Name:...

Class:...

Fpt University

Mathematics for Engineering

Exercise Book

CALCULUS

Chapter 1: Function and Limit

1. Find the domain of each function:

a. $f(x)=\sqrt{x+2}$ b. $f(x)=\dfrac{1}{x^2-x}$ c. $f(x)=\ln(x+1)-\dfrac{x}{\sqrt{x-1}}$

2. Find the range of each function:

a. $f(x)=\sqrt{x-1}$ b. $f(x)=x^2-2x$ c. $f(x)=\sin x$

3. Determine whether is even, odd, or neither

a. $f(x)=\dfrac{x}{x^2+1}$ b. $f(x)=\dfrac{x^2}{x^4+1}$ c. $f(x)=\dfrac{x}{x+1}$

4. Explain how the following graphs are obtained from the graph of *f(x)*

a. $f(x-4)$ b. $f(x)+3$ c. $f(x-2)-3$ d. $f(x+5)-4$

5. Suppose that the graph of $f(x)=\sqrt{x}$ is given. Describe how the graph of the function $y=\sqrt{x-1}+2$ can be obtained from the graph of f.

6. Let $f(x)=\sqrt{x}$ and $g(x)=\sqrt{2-x}$. Find each function

a. $f_o g$ b. $g_0 f$ c. $g_o g$ d. $f_o f$

7. Let $f(x)=\dfrac{x^2+x+1}{x}$. Find

a. $f\left(x+\dfrac{1}{x}\right)$ b. $f(2x-1)$

8. Use the table to evaluate each expression

a. f(g(1)) b. g(f(1)) c. f(f(1)) d. g(g(1))

e. $g_o f(3)$ f. $g_o f(6)$

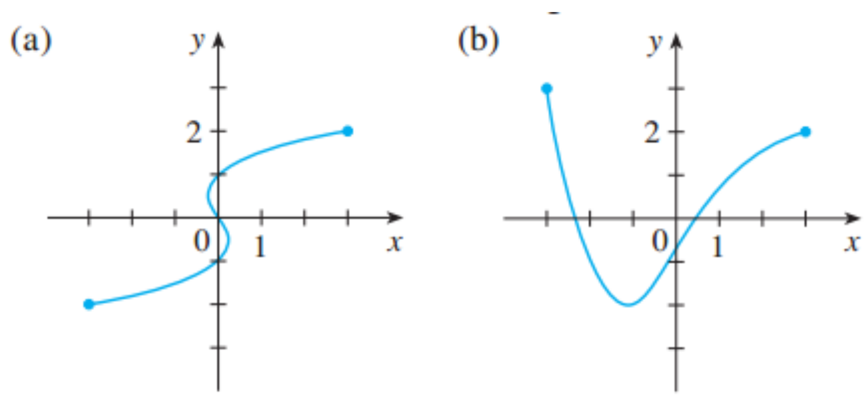
x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

9. Evaluate the following limits

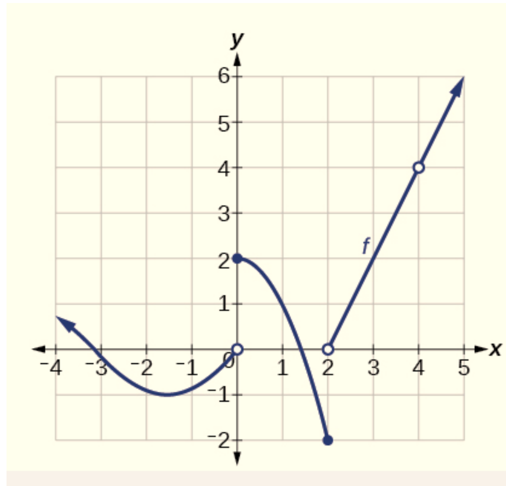
a. $\lim\limits_{x\to 3}\dfrac{x^2+x-12}{x-3}$ b. $\lim\limits_{x\to 1}\dfrac{x^6-1}{x^{10}-1}$ c. $\lim\limits_{x\to 0}\dfrac{\tan 3x}{\tan 5x}$ d. $\lim\limits_{h\to 0}\dfrac{(3+h)^2-9}{h}$

e. $\lim\limits_{t\to 0}\dfrac{\sqrt{t^2+9}-3}{t^2}$ f. $\lim\limits_{x\to +\infty}\dfrac{x^2+x-12}{x^3-3}$ g. $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{|x|}\right)$ h. $\lim\limits_{x\to 1}\dfrac{x^2-1}{|x-1|}$

10. Determine whether each curve is the graph of a function of x. If it is, state the domain and range of the function.



11. The graph of f is given.



a. Find each limit, or explain why it does not exist.

i. $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0} f(x)$

ii. $\lim_{x\to 1} f(x)$ and $\lim_{x\to 4} f(x)$

b. At what numbers is discontinuous?

12. Determine where the function $f(x)$ is continuous

a. $f(x)=\dfrac{2x^2+x-1}{x-2}$ b. $f(x)=\dfrac{x-9}{\sqrt{4x^2+4x+1}}$ c. $f(x)=\ln(2x+5)$

13. Find the constant m that makes f continuous on R

a. $f(x)=\begin{cases} x^2-m^2, & x<4 \\ mx+20, & x\geq 4 \end{cases}$ b. $f(x)=\begin{cases} mx^2+2x, & x<2 \\ x^3-mx, & x\geq 2 \end{cases}$

c. $f(x)=\begin{cases} \dfrac{e^{2x}-1}{x}, & x\neq 0 \\ m, & x=0 \end{cases}$ d. $f(x)=\begin{cases} \dfrac{x^2-1}{\sqrt{x}-1}, & x\neq 1 \\ mx+1, & x=1 \end{cases}$

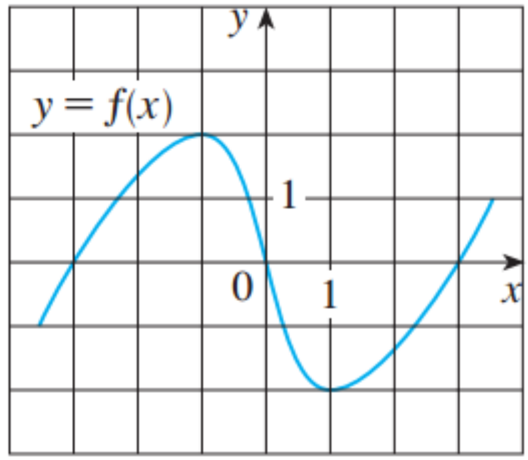
14. Find the numbers at which the function $f(x)=\begin{cases} x+2, & x<0 \\ 2x^2, & 1\geq x\geq 0 \\ 2-x, & x>1 \end{cases}$ is discontinuous.

Chapter 2: Derivatives

1. Use the given graph to estimate the value of each derivative

a. $f'(-3)$ b. $f'(-1)$

c. $f'(0)$ d. $f'(3)$



2. Find an equation of the tangent line to the curve at the given point:

a. $y = \dfrac{x-1}{x-2}, \quad (3,2)$ b. $y = \dfrac{2x}{x^2+1}, \quad (0,0)$

c. $y = 3 - 2x + x^2, \quad x = 1$ d. $y = \dfrac{3-2x}{x-1}, \quad y = -1$

3. Find y'

a. $y = x^2 - x\sqrt{x} + \dfrac{1}{x} + 2$ b. $y = \sqrt{x + \sqrt{x}}$ c. $y = \dfrac{x^2}{x+1}$

d. $y = x\sqrt{x+2}$ e. $y = \ln\left(x^2+1\right) - \dfrac{1}{x}$ f. $y = e^x \sin(2x+1)$

4. Find y''

a. $y = xe^{3x-1}$ b. $y = \sqrt[3]{2x+1}$ c. $y = e^{-x}\cos x$

5. Find dy/dt for:

a. $y = x^3 + x + 2, dx/dt = 2$ and $x = 1$ b. $y = \ln x, dx/dt = 1$ and $x = e^2$

c. $y = \tan\sqrt{t}$ and $t = \dfrac{\pi^2}{16}$

d. $\begin{cases} y = \sin\varphi \\ t = \cos\varphi \end{cases}$ and $\varphi = \dfrac{\pi}{3}$

6. Find dy for:

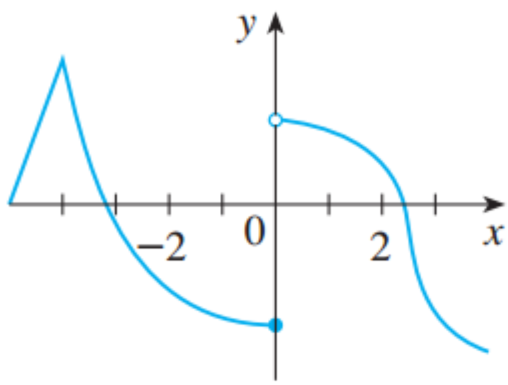
a. $y = \dfrac{1}{x^2+1}$

b. $y = \sqrt{x+1}, x = 3$

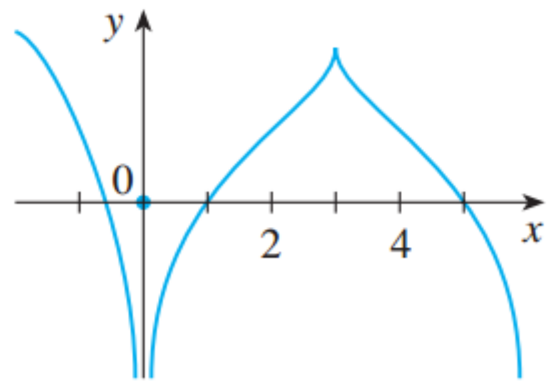
c. $y = \ln\left(x^2+1\right)$, $x = 1$ and $dx = 2$

7. The graph of is given. State the numbers at which is not differentiable

a.



b.



8. A table of values for f, f', g and g' is given

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

a. If $h(x) = f\left(g(x)\right)$, find $h'(1)$

b. If $H(x) = g_o f(x)$, find $H'(1)$

c. If $F(x) = f_o f(x)$, find $F'(2)$

d. If $G(x) = g_o g(x)$, find $G'(3)$

9. If $h(x) = \sqrt{4+3f(x)}$, where $f(1) = 7, f'(1) = 4$, find $h'(1)$.

10. For the circle $x^2 + y^2 = 25$.

a. Find dy/dx

b. Find an equation of the tangent to the circle at the point (3, 4).

11. Let $(L): x^3 + y^3 = 6xy$

a. Find dy/dx

b. Find an equation of tangent to the curve (L) at the point (3, 3)

12. Find y' by implicit differentiation

a. $x^4 + y^4 = 16x + y$ b. $\sqrt{x} + \sqrt{y} = 4$ c. $x^3 + xy = y^2$

13. Find f' in terms of g'

a. $f(x) = g(\sin 2x)$ b. $f(x) = g\left(e^{1-3x}\right)$

14. Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2?

15. If $x^2 + y^2 = 25$ and $dy/dt = 6$, find dx/dt when y = 4 and x > 0.

16. If $z^2 = x^2 + y^2$ $(z > 0)$, $dx/dt = 2, dy/dt = 3$, find dz/dt when $x = 5, y = 12$

17. Find the linearization L(x) of the function at a.

a. $f(x) = \dfrac{1}{\sqrt{2+x}}$, $a = 2$ b. $f(x) = \sqrt[3]{5-x}$, $a = -3$

18. The equation of motion is $s(t) = 3\sin t - 4\cos t + 1$ for a particle, where s is in meters and t is in seconds. Find the acceleration (in m/s^2) after 3 seconds.

Chapter 3: Applications of Differentiation

1. Find the absolute maximum and absolute minimum values of the function on the given interval

a. $f(x)=3x^2-12x+5,[0;3]$ b. $f(x)=x^3-3x+5,\quad [0;3]$

c. $f(x)=x\sqrt{4-x^2},\quad [-1;2]$ d. $f(x)=x-\ln x,\quad \left[\frac{1}{2};2\right]$

2. Find the critical numbers of the function

a. $f(x)=5x^2+4x$ b. $f(x)=\dfrac{x-1}{x^2-x+1}$ c. $f(x)=x\ln x$

3. Find all numbers that satisfy the conclusion of the Rolle's Theorem

a. $f(x)=x\sqrt{x+2},\quad [-2;0]$ b. $f(x)=(x-2)x^2,\quad [0;2]$

4. Find all numbers that satisfy the conclusion of the Mean Value Theorem

a. $f(x)=3x^2+2x+5,\ [-1;1]$ b. $f(x)=e^{-2x},\quad [0;3]$

5. If $f(1)=10$ and $f'(x)\geq 2, \forall x\in[1;4]$, how small can $f(4)$ possibly be?

6. Find where the function $f(x)=3x^4-4x^3-12x^2+1$ is increasing and where it is decreasing.

7. Find the inflection points for the function

a. $f(x)=x^4-4x+1$ b. $f(x)=x^6$ c. $f(x)=xe^x$

8. Find $f(x)$ for $f'(x)=\sqrt{2x+1}$ and $f(0)=1$.

9. Find the point on the parabola $y^2=2x$ that is closest to the point $(1;4)$

10. Find two numbers whose difference is 100 and whose product is a minimum.

11. Find two positive numbers whose product is 100 and whose sum is a minimum.

12. Use Newton's method with the specified initial approximation x_1 to find x_3

a. $x^3+2x-4=0,\ x_1=1$

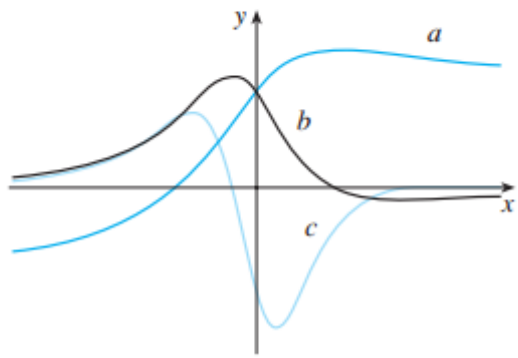
b. $x^5+2=0,\ x_1=-1$

c. $\ln\left(x^2+1\right)-2x-1=0,\ \ x_1=1$

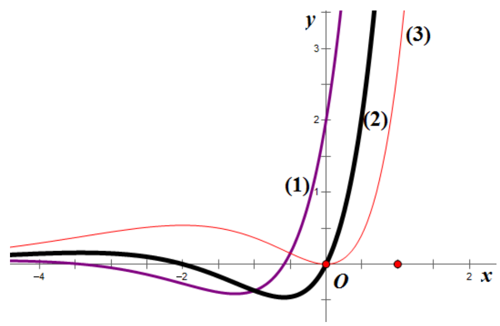
d. $\ln\left(4-x^2\right)=x,\ x_1=1$

13. The figure shows the graphs of f, f' and f''. Identify each curve, and explain your choices

a.



b.



14. Find the most general anti-derivative of the function.

a. $f(x)=6x^2-2x+3$

b. $f(x)=\sqrt[6]{x}+\dfrac{1}{x^2}$

c. $f(x)=\dfrac{x^2+x+2}{x}$

d. $f(x)=2x\left(x^2+1\right)$

15. Find the anti-derivative of that satisfies the given condition

a. $f(x)=5x^4-2x^5, F(0)=4$

b. $f(x)=4-\dfrac{2x}{x^2+1}, F(0)=1$

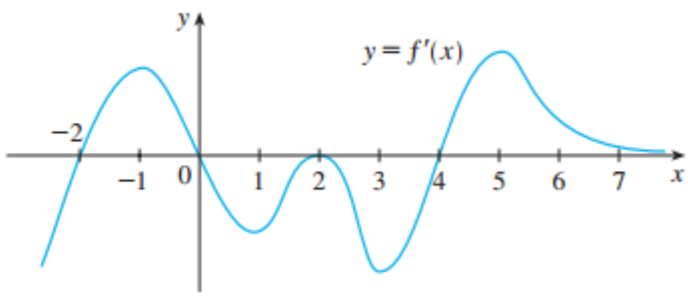
16. A particle is moving with the given data. Find the position of the particle

a. $v(t)=\sin t-\cos t,\ s(0)=0$

b. $v(t)=10\sin t+3\cos t,\ s(\pi)=0$

c. $v(t)=10+3t-3t^2,\ s(2)=10$

17. The figure shows the graph of the derivative f' of a function f



a. On what intervals is f increasing or decreasing?

b. For what values of x does f have a local maximum or minimum?

Chapter 4 - 6: Integration

1. Estimate the area under the graph of $y = f(x)$ using 6 rectangles and left endpoints

a. $f(x) = \frac{1}{x} + x$, $x \in [1,4]$ b. $f(x) = x^2 - 2$, $x \in [-1,2]$

c. A table of values for f is given

x	1	2	3	4	5	6	7
$f(x)$	5	6	3	2	7	1	2

3. Repeat part (1) using right endpoints

4. For the function $f(x) = x^3, x \in [-2,2]$. Estimate the area under the graph of using four approximating rectangles and taking the sample points to be

a. Right endpoints

b. Left endpoints

c. Midpoints

5. Use (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with the specified value of *n.*

a. $\int_0^3 \sqrt{x}dx,$ $n = 4$ b. $\int_1^3 \frac{\sin x}{x} dx$,n = 6

6. Let $I = \int_0^2 \frac{dx}{x^2+1}$. Find the approximations L_4, R_4, M_4, T_4 and S_4 for I.

7. Find the derivative of the function $g(x) = \int_0^x \sqrt{t^2+1}\, dt$

8. Find g'

a. $g(x)=\int_1^{x^4}\frac{1}{\cos t}dt$ b. $g(x)=\int_1^{\sqrt{x}}\frac{\sin u}{u}du$

c. $g(x)=\int_{2x}^{x^2+x+2}\frac{e^t}{t}dt$ d. $g(x)=\int_{\sin x}^{\cos x}\left(1+v^2\right)^{10}dv$

9. Find the average value of the function on the given interval

a. $f(x)=x^2, \quad [-1,1]$ b. $f(x)=\frac{1}{x}, \quad [1,5]$

c. $f(x)=x\sqrt{x}, \ [1,4]$ d. $f(x)=x\ln x, \left[1, \mathrm{e}^2\right]$

10. A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (m/s)

a. Find the displacement of the particle during the time period $1 \le t \le 4$

b. Find the distance traveled during this time period

11. Suppose the acceleration function and initial velocity are $a(t)= t + 3$ (m/s^2), $v(0)=5$ (m/s). Find the velocity at time t and the distance traveled when $0 \le t \le 5$.

12. A particle moves along a line with velocity function $v(t)=t^2-t$, where is measured in meters per second. Find the displacement and the distance traveled by the particle during the time interval $t\in[0,2]$.

13. Evaluate the integral

a. $\int_0^2 x^2.\sqrt{x^3+1}\,dx$ b. $\int xe^{x^2}dx$ c. $\int\left(\frac{1}{x}+\sqrt{x}-3x^2\right)dx$

d. $\int_0^1 y\left(1+y^2\right)^5 dy$ e. $\int\frac{\ln x}{x}dx$ f. $\int\frac{t}{t^2+1}dt$

14. Evaluate the integral

a. $\int xe^x dx$ b. $\int_0^1 x^2 e^{-x} dx$ c. $\int x \sin x dx$

d. $\int \ln x dx$ e. $\int_1^e x \ln x dx$ f. $\int e^{\sqrt{x}} dx$

15. Suppose f(x) is differentiable, f(1) = 4 and $\int_0^1 f(x)dx = 5$. Find $\int_0^1 xf'(x)dx$

16. Suppose *f(x)* is differentiable, *f(1) = 3, f(3) = 1* and $\int_1^3 xf'(x)dx = 13$. What is the average value of *f* on the interval *[1,3]*?

17. Let $f(x) = \begin{cases} -x-1, & -3 \le x \le 0 \\ -\sqrt{1-x^2}, & 0 < x \le 1 \end{cases}$. Evaluate $\int_{-3}^{1} f(x)dx$

18. Find $g'(0)$ for

a. $g(x) = \int_x^{x^2} e^{2x+1} dx$ b. $\int_{2x-1}^{x^3} x\sqrt{x+1}dx$

19. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

a. $\int_1^\infty \frac{dx}{(3x+1)^2}$ b. $\int_{-\infty}^0 \frac{dx}{2x-5}$ c. $\int_{-\infty}^{-1} \frac{dx}{\sqrt{2-x}}$ d. $\int_0^\infty \frac{xdx}{(x^2+2)^2}$

e. $\int_4^\infty e^{-\frac{y}{2}} dy$ f. $\int_{-\infty}^{-1} e^{-2t} dt$ g. $\int_{2\pi}^{\infty} \sin\varphi d\varphi$ h. $\int_{-\infty}^{\infty} xe^{-x^2} dx$

i. $\int_0^1 \frac{dx}{4x-1}$ j. $\int_3^4 \frac{dx}{\sqrt{x-3}}$ k. $\int_{-1}^1 \frac{1}{\sqrt[3]{x^2}} dx$ l. $\int_0^1 \frac{dx}{\sqrt{x}}$

20. Use the Comparison Theorem to determine whether the integral is convergent or divergent

a. $\int_{1}^{\infty} \frac{\cos^2 x dx}{1+x^2}$

b. $\int_{1}^{\infty} \frac{2+e^{-x}}{x} dx$

c. $\int_{1}^{\infty} \frac{dx}{x+e^{2x}}$

d. $\int_{1}^{\infty} \frac{x dx}{\sqrt{1+x^6}}$

e. $\int_{0}^{\frac{\pi}{2}} \frac{\cos x dx}{\sqrt{\sin x}}$

f. $\int_{0}^{1} \frac{2dx}{\sqrt{x^3}}$

Chapter 8: Series

1. Determine whether the sequence converges or diverges. If it converges, find the limit

a. $a_n = \dfrac{3+2n^2}{n+n^2}$ b. $a_n = \dfrac{\sqrt{n}}{\sqrt{2n+1}+3}$ c. $a_n = \dfrac{n}{\sqrt{n}+1}$ d. $a_n = \left(1+\dfrac{2}{n}\right)^n$

e. $\left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$ f. $\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\right\}$

g. $\{0.12,\ 0.1212,\ 0.121212, \ldots\}$

2. Find the limit of the sequence $\{a_n\}$

a. $a_1 = \sqrt{5},\ a_{n+1} = \sqrt{5+a_n}$ b. $a_1 = 2,\ a_{n+1} = \dfrac{1}{3-a_n}$ c. $a_1 = 1,\ a_{n+1} = \dfrac{1}{1+a_n}$

3. Determine whether the sequence is increasing, decreasing or not monotonic

a. $u_n = \dfrac{1}{2n^2-n+1}$ b. $u_n = \dfrac{\sqrt{n+5}}{n+1}$ c. $\begin{cases} u_1 = 1 \\ u_{n+1} = \dfrac{u_n}{3-u_n} \end{cases}$

4. Find the formula for the nth term of the sequence

a. $\{1,3,5,7,\ldots\}$ b. $\begin{cases} u_1 = 1 \\ u_n = 2u_{n-1}+1 \end{cases}$ c. $\begin{cases} u_1 = u_2 = 1 \\ u_{n+2} = u_{n+1}+u_n \end{cases}$

5. Suppose that $f(1)=1, f(2)=-2$ and $f(n+2)=-2f(n+1)+3f(n)$.

a. Find $f(5)$

b. Determine the formula for $f(n)$

6. Determine whether the series is convergent or divergent. If it is convergent, find its sum

a. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ b. $\sum_{n=2}^{\infty}\frac{n^2+n-1}{n(n-1)}$ c. $\sum_{n=2}^{\infty}\frac{1}{3.2^{n-1}}$

d. $\sum_{n=1}^{\infty}\sin n$ e. $\sum_{n=1}^{\infty}\frac{1+3^n}{2^n}$ f. $\sum_{n=1}^{\infty}\left(0,8^n+0,3^{n-1}\right)$

7. Determine whether the series is convergent or divergent

a. $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ b. $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n+1}$ c. $\sum_{n=1}^{\infty}\left(\frac{1}{n^6}+\frac{4}{n\sqrt{n}}\right)$ d. $\sum_{n=1}^{\infty}ne^{-n}$

e. $\sum_{n=1}^{\infty}\frac{1}{2n+3}$ f. $\sum_{n=1}^{\infty}\frac{4+3^n}{2^n}$ g. $\sum_{n=1}^{\infty}\frac{n!}{n^2 2^n}$ h. $\sum_{n=1}^{\infty}\frac{\cos n}{n^2+1}$

8. Determine whether the series is convergent or divergent

a. $\sum_{n=1}^{\infty}\frac{n}{2^n}$ b. $\sum_{n=1}^{\infty}\frac{(-3)^{n-1}}{\sqrt{n}}$ c. $\sum_{n=1}^{\infty}\frac{\sqrt{n+1}}{n^2+n+1}$ d. $\sum_{n=1}^{\infty}\frac{(-2)^n}{n!}$

e. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n+3}$ f. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\ln n}{n}$ g. $\sum_{n=1}^{\infty}\frac{\cos\pi n}{\sqrt{n+1}}$ h. $\sum_{n=1}^{\infty}\left(\frac{n^2+1}{2n^2+2n+3}\right)^n$

9. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

a. $\sum_{n=1}^{\infty}\frac{(-3)^{n+1}}{n^3}$ b. $\sum_{n=1}^{\infty}\frac{n^2}{2^n}$ c. $\sum_{n=1}^{\infty}\frac{(-10)^n}{n!}$ d. $\sum_{n=1}^{\infty}\frac{(-1)^n n}{n^2+1}$

e. $\sum_{n=1}^{\infty}\frac{\sin 4n}{n^2}$ f. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\sqrt{n+1}}$ g. $\sum_{n=2}^{\infty}\frac{\cos\pi n}{\ln n}$ h. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+1}}$

10. Find the radius of convergence and interval of convergence of the series

a. $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$ b. $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$ c. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ d. $\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^n}{2n+1}$

e. $\sum_{n=0}^{\infty} n!(2x-1)^n$ f. $\sum_{n=0}^{\infty} \frac{x^n}{n^2 3^n}$ g. $\sum_{n=0}^{\infty} \sqrt{n+1}\, x^n$ h. $\sum_{n=0}^{\infty} \frac{(-2)^n (x-3)^n}{\sqrt[4]{n}}$

11. Find the first *n* terms in the Maclaurin series for the given function

a. $f(x) = x\sin x,\ n = 4$ b. $f(x) = x\cos 2x,\ n = 3$

c. $f(x) = \ln(1+x^2),\ n = 4$ d. $f(x) = e^x \sin x,\ n = 3$

12. Approximate f by a Taylor polynomial with degree at the number a

a. $f(x) = \sqrt{x+1},\ n = 1,\ a = 0$ b. $f(x) = \frac{1}{x},\ n = 3,\ a = 1$

c. $f(x) = e^{x^2},\ n = 3,\ a = 0$ d. $f(x) = \cos x,\ n = 4,\ a = \frac{\pi}{3}$

LINEAR ALGEBRA

Chapter 1: Systems of Linear Equations

1. Write the augmented matrix for each of the following systems of linear equations and then solve them.

a. $\begin{cases} -x+y+2z=1 \\ 2x+3y+z=-2 \\ 5x+4y+2z=4 \end{cases}$ b. $\begin{cases} 2x+3y+z=10 \\ 2x-3y-3z=22 \\ 4x-2y+3z=-2 \end{cases}$

c. $\begin{cases} x+y+z=0 \\ 2x-y+2z=0 \\ x+z=0 \end{cases}$ d. $\begin{cases} x_1+2x_2-x_3+x_4=0 \\ 2x_1+3x_2-2x_3+3x_4=0 \\ x_1+x_2-3x_3+x_4=0 \end{cases}$

2. Compute the rank of each of the following matrices.

a. $A=\begin{pmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ b. $B=\begin{pmatrix} -2 & 3 & 3 \\ 3 & -4 & 1 \\ -5 & 7 & 2 \end{pmatrix}$

c. $C=\begin{pmatrix} 1 & 1 & -1 & 4 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & 5 & 8 \end{pmatrix}$ d. $D=\begin{pmatrix} 1 & 1 & -1 & 3 \\ -1 & 4 & 5 & -2 \\ 1 & 6 & 3 & 4 \end{pmatrix}$

3. Find all values of k for which the system has nontrivial solutions and determine all solutions in each case.

a. $\begin{cases} x-y+2z=0 \\ -x+y-z=0 \\ x+ky+z=0 \end{cases}$ b. $\begin{cases} x-2y+z=0 \\ x+ky-3z=0 \\ x-6y+5z=0 \end{cases}$

c. $\begin{cases} x+y+z=0 \\ x+y-z=0 \\ x+y+kz=0 \end{cases}$ d. $\begin{cases} x+y-z=0 \\ ky-z=0 \\ x+y+kz=0 \end{cases}$

4. Determine the values of m such that the system of linear equations has exactly one solution.

a. $\begin{cases} x-y+2z=m \\ -x+y-z=0 \\ -x+my-z=1-m \end{cases}$ b. $\begin{cases} mx+y+z=1 \\ x+my+z=m \\ x+y+mz=m^2 \end{cases}$

c. $\begin{cases} x+y-z=1 \\ x+my+2z=m \\ x+2y+z=2 \end{cases}$ d. $\begin{cases} x+my-mz=m \\ 2x+y-z=2 \\ x+y+z=0 \end{cases}$

5. Determine the values of m such that the system of linear equations is inconsistent.

a. $\begin{cases} x-y+2z=m \\ -x+y-z=0 \\ x-y+3z=1-m \end{cases}$ b. $\begin{cases} x-2y+2z=m \\ x+my-z=0 \\ 2x+y+mz=2-m \end{cases}$

6. Find a, b and c so that the system $\begin{cases} x+ay+cz=0 \\ bx+cy-3z=1 \\ ax+2y+bz=5 \end{cases}$ has the solution $(3,-1,2)$

7. Consider the matrix $A=\begin{pmatrix} 2 & -1 & 3 \\ -4 & 2 & k \\ 4 & -2 & 6 \end{pmatrix}$

a. If A is the augmented matrix of a system of linear equations, determine the number of equations and the number of variables.

b. If A is the augmented matrix of a system of linear equations, find the value(s) of k such that the system is consistent.

8. Find all values of k so that the system of equations has no solution.

a. $\begin{cases} x+y-z=2 \\ -2y+z=3 \\ 4y-2z=k \end{cases}$

b. $\begin{cases} x+y-z=1 \\ 2x+(k+5)y-2z=4 \\ x+(k+3)y+(k-1)z=k+3 \end{cases}$

9. Find all values of a and b for which the system of equations $\begin{cases} x+y+3z=2 \\ x+2y+5z=1 \\ 2x+2y+az=b \end{cases}$ is inconsistent.

10. Solve the system of linear equation corresponding to the given augmented matrix

a. $A=\begin{pmatrix} 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$

b. $B=\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

11. Determine the values of m such that the rank of the matrix is 2

A. $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 5 \\ 1 & 2 & m \end{pmatrix}$

b. $\begin{pmatrix} 1 & 2 & 1 & 4 \\ 2 & 1 & 1 & 5 \\ -3 & 6 & 1 & m \end{pmatrix}$

c. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & -1 \\ 3 & 1 & 2 \\ m & 3 & 5 \end{pmatrix}$

12. Solve the system $\begin{cases} x+2y=12 \\ 3x-y=8 \\ -x+5y=16 \end{cases}$

Chapter 2: Matrix Algebra

1. Let $A=\begin{pmatrix}1 & 4\\ 2 & 3\end{pmatrix}, B=\begin{pmatrix}-1 & 1\\ 3 & 2\end{pmatrix}$ and $C=\begin{pmatrix}1 & 3 & -4\\ -1 & 2 & 1\end{pmatrix}$. Compute the matrix

a. $2A-B^T$ b. AB c. BA d. AC

e. CC^T f. C^TC g. A^3 h. B^2A^T

2. Suppose that A and B are nxn matrices. Simplify the expression

a. $(A+B)^2-(A-B)^2$ b. $A(BC-CD)+A(C-B)D-AB(C-D)$

3. Let $A=\begin{pmatrix}3 & 1 & 2\\ 4 & 8 & 0\\ 0 & 1 & 2\end{pmatrix}$ and $B=\begin{pmatrix}0 & 5 & 2 & 1\\ 1 & 8 & 0 & -6\\ 1 & 4 & 3 & 7\end{pmatrix}$.

a. Compute AB

b. Compute $f(A)$ if $f(x)=x^2-3x+2$

4. Find the inverse of each of the following matrices.

a. $\begin{pmatrix}1 & 5\\ 2 & -1\end{pmatrix}$ b. $\begin{pmatrix}2 & 1\\ 2 & -4\end{pmatrix}$ c. $\begin{pmatrix}1 & -1 & 2\\ -5 & 7 & -11\\ -2 & 3 & -5\end{pmatrix}$ d. $\begin{pmatrix}1 & -1 & 3\\ 2 & 0 & 5\\ -1 & 1 & 0\end{pmatrix}$

5. Given $A^{-1}=\begin{pmatrix}1 & -1 & 3\\ 2 & 0 & 5\\ -1 & 1 & 0\end{pmatrix}$. Find a matrix X such that

a. $AX=\begin{pmatrix}1\\ -1\\ 3\end{pmatrix}$ b. $AX=\begin{pmatrix}1 & -1 & 2\\ 0 & 1 & 1\\ 1 & 0 & 0\end{pmatrix}$ c. $XA=\begin{pmatrix}1 & 2 & -1\\ 3 & 1 & 1\end{pmatrix}$

6. Find A when

a. $(3A)^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$ b. $(I+2A)^{-1} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ c. $(A^{-1}-2I)^T = -2\begin{pmatrix} 1 & 4 \\ 3 & 11 \end{pmatrix}$

7. Write the system of linear equations in matrix form and then solve them.

a. $\begin{cases} 2x - y = 4 \\ 3x + 2y = -4 \end{cases}$ b. $\begin{cases} 2x + 3y + z = 10 \\ 2x - 3y - 3z = 22 \\ 4x - 2y + 3z = -2 \end{cases}$ c. $\begin{cases} x + y = a \\ 2x + 3y = 1 - 2a \end{cases} (a \in R)$

8. Find A^{-1} if

a. $A^2 - 6A + 5I = 0$ b. $A^2 + 3A - I = 0$ c. $A^4 = I$

9. Solve for X

a. $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} X = \begin{pmatrix} 1 & -1 \\ 3 & 3 \end{pmatrix}$ b. $ABXC = B^T$ c. $AX^T BC = B$

(where A, B and C are nxn invertible matrices)

10. Compute $\begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}^{101}$

11. Let $T: R^2 \to R^2$ be a linear transformation, and assume that $T(1,2) = (-1,1)$ and $T(0,3) = (-3,3)$

a. Compute $T(11,-5)$ b. Compute $T(1,11)$

c. Find the matrix of T d. Compute $T^{-1}(2,3)$

12. Let $T: R^2 \to R^2$ be a linear transformation such that the matrix of T is $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$.

Find $T(3,-2)$

13. The (2;1)-entry of the product $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 5 & 1 \\ 4 & -1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 5 & 1 & 0 \\ 0 & 4 & 3 \end{pmatrix}$

Chapter 3: Determinants and Diagonalization

1. Evaluate the determinant

a. $\begin{vmatrix} x-2 & -1 \\ -3 & x \end{vmatrix}$ b. $\begin{vmatrix} -2 & 0 & 0 \\ 4 & 6 & 0 \\ -3 & 7 & 2 \end{vmatrix}$ c. $\begin{vmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{vmatrix}$ d. $\begin{vmatrix} 2 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{vmatrix}$

e. $\begin{vmatrix} x & y & 1 \\ -1 & -2 & 1 \\ 1 & 5 & 1 \end{vmatrix}$ f. $\begin{vmatrix} m & -1 & 0 \\ 1 & 2 & 1 \\ 2 & m & -3 \end{vmatrix}$

2. Find the minors and the cofactors of the matrix

a. $A = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$ b. $B = \begin{pmatrix} -3 & 4 & 2 \\ 6 & 3 & 1 \\ 4 & -7 & -8 \end{pmatrix}$ c. $C = \begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & m \end{pmatrix}$

3. Find the adjugate and the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$

4. Let $A = \begin{pmatrix} 1 & * & * & * \\ 0 & -1 & * & * \\ 0 & 0 & 2 & * \\ 0 & 0 & 0 & 2 \end{pmatrix}$. Find

a. $\left|2A^{-1}\right|$ b. $\left|AA^T\right|$ c. $\left|\text{adj}\,A\right|$

d. $\left|-A^3\right|$ e. $\left|(2A)^{-1}\right|$ f. $\left|A^{-1} - 2\text{adj}\,A\right|$

5. Let A and B be square matrices of order 4 such that $|A| = -5$ and $|B| = 3$. Find

a. $|2AB|$ b. $|adj(AB)|$ c. $\left|5A^{-1}B^T\right|$ d. $\left|A^T B^{-1} A^2\right|$

6. Find all values of m for which the matrix is not invertible

a. $A = \begin{pmatrix} 1 & 3 \\ m & 2 \end{pmatrix}$ b. $B = \begin{pmatrix} m & 1 & 3 \\ 1 & 3 & 2 \\ -1 & 4 & 5 \end{pmatrix}$ c. $C = \begin{pmatrix} m & 2 & 0 \\ 1 & m & 1 \\ 2 & 3 & 1 \end{pmatrix}$

7. Find the characteristic polynomial of the matrix

a. $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ b. $B = \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}$

c. $C = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ d. $D = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ -1 & 1 & 1 \end{pmatrix}$

8. Find the eigenvalues and corresponding eigenvectors of the matrix

a. $A = \begin{pmatrix} -3 & 5 \\ 10 & 2 \end{pmatrix}$ b. $B = \begin{pmatrix} 5 & 4 \\ 2 & 1 \end{pmatrix}$

c. $C = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ d. $D = \begin{pmatrix} -3 & 2 & -1 \\ 0 & 1 & 0 \\ 4 & 1 & 1 \end{pmatrix}$

9. Find the determinant of the matrix $A = \begin{pmatrix} 5 & 1 & 2 & 4 \\ 1 & 0 & -1 & -3 \\ 1 & 1 & 6 & 1 \\ 1 & 0 & 0 & -4 \end{pmatrix}$

10. Find the (1, 2)-cofactor and (3,1) - cofactor of the matrix $\begin{bmatrix} -1 & 3 & -2 \\ 4 & 5 & -7 \\ 7 & 8 & 1 \end{bmatrix}$

11. Let $A = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & x \end{pmatrix}$. For which values of x is A invertible ?

Chapter 5: The Vector Space R^{n}

1. Let $x=(-1,-2,-2), u=(0,1,4), v=(-1,1,2)$ and $w=(3,1,2)$ in R^3. Find scalars a, b and c such that $x=au+bv+cw$

2. Write v as a linear combination of u and w, if possible, where $u=(1,2), w=(1,-1)$

a. $v=(0,1)$ b. $v=(2,3)$ c. $v=(1,4)$ d. $(-5,1)$

3. Determine whether the set S is linearly independent or linearly dependent

a. $S=\{(-1,2),(3,1),(2,1)\}$ b. $S=\{(-1,2,3),(1,3,5)\}$

c. $S=\{(1,-2,2),(2,3,5),(3,1,7)\}$ d. $S=\{(-1,2,1),(2,4,0),(3,1,1)\}$

e. $S=\{(1,-2,2,1),(1,2,3,5),(-1,3,1,7)\}$

4. For which values of k is each set linearly independent?

a. $S=\{(-1,2,1),(k,4,0),(3,1,1)\}$ b. $S=\{(-1,k,1),(1,1,0),(2,-1,1)\}$

c. $S=\{(k,1,1),(1,k,1),(1,1,k)\}$ d. $S=\{(1,2,1,0),(-2,1,1,-1),(-1,3,2,k)\}$

5. Find all values of m such that the set S is a basis of R^3

a. $S=\{(1,2,1),(m,1,0),(-2,1,1)\}$ b. $S=\{(-1,m,1),(1,1,0),(m,-1,-1)\}$

6. Find a basis for and the dimension of the subspace U

a. $U=\{(2s-t,s,s+t) \mid s,t\in R\}$ b. $U=\{(s-t,s,t,s+t) \mid s,t\in R\}$

c. $U=\{(0,t,-t) \mid t\in R\}$ d. $U=\{(x,y,z) \mid x+y+z=0\}$

e. $U=\{(x,y,z) \mid x+y+z=0, x-y=0\}$ f. $U=span\{(1,2,3),(2,3,4),(3,5,7)\}$

g. $U=span\{(1,2,4),(-1,3,4),(2,3,1)\}$ h. $U=span\{(1,2,1,1),(2,1,-1,0),(3,3,0,1)\}$

7. Find a basis for and the dimension of the solution space of the homogeneous system of linear equations.

a. $\begin{cases} -x+y+z=0 \\ 3x-y=0 \\ 2x-4y-5z=0 \end{cases}$ b. $\begin{cases} x+2y-4z=0 \\ -3x-6y+12z=0 \end{cases}$ c. $\begin{cases} x+y+z+t=0 \\ 2x+3y+z=0 \\ 3x+4y+2z+t=0 \end{cases}$

8. Find all values of m for which x lies in the subspace spanned by S

a. $x=(-3,2,m)$ and $S=\{(-1,-1,1),(2,-3,-4)\}$

b. $x=(4,5,m)$ and $S=\{(1,-1,1),(2,-3,4)\}$

c. $x=(m+1,5,m)$ and $S=\{(1,1,1),(2,3,1),(3,4,2)\}$

d. $x=(3,5,7,m)$ and $S=\{(1,1,1,-1),(1,2,3,1),(2,3,4,0)\}$

9. Find the dimension of the subspace
$U=span\{(\text{-}2,\ 0,\ 3),(1,\ 2,\ \text{-}1),(\text{-}2,\ 8,\ 5),(\text{-}1,\ 2,\ 2)\}$

10. Let $A=\begin{pmatrix} 1 & 2 & 2 & -1 \\ 3 & 6 & 5 & 0 \\ 2 & 2 & 1 & 2 \end{pmatrix}$. Find $\dim(\operatorname{col} A)$ and $\dim(\operatorname{row} A)$

11. Which of the following are subspaces of R^3 ?

(i) $\{(2+a,b-a,b) \mid a,b\in R\}$

(ii) $\{(a+b,a,b) \mid a,b\in R\}$

(iii) $\{(2a+b,0,ab) \mid a,b\in R\}$

12. Let $u=(1,-3,-2), v=(-1,1,0)$ and $w=(1,2,-3)$. Compute $\|u-v+w\|$

13. Let $u,v\in\mathbf{R}^3$ such that $\|u\|=3, \|v\|=4$ and $u\bullet v=-2$. Find

a. $\|u+v\|$ b. $\|2u+3v\|$