Self- and cross- phase modulation in the generation of bright twin beams using SPDC

Nicolás Quesada[1] and J.E Sipe [2]

[1]Department of Physics and Astronomy, Macquarie University
[2]Department of Physics, University of Toronto

Abstract

We introduce a simple methodology to calculate the effects of self- and cross-phase modulation in SPDC photon generation. We show that these processes make SPDC less efficient in the low spatio-temporal mode number limit.

Phase matching and energy conservation in photon generation

In recent experiments [1, 2] bright sources of squeezed light have been engineered using $\chi^{(2)}$ nonlinearities. These sources cannot be described perturbatively and one needs to worry about higher order effects (e.g. χ^3) that could also affect the states generated.

χ^2 nonlinearities

Three modes A, B, C interact via a $\chi^{(2)}$ nonlinear susceptibility

$$H_2 \sim \int dz \chi^{(2)}(z) \left(\sum_{M=A,B,C} a_M(\omega_M) e^{i(\bar{k}_M + \delta k_M)z - i(\bar{\omega}_M + \delta\omega_M)t} + \text{H.c.} \right)^3$$

where $\bar{k}_M = \bar{\omega}_M / \underbrace{v_M^{(p)}}_{\text{phase vel.}}$ and $\delta k_M = \delta\omega_M / \underbrace{v_M^{(g)}}_{\text{group vel.}}$. For

$$\bar{\omega}_A + \bar{\omega}_B = \bar{\omega}_C \text{ and } \bar{k}_A + \bar{k}_B = \bar{k}_C,$$

the only term that is relevant is $H_{\text{SPDC}} \sim a_A^\dagger a_B^\dagger a_C + \text{H.c.}$

χ^3 nonlinearities

Three modes A, B, C interact via a $\chi^{(3)}$ nonlinear susceptibility

$$H_3 \sim \int dz \chi^{(3)}(z) \left(\sum_{M=A,B,C} a_M(\omega_M) e^{i(\bar{k}_M + \delta k_M)z - i(\bar{\omega}_M + \delta\omega_M)t} + \text{H.c.} \right)^4$$

- Terms like $a_A^\dagger a_A a_C^\dagger a_C$ always survive.
- If mode C is a strong coherent state it will create a nonlinear index of refraction for itself $H_{\text{SPM}} \sim a_C^\dagger a_C a_C^\dagger a_C$: *self-phase modulation (SPM).*
- It will also cause a nonlinear index of refraction for the other modes $H_{\text{XPM}} \sim a_A^\dagger a_A a_C^\dagger a_C$: *cross-phase modulation (XPM)*

Accounting for the effects of SPM and XPM

In the strong driven regime one needs to consider both the photon generation via SPDC and the SPM and XPM:

$$H = H_{\text{SPDC}} + H_{\text{XPM}} + H_{\text{SPM}}.$$

If the pump is a strong coherent state $a_C(\omega_C) \to \langle a_C(\omega_C) \rangle = \alpha_C(\omega_C)$

$$H_{\text{SPDC}} \sim \int dz \chi^{(2)}(z) \int d\omega_A d\omega_B d\omega_P e^{i(\delta k_P - \delta k_A - \delta k_B)z} e^{i(\delta\omega_A + \delta\omega_B - \delta\omega_P)t} \alpha_C(\omega_C) a_A^\dagger(\omega_A) a_B^\dagger(\omega_B),$$

- Free propagation is accounted for by the phases $e^{i(\delta k z - \delta\omega t)}$.
- Go to an interaction picture with respect to H_{XPM} and H_{SPM}.
- This is done by including an extra propagation phase

$$\Gamma_{A/B}(z,t) = 2 \int^t dt'' \chi^{(3)}(z - v_{A/B}(t - t'')) |\phi_C(z - v_{A/B}(t - t''), t'')|^2,$$

- $\chi^3(z)$ describes the spatial profile of the XPM nonlinearity.
- For the pump mode undergoing SPM, one should replace $v_{A/B}$ by v_P and remove the prefactor of 2.
- $\phi_P(z,t) = \int \frac{d\omega_P}{\sqrt{2\pi v_P}} \alpha(\omega_P) e^{-i(\omega_P - \bar{\omega}_P)(t - z/v_P)}$ is the pump profile in real space.

Upon the inclusion of SPM and XPM and taking

$$\alpha(\omega_c) = \frac{\tau}{\sqrt{\pi}} \exp(-\tau^2 \delta\omega_c^2)$$

we can rewrite the interaction picture Hamiltonian as

$$H_I(t) = -\hbar\varepsilon \int d\omega_A d\omega_B d\omega_P F(\omega_A, \omega_B, \omega_P) e^{i(\delta\omega_A + \delta\omega_B - \delta\omega_P)t} a^\dagger(\omega_A) b^\dagger(\omega_B) + \text{H.c.}$$

where $F(\omega_A, \omega_B, \omega_P) = \mathcal{F}\left(-(\delta\omega_A/v_A + \delta\omega_B/v_B), \omega_P\right)$, and

$$\mathcal{F}(q, \omega_P) = \int \frac{dz}{\sqrt{2\pi}} \frac{dt}{\sqrt{2\pi}} \mathcal{G}(z,t) e^{i(qz + \delta\omega_P t)},$$

$$\mathcal{G}(z,t) = \frac{\chi^{(3)}(z)}{L} e^{-(z - v_P t)^2/(4 v_P^2 \tau^2)} e^{-i\Gamma(z,t)}$$

are a 2D Fourier transform pair in the spaces (q, ω_P), (z, t) and

$$\Gamma = \Gamma_A + \Gamma_B - \Gamma_P.$$

The joint spectral amplitude is:

$$J_1(\omega_A, \omega_B) = F(\omega_A, \omega_B, \omega_A + \omega_P).$$

Consequences for SPDC

- The spectral structure: the Schmidt decomposition $J_1(\omega_A, \omega_B) = \Sigma_\lambda s_\lambda f_\lambda(\omega_A) j_\lambda(\omega_B)$.
- The Schmidt coefficients satisfy $\Sigma_\lambda s_\lambda^2 = \int d\omega_A d\omega_B |J_1(\omega_A, \omega_B)|^2$.
- XPM and SPM will not affect the norm of the JSA since

$$\begin{aligned} \int d\omega_A d\omega_B |J_1(\omega_A, \omega_B)|^2 &= \mathcal{J} \int d\omega_P dq |\mathcal{F}(q, \omega_P)|^2 \\ &= \mathcal{J} \int dz dt |\mathcal{G}(q, \omega_P)|^2, \end{aligned}$$

  where $\mathcal{J} = \frac{v_A v_B}{|v_A - v_B|}$ is the Jacobian of $(\omega_P = \omega_A + \omega_B, q = -(\delta\omega_A/v_A + \delta\omega_B/v_B))$ and we used Parseval's theorem.
- The number of photons will be smaller than than would be predicted if XPM and SPM where ignored since

$$\begin{aligned} \langle \tilde{N}_{A/B}^{\text{no XPM-SPM}} \rangle = \sum_\lambda \sinh^2\left(\sqrt{s_\lambda^2}\right) &\leq \sinh^2\left(\sqrt{\sum_\lambda s_\lambda^2}\right) \\ &= \sinh^2\left(\sqrt{\tilde{s}_0^2}\right) = \langle N_{A/B} \rangle. \end{aligned}$$

  In the last derivation time-ordering effects have been assumed to be negligible [3].
  XPM and SPM leads to additional Schmidt modes and reduces estimulation

References

[1] G. Harder *et al.*, "Single-Mode Parametric-Down-Conversion States with 50 Photons as a Source for Mesoscopic Quantum Optics", Phys. Rev. Lett. **116**, 143601 (2016).

[2] S. Lemieux *et al.*, "Engineering the Frequency Spectrum of Bright Squeezed Vacuum via Group Velocity Dispersion in an SU(1,1) Interferometer", Phys. Rev. Lett. **117**, 183601 (2016).

[3] N. Quesada and J.E. Sipe, "Time-ordering effects in the generation of entangled photons using nonlinear optical processes", Phys. Rev. Lett. **114** , 093903 (2015).