Second Year B.S. (Honors) 2023-2024

Department of Applied Mathematics, University of Dhaka

Course Title: Math Lab II (Fortran), Course No.: AMTH 250

Assignment 06

Name: Naymun Rahman **Roll:** AE-123-039 **Group:** B

Use FORTRAN to solve each of the following problems.

1. Solve the following linear system

$$
\begin{array}{rrrrrrl}
4x_1 & -x_2 & & -x_4 & & & = 0 \\
-x_1 & +4x_2 & -x_3 & & -x_5 & & = 5 \\
& -x_2 & +4x_3 & & & -x_6 & = 0 \\
-x_1 & & & +4x_4 & -x_5 & & = 6 \\
& -x_2 & & -x_4 & +4x_5 & -x_6 & = -2 \\
& & -x_3 & & -x_5 & +4x_6 & = 6
\end{array}
$$

using Jacobi iterative method with $\bar{x}^{(0)} = \bar{0}$ and $TOL = 10^{-3}$ in the l_∞ norm. Show your result in a tabular form with proper headings for each column.

2. The linear system

No x1 x2 x3
1
2
3

$$
\begin{aligned}
2x_1 - x_2 + x_3 &= -1 \\
2x_1 + 2x_2 + 2x_3 &= 4 \\
-x_1 - x_2 + 2x_3 &= -5
\end{aligned}
$$

has the solution $(1, 2, -1)^t$.

(a) Show that the Jacobi method with $\bar{x}^{(0)} = \bar{0}$ fails to give a good approximation after 25 iterations.

(b) Use the Gauss-Seidel method with $\bar{x}^{(0)} = \bar{0}$ to approximate the solution to the linear system within 10^{-5} in the l_∞ norm.

3. Solve the following system of equations by using SOR iterative method with $\omega = 1.1$ (correct up to 5 decimal places):

$$
\begin{aligned}
4x_1 - x_2 &= 3 \\
-x_1 + 4x_2 - x_3 &= 2 \\
-x_2 + 4x_3 &= 3 \\
4x_4 - x_5 &= 3 \\
-x_4 + 4x_5 - x_6 &= 2 \\
-x_5 + 4x_6 &= 3
\end{aligned}
$$

Show your result in a tabular form with proper headings for each column.

4. Consider the vector space R^3 with the Euclidean inner product. Apply the Gram-Schmidt orthogonalization process to transform the basis $u_1 = (1, 1, 1)$, $u_2 = (1, 1, 0)$ and $u_3 = (1, 0, 0)$ into an orthonormal basis for R^3.

5.

$$\begin{aligned} 2x_1 - x_2 + x_3 &= -1 \\ 2x_1 + 2x_2 + 2x_3 &= 4 \\ -x_1 - x_2 + 2x_3 &= -5 \end{aligned}$$

(a) Use Gaussian elimination method to find the solution of the above system.

(b) Write the system in $A\bar{x} = \bar{b}$ form, and find the LU decomposition of matrix A, and hence solve the system.