CENG 462 - Artificial Intelligence
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Homework 2

1. **a.** We define the variables to be the 6 courses,

$$X = \{CENG111, CENG213, CENG223, CENG315, CENG331, CENG351\}$$

. There are 6 possible classroom-time pairs available:

$$\begin{gathered} BMB1-09:30, BMB1-13:30, \\ BMB2-09:30, BMB2-13:30, \\ BMB3-13:30, BMB3-16:30. \end{gathered}$$

The exam of each course can be assigned any one of these pairs. So, for the domain values of each course we can choose a representation like this: take the digit part of the classroom name and combine it with the hour part of the time slot. For example, domain value denoting BMB1-09:30 would be 109. This numeric representation is chosen so that constraints can be written in a concise way.
As a result the domain of each variable is the set

$$\begin{aligned} D_i = \{&109, 113, \\ &209, 213, \\ &313, 316\}. \end{aligned}$$

We can divide all the course pairs into 2 categories:
1. Different-grade course pairs, i.e. those whose codes start with different digits
The only constraint for a pair of different-grade courses is that their exams not take place in the same classroom at the same time. Thus the possible assignments for each pair are permutations of the domain values, resulting in a total of $6 \times 5 = 30$ possible assignments.

There are 11 such pairs, hence 11 constraints associated with them.

$$\begin{aligned} C_{diff} = \{&CENG111 \neq CENG213, CENG111 \neq CENG223, CENG111 \neq CENG315, \\ &CENG111 \neq CENG331, CENG111 \neq CENG351, \\ &CENG213 \neq CENG315, CENG213 \neq CENG331, CENG213 \neq CENG351, \\ &CENG223 \neq CENG315, CENG223 \neq CENG331, CENG223 \neq CENG351\}. \end{aligned}$$

2. Pairs of courses of the same grade: (CENG213, CENG223),
(CENG315, CENG331), (CENG315, CENG351), (CENG331, CENG351)

Clearly the aforementioned constraint applies to these course pairs as well. However, the constraint specific to these pairs covers that constraint, so there is no need for repetition.

$$
\begin{aligned}
C_{same} = \{ & CENG213\, mod\, 10 \neq CENG223\, mod\, 10, \\
& CENG315\, mod\, 10 \neq CENG331\, mod\, 10, \\
& CENG315\, mod\, 10 \neq CENG351\, mod\, 10, \\
& CENG331\, mod\, 10 \neq CENG351\, mod\, 10\}.
\end{aligned}
$$

Due to these constraints, exams of the same grade cannot be held at the same time (For example, CENG223 cannot be assigned 109 while CENG213 is assigned 209, as $109\, mod\, 10 = 209\, mod\, 10$).

For these course pairs, the set of possible assignments are permutations of domains, excluding the pairs

(109, 209), (209, 109),
(113, 213), (213, 113),
(313, 213), (213, 313),
(113, 313), (313, 113),

thus allowing a total of 30 - 8 = 22 possible assignments for each pair.

For parts b and c, let's use indices from 1 to 6 for domain values in the order they are given in part a for the sake of brevity (1 for 109, 2 for 113 and so on).

b. Forward-checking (No more possible assignments are left for CENG351.)

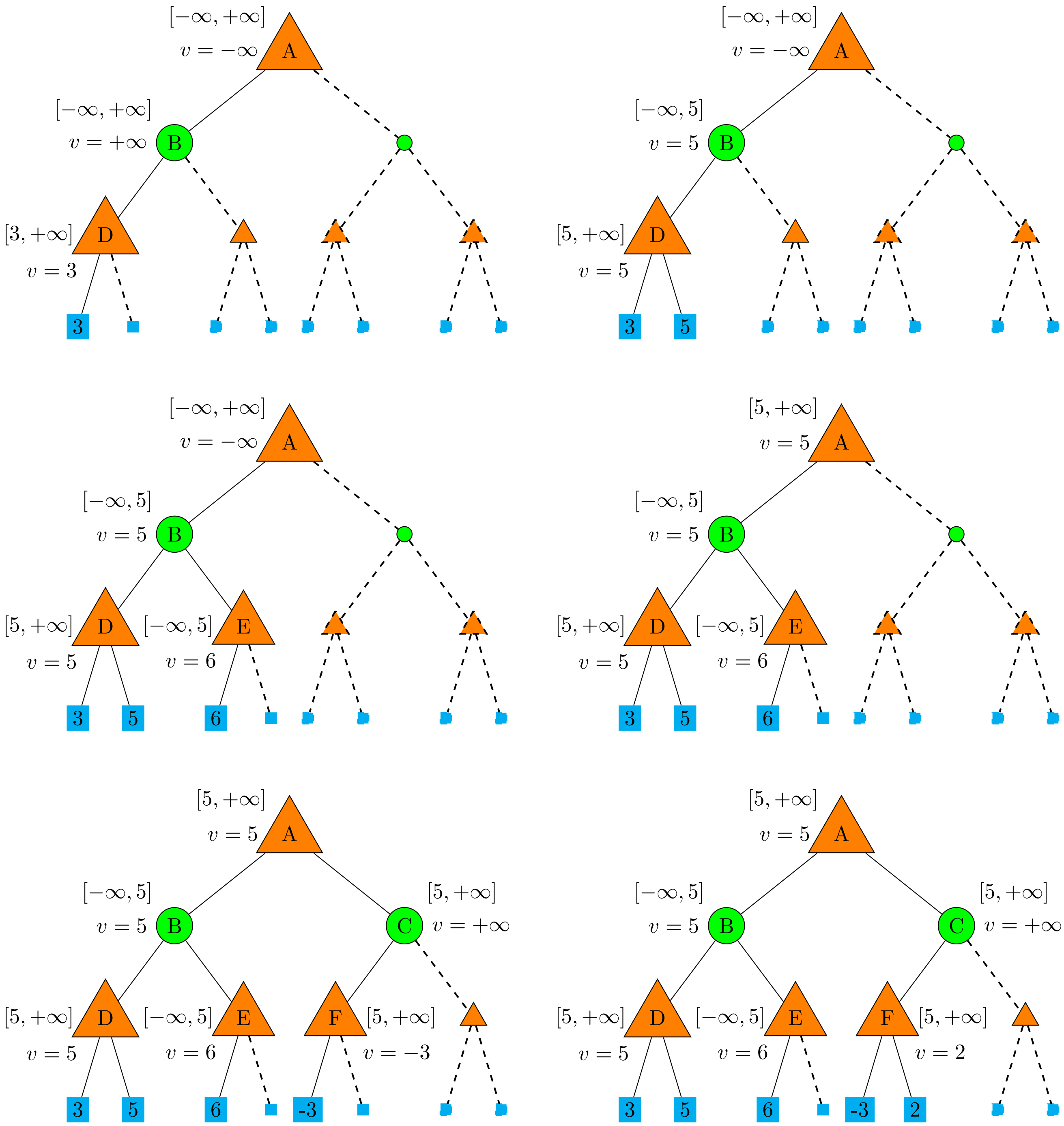
CENG111	CENG213	CENG223	CENG315	CENG331	CENG351
123456	123456	123456	123456	123456	123456
6	12345x	12345x	12345x	12345x	12345x
6	1x345x	1x345x	2	1x3xxx	1x3xxx
6	xx345x	xx345x	2	1	xxxxxx

c. Arc-consistency

CENG111	CENG213	CENG223	CENG315	CENG331	CENG351
123456	123456	123456	123456	123456	123456
6	12345x	12345x	12345x	12345x	12345x
6	1x345x	1x345x	2	1x3xxx	1x3xxx

When we apply the arc-consistency check to the last row, for the pair CENG331 and CENG351, there is no value left for CENG351 that makes allowable assignments with any of the values left for CENG331 and vice versa. Therefore, with arc-consistency we detected failure for this example one step earlier than we did with forward-checking.

2.



$[-\infty, +\infty]$
$v = -\infty$
A
$[-\infty, +\infty]$
$v = +\infty$
B
$[3, +\infty]$
D
$v = 3$
3
$[-\infty, +\infty]$
$v = -\infty$
A
$[-\infty, 5]$
$v = 5$
B
$[5, +\infty]$
D
$v = 5$
3
5
$[-\infty, +\infty]$
$v = -\infty$
A
$[-\infty, 5]$
$v = 5$
B
$[5, +\infty]$
D
$v = 5$
$[-\infty, 5]$
E
$v = 6$
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6
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$v = 5$
A
$[-\infty, 5]$
$v = 5$
B
$[5, +\infty]$
D
$v = 5$
$[-\infty, 5]$
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$[5, +\infty]$
$v = 5$
A
$[-\infty, 5]$
$v = 5$
B
C
$[5, +\infty]$
$v = +\infty$
$[5, +\infty]$
D
$v = 5$
$[-\infty, 5]$
E
$v = 6$
F
$[5, +\infty]$
$v = -3$
3
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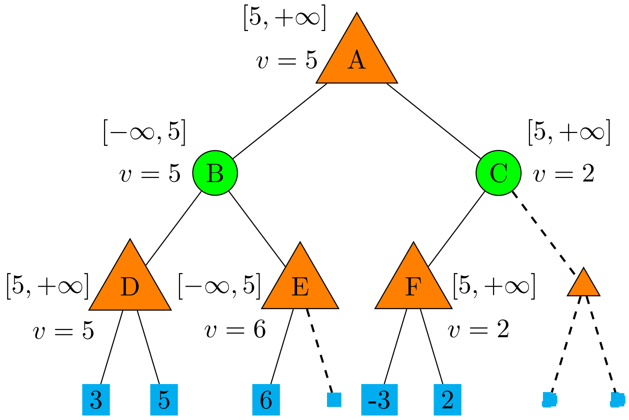
6
-3
$[5, +\infty]$
$v = 5$
A
$[-\infty, 5]$
$v = 5$
B
C
$[5, +\infty]$
$v = +\infty$
$[5, +\infty]$
D
$v = 5$
$[-\infty, 5]$
E
$v = 6$
F
$[5, +\infty]$
$v = 2$
3
5
6
-3
2



MAX's decision at the root would be to move to the node with $v = 5$, namely B.

3. From the definition of implication, we know that $K \implies L$ is true unless K is true and L is false. Using the knowledge base at hand, K can, in fact, be proven to be true. Therefore, truth of the statement $K \implies L$ depends on the value of L. However, since there is no information about L in the KB, proving this statement is impossible. Regardless, for the purpose of solving the question, I will make the assumption that K $\implies$ L is given as a rule in the KB, and we are actually supposed to prove KB $\models$ L (The formulation of the problem in this way actually seems to be more exemplary of the typical usage of forward-chaining and backward-chaining algorithms. In fact, the textbook states, "The forward-chaining algorithm PL-FC-ENTAILS?(KB, q) determines if a **single proposition symbol** q–the query–is entailed by a knowledge base of definite clauses." [1]).

a.

clause												
$K \implies L$	1	1	1	1	1	1	1	1	1	1	1	1
$I \land J \implies K$	2	2	2	2	2	2	2	2	1	1	1	0
$G \land H \implies I$	2	2	2	2	2	2	2	1	2	1	0	0
$H \land D \implies J$	2	2	2	2	1	1	1	0	0	0	0	0
$E \land H \implies G$	2	2	2	2	2	1	1	0	0	0	0	0
$E \land F \implies H$	2	2	2	2	2	1	0	0	0	0	0	0
$G \land A \implies E$	1	1	1	1	1	1	1	1	0	0	0	0
$A \land B \implies E$	2	1	0	0	0	0	0	0	0	0	0	0
$B \land C \implies F$	2	2	1	0	0	0	0	0	0	0	0	0
agenda	$ABCD$	BCD	CDE	DEF	EF	F	H	JG	JI	J	K	L

Lastly, we pop the final element from the agenda and it is L, the proposition that we had been trying to infer. Therefore, we have shown that $KB \models L$.

b.

$\color{green}{\boldsymbol{K \Longrightarrow L}}$ $I \wedge J \Longrightarrow K$ $G \wedge H \Longrightarrow I$ $H \wedge D \Longrightarrow J$ $E \wedge H \Longrightarrow G$ $E \wedge F \Longrightarrow H$ $G \wedge A \Longrightarrow E$ $A \wedge B \Longrightarrow E$ $B \wedge C \Longrightarrow F$	$\boldsymbol{K \Longrightarrow L}$ $\color{green}{\boldsymbol{I \wedge J \Longrightarrow K}}$ $G \wedge H \Longrightarrow I$ $H \wedge D \Longrightarrow J$ $E \wedge H \Longrightarrow G$ $E \wedge F \Longrightarrow H$ $G \wedge A \Longrightarrow E$ $A \wedge B \Longrightarrow E$ $B \wedge C \Longrightarrow F$	$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\color{green}{\boldsymbol{G \wedge H \Longrightarrow I}}$ $H \wedge D \Longrightarrow J$ $E \wedge H \Longrightarrow G$ $E \wedge F \Longrightarrow H$ $G \wedge A \Longrightarrow E$ $A \wedge B \Longrightarrow E$ $B \wedge C \Longrightarrow F$	$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\boldsymbol{G \wedge H \Longrightarrow I}$ $H \wedge D \Longrightarrow J$ $\color{green}{\boldsymbol{E \wedge H \Longrightarrow G}}$ $E \wedge F \Longrightarrow H$ $G \wedge A \Longrightarrow E$ $A \wedge B \Longrightarrow E$ $B \wedge C \Longrightarrow F$	$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\boldsymbol{G \wedge H \Longrightarrow I}$ $H \wedge D \Longrightarrow J$ $\boldsymbol{E \wedge H \Longrightarrow G}$ $E \wedge F \Longrightarrow H$ $\color{green}{\boldsymbol{G \wedge A \Longrightarrow E}}$ $A \wedge B \Longrightarrow E$ $B \wedge C \Longrightarrow F$
$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\boldsymbol{G \wedge H \Longrightarrow I}$ $H \wedge D \Longrightarrow J$ $\boldsymbol{E \wedge H \Longrightarrow G}$ $E \wedge F \Longrightarrow H$ $G \wedge A \Longrightarrow E$ $A \wedge B \Longrightarrow E$ $B \wedge C \Longrightarrow F$	$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\boldsymbol{G \wedge H \Longrightarrow I}$ $H \wedge D \Longrightarrow J$ $\boldsymbol{E \wedge H \Longrightarrow G}$ $E \wedge F \Longrightarrow H$ $G \wedge A \Longrightarrow E$ $\color{red}{\boldsymbol{A \wedge B \Longrightarrow E}}\checkmark$ $B \wedge C \Longrightarrow F$	$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\boldsymbol{G \wedge H \Longrightarrow I}$ $H \wedge D \Longrightarrow J$ $\boldsymbol{E \wedge H \Longrightarrow G}$ $\color{green}{\boldsymbol{E \wedge F \Longrightarrow H}}$ $G \wedge A \Longrightarrow E$ $\boldsymbol{A \wedge B \Longrightarrow E}\checkmark$ $B \wedge C \Longrightarrow F$	$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\boldsymbol{G \wedge H \Longrightarrow I}$ $H \wedge D \Longrightarrow J$ $\boldsymbol{E \wedge H \Longrightarrow G}$ $\boldsymbol{E \wedge F \Longrightarrow H}$ $G \wedge A \Longrightarrow E$ $\boldsymbol{A \wedge B \Longrightarrow E}\checkmark$ $\color{red}{\boldsymbol{B \wedge C \Longrightarrow F}}\checkmark$	$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\boldsymbol{G \wedge H \Longrightarrow I}$ $H \wedge D \Longrightarrow J$ $\boldsymbol{E \wedge H \Longrightarrow G}$ $\color{red}{\boldsymbol{E \wedge F \Longrightarrow H}}\checkmark$ $G \wedge A \Longrightarrow E$ $\boldsymbol{A \wedge B \Longrightarrow E}\checkmark$ $\boldsymbol{B \wedge C \Longrightarrow F}\checkmark$
$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\boldsymbol{G \wedge H \Longrightarrow I}$ $H \wedge D \Longrightarrow J$ $\color{red}{\boldsymbol{E \wedge H \Longrightarrow G}}\checkmark$ $\boldsymbol{E \wedge F \Longrightarrow H}\checkmark$ $G \wedge A \Longrightarrow E$ $\boldsymbol{A \wedge B \Longrightarrow E}\checkmark$ $\boldsymbol{B \wedge C \Longrightarrow F}\checkmark$	$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\color{red}{\boldsymbol{G \wedge H \Longrightarrow I}}\checkmark$ $H \wedge D \Longrightarrow J$ $\boldsymbol{E \wedge H \Longrightarrow G}\checkmark$ $\boldsymbol{E \wedge F \Longrightarrow H}\checkmark$ $G \wedge A \Longrightarrow E$ $\boldsymbol{A \wedge B \Longrightarrow E}\checkmark$ $\boldsymbol{B \wedge C \Longrightarrow F}\checkmark$	$\boldsymbol{K \Longrightarrow L}$ $\boldsymbol{I \wedge J \Longrightarrow K}$ $\boldsymbol{G \wedge H \Longrightarrow I}\checkmark$ $\color{red}{\boldsymbol{H \wedge D \Longrightarrow J}}\checkmark$ $\boldsymbol{E \wedge H \Longrightarrow G}\checkmark$ $\boldsymbol{E \wedge F \Longrightarrow H}\checkmark$ $G \wedge A \Longrightarrow E$ $\boldsymbol{A \wedge B \Longrightarrow E}\checkmark$ $\boldsymbol{B \wedge C \Longrightarrow F}\checkmark$	$\boldsymbol{K \Longrightarrow L}$ $\color{red}{\boldsymbol{I \wedge J \Longrightarrow K}}\checkmark$ $\boldsymbol{G \wedge H \Longrightarrow I}\checkmark$ $\boldsymbol{H \wedge D \Longrightarrow J}\checkmark$ $\boldsymbol{E \wedge H \Longrightarrow G}\checkmark$ $\boldsymbol{E \wedge F \Longrightarrow H}\checkmark$ $G \wedge A \Longrightarrow E$ $\boldsymbol{A \wedge B \Longrightarrow E}\checkmark$ $\boldsymbol{B \wedge C \Longrightarrow F}\checkmark$	$\color{red}{\boldsymbol{K \Longrightarrow L}}\checkmark$ $\boldsymbol{I \wedge J \Longrightarrow K}\checkmark$ $\boldsymbol{G \wedge H \Longrightarrow I}\checkmark$ $\boldsymbol{H \wedge D \Longrightarrow J}\checkmark$ $\boldsymbol{E \wedge H \Longrightarrow G}\checkmark$ $\boldsymbol{E \wedge F \Longrightarrow H}\checkmark$ $G \wedge A \Longrightarrow E$ $\boldsymbol{A \wedge B \Longrightarrow E}\checkmark$ $\boldsymbol{B \wedge C \Longrightarrow F}\checkmark$

Green statements are the ones explored at a given step, red statements are those whose consequents get confirmed.

References

[1] Russell, S. J., Norvig, P., & Davis, E. (2010). *Artificial intelligence: a modern approach.* 3rd ed. Upper Saddle River, NJ: Prentice Hall.