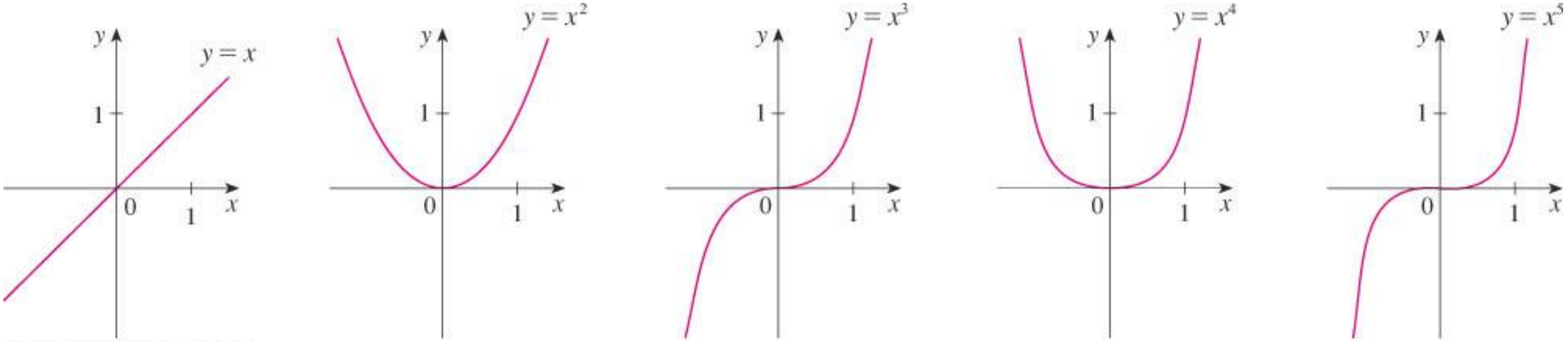
$$y = f(x) = mx + b$$

where m is the slope of the line and b is the y-intercept.

ALGEBRAIC FUNCTIONS

POWER FUNCTIONS

A function of the form $f(x) = x^a$, where a is constant, is called **a power function**.



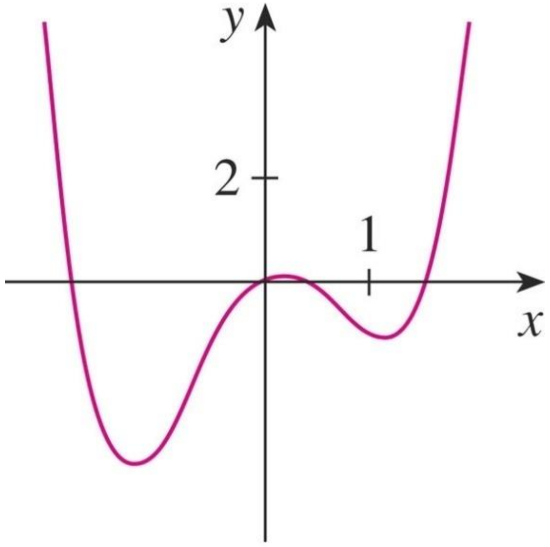
ALGEBRAIC FUNCTIONS

POLYNOMIALS

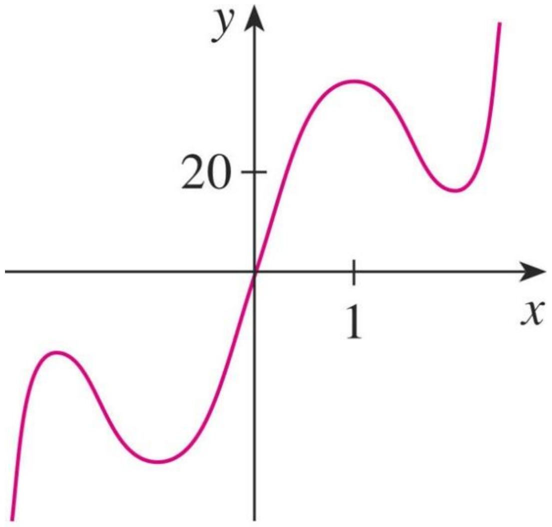
A function P is called a polynomial if

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where n is a nonnegative integer and the numbers a_0, a_1, a_2, …, a_n are constants called the coefficients of the polynomial.



(b) $y = x^4 - 3x^2 + x$



(c) $y = 3x^5 - 25x^3 + 60x$

ALGEBRAIC FUNCTIONS

RATIONAL FUNCTIONS

A rational function *f* is a ratio of two polynomials

$$f(x) = \frac{P(x)}{Q(x)}$$

where *P* and Q are polynomials.

- The domain consists of all values of *x* such that $Q(x) \neq 0$.

TRANSCENDENTAL FUNCTIONS

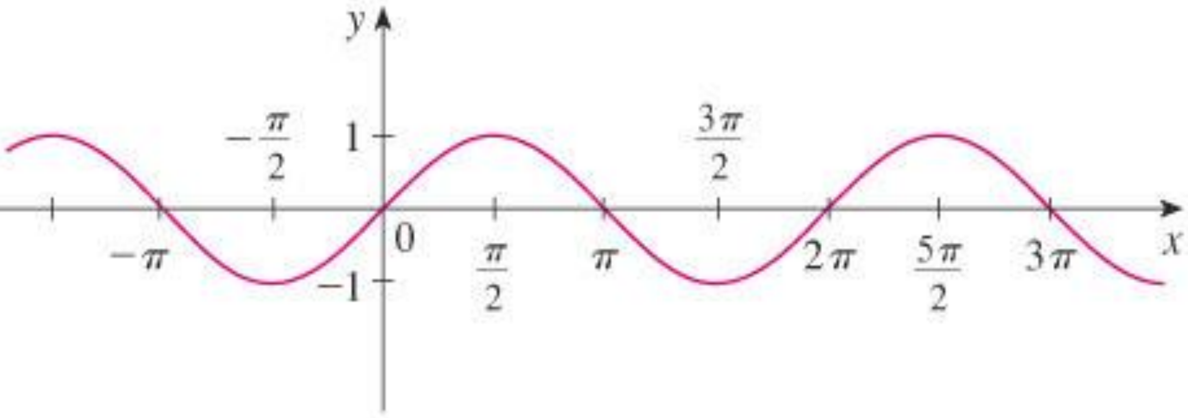
TRIGONOMETRIC FUNCTIONS

$$f(x) = \sin x$$

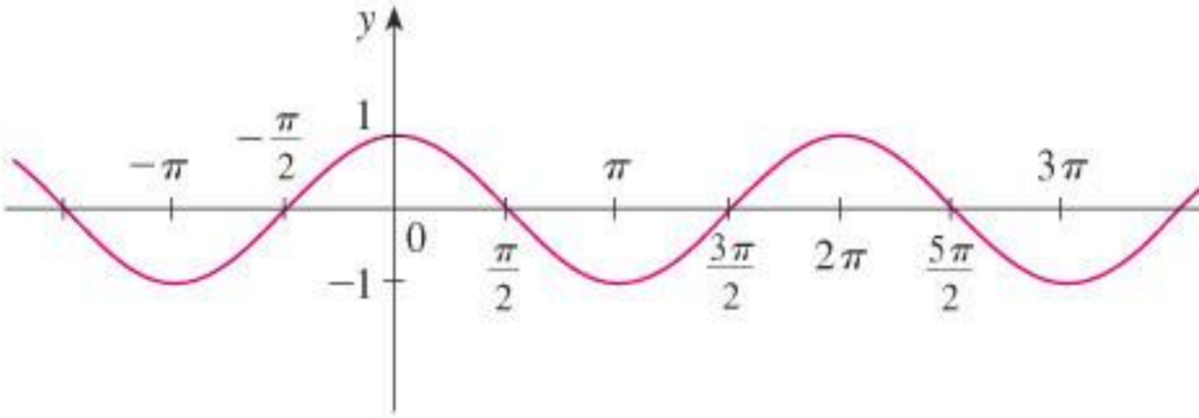
$$g(x) = \cos x$$

$$D = (-\infty, \infty) \qquad R = [-1, 1]$$

$$\sin(x + k2\pi) = \sin x \qquad \cos(x + k2\pi) = \cos x; \quad k \in Z$$



(a) $f(x) = \sin x$



(b) $g(x) = \cos x$

TRANSCENDENTAL FUNCTIONS

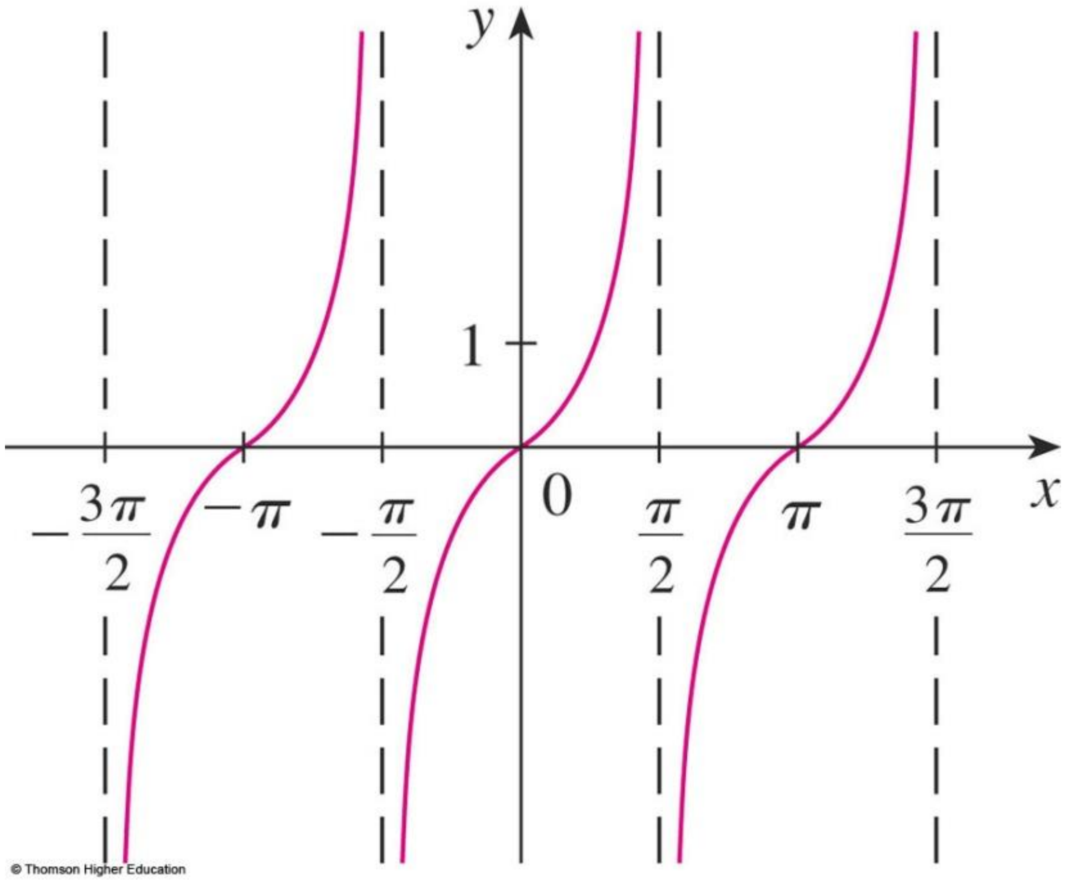
TRIGONOMETRIC FUNCTIONS

$$\tan x = \frac{\sin x}{\cos x}$$

$$đk:\ x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots$$

$$R = (-\infty, \infty)$$

$$\tan(x + k\pi) = \tan x; \quad k \in Z$$



TRANSCENDENTAL FUNCTIONS

TRIGONOMETRIC FUNCTIONS

The reciprocals of the sine, cosine, and tangent functions are

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

TRANSCENDENTAL FUNCTIONS

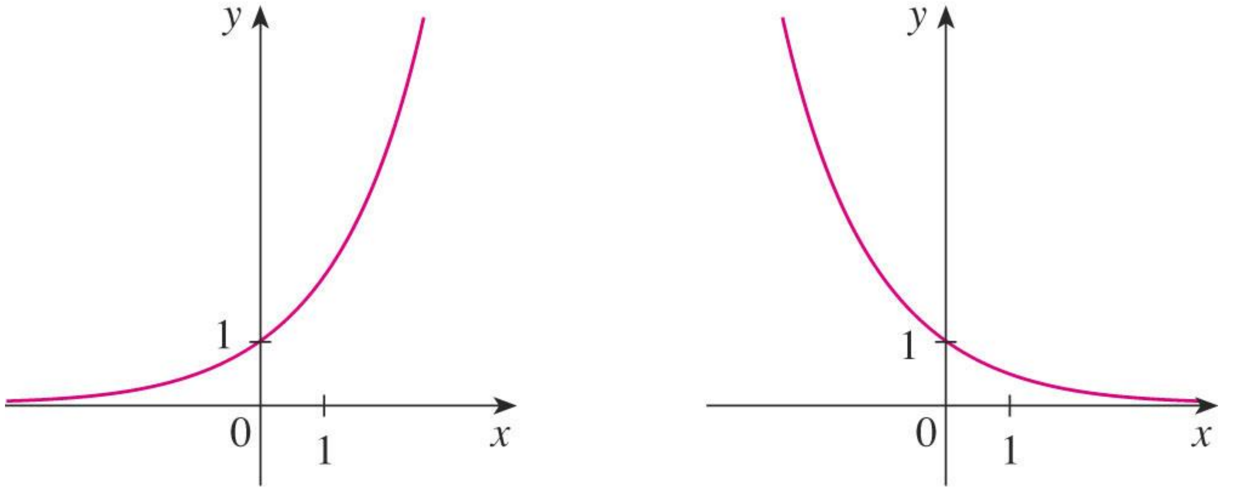
EXPONENTIAL FUNCTIONS

The **exponential functions** are the functions of the form $f(x) = a^x$,

where the base *a* is a positive constant.

- The graphs of $y = 2^x$ and $y = (0.5)^x$ are shown.
- In both cases, the domain is $(-\infty, \infty)$

  and the range is $(0, \infty)$.



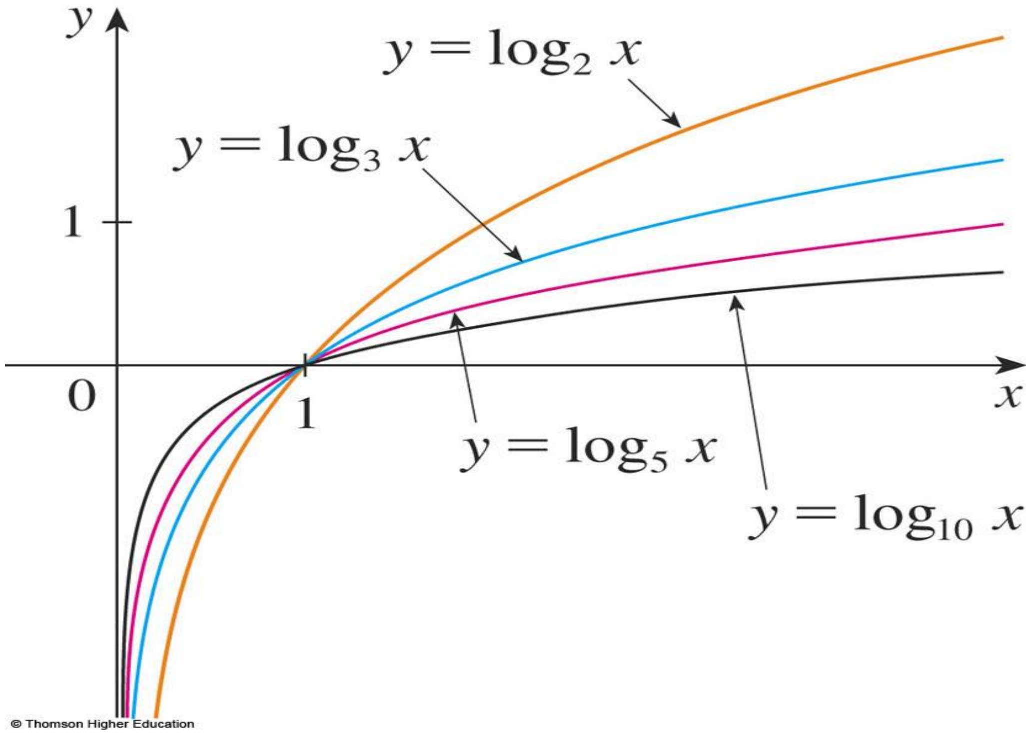
(a) $y = 2^x$ (b) $y = (0.5)^x$

TRANSCENDENTAL FUNCTIONS

LOGARITHMIC FUNCTIONS

The logarithmic functions $f(x) = \log_a x$, where the base *a* is a positive constant, are the inverse functions of the exponential functions.

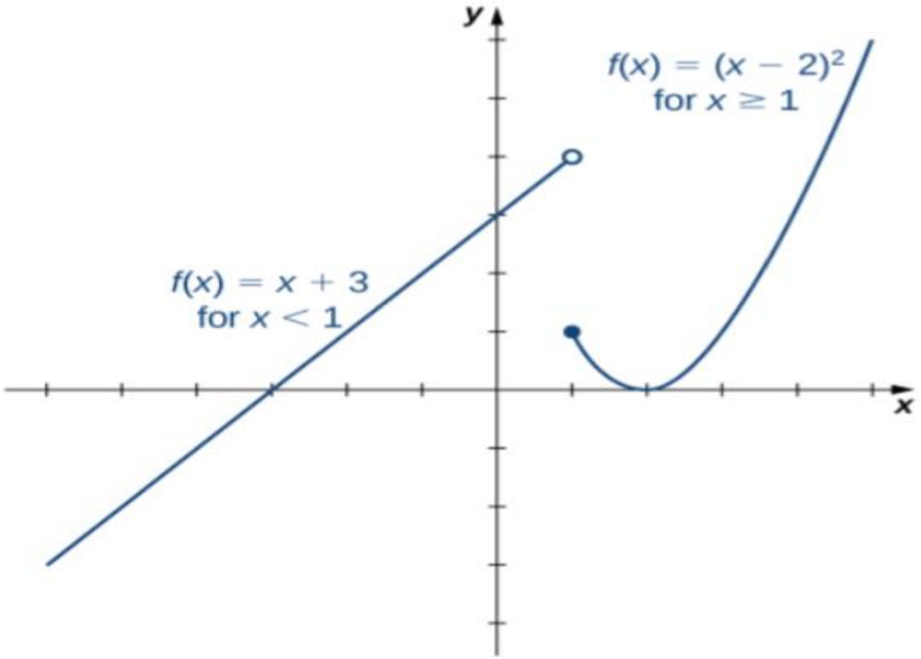
The figure shows the graphs of four logarithmic functions with various bases.



PIECEWISE-DEFINED FUNCTIONS

Example:

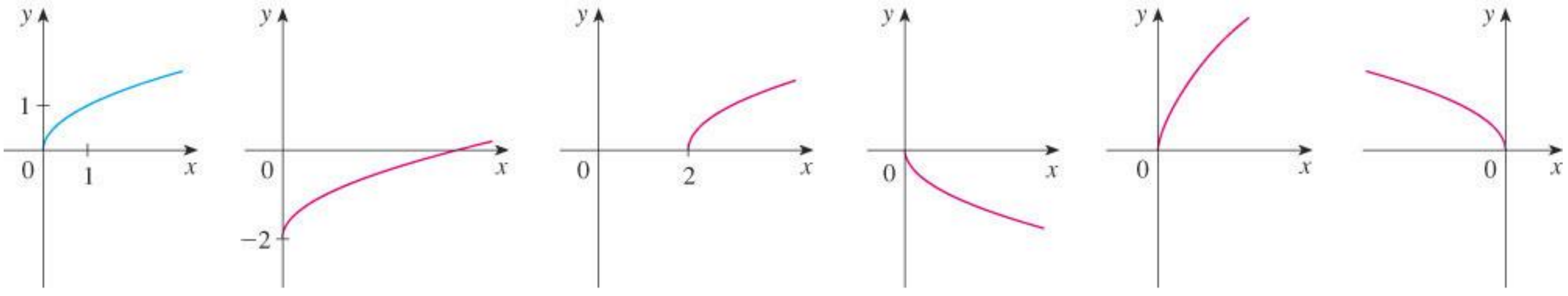
$$f(x) = \begin{cases} x+3, & x < 1 \\ (x-2)^2, & x \geq 1 \end{cases}.$$



TRANSFORMATIONS OF FUNCTION

Label the following graph from the graph of the function y=f(x) shown in the part (a)

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?



(a) $y=\sqrt{x}$ (b) $y=\sqrt{x}-2$ (c) $y=\sqrt{x-2}$ (d) $y=-\sqrt{x}$ (e) $y=2\sqrt{x}$ (f) $y=\sqrt{-x}$

SHIFTING

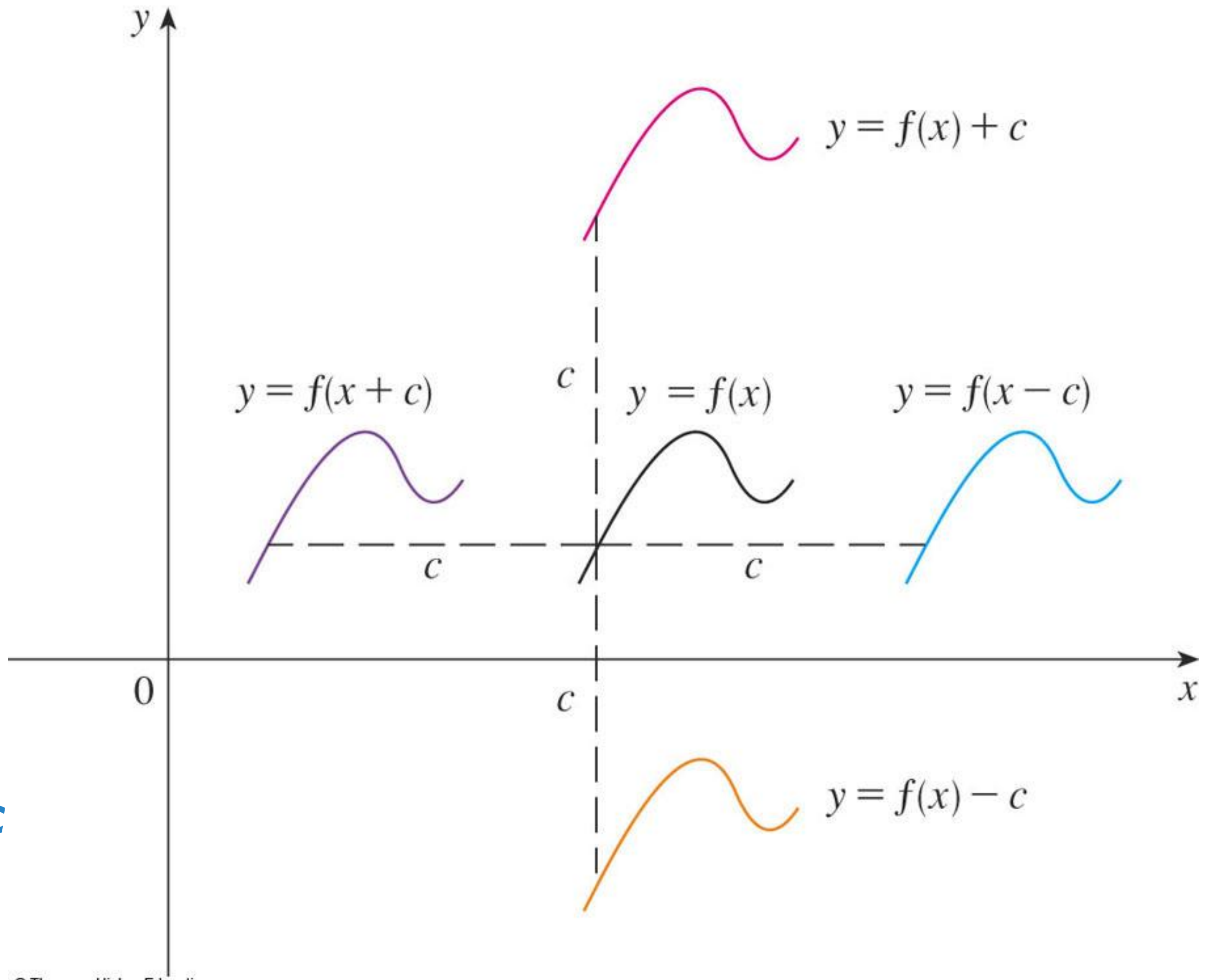
Phép dời hình

Suppose c > 0.

- To obtain the graph of $y = f(x) + c$, shift the graph of $y = f(x)$ a distance units upward.
- To obtain the graph of $y = f(x) - c$,

shift the graph of $y = f(x)$ a distance c units downward.

Why don't we consider the case c<0?

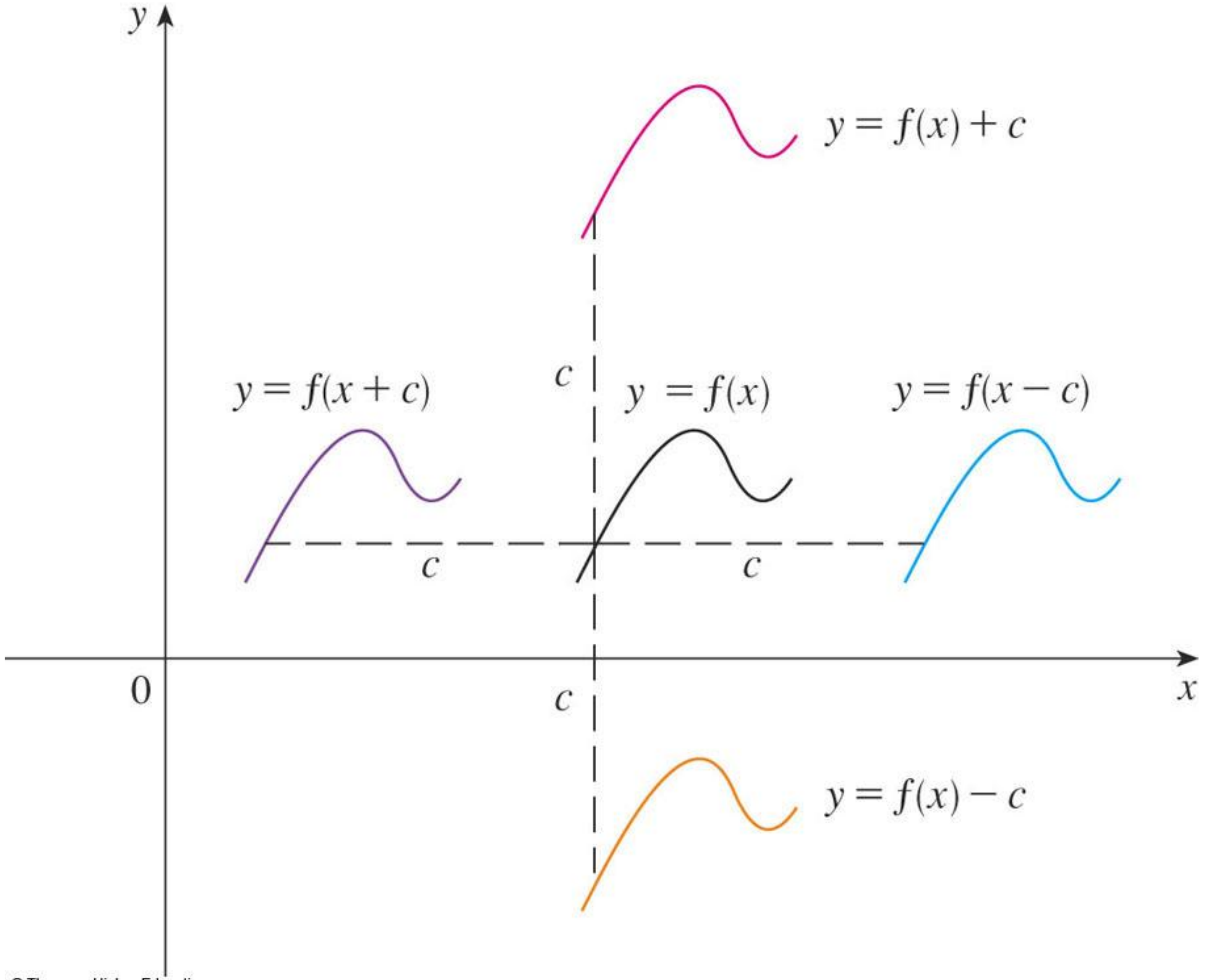


SHIFTING

Phép dời hình

- To obtain the graph of $y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right.

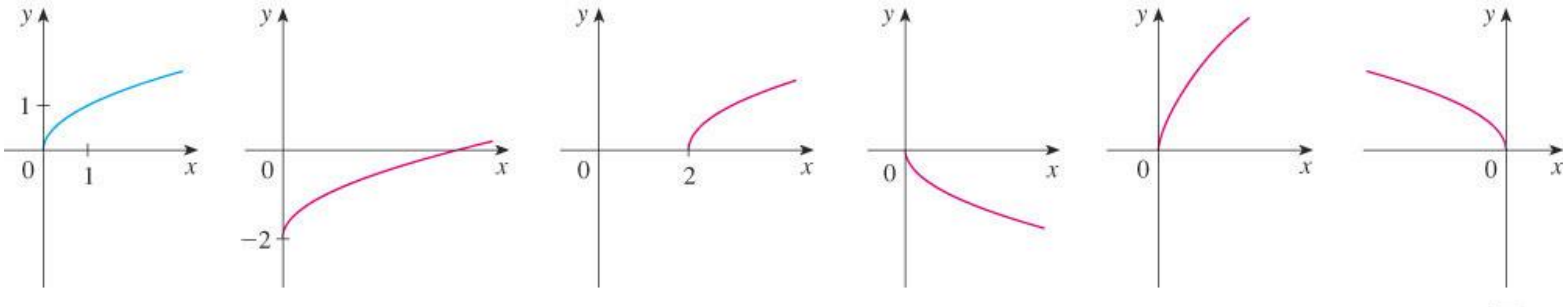
- To obtain the graph of $y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left.



NEW FUNCTIONS FROM OLD FUNCTIONS

- Label the following graph from the graph of he function y=f(x) shown in the part (a)

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?



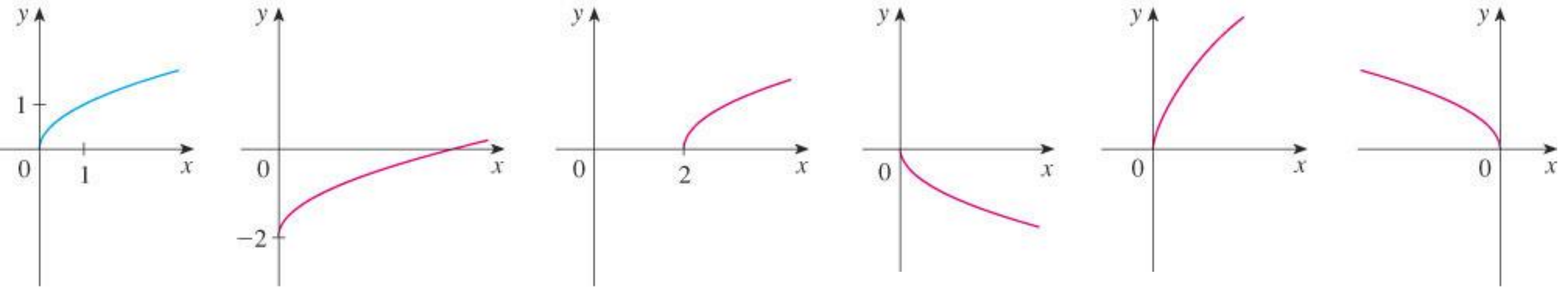
(a) $y=\sqrt{x}$ (b) $y=\sqrt{x}-2$ (f) $y=\sqrt{-x}$

NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function $y = \sqrt{x}$ shown in the part (a):

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?

- $y = \sqrt{x} - 2$ by shifting 2 units downward.
- $y = \sqrt{x - 2}$ by shifting 2 units to the right.



(a) $y = \sqrt{x}$ (b) $y = \sqrt{x} - 2$ (c) $y = \sqrt{x - 2}$ (d) $y = -\sqrt{x}$ (e) $y = 2\sqrt{x}$ (f) $y = \sqrt{-x}$

TRANSFORMATIONS

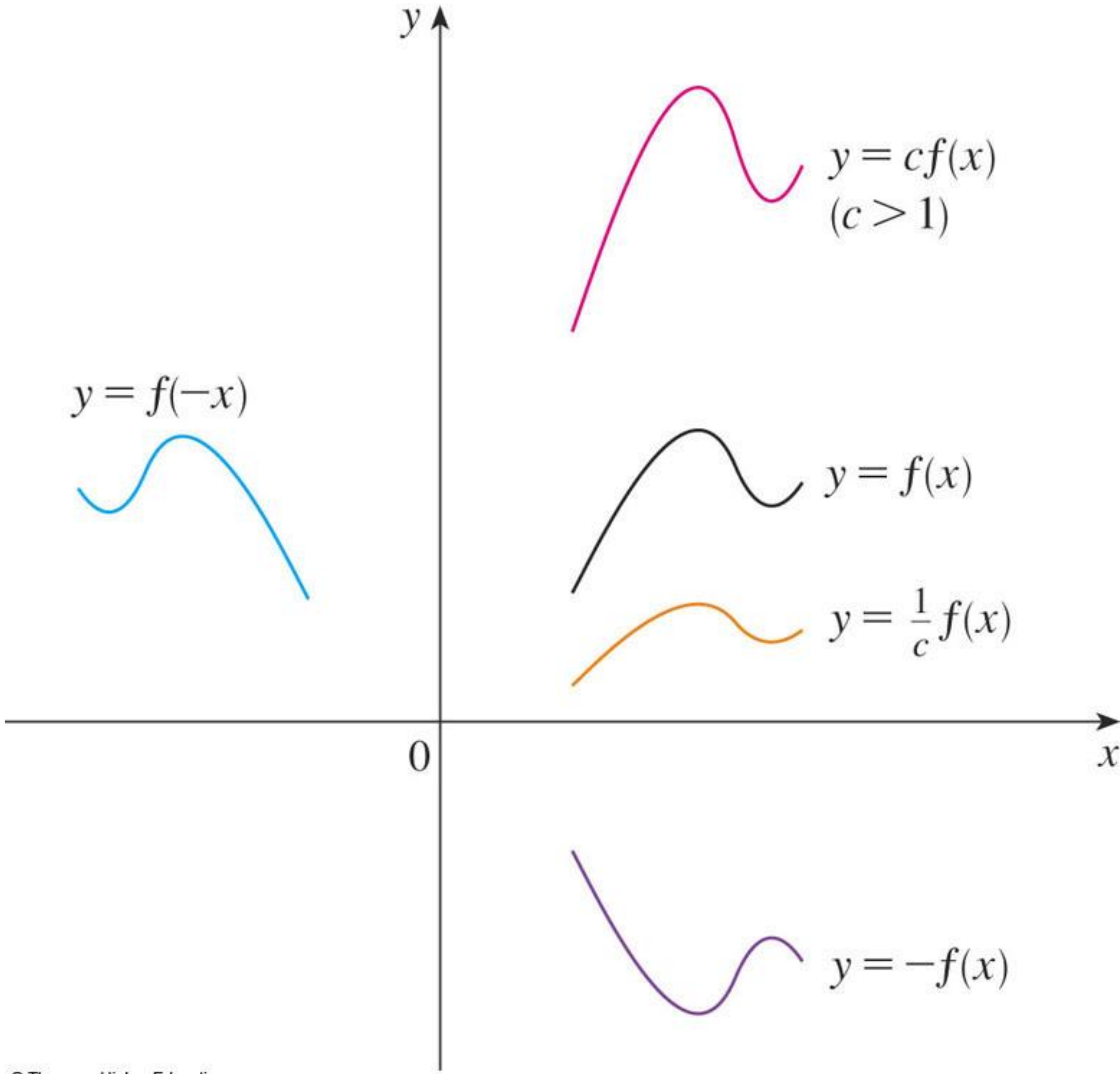
How about the case c<1?

Suppose c > 1.

- To obtain the graph of $y = cf(x)$,

stretch (giãn) the graph of $y = f(x)$ vertically by a factor of c.

- To obtain the graph of $y = (1/c)f(x)$, compress (co) the graph of $y = f(x)$ vertically by a factor of c.

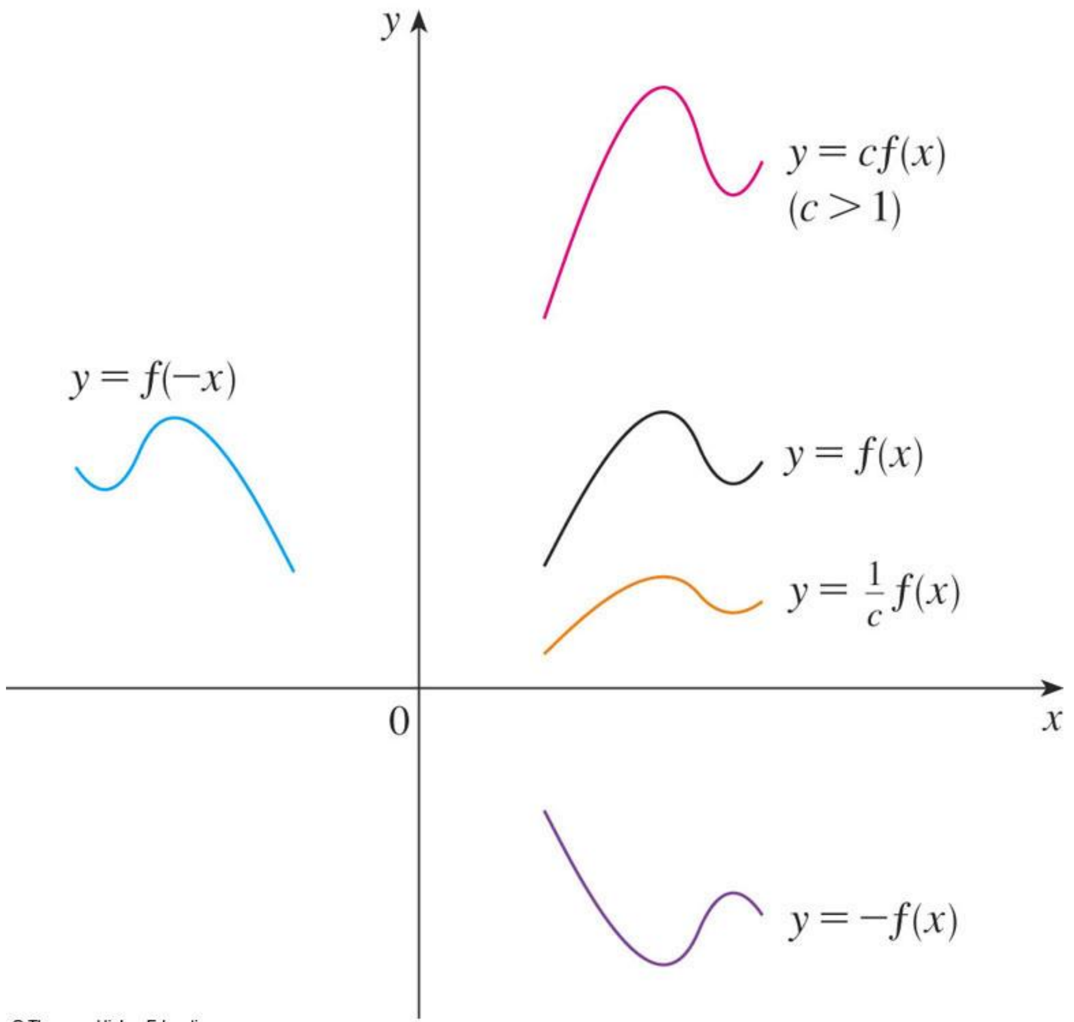


TRANSFORMATIONS

- In order to obtain the graph of $y = f(cx)$, compress the graph of $y = f(x)$ horizontally by a factor of c.

- To obtain the graph of $y = f(x/c)$,

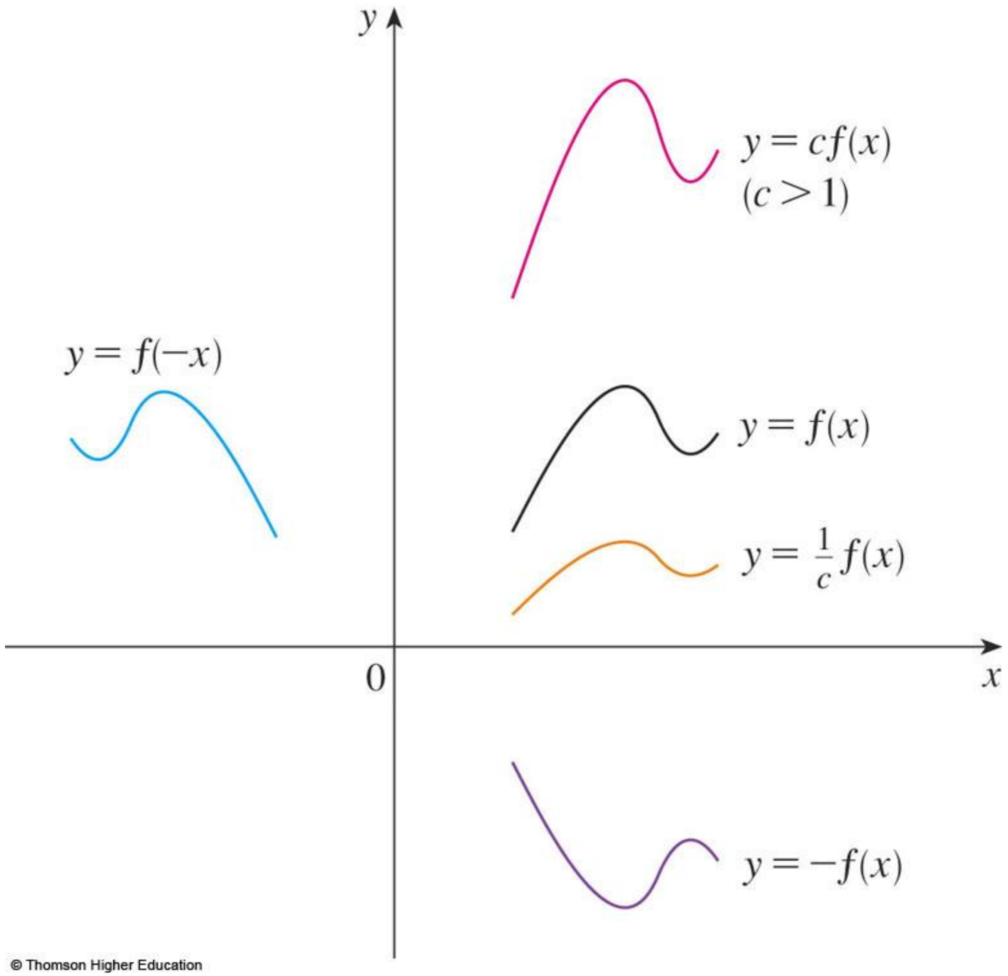
stretch the graph of $y = f(x)$ horizontally by a factor
of c.



TRANSFORMATIONS

- In order to obtain the graph of $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis.

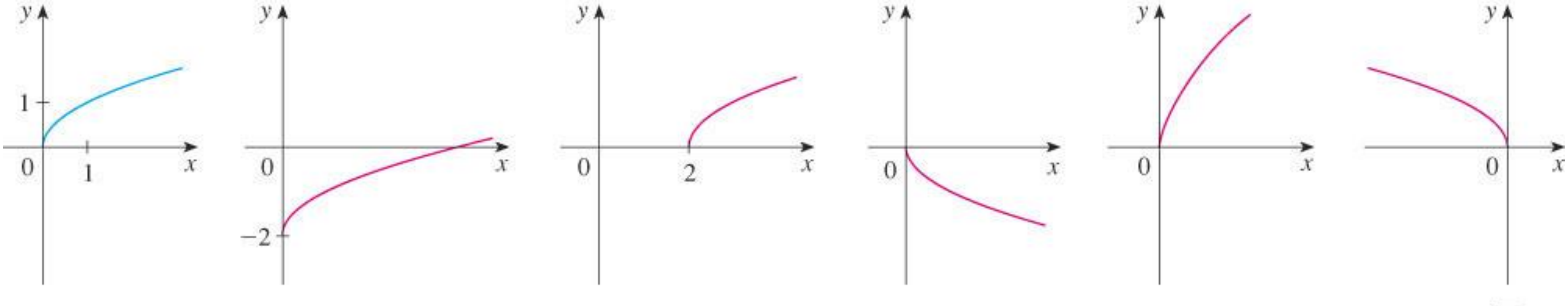
- To obtain the graph of $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis.



NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function $y = \sqrt{x}$ shown in the part (a):

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?



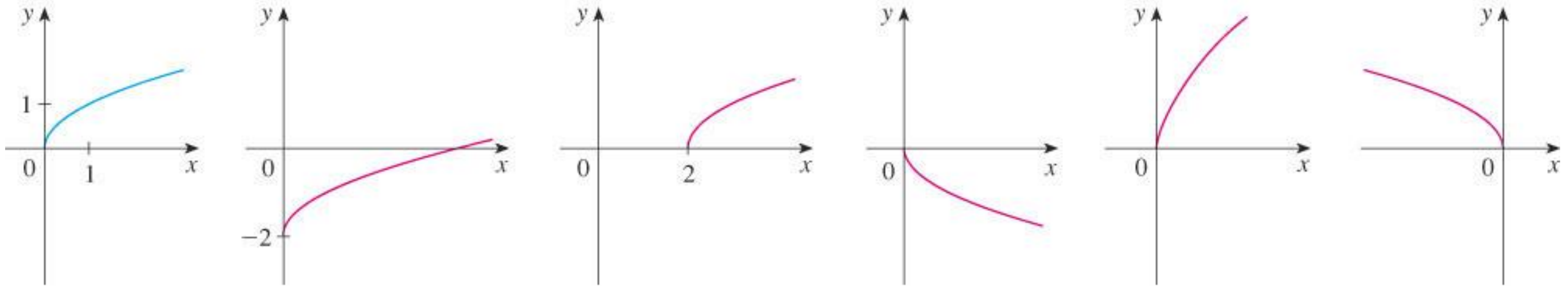
(a) $y=\sqrt{x}$ (b) $y=\sqrt{x}-2$ (c) $y=\sqrt{x-2}$ (f) $y=\sqrt{-x}$

NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function $y = \sqrt{x}$ shown in the part (a):

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?

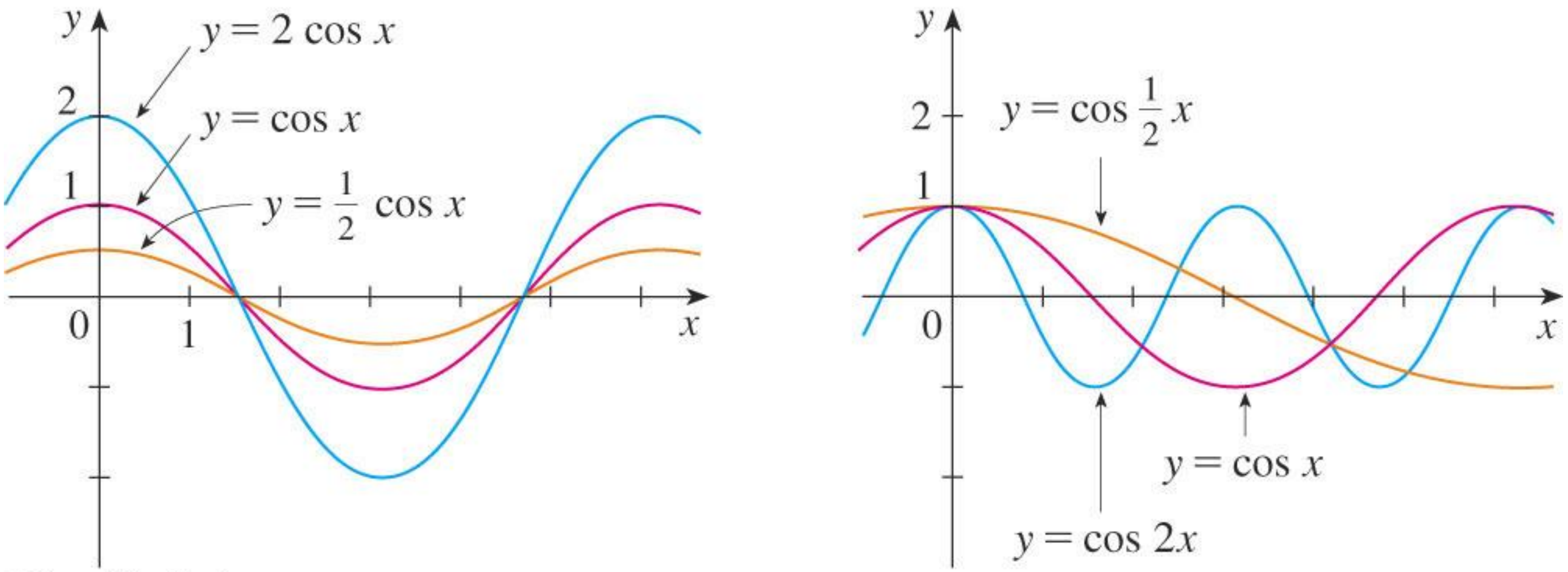
- $y = -\sqrt{x}$ by reflecting about the *x*-axis.
- $y = 2\sqrt{x}$ by stretching vertically by a factor of 2.
- $y = \sqrt{-x}$ by reflecting about the *y*-axis



(a) $y=\sqrt{x}$ (b) $y=\sqrt{x}-2$ (c) $y=\sqrt{x-2}$ (d) $y=-\sqrt{x}$ (e) $y=2\sqrt{x}$ (f) $y=\sqrt{-x}$

TRANSFORMATIONS

The figure illustrates these stretching transformations when applied to the cosine function with c = 2.



Example

Suppose that the graph of f is given. Describe how the graph of the function $f(x-2)+2$ can be obtained from the graph of f.

Select the correct answer.

a. Shift the graph 2 units to the left and 2 units down.

b. Shift the graph 2 units to the right and 2 units down.

c. Shift the graph 2 units to the right and 2 units up.

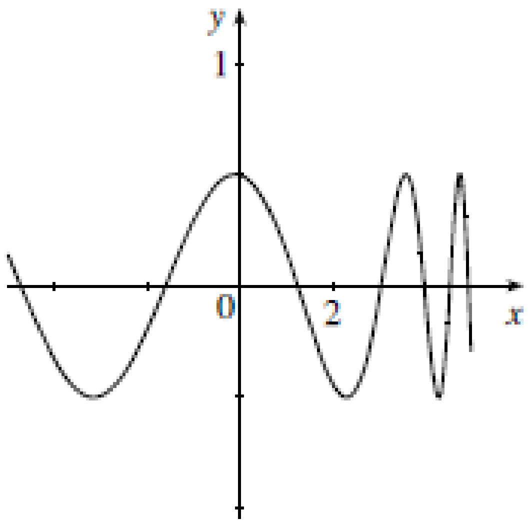
d. Shift the graph 2 units to the left and 2 units up.

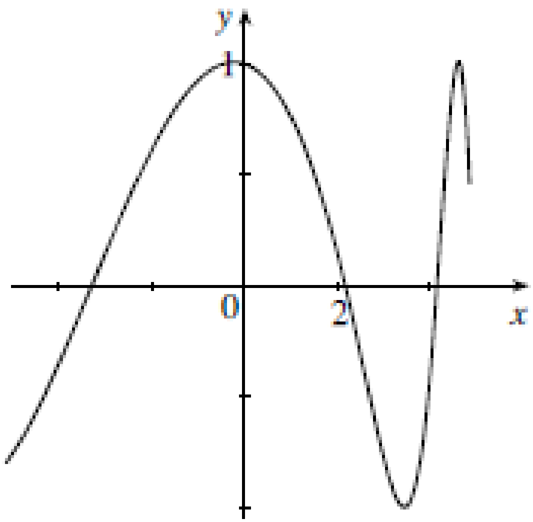
e. none of these

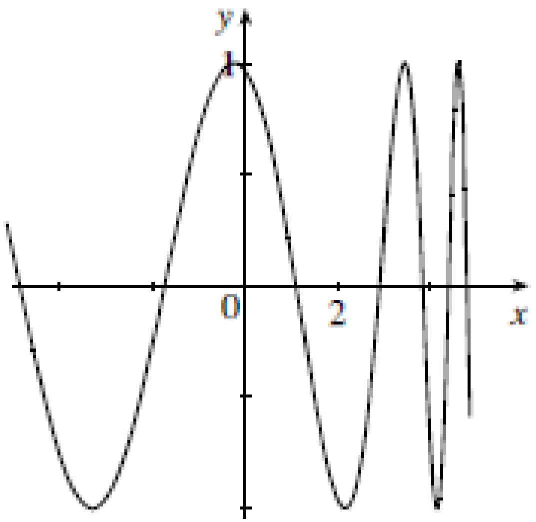
Answer: c

QUIZ QUESTIONS

Label the following graphs: $f\,(x)$, $\frac{1}{2}f\,(x)$, $f\left(\frac{1}{2}x\right)$.







Answer: ½ f(x), f(½ x), f(x)

Thanks