Shu's notes on nominal and real APG's

1 Selection under log-normal abilities: Some formulae

Let the ability $\log(\mathbf{z}) \sim \boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu} = \begin{pmatrix} \mu_a \\ \mu_n \end{pmatrix}$, $\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_a^2 & \rho \cdot \sigma_a \sigma_n \\ \rho \cdot \sigma_a \sigma_n & \sigma_n^2 \end{pmatrix}$, $\sigma_a < \sigma_n$.

Define $\hat{w} = w_n / w_a$, $\hat{w} \cdot z_n > z_a \rightarrow$ non-agriculture; otherwise, agriculture.

Define $\sigma^* = \sqrt{\sigma_a^2 + \sigma_n^2 - 2\rho \cdot \sigma_a \sigma_n}$, $d_n = \frac{log\hat{w} + \mu_n - \mu_a}{\sigma^*}$, $d_a = -d_n = \frac{-log\hat{w} - \mu_n + \mu_a}{\sigma^*}$.

The non-agricultural employment share is $\ell_n = \Phi(d_n)$; and the agricultural employment share is $\ell_a = \Phi(d_a)$.

Φ is the CDF of normal distribution.

The expected log abilities are as follow:

$E\left[log(z_n) | \hat{w} \cdot z_n \geq z_a\right] = \mu_n + \frac{\sigma_n^2 - \rho \cdot \sigma_a \sigma_n}{\sigma^*} \cdot \lambda(d_n)$;

$E\left[log(z_a) | \hat{w} \cdot z_n < z_a\right] = \mu_a + \frac{\sigma_a^2 - \rho \cdot \sigma_a \sigma_n}{\sigma^*} \cdot \lambda(d_a)$.

$\lambda(x) = \frac{\phi(x)}{\Phi(x)}$ is the inverse Mills ratio.

To derive the expected abilities, we need the following approximation:

$log(E\left[z_n | \hat{w} \cdot z_n \geq z_a\right]) \approx E\left[log(z_n) | \hat{w} \cdot z_n \geq z_a\right] + \frac{1}{2} Var\left[log(z_n) | \hat{w} \cdot z_n \geq z_a\right] = \mu_n + \frac{\sigma_n^2 - \rho \cdot \sigma_a \sigma_n}{\sigma^*} \cdot \lambda(d_n) + \frac{1}{2}\left[\sigma_n^2 + \left(\frac{\sigma_n^2 - \rho \cdot \sigma_a \sigma_n}{\sigma^*}\right)^2 \cdot \lambda'(d_n)\right]$;

$log(E\left[z_a | \hat{w} \cdot z_n < z_a\right]) \approx E\left[log(z_a) | \hat{w} \cdot z_n < z_a\right] + \frac{1}{2} Var\left[log(z_a) | \hat{w} \cdot z_n < z_a\right] = \mu_a + \frac{\sigma_a^2 - \rho \cdot \sigma_a \sigma_n}{\sigma^*} \cdot \lambda(d_a) + \frac{1}{2}\left[\sigma_a^2 + \left(\frac{\sigma_a^2 - \rho \cdot \sigma_a \sigma_n}{\sigma^*}\right)^2 \cdot \lambda'(d_a)\right]$.

$\lambda'(x) = \frac{-x \cdot \phi(x) \cdot \Phi(x) - \phi^2(x)}{\Phi^2(x)}$ is the first order derivative of λ.

Next, take the exponents and we get the approximate expected abilities.

2 Selection and sectoral effective units of labor: Comparing the approximation to the exact solution.

There are two ways to compare the two approaches: approximation and large population simulation.

Since the gradient optimization algorithm works poorly on the large population simulation, the first method of comparison is to apply the time series of relative wage and capital from the approximation approach to the large population simulation without further optimization.

This method works very fast, so I choose the population size to be 1 million, iterate the code for 100 times, and take the averages of the 7 time series.

The results of this method is shown in Figure 1. The results from the approximation approach are in blue and those from the large population are in orange. In most cases, the blue parts are completely covered by the orange part, indicating the similarity between the results from the two approach. Table 1 shows the begin and end values of those series. The percentage difference is very small and less than 1% in almost all cells except for the end value for real APG, which is close to 1% though.

Figure 1: Method 1

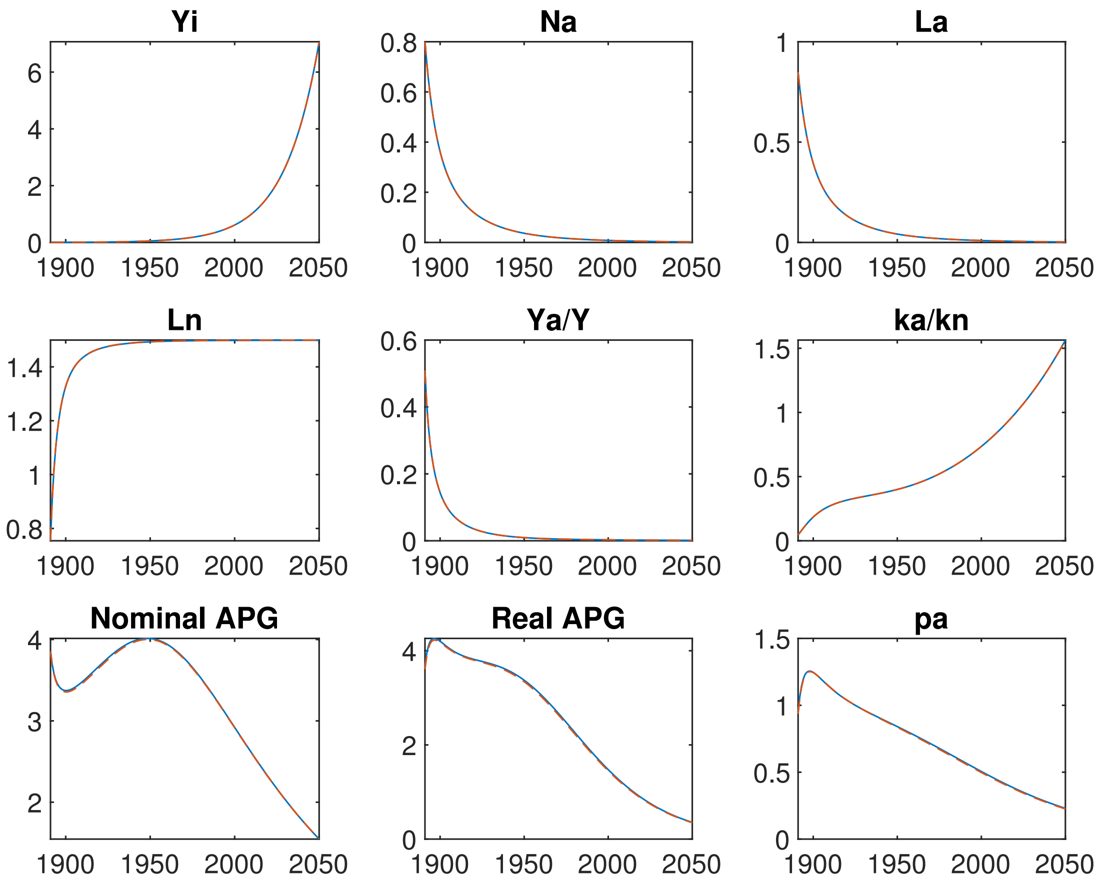


Table 1: Method1

Approach	Approx.		Large		% Diff.	
Series	Begin	End	Begin	End	Begin	End
Y_i, real GDP per capita (2010=1)	0.0006	7.0683	0.0006	7.0521	0.26%	-0.23%
N_a, agric. emp. share	0.8000	0.0019	0.8000	0.0019	0.00%	0.16%
Y_a/Y, agric. value added share	0.5088	0.0012	0.5093	0.0012	0.09%	0.06%
k_a/k_n, per-worker capital ratio	0.0454	1.5638	0.0454	1.5623	0.02%	-0.10%
Nominal APG	3.8612	1.5468	3.8535	1.5482	-0.20%	0.10%
Real APG	3.6609	0.3556	3.6053	0.3493	-1.52%	-1.75%
p_a (1935=1)	0.9481	0.2299	0.9356	0.2256	-1.32%	-1.84%

The second method is to compare the optimized outcomes from the two approach. To accelerate the speed of the code, I reduce the population size to be 0.1 million. The iteration is still 100. The results are shown in Figure 2 and Table 2. The differences between the two approaches become larger, but still all below 2%. From my point of view, this approximation works very well. When the population size is small, the deviation from these average results could be also larger than 1%. For example, one of large population simulations gives the beginning agricultural employment share at 10.73%, which is 1.36% lager than the average.

Figure 2: Method 2

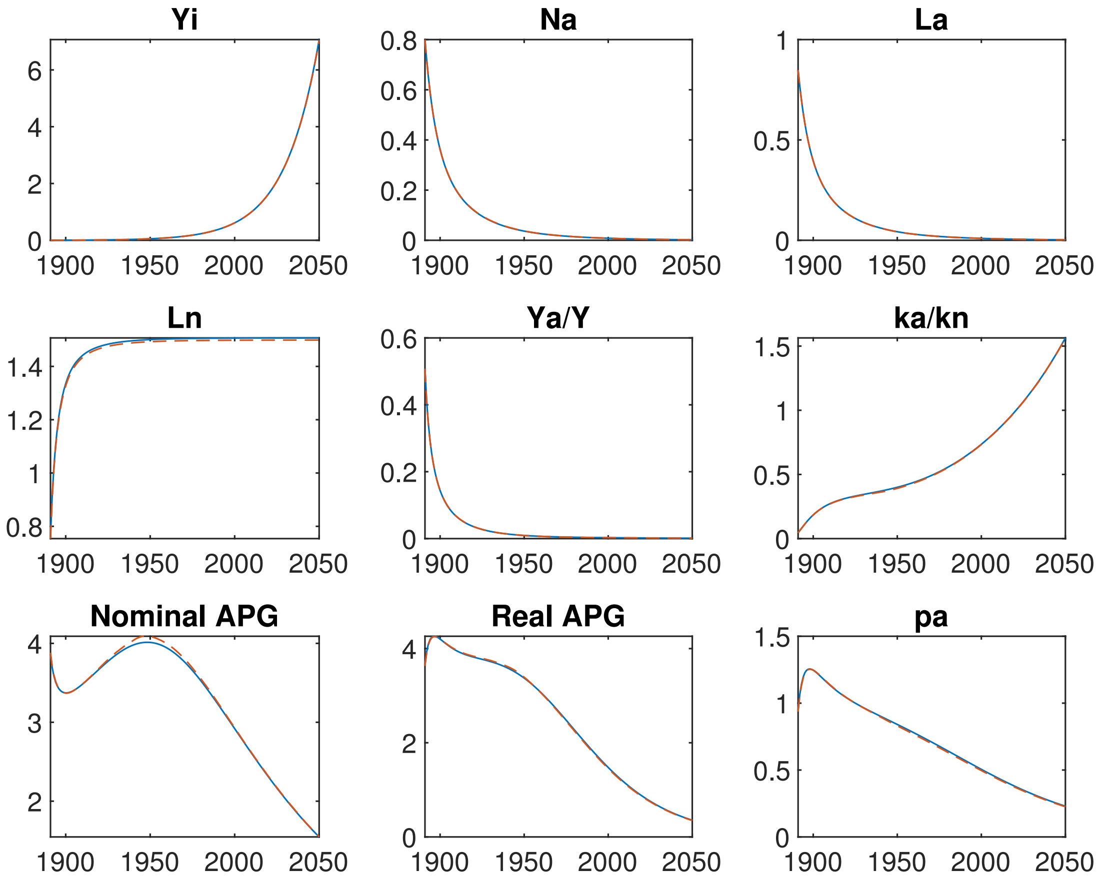


Table 2: Method 2

Approach	Approx.		Large		% Diff.	
Series	Begin	End	Begin	End	Begin	End
Y_i, real GDP per capita (2010=1)	0.0006	7.0683	0.0006	7.0396	-0.10%	-0.41%
N_a, agric. emp. share	0.8000	0.0019	0.8002	0.0019	0.02%	0.21%
Y_a/Y, agric. value added share	0.5088	0.0012	0.5078	0.0012	-0.20%	-0.06%
k_a/k_n, per-worker capital ratio	0.0454	1.5638	0.0451	1.5610	-0.67%	-0.18%
Nominal APG	3.8612	1.5468	3.8810	1.5510	0.51%	0.27%
Real APG	3.6609	0.3556	3.6330	0.3503	-0.76%	-1.48%
p_a (1935=1)	0.9481	0.2299	0.9361	0.2259	-1.27%	-1.74%

3 Comparison between Ramsey and Solow Model

This section compares the results from the Ramsey model and the Solow model. The difference between the two models is that in the Ramsey model the saving decision is a internal problem and households decide it every period; in the Solow model the saving rate is given exogenously. To make the Ramsey model work, I changed An0 from 2 to 1.5 and g(An) from 0.05 to 0.04. Otherwise, $N_a = 0.8$ cannot be achieved by any initial capital, K_0.

Figure 3 compares the Ramsey model and the Solow model with the saving rate equal to 30%. They are very close to each other in this scenario, except for the nominal APG. The trends in every subplot are almost the same for both models.

Figure 4 includes more variants of the Solow model with different saving rates. We can see the large effect of saving rates on the Solow model, but the differences exist mostly in levels not trends. So the differences in Y_i and P_a are not as large as in other variables, because they are somewhat normalized.

To see the reason behind the similarity between the two model, I calculate the saving rate in the Ramsey model as shown in Figure 5. The saving rate is 25.5% at the beginning ($N_a = 80\%$), quickly increases to 32.5%, then decrease to 29%, and finally decreases at a very slow speed. Thus, in the Ramsey model, the saving rate varies around 30% for the most of time.

Figure 3: Ramsey (solid) and Solow (dashed, saving rate = 30%)

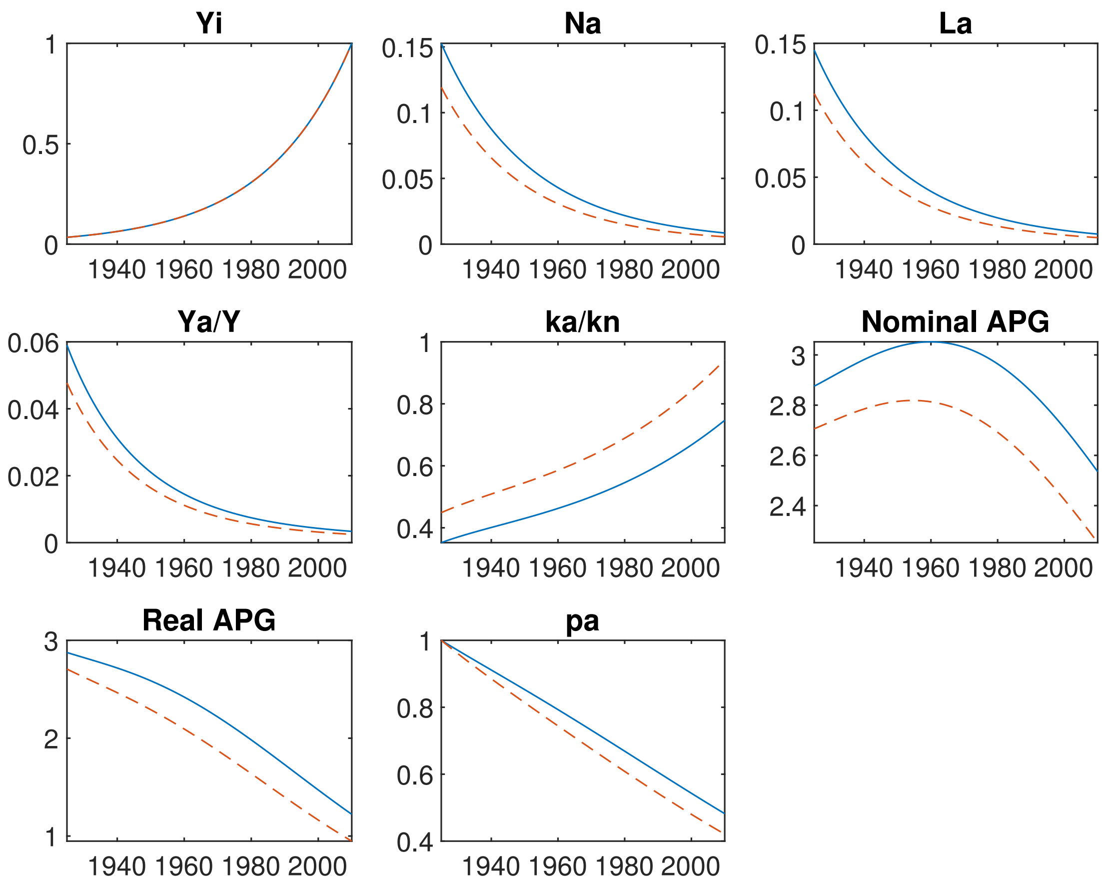


Figure 4: Ramsey (solid) and Solow (dashed, sr = 30% / dashed-dot, sr = 20% / dot-dot, sr = 40%)

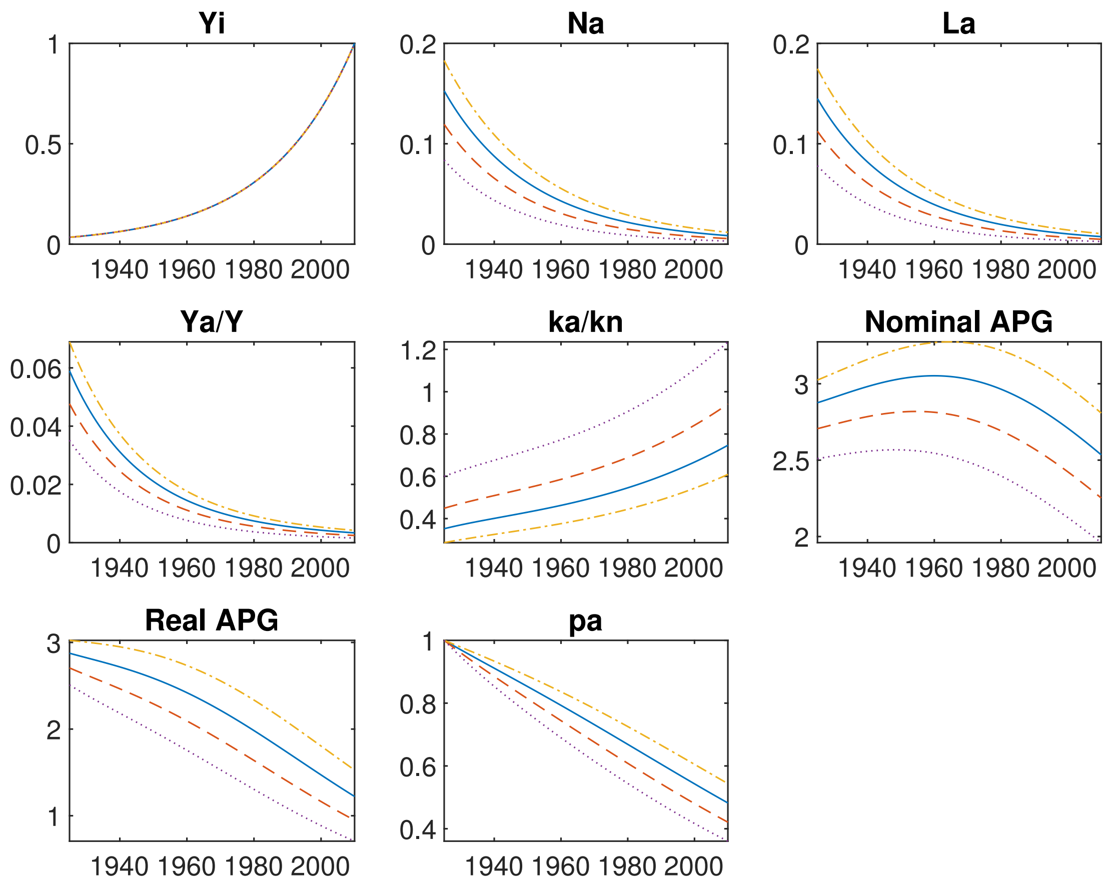
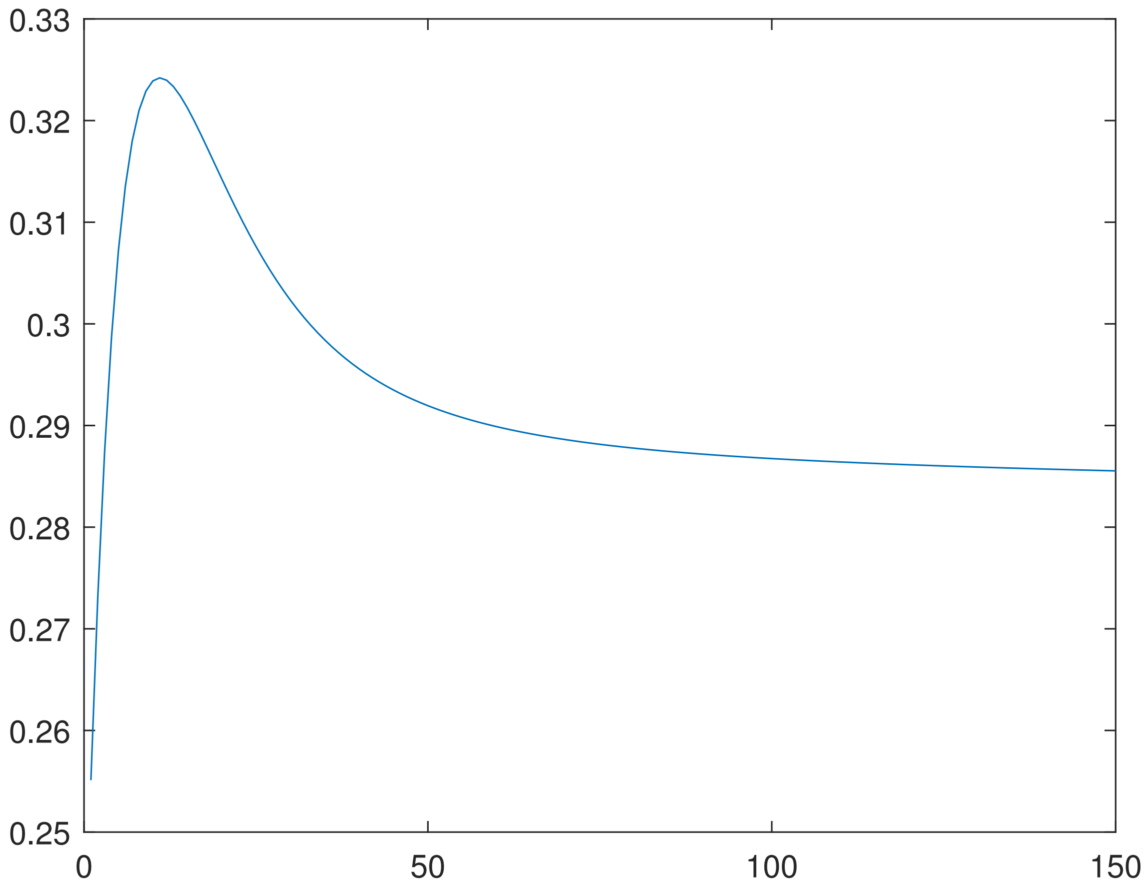


Figure 5: Saving Rate in the Ramsey Model (longer scope than Previous Figures)



4 Estimation of Non-Agricultural Value-Added in the PPP

To calculate the non-agricultural value-added in the PPP, we need to first understand how the aggregate and sectoral value-addeds are calculated in the databases.

First, I don't understand the documentation for the PLD very clearly. My impression is that the GGDC PLD is based on rich data. It uses different sources of data for different sectors. Their method is not very helpful for our project, because there are only three years having so detailed cross-country comparison in sectoral price levels in the ICP. Also, their aggregation method requires all the sectoral PPPs in the first place and compare each pair of countries to get the bilateral index and take the averages. This method is usually irreversible, so we cannot use the reverse of this method to disaggregate the economy-level PPPs from the PWT in to sectoral PPPs.

Next, the PWT provides a formula to aggregate consumption, export and import PPPs. To make the sectoral VAs in PPP add up to the aggregate one, it recalculate the sectoral VAs in a way that the PPP sectoral VAs are equal to nominal sectoral VAs divided by the aggregate PPP. And the

Table 3: List of Data

Variable	Description	Source
$\cancel{GO_a^{nom}}$	~~Gross agricultural output in current price (US$)~~	~~FAO Value of Agricultural Production~~
$\cancel{GO_a^{real}}$	~~Gross agricultural output in constant price (US$)~~	
$\cancel{GO_a^{PPP}}$	~~Gross agricultural output in PPP (constant I$)~~	
$\cancel{PPP_{att}}$	~~Current PPP for a specific year~~	~~pl_gdpo in the PWT 10.1~~
VA_a^{nom}, VA_a^{real} VA_n^{nom}, VA_n^{real}	Sectoral value-added in current and constant prices (US$)	UNSD AMA–Downloads (ISIC A-B = agriculture) All other ISIC codes = non-agriculture Note: ISIC D (manufacture) is included in ISIC C-E Thus, just use ISIC C-E and ignore ISIC D
VA_{agr}^{LCU} $VA_{min}^{LCU} - VA_{oth}^{LCU}$	Sectoral value added in current local currency	GGDC PLD VA
PPP_{agr}^{LCU} $PPP_{min}^{LCU} - PPP_{oth}^{LCU}$	PPP price for value added in current LCU/USD	GGDC PLD PPP_va
XR	Exchange rate (LCU/USD)	GGDC PLD xr

ratio between PPP sectoral VAs is equal to the nominal one, which is not good for our project. The formula Boppart uses is the linearized version of $PPP_a \cdot VA_a^{PPP} + PPP_n \cdot VA_n^{PPP} = PPP_{all} \cdot VA_{all}^{PPP}$. In the linearized formular, we can use nominal shares instead of real quantities, which I think would be a good feature; because PPP_n and VA_n^{PPP} are non-separable in $PPP_a \cdot VA_a^{PPP} + PPP_n \cdot VA_n^{PPP} = PPP_{all} \cdot VA_{all}^{PPP}$ and $VA_n^{PPP} \neq VA_{all}^{PPP} - VA_a^{PPP}$ in general.

4.1 A Description of Data Construction

Cross-country comparison at a specific year (suppress subscript t for simplicity) Mathematically, it is possible to decompose the aggregate PPP from the PWT into agriculture and non-agriculture. We can follow the same procedure as the PWT to calculate the "reference price", "implicit quantity", and hence the additive value added in PPP for agriculture. Then subtracting the agricultural value add in PPP from the aggregate will give us the non-agriculture value added in PPP. However, there is no precedent for this method in the literature. A possible reason is that when people want to estimate the non-agriculture value added in PPP, they could simply calculate it as the difference between the aggregate VA in PPP and the agricultural VA in PPP from difference sources and ignore the non-additivity (ChatGPT suggests me to do so as an estimate). Also, there is a theory arguing that the PWT's method has advantages in aggregation, but there is no evidence supporting that it has similar advantages in disaggregation—in the process of disaggregation, the problem becomes the comparison between the data from two sources or more, which is a different problem from aggregation. In other words, this method may not be better than directly subtracting the agricultural VA in PPP from the aggregate VA in PPP.

A more classic way to calculate the non-agricultural value add in PPP is to aggregate the sectoal VAs in the GGDC PLD. Although the PLD only have 84 countries, it covers 88% of the world population. Moreover, the PLD provides information on agricultural VA instead of gross output. The procedure for calculating the VAs in PPP is as follow:

1. Calculate the sectoral VAs in PPP from the PLD data. (Let's choose 2005 as the base year for now)

$$VA_{sec}^{x,PPP} = VA_{sec}^{x,LCU}/PPP_{sec}^{x,LCU}$$

   where x stands for country and sec stands for sector (agr–oth).

2. Adjust the sectoral PPP with exchange rates:

$$PPP_{sec}^{x} = PPP_{sec}^{x,LCU}/XR^{x}$$

3. Calculate the GEKS index for non-agriculture. First, following the PWT, define the Fisher price index

$$\begin{aligned} p_n^{jh,F} &= \left[\frac{\sum^{sec\in n} PPP_{sec}^{j} \cdot VA_{sec}^{j,PPP}}{\sum^{sec\in n} PPP_{sec}^{h} \cdot VA_{sec}^{j,PPP}} \cdot \frac{\sum^{sec\in n} PPP_{sec}^{j} \cdot VA_{sec}^{h,PPP}}{\sum^{sec\in n} PPP_{sec}^{h} \cdot VA_{sec}^{h,PPP}} \right]^{\frac{1}{2}} \\ &= \left[\frac{PPP_{min}^{j} \cdot VA_{min}^{j,PPP} + \cdots + PPP_{oth}^{j} \cdot VA_{oth}^{j,PPP}}{PPP_{min}^{h} \cdot VA_{min}^{j,PPP} + \cdots + PPP_{oth}^{h} \cdot VA_{oth}^{j,PPP}} \cdot \frac{PPP_{min}^{j} \cdot VA_{min}^{h,PPP} + \cdots + PPP_{oth}^{j} \cdot VA_{oth}^{h,PPP}}{PPP_{min}^{h} \cdot VA_{min}^{h,PPP} + \cdots + PPP_{oth}^{h} \cdot VA_{oth}^{h,PPP}} \right]^{\frac{1}{2}} \end{aligned}$$

4. Next, define the GEKS index

$$p_n^{jk,G} = \prod^{C} (p_n^{jh,F} \cdot p_n^{hk,F})^{1/C}$$

   where C is the set of all countries other than j, k. Always use the US as country k and use every country as j once at a time, including the US.

5. Divide $p_n^{jk,G}$ for every country by $p_n^{US,G}$ to get $p_n^{j,G}$.

6. Repeat Step 2-4 for agriculture and aggregate.

7. Calculate the sectoral value-added prices, p_a and p_n for each country x using $p_a^x = VA_a^{x,nom}/VA_a^{x,real}$ and $p_n^x = VA_n^{x,nom}/VA_n^{x,real}$ as well as the aggregate price $p^x = (VA_a^{x,nom}+VA_n^{x,nom})/(VA_a^{x,real}+VA_n^{x,real})$.

8. Multiply the GEKS index by the US domestic price to get the PPPs:

$$PPP_a^x = p_a^{x,G} \cdot p_a^{US}$$

$$PPP_n^x = p_n^{x,G} \cdot p_n^{US}$$

9. Calculate the VAs in PPP:

$$
VA_a^{x,PPP} = \frac{VA_a^{x,nom}}{PPP_a^x}
$$

$$
VA_n^{x,PPP} = \frac{VA_n^{x,nom}}{PPP_n^x}
$$

10. Next, let us apply the GK method to make the sectoral VAs additive. This can be done by iteration or the optimization algorithm. The optimization algorithm can be implemented as follows: (it is recommended to use matrices when actually writing the code. I use scalars to make the explanation clearer.)

    (a) We need to find the root for the function of aggregate PPP, $f(\boldsymbol{PPP_{all}})$. $\boldsymbol{PPP_{all}}$ is the vector that contains all the aggregate PPPs for all the countries.

    (b) Guess the aggregate PPP for country x, PPP_{all}^x; and we get the guessed aggregate PPP vector, $\boldsymbol{PPP_{all}}$ (The package will do it.)

    (c) Calculate the sectoral reference prices:

    $$
    \pi_a = \frac{\sum^j (VA_a^{j,nom}/PPP_{all}^j)}{\sum^j (VA_a^{j,nom}/PPP_a^j)} \text{ and } \pi_n = \frac{\sum^j (VA_n^{j,nom}/PPP_{all}^j)}{\sum^j (VA_n^{j,nom}/PPP_n^j)}
    $$

    where j means all the countries in the sample.

    (d) Calculate the aggregate PPP vector for country x using the derived reference prices:

    $$
    PPP_{all}^{x,derived} = \frac{VA_a^{x,nom} + VA_n^{x,nom}}{\pi_a \cdot VA_a^{x,nom}/PPP_a^x + \pi_n \cdot VA_n^{x,nom}/PPP_n^x}.
    $$

    Do this for all country x. And construct the vector from the derived aggregate PPP vector, $\boldsymbol{PPP_{all}^{derived}}$.

    (e) Compare $\boldsymbol{PPP_{all}}$ and $\boldsymbol{PPP_{all}^{derived}}$. Repeat Step (b)–(d), until the difference is close to 0. (The package will do it.)

11. Alternatively, the iteration method can be done as follows:

    (a) Set $diff = 100$ and the initial guess for the aggregate PPP for country x, $PPP_{all}^x = 1$. Start a while loop until $diff < 10^{-10}$.

    (b) In each loop calculate the sectoral reference prices:

    $$
    \pi_a = \frac{\sum^j (VA_a^{j,nom}/PPP_{all}^j)}{\sum^j (VA_a^{j,nom}/PPP_a^j)} \text{ and } \pi_n = \frac{\sum^j (VA_n^{j,nom}/PPP_{all}^j)}{\sum^j (VA_n^{j,nom}/PPP_n^j)}
    $$

    where j means all the countries in the sample.

    (c) Calculate the aggregate PPP vector for country x using the derived reference prices:

    $$
    PPP_{all}^{x,derived} = \frac{VA_a^{x,nom} + VA_n^{x,nom}}{\pi_a \cdot VA_a^{x,nom}/PPP_a^x + \pi_n \cdot VA_n^{x,nom}/PPP_n^x}.
    $$

(d) update $diff = \sum^{j} \left| log(PPP^{j}_{all}) - log(PPP^{j,derived}_{all}) \right|$, and $PPP^{x}_{all} = PPP^{x,derived}_{all}$.

(e) Repeat (b)–(d) until the criteria is meet.

12. Take the $\boldsymbol{PPP_{all}}$ from Step 10 or 11, and change the base to the US:

$$PPP^{x,fin}_{all} = PPP^{x}_{all} / PPP^{US}_{all} \cdot p^{US}_{all}$$

where p^{US}_{all} is the ratio between the nominal GDP and real GDP of the US from UNSD.

13. Calculate the sectoral reference prices:

$$\pi_a = \frac{\sum^{j}(VA^{j,nom}_{a}/PPP^{j,fin}_{all})}{\sum^{j}(VA^{j,nom}_{a}/PPP^{j}_{a})} \text{ and } \pi_n = \frac{\sum^{j}(VA^{j,nom}_{n}/PPP^{j,fin}_{all})}{\sum^{j}(VA^{j,nom}_{n}/PPP^{j}_{n})}$$

where j means all the countries in the sample.

14. Calculate the aggregate VA for country x:

$$cVA^{x}_{all} = \pi_a \cdot VA^{x,nom}_{a}/PPP^{x}_{a} + \pi_n \cdot VA^{x,nom}_{n}/PPP^{x}_{n}$$

15. Calculate the sectoal VAs (these VAs can be used to calculate VA shares)

$$cVA^{x,PPP}_{a} = \pi_a \cdot VA^{x,nom}_{a}/PPP^{x}_{a} \text{ and } cVA^{x,PPP}_{n} = \pi_n \cdot VA^{x,nom}_{n}/PPP^{x}_{n}$$

16. Calculate the price levels

$$PPP^{x,fin}_{a} = \frac{VA^{x,nom}_{a}}{cVA^{x,PPP}_{a}} \text{ and } PPP^{x,fin}_{n} = \frac{VA^{x,nom}_{n}}{cVA^{x,PPP}_{n}}$$

17. Derive the aggregate PPP again (the PWT does this, I don't know if this is different from $PPP^{x,fin}_{all}$. We can do a comparison.):

$$PPP^{x,fin2}_{all} = \frac{VA^{x,nom}_{all}}{cVA^{x}_{all}}$$

~~First, we derive the PPP for agricultural gross output, $PPP^{x}_{a,GO}$, for each country x using $PPP^{x}_{a,GO} = GO^{x,nom}_{a}/$~~
~~And we will use the PPP for agricultural gross output as the PPP for agricultural value-added, PPP^{x}_{a}.~~

~~Extra: we get the sectoral value-added prices, p_a and p_n for each country x using $p^{x}_{a} = VA^{x,nom}_{a}/VA^{x,real}_{a}$ and $p^{x}_{n} = VA^{x,nom}_{n}/VA^{x,real}_{n}$ as well as the aggregate price $p^{x} = (VA^{x,nom}_{a} + VA^{x,nom}_{n})/(VA^{x,real}_{a} + VA^{x,real}_{n})$. Also, we can calculate the agricultural gross out put price using $p^{x}_{a,GO} = GO^{nom}_{a}/GO^{real}_{a}$, and compare p_a and $p_{a,GO}$.~~

~~The nominal VA shares can be calculated as $s^{x}_{a} = VA^{x,nom}_{a}/(VA^{x,nom}_{a} + VA^{x,nom}_{n})$ and $s^{x}_{n} = VA^{x,nom}_{n}/(VA^{x,nom}_{a}$~~
~~for agriculture and non-agriculture respectively.~~

~~The change rates in the non-agricultural price level between two countries, x and y, can be calculated using~~

$$dlog(PPP_n) = \frac{dlog(PPP_{all}) - \frac{1}{2}(s_a^x + s_a^y)dlog(PPP_a)}{\frac{1}{2}(s_n^x + s_n^y)}$$

~~where $dlog(PPP) = log(PPP^x) - log(PPP^y)$. It is recommended to first sort the data before deriving the change rates.~~

~~Construct the index for PPP_n from $dlog(PPP_n)$ and divide it by the US' price level PPP_n^{US}. Also, normalize PPP_a^x by dividing it by PPP_a^{US}. Then the sectoral VAs in the PPP at a certain year are $VA_a^{x,PPP} = VA_a^{x,nom}/PPP_a^x$ and $VA_n^{x,PPP} = VA_n^{x,nom}/PPP_n^x$.~~

~~Repeat Step 1-5 for each year available in the data.~~

Time series of sectoral VA^{PPP} In principle there are two potential methods each with advantages and disadvantages.

Method 1 calculates sectoral PPPs for every country and every year. This allows for cross-country comparisons in a cross-section, at a point in time, and through time, comparing the time-series of different countries. Nonetheless, Shu is not confident that this method generates reliable within country time series.

Method 2 calculates sectoral PPPs for a base year and then it uses a domestic sectoral price deflator to generate the time series within each country. This allows for cross-country comparison in the base year and for consistent within-country time series. With this method you can't make comparisons of the time-series across countries.

Method 1 ~~In the first step, we use the current US price as the price base. Thus, for a country x, its VAs in different year are not comparable due to the change in the price base. To construct the time series for the VAs, we do the following steps:~~

1. ~~Calculate $PPP_{a,t}^{x,constant} = PPP_{a,t}^x \cdot p_{a,t}^{US}$ and $PPP_{n,t}^{x,constant} = PPP_{n,t}^x \cdot p_{n,t}^{US}$, where $PPP_{a,t}^x$ and $PPP_{n,t}^x$ are the normalized PPPs from Step 5 above. $p_{a,t}^{US}$ and $p_{n,t}^{US}$ are the ratio between the US nominal and constant-price VAs from Step 2 above.~~

2. ~~Following the PWT, let us call the VAs calculated using the new prices $cVAs$. Calculate the sectoral cVAs in the PPP as $cVA_{a,t}^{x,PPP} = VA_{a,t}^{x,nom}/PPP_{a,t}^{x,constant}$ and $cVA_{n,t}^{x,PPP} = VA_{n,t}^{x,nom}/PPP_{n,t}^{x,constan}$~~

3. ~~(I'm not sure if this step is correct) Link the VAs in t and $t-1$ in a country with the Fisher index. Following the PWT, let us call the linked VAs as $rVAs$. The growth rate of rVA between t and $t-1$ can be calculated as~~

$$\frac{rVA_{a,t}^{x,PPP}}{rVA_{a,t-1}^{x,PPP}} = \left[\frac{PPP_{a,t-1}^x \cdot cVA_{a,t}^{x,PPP}}{PPP_{a,t-1}^x \cdot cVA_{a,t-1}^{x,PPP}} \cdot \frac{PPP_{a,t}^x \cdot cVA_{a,t}^{x,PPP}}{PPP_{a,t}^x \cdot cVA_{a,t-1}^{x,PPP}}\right]^{\frac{1}{2}}$$

~~and~~

$$g(rVA_{a,t}^{x,PPP}) = log\left(\frac{rVA_{a,t}^{x,PPP}}{rVA_{a,t-1}^{x,PPP}}\right).$$

~~Using this growth rate, we can construct the index for $rVA_a^{x,PPP}$ across time. Times this index by the $cVA_a^{x,PPP}$ in the base year (2015), and we will get the time series for $rVA_a^{x,PPP}$.~~

4. ~~Follow the same method in Step 3 to calculate the time series for the non-agricultural sector.~~

5. ~~Repeat Step 1-4 for each country.~~

Method 2 Actually, if we only want to compare the changes in a country's RAPG, it does not need to involve the complex math that makes China in 1990 comparable to the US in 2000. In my understanding, the comparison between China and the US both in 2000, and between China in 1990 and 2000 should be good enough. Thus, using constant-price GDP to extend each country's time series compared to the base year (say 2015) should be a better solution.

1. Take the $PPP_{a,2005}^{x,fin}$, $PPP_{n,2005}^{x,fin}$, $p_{a,t}^x$ and $p_{n,t}^x$ from the cross-country case.

2. Deflate the PPP prices using the nominal prices.

$$PPP_{a,t}^{x,na} = PPP_{a,2005}^{x,fin} \cdot \frac{p_{a,t}^x}{p_{a,2005}^x}$$

   and

$$PPP_{n,t}^{x,na} = PPP_{n,2015}^{x,fin} \cdot \frac{p_{n,t}^x}{p_{n,2005}^x}$$

3. Calculate the country time series of value addeds using the deflated PPP prices. Using the notation in the PWT, let us call them $rVA_a^{x,na}$ and $rVA_n^{x,na}$.

$$rVA_{a,t}^{x,na} = \frac{VA_{a,t}^{nom}}{PPP_{a,t}^{x,na}}$$

   and

$$rVA_{n,t}^{x,na} = \frac{VA_{n,t}^{nom}}{PPP_{n,t}^{x,na}}.$$

4. Do the same for the aggregate.