CENG 460 - Introduction to Robotics Narmin Aliyeva
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Homework 2 - Mobile Robot Kinematics

1. The robot body pose relative to the world frame O can be denoted by the transformation matrix ${}^{O}T_F$:

$$
{}^{O}T_F = \begin{bmatrix} cos\theta & -sin\theta & x \\ sin\theta & cos\theta & y \\ 0 & 0 & 1 \end{bmatrix}
$$

The positions of the centers of the three wheels in the body frame F:

$$
{}^{F}p_A = \begin{bmatrix} Dcos60° \\ Dsin60° \end{bmatrix} \qquad {}^{F}p_B = \begin{bmatrix} -D \\ 0 \end{bmatrix} \qquad {}^{F}p_C = \begin{bmatrix} Dcos60° \\ -Dsin60° \end{bmatrix}
$$

The positions of the centers of the three wheels in the world frame O:

$$
\begin{aligned}
p_A &= {}^{O}T_F\, {}^{F}p_A = \begin{bmatrix} \frac{1}{2}Dcos\theta - \frac{\sqrt{3}}{2}Dsin\theta + x \\ \frac{1}{2}Dsin\theta + \frac{\sqrt{3}}{2}Dcos\theta + y \end{bmatrix} \\
p_B &= {}^{O}T_F\, {}^{F}p_B = \begin{bmatrix} -Dcos\theta + x \\ -Dsin\theta + y \end{bmatrix} \\
p_C &= {}^{O}T_F\, {}^{F}p_C = \begin{bmatrix} \frac{1}{2}Dcos\theta + \frac{\sqrt{3}}{2}Dsin\theta + x \\ \frac{1}{2}Dsin\theta - \frac{\sqrt{3}}{2}Dcos\theta + y \end{bmatrix}
\end{aligned}
$$

2. Taking the derivatives of the wheel positions with respect to time, we get

$$\dot{p}_A = \begin{bmatrix} -\frac{1}{2}D\dot{\theta}(sin\theta + \sqrt{3}cos\theta) + \dot{x} \\ \frac{1}{2}D\dot{\theta}(cos\theta - \sqrt{3}sin\theta) + \dot{y} \end{bmatrix}$$
$$\dot{p}_B = \begin{bmatrix} D\dot{\theta}sin\theta + \dot{x} \\ -D\dot{\theta}cos\theta + \dot{y} \end{bmatrix}$$
$$\dot{p}_C = \begin{bmatrix} -\frac{1}{2}D\dot{\theta}(sin\theta - \sqrt{3}cos\theta) + \dot{x} \\ \frac{1}{2}D\dot{\theta}(cos\theta + \sqrt{3}sin\theta) + \dot{y} \end{bmatrix}$$

Projections of x_F and y_F axes of the body frame F onto wheel velocity vectors $\overrightarrow{v_A}$, $\overrightarrow{v_B}$ and $\overrightarrow{v_C}$:

$$^{A}P_F = \begin{bmatrix} -cos30^\circ \\ cos60^\circ \end{bmatrix} \qquad ^{B}P_F = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad ^{C}P_F = \begin{bmatrix} cos30^\circ \\ cos60^\circ \end{bmatrix}$$

(Directions of wheel velocities are chosen so that positive wheel speeds will make the robot spin counterclockwise.)

To find the linear speed of each wheel, we project the previously found $\dot{p}_A$, $\dot{p}_B$ and $\dot{p}_C$ velocities (in the world frame) onto the vectors $\overrightarrow{v_A}$, $\overrightarrow{v_B}$, and $\overrightarrow{v_C}$, respectively:

$$\begin{aligned} v_A = \; & (^{A}P_F)^T \, {}^{F}R_O \, \dot{p}_A = \begin{bmatrix} -cos30^\circ cos60^\circ \end{bmatrix} \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \dot{p}_A \\ & = (-\frac{\sqrt{3}}{2}cos\theta - \frac{1}{2}sin\theta)\dot{x} + (-\frac{\sqrt{3}}{2}sin\theta + \frac{1}{2}cos\theta)\dot{y} + D\dot{\theta} \\ v_B = \; & (^{B}P_F)^T \, {}^{F}R_O \, \dot{p}_B = D\dot{\theta}sin^2\theta + \dot{x}sin\theta + D\dot{\theta}cos^2\theta - \dot{y}cos\theta \\ & = \dot{x}sin\theta - \dot{y}cos\theta + D\dot{\theta} \\ v_C = \; & (^{C}P_F)^T \, {}^{F}R_O \, \dot{p}_C = (\frac{\sqrt{3}}{2}cos\theta - \frac{1}{2}sin\theta)\dot{x} + (\frac{\sqrt{3}}{2}sin\theta + \frac{1}{2}cos\theta)\dot{y} + D\dot{\theta} \end{aligned}$$

Rotational speeds are thus

$$\begin{bmatrix} w_A \\ w_B \\ w_C \end{bmatrix} = \frac{1}{R} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{3}}{2R}cos\theta - \frac{1}{2R}sin\theta & -\frac{\sqrt{3}}{2R}sin\theta + \frac{1}{2R}cos\theta & \frac{D}{R} \\ \frac{1}{R}sin\theta & -\frac{1}{R}cos\theta & \frac{D}{R} \\ \frac{\sqrt{3}}{2R}cos\theta - \frac{1}{2R}sin\theta & \frac{\sqrt{3}}{2R}sin\theta + \frac{1}{2R}cos\theta & \frac{D}{R} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

3. See *hw3_script3.m*.

4. Integrating the returned result from the function implemented in Q3 using ode45, and plotting x, y, θ with respect to time for different values of w_A, w_B, w_C:
(a) all wheels rotate at the same speed ($w_A = w_B = w_C = 1$)

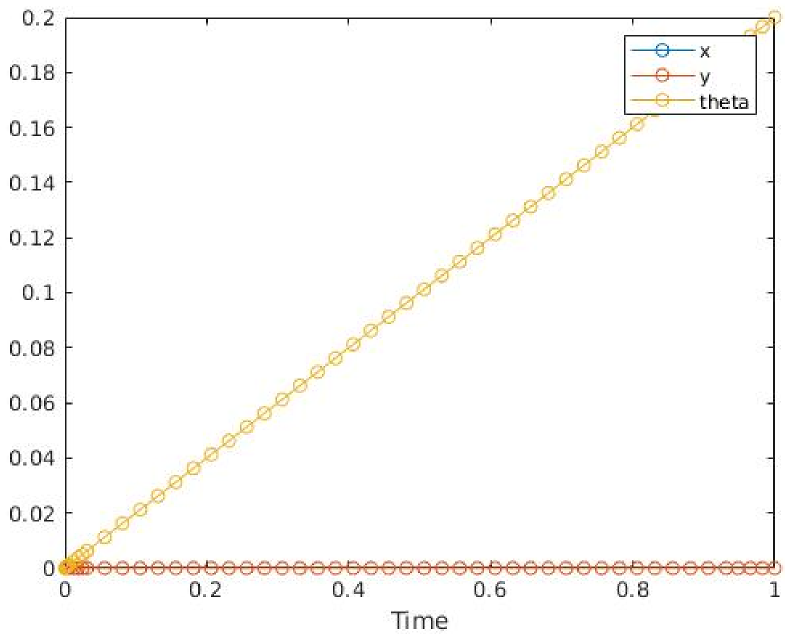

Figure 1: all wheels rotate at the same speed

As seen from Figure 1, only θ changes in value over time, translational velocity of the robot is zero. The robot is spinning with its position fixed.

(b) $w_A = 1$, $w_B = 0$, $w_C = -w_A = -1$

As seen from Figure 2, when two of the wheels have the opposite speed values, and the third one is not rotating, the robot follows a straight path with zero rotation.

(c) only one of the three wheels is rotating ($w_A = 1$, $w_B = w_C = 0$)

The robot follows a circular trajectory.

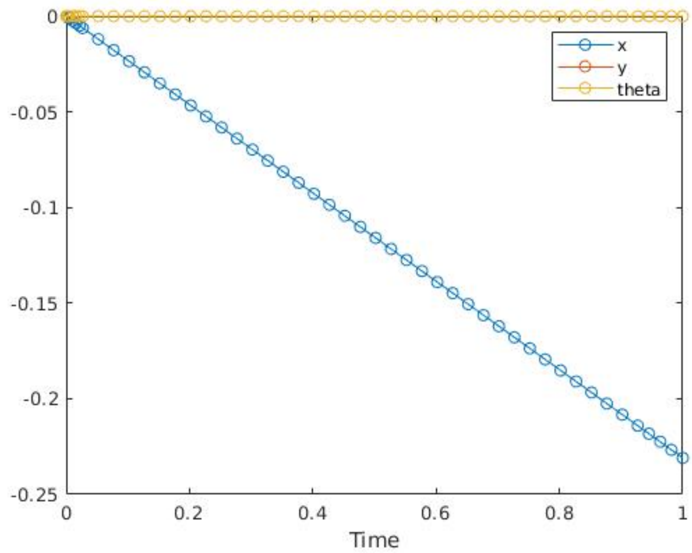

Figure 2: forward motion, no change in orientation

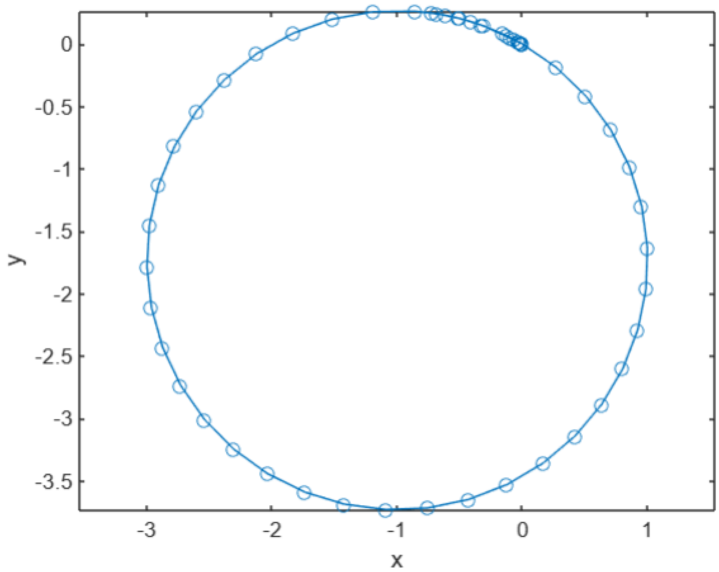

Figure 3: only one wheel is rotating

5. The pose of the end effector relative to frame O is derived by applying a sequence of transformations given by a rotation in 2D followed by a translation along the rotated x axis:

$$f(\theta_1, \theta_2, \theta_3) = {}^{O}T_E = R(\theta_1)t_x(a_1)R(\theta_2)t_x(a_2)R(\theta_3)t_x(a_3)$$

$$= \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 \\ sin\theta_1 & cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R(\theta_2)t_x(a_2)R(\theta_3)t_x(a_3)$$

$$= \begin{bmatrix} cos\theta_1 & -sin\theta_1 & a_1cos\theta_1 \\ sin\theta_1 & cos\theta_1 & a_1sin\theta_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_2 & -sin\theta_2 & a_2cos\theta_2 \\ sin\theta_2 & cos\theta_2 & a_2sin\theta_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_3 & -sin\theta_3 & a_3cos\theta_3 \\ sin\theta_3 & cos\theta_3 & a_3sin\theta_3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos(\theta_1+\theta_2+\theta_3) & -sin(\theta_1+\theta_2+\theta_3) & a_2cos(\theta_1+\theta_2) + a_1cos(\theta_1) + a_3cos(\theta_1+\theta_2+\theta_3) \\ sin(\theta_1+\theta_2+\theta_3) & cos(\theta_1+\theta_2+\theta_3) & a_2sin(\theta_1+\theta_2) + a_1sin(\theta_1) + a_3sin(\theta_1+\theta_2+\theta_3) \\ 0 & 0 & 1 \end{bmatrix}$$