ASSIGNMENT 1

Problem 1. *(2 points)* Which of these are propositions? What are the truth values of those that are propositions?

a) Do not pass go.

b) What time is it?

c) There are no cockroaches in Vietnam

d) 4 + x = 5

e) The moon is made of green cheese.

f) $2^n \geq 100$.

Problem 2. *(1 points)* What is the negation of each of these propositions?

a) Jennifer and Teja are friends.

b) There are 13 items in a baker's dozen.

c) Abby sent more than 100 text messages every day.

d) 121 is a perfect square.

Problem 3. *(1 points)* Let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog," and let F(x) be the statement "x has a ferret." Express each of these statements in terms of C(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) There is a student in your class which has all three animals as pets.

b) All students in your class have a cat, a dog, or a ferret.

c) Some student in your class has a cat and a ferret, but not a dog.

d) No student in your class has a cat, a dog, and a ferret.

Problem 4. *(2 points)* Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

Problem 5. *(2 points)* State whether the following are true or false, where the universe for all variables consists of all integers.

a) $\forall x \exists y (2x - y = 0)$

b) $\exists y \forall x (2x - y = 0)$

c) $\forall x \exists y (x - 2y = 0)$

d) $\forall x \forall y (x + y = y + x)$

e) $\forall x \Big((x < 10) \rightarrow \forall y \big((y < x) \rightarrow (y < 9) \big) \Big)$

f) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$

g) $\forall x \exists y \big(y > x \wedge \exists z (y + z = 100) \big)$

h) $\forall x \forall y \exists z \left(z = \frac{x+y}{2} \right)$

Problem 6. *(2.25 points)* Let L(x, y) be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody loves Jerry.

b) Everybody loves somebody.

c) There is somebody whom everybody loves.

d) Nobody loves everybody.

e) There is somebody whom Lydia does not love.

f) There is somebody whom no one loves.

g) There is exactly one person whom everybody loves.

h) There are exactly two people whom Lynn loves.

i) Everyone loves himself or herself.

Problem 7. *(2 points)* Write each of these statements in the form "if p, then q" in English. For example: "It is necessary to wash the boss's car to get promoted." $\rightarrow$ "If you want to get promotion, then you should wash the boss's car"

a) It snows whenever the wind blows from the northeast.

b) The apple trees will bloom if it stays warm for a week.

c) That the Pistons win the championship implies that they beat the Lakers.

d) It is necessary to walk 8 miles to get to the top of Long's Peak.

e) To get tenure as a professor, it is sufficient to be world-famous.

f) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

g) Your guarantee is good only if you bought your CD player less than 90 days ago.

h) Jan will go swimming unless the water is too cold.